NLO QCD Corrections to inclusive $B \to X_c \tau \bar{\nu}_\tau$ decay rate and spectrum up to $1/m_Q^3$

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based on

D. Moreno, hep-ph/2207.14245

T. Mannel, D. Moreno and A. A. Pivovarov, hep-ph/2112.03875

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Context

High precision studies of inclusive weak decays of H_Q are important for testing the flavour sector of the SM

- Pattern of lifetimes and branching fractions is a **solid test of QCD/EW**.
- Allow **precise extraction of V_{CKM}**.
- Required to understand **B-anomalies** ⇒ hints of **new physics**

They require

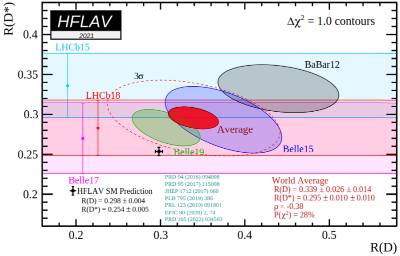
(a) **Precise measurements**: ongoing BelleII, LHCb experiments

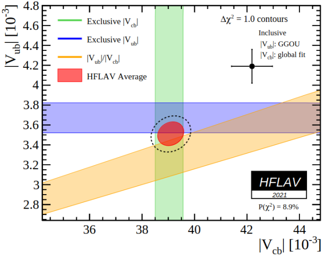
(b) **Precise theoretical calculations**: in the context of the **HQE** means to push for higher orders in Λ_{QCD}/m_b and $\alpha_s(m_b)$.
[Shifman and Voloshin, SJNP 47 (1988)], [Eichten and Hill, PLB 234 (1990)],
[Isgur and Wise, PLB 232 (1989)], [Grinstein, NPB 339 (1990)],
[Blok, Koyrakh, Shifman and Vainshtein, PRD 50, 3572 (1994)]

Context

B-anomalies

[Y. S. Amhis *et al.* [HFLAV], PRD **107** (2023), 052008]





- 3.3σ tension in $R(D^{(*)}) = \mathcal{B}(B \to D^{(*)}\tau\bar{\nu}_\tau)/\mathcal{B}(B \to D^{(*)}e\bar{\nu}_e)$.
- Inclusive decays provide valuable complementary information.
  [LEP and recently Belle 18']
- $\Gamma(B \to X_c\tau\bar{\nu}_\tau)/\Gamma(B \to X_c e\bar{\nu}_e)$ or moments (ratios) of $d\Gamma/dq^2$.
- 3.3σ tension between $|V_{cb}|^{\text{in./ex.}}$.
- $|V_{cb}|^{\text{in.}}$ can be precisely extracted from $d\Gamma(B \to X_c e\bar{\nu}_e)/dq^2$.

HQE for inclusive semileptonic decays

The $\Gamma(B \to X_c \tau \bar{\nu}_\tau)$ obtained from

$$\Gamma(B \to X_c \tau \bar{\nu}_\tau) \sim \mathrm{Im}\, \langle B| i \int dx\, T\, \{\mathcal{L}_{\rm eff}(x)\mathcal{L}_{\rm eff}(0)\}\, |B\rangle$$

$$\mathcal{L}_{\rm eff}^{b\to c\tau\bar{\nu}_\tau} = 2\sqrt{2} G_F V_{cb} (\bar{b}_L \gamma_\mu c_L)(\bar{\nu}_{\tau,L}\gamma^\mu \tau_L) + \mathrm{h.c.}$$

Since $m_b \gg \Lambda_{\rm QCD}$ one can set up an expansion in $\Lambda_{\rm QCD}/m_b$ (HQE)

$$\Gamma(B \to X_c \tau \bar{\nu}_\tau) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[C_0 \left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) + C_{\mu_G} \left(\frac{\mu_G^2}{2m_b^2} - \frac{\rho_{LS}^3}{2m_b^3}\right) - C_{\rho_D} \frac{\rho_D^3}{2m_b^3}\right]$$

perturbative and non-perturbative contributions are factorized in:

- **Wilson coefficients**: $C_i(\rho = m_c^2/m_b^2, \eta = m_\tau^2/m_b^2)$ have a perturbative expansion in $\alpha_s(m_b)$, obtained by matching to QCD.

  C_0 at N^{2}LO [S. Biswas and K. Melnikov, JHEP **02** (2010), 089]

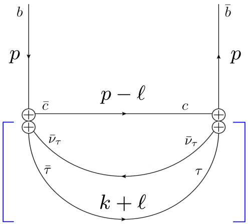
  C_{μ_G} at LO [S. Balk *et al.*, ZPC **64** (1994)] [L. Koyrakh, PRD **49** (1994)]

  C_{ρ_D} at LO [T. Mannel, A. V. Rusov and F. Shahriaran, NPB **921** (2017)]

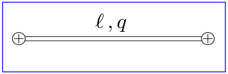
- **Forward ME of HQET operators**: called hadronic parameters μ_π^2, μ_G^2, ρ_{LS}^3 and $\rho_D^3 \sim \langle B|\bar{h}_v [D_{\perp\,\mu}, [D_\perp^\mu, v\cdot D]] h_v|B\rangle$.

HQE for inclusive semileptonic decays

The Γ can be written as an integral in $d(q^2)$ by using a **dispersion relation** for the $\tau\bar{\nu}_\tau$ loop



T. Mannel, D. Moreno and A. A. Pivovarov, Phys. Rev. D **104** (2021), 114035 [hep-ph/2104.13080]



$$\Pi^{\rho\sigma} \equiv i \int \frac{d^D k}{(2\pi)^D} \frac{-\operatorname{Tr}(\Gamma^\sigma i(\not{k} + \not{\ell} + m_\tau)\Gamma^\rho i\not{k})}{k^2((k+\ell)^2 - m_\tau^2)} = \frac{1}{\pi}\int_{m_\tau^2}^{\infty} d(q^2)\, \underbrace{\frac{\operatorname{Im}\Pi^{\rho\sigma}}{q^2 - \ell^2 - i\eta}}_{\text{“massive propagator” of mass } q}$$

HQE for inclusive semileptonic decays

The HQE of the decay spectra is writen as follows

$$\frac{d\Gamma(B \to X_c \tau \bar{\nu}_\tau)}{dr} = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \left[\mathcal{C}_0 \left(1 - \frac{\mu_\pi^2}{2m_b^2} \right) + \mathcal{C}_{\mu_G} \left(\frac{\mu_G^2}{2m_b^2} - \frac{\rho_{LS}^3}{2m_b^3} \right) - \mathcal{C}_{\rho_D} \frac{\rho_D^3}{2m_b^3} \right]$$

where $r = q^2/m_b^2$. The $\mathcal{C}_i(r, \rho, \eta)$ are related to the $C_i(\rho, \eta)$ by

$$C_i(\rho, \eta) = \int_\eta^{(1-\sqrt{\rho})^2} dr\, \mathcal{C}_i(r, \rho, \eta)\,.$$

Now we can investigate moments of the distribution

$$M_n(\rho, \eta) = \int_\eta^{(1-\sqrt{\rho})^2} dr\, r^n \frac{d\Gamma(r, \rho, \eta)}{dr}\,,$$

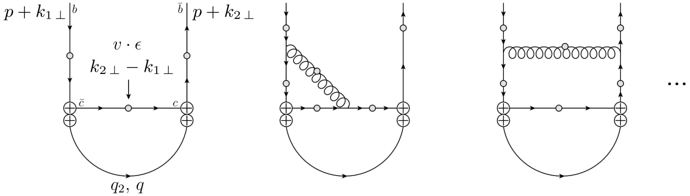
and related observables like normalized moments to the width and ratios between different channels

$$\hat{M}_n = \frac{M_n(B \to X_q \ell \bar{\nu}_\ell)}{M_0(B \to X_q \ell \bar{\nu}_\ell)}\,, \qquad R_n^{q\ell/q'\ell'} = \frac{|V_{q'b}|^2}{|V_{qb}|^2} \frac{M_n(B \to X_q \ell \bar{\nu}_\ell)}{M_n(B \to X_{q'} \ell' \bar{\nu}_{\ell'})}\,.$$

Differential rate in the lepton invariant mass at $\mathcal{O}(\alpha_s/m_b^3)$

At α_s/m_b^3 we only need to determine the coefficient of ρ_D (Darwin term)

- **Take the amplitude** of quark to quark-gluon scattering with kin. conf.



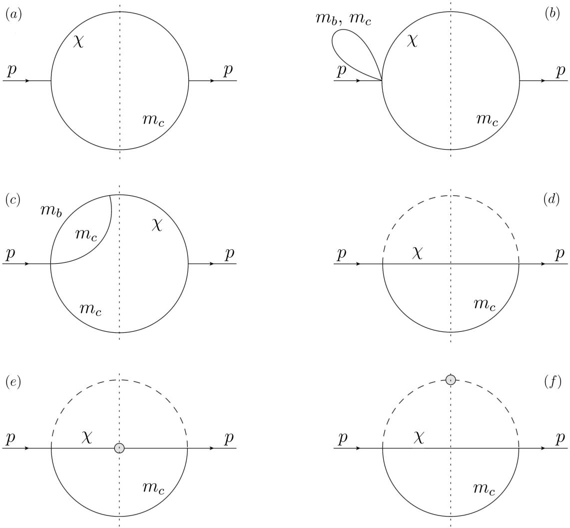
with $p^2 = m_b^2$ and $k_\perp^\mu = k^\mu - v^\mu(v \cdot k)$.

- **Expand** to quadratic order in the small momenta $k_{1\perp}$, $k_{2\perp}$.

- **Project** to the Darwin operator, i. e pick up $k_{1\perp}^{(\alpha} k_{2\perp}^{\beta)}$ structure.

Be careful! We must disantangle contributions to dim. 6 operators $\bar{h}_v(v \cdot D)D_\perp^2 h_v$, ..., that contribute to higher orders after using the EOM.

Differential rate in the lepton invariant mass at $\mathcal{O}(\alpha_s/m_b^3)$



$$
\begin{aligned}
x_- &= \frac{1}{2}(1 - r + \rho - A) \\
x_+ &= \frac{1}{2}(1 - r + \rho + A) \\
A &= \sqrt{(1-(\sqrt{r}-\sqrt{\rho})^2)(1-(\sqrt{r}+\sqrt{\rho})^2)}
\end{aligned}
$$

T. Mannel, D. Moreno and A. A. Pivovarov, PRD 104 (2021), 114035 [hep-ph/2104.13080]

IBP

dim. reg. $D = 4 - 2\epsilon$

Differential rate in the lepton invariant mass at $\mathcal{O}(\alpha_s/m_b^3)$

Other important remarks:

- **Renormalization** can be performed **at differential level** ($\epsilon \to 0$ finite).
- **Cancellation of poles** is delicated and provides a solid check:
  - (a) Requires to consider the **mixing** under renormalization between **HQET operators** of different dimension, like [Bauer and Manohar, PRD **57**, 337 (1998)]

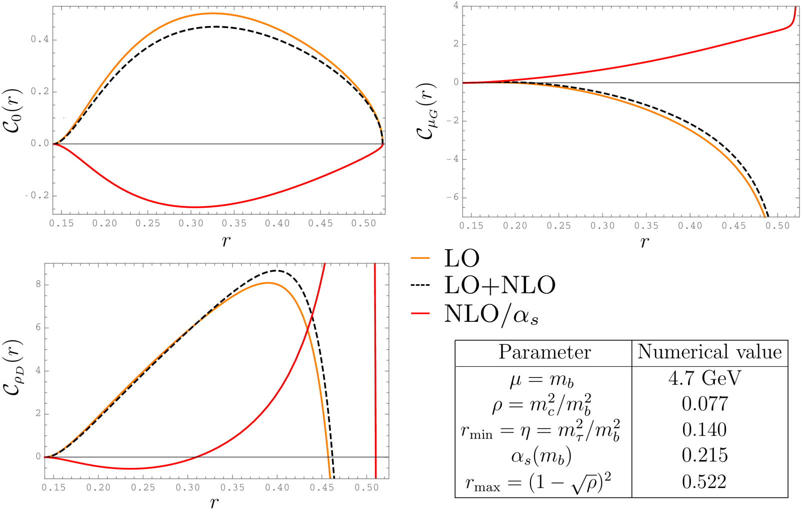
$$\mathcal{O}_\pi^B = \mathcal{O}_\pi^R + \gamma_{\pi D} \frac{\alpha_s}{\pi} \frac{1}{m_b} \mathcal{O}_D$$

  - (b) γ_{iD} obtained from the combined insertion of operators of the HQE and operators of the HQET Lagrangian.
- $\mathcal{C}_{\rho_D}(\epsilon = 0)$ finite, but **integration over r is IR singular** at $r_{\max}$ (ϵ dep. must be restored in the IR singular terms).

$$C_{\rho_D}^{\mathrm{IR}} \sim \int_\eta^{r_{\max}} dr \frac{1}{(r_{\max} - r)^{3/2}} \to \int_\eta^{r_{\max}} dr \frac{1}{(r_{\max} - r)^{3/2+\epsilon}}$$

Numerical analysis

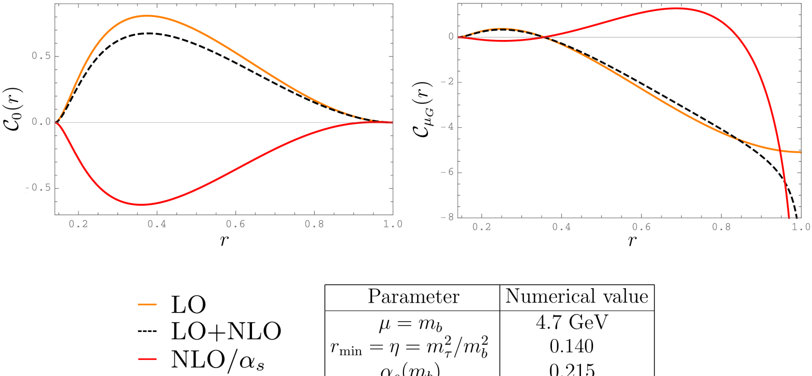
$$B \to X_c \tau \bar{\nu}_\tau$$



Parameter	Numerical value
$\mu = m_b$	4.7 GeV
$\rho = m_c^2/m_b^2$	0.077
$r_{\min} = \eta = m_\tau^2/m_b^2$	0.140
$\alpha_s(m_b)$	0.215
$r_{\max} = (1-\sqrt{\rho})^2$	0.522

Numerical analysis

$$B \to X_u \tau \bar{\nu}_\tau$$



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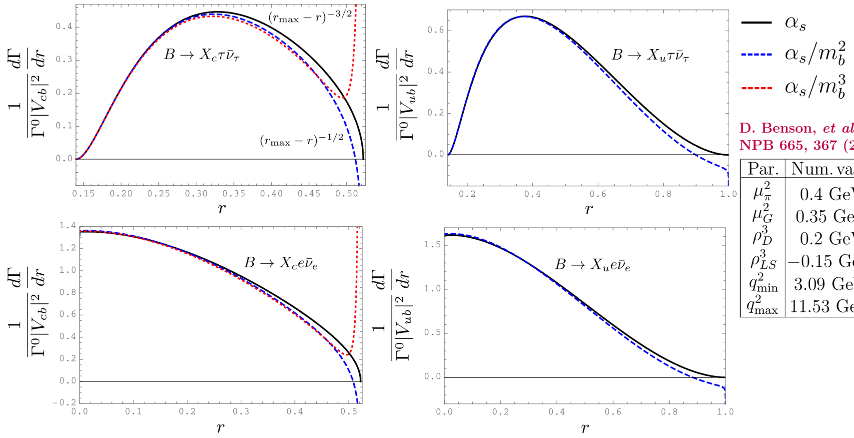
Numerical analysis



D. Benson, *et al.*
NPB 665, 367 (2003)

Par.	Num. value
μ_π^2	0.4 GeV^2
μ_G^2	0.35 GeV^2
ρ_D^3	0.2 GeV^3
ρ_{LS}^3	−0.15 GeV^3
$q^2_{\min}$	3.09 GeV^2
$q^2_{\max}$	11.53 GeV^2

Final remarks

$$B \to X_c \tau \bar{\nu}_\tau$$

- We have computed the α_s/m_b^2 and α_s/m_b^3 corrections to $\Gamma(B \to X_c \tau \bar{\nu}_\tau)$ and $d\Gamma(B \to X_c \tau \bar{\nu}_\tau)/dr$ with full dependence on m_c and m_τ analytically.
[D. Moreno, PRD **106** (2022), 114008]

- Current knowledge of the HQE for $B \to X_c \tau \bar{\nu}_\tau$ decay rate and q^2-distribution: $(\alpha_s^2, \alpha_s/m_b^3)$.

- We propose to analyze the Γ, $\hat{M}_n$ and $R_n^{q\ell/q'\ell'}$.

- May provide valuable complementary information to $R(D^{(*)})$ where a more than 3σ deviation from the SM is present.
[Z. Ligeti and F. J. Tackmann, PRD **90** (2014), 034021]
[M. Rahimi and K. K. Vos, JHEP **11** (2022), 007]

- The LO $1/m_b^3$ corrections ($\sim 10\%$) correction to the leading term.

- The α_s/m_b^2 and α_s/m_b^3 corrections ($\sim 1\%$) correction.

Final remarks

$$B \to X_u\tau\bar{\nu}_\tau$$

- In the $m_c = 0$ case the α_s/m_b^2 results can be applied to $B \to X_u\tau\bar{\nu}_\tau$.
- Precise predictions are interesting because
  [Z. Ligeti, M. Luke and F. J. Tackmann, PRD **105**, 073009 (2022)]

  (*i*) It is a signal channel to measure in the future.
  Smaller room for the application of the HQE (Dominant $B \to X_c$ overwhelms 1/2 of the PT spectrum)
  $\Rightarrow$ shape functions important.

  (*ii*) Important for modelling this decay as a background with impact on precise measurements of $R(D^{(*)})$.

Final remarks

$$B \to X_c e \bar{\nu}_e$$

- The $m_\tau = 0$ case important for $|V_{cb}|$ extraction from $\hat{M}_n(r_{\text{cut}})$.

- The α_s/m_b^3 corrections are ($\sim 1\%$), and we expect a small but visible impact on $|V_{cb}|$.
  [T. Mannel, D. Moreno and A. A. Pivovarov, PRD **105** (2022), 054033]

- Overall, this will allow to increase the precision of $|V_{cb}|$ by using $M_n(r_{\text{cut}})$, where a first analysis have given $|V_{cb}| = (41.69 \pm 0.63) \cdot 10^{-3}$.
  [M. Fael, T. Mannel and K. Keri Vos, JHEP **02** (2019), 177]
  [R. van Tonder *et al.* [Belle], PRD **104** (2021), 112011]
  [F. Bernlochner *et al.*, JHEP **10** (2022), 068]
  [M. Bordone, B. Capdevila and P. Gambino, PLB **822** (2021), 136679]

- High precision in $B \to X_c e \bar{\nu}_e$ makes it an interesting channel to explore NP. (see next talk by M. Fael)