$B \to \pi$ and $B \to D^*$ from JLQCD

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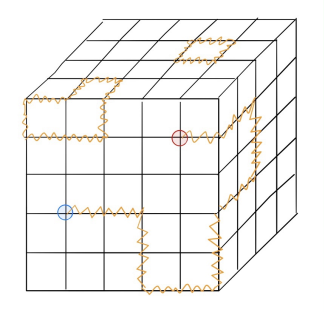
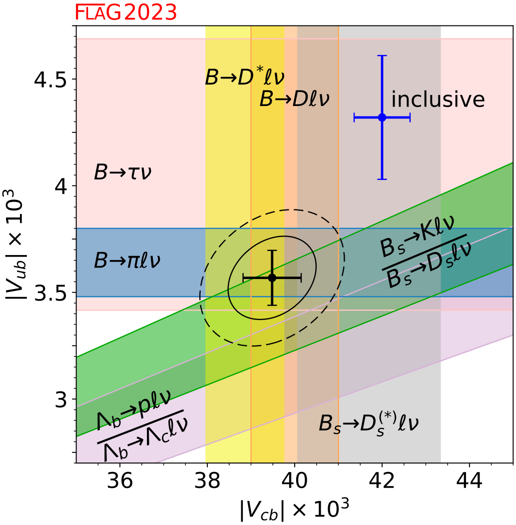
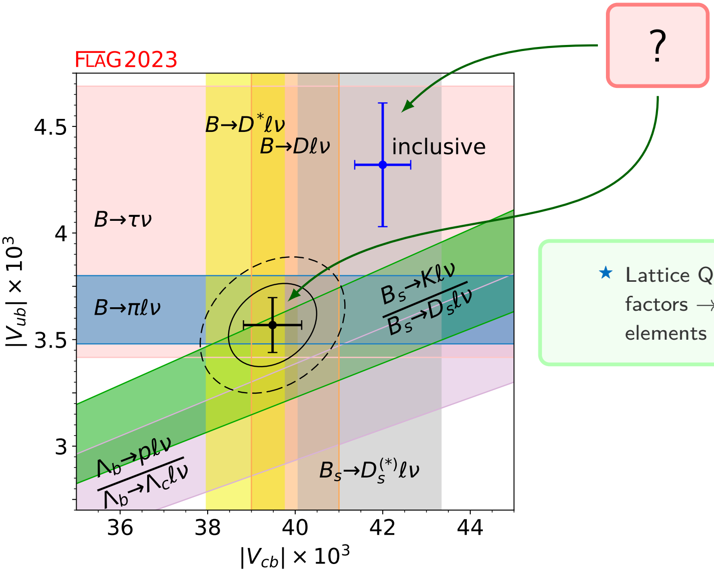
for the JLQCD Collaboration

21 September 2023





★ Lattice QCD → form factors → CKM matrix elements



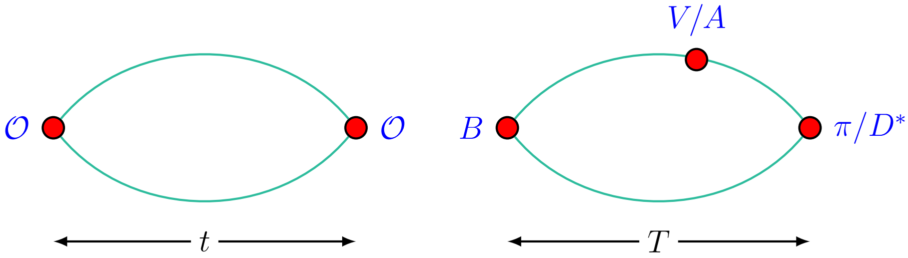
★ Lattice QCD → form factors → CKM matrix elements

Calculations in this talk

- We use the Möbius Domain Wall Fermion (MDWF) Action
  - Pros: fully relativistic & good chiral symmetry
  - Con: 5th dimension makes MDWF expensive
- MDWF used for all valence & sea quarks
  - Includes effect of 2+1 quarks in the sea
- Lattice spacings: 0.08, 0.055, 0.44 fm
- Pion masses from 500 MeV down to 230 MeV
- m_b from m_c to $\approx 3m_c$ ($2.44m_c$ for $B \to \pi$)
- $B \to \pi\ell\nu$ first & only fully relativistic calculation
  - Other collaborations will have fully relativistic results in near future!

Full details in Phys.Rev.D **106** (2022) 054502 [arXiv:2203.04938] & [arXiv:2306.05657]

- ★ 2-point correlators have meson interpolating operators with t separation
- ★ For 3-point correlators:
  - Meson interpolating operators separated by T
  - Currents inserted at times t
  - B meson at rest; momentum given to π/D
  - Multiple heavy quark masses used to extrapolate to physical b
    - Similarly for chiral extrap. to physical m_π
    - c has physical mass



$$B \to \pi \ell \nu$$

Need $f_+(q^2)$ for $|V_{ub}|$

$$\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3}|\mathbf{p}_\pi(q^2)|^3|f_+(q^2)|^2$$

$$\langle\pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\,\frac{M_B^2 - M_\pi^2}{q^2}\,q^\mu$$

On the lattice, useful to define[1] f_1 and f_2 (that relate more directly to matrix elements), then:

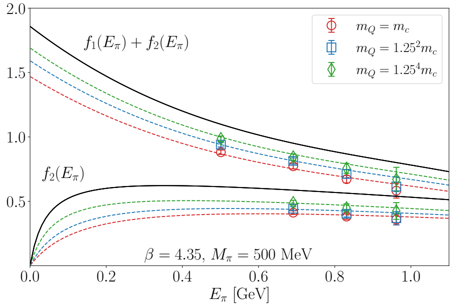
$$\langle\pi(p_\pi)|V^\mu|B(v)\rangle = 2\left[f_1\,(v\cdot p_\pi)\,v^\mu + f_2\,(v\cdot p_\pi)\,\frac{p_\pi^\mu}{v\cdot p_\pi}\right],$$

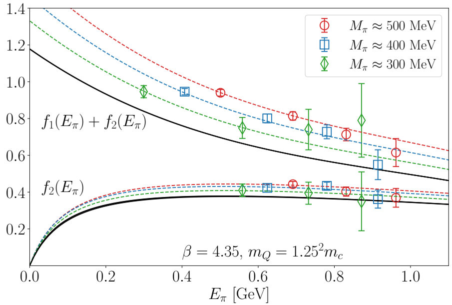
$$f_+(q^2) = \sqrt{M_B}\left\{\frac{f_2(v\cdot p_\pi)}{v\cdot p_\pi} + \frac{f_1(v\cdot p_\pi)}{M_B}\right\},$$

[1] $f_{||}$ and $f_\perp$ more common

Extrapolations to physical b and m_π

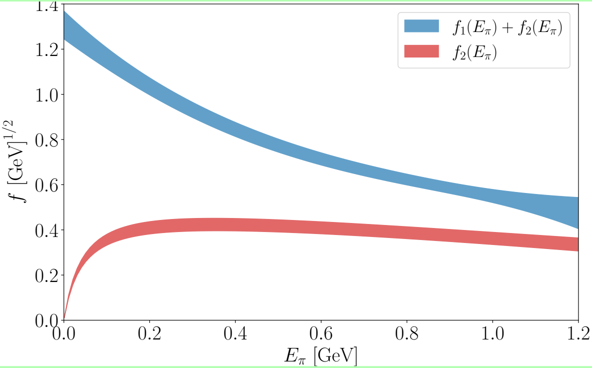
- ★ Different colours/points are lattice data for lattice form factors
- ★ Black solid lines are extrapolation to physical m_b (left) and m_π (right)





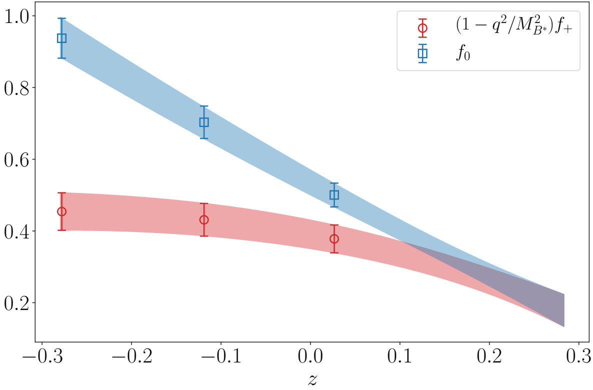
Global fit

$$f_1(v\cdot p_\pi)+f_2(v\cdot p_\pi)=C_0\left(1+\sum_{n=1}^{3}C_{E^n}N_E^nE_\pi^n\right)\left(1+C_{\chi\log}\delta f^{B\to\pi}+C_{M_\pi^2}N_{M_\pi^2}M_\pi^2\right)$$
$$\times\left(1+\frac{C_{m_Q}N_{m_Q}}{m_Q}\right)\left(1+C_{m_{s\bar{s}}^2}\delta m_{s\bar{s}}^2\right)\left(1+C_{a^2}(\Lambda_{QCD}a)^2+C_{(am_Q)^2}(am_Q)^2\right)$$



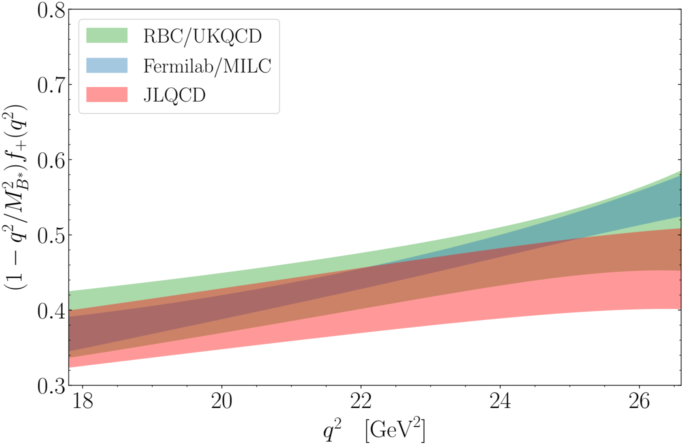
$$f_2(v\cdot p_\pi)=D_0\left[\frac{E_\pi}{E_\pi+\Delta_B}\left(1+D_{E_\pi}N_EE_\pi\right)\right]\left(1+D_{\chi\log}\delta f^{B\to\pi}+D_{M_\pi^2}N_{M_\pi^2}M_\pi^2\right)$$
$$\times\left(1+\frac{D_{m_Q}N_{m_Q}}{m_Q}\right)\left(1+D_{m_{s\bar{s}}^2}\delta m_{s\bar{s}}^2\right)\left(1+D_{a^2}(\Lambda_{QCD}a)^2+D_{(am_Q)^2}(am_Q)^2\right)$$

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ + t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ + t_0}} \qquad |z| < 0.3$$



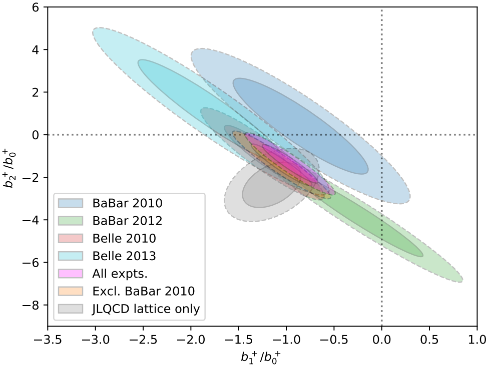
$$f_+(q^2) = \frac{1}{1 - q^2/M^2_{B^*}} \sum_{k=0}^{N_z-1} b_k^+ \left[z^k - (-1)^{k-N_z} \frac{k}{N_z} z^{N_z} \right]$$

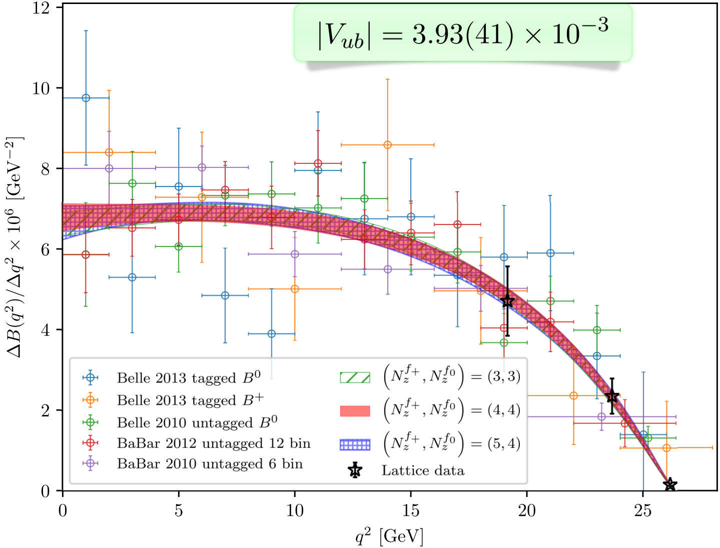
Comparison with other lattice calculations



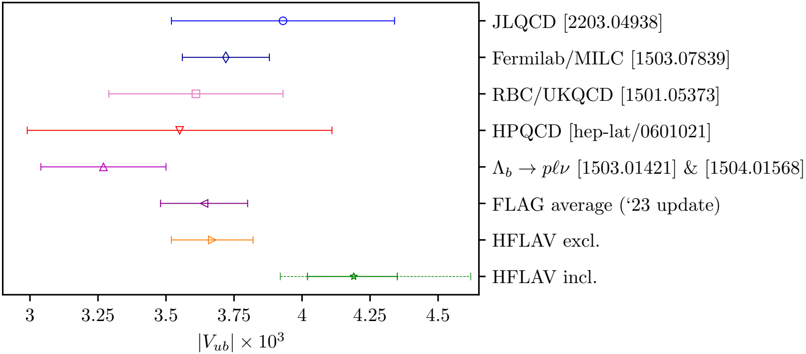
Experimental data for $B \to \pi\ell\nu$

- BaBar 2010 untagged 6 bins
- Belle 2010 untagged 13 bins
- BaBar 2012 untagged 12 bins
- Belle 2013 tagged
  - $B^0 \to \pi^+\ell\nu$: 13 bins
  - $B^-\pi^0\ell\nu$: 7 bins




$|V_{ub}| = 3.93(41) \times 10^{-3}$
$\Delta B(q^2)/\Delta q^2 \times 10^6$ [GeV^{-2}]
q^2 [GeV]
0
5
10
15
20
25
0
2
4
6
8
10
12
Belle 2013 tagged B^0
Belle 2013 tagged B^+
Belle 2010 untagged B^0
BaBar 2012 untagged 12 bin
BaBar 2010 untagged 6 bin
$(N_z^{f_+}, N_z^{f_0}) = (3,3)$
$(N_z^{f_+}, N_z^{f_0}) = (4,4)$
$(N_z^{f_+}, N_z^{f_0}) = (5,4)$
Lattice data

$|V_{ub}|$ comparison summary



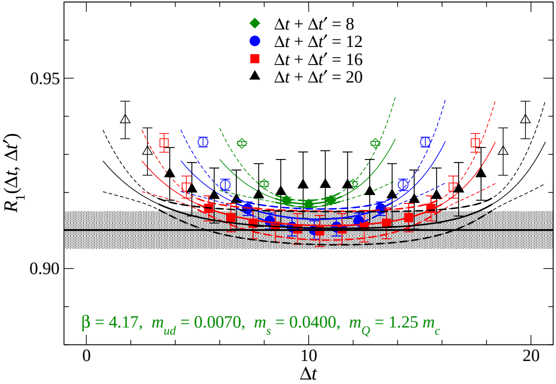
JLQCD now working on $B \to \pi\ell\nu$ update: increased statistics, additional m_b values to improve control of extrapolation...

$$B \to D^* \ell \nu$$

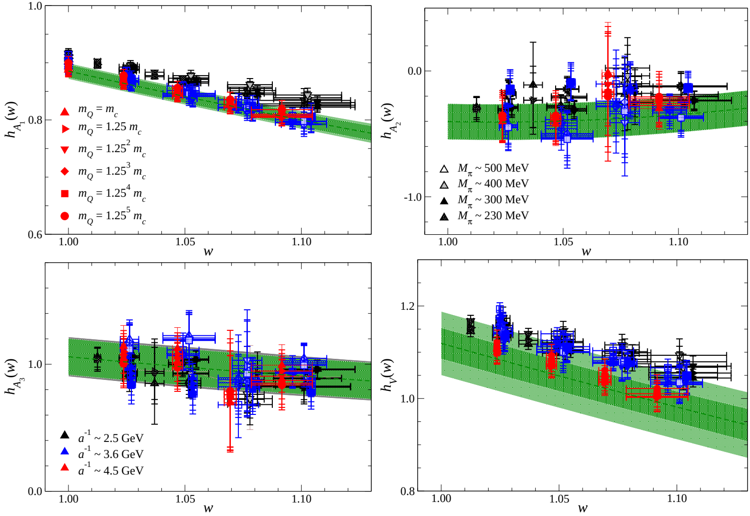
Lattice calculation

$$\sqrt{M_B M_{D^*}}^{-1} \langle D^*(\varepsilon', p')|V_\mu|B(p)\rangle = i\epsilon_{\mu\nu\rho\sigma}\varepsilon'^{\nu} v'^{\rho} v^{\sigma} h_V(w)$$

$$\sqrt{M_B M_{D^*}}^{-1} \langle D^*(\varepsilon', p')|A_\mu|B(p)\rangle = (w+1)\varepsilon'^{*}_{\mu} h_{A_1}(w) - \left(\varepsilon'^{*} v\right) \left\{ v_\mu h_{A_2}(w) + v'_\mu h_{A_3}(w) \right\}$$



Form factors from the lattice



$$\frac{h_X}{\eta_X} = c + \frac{g^2_{D^*D\pi}}{2\Lambda^2_\chi}\bar{F}_{\log}\left(\frac{M^2_\pi}{\Lambda^2_\chi}, \Delta_D, \Lambda_\chi\right) + c_\pi \frac{M^2_\pi}{\Lambda^2_\chi} + c_{\eta_s}\frac{M^2_{\eta_s}}{\Lambda^2_\chi} + c_Q \frac{\bar{\Lambda}}{2m_Q}$$
$$+ c_a(\Lambda a)^2 + c_{am_Q}(am_Q)^2 + c_w(w-1) + d_w(w-1)^2$$

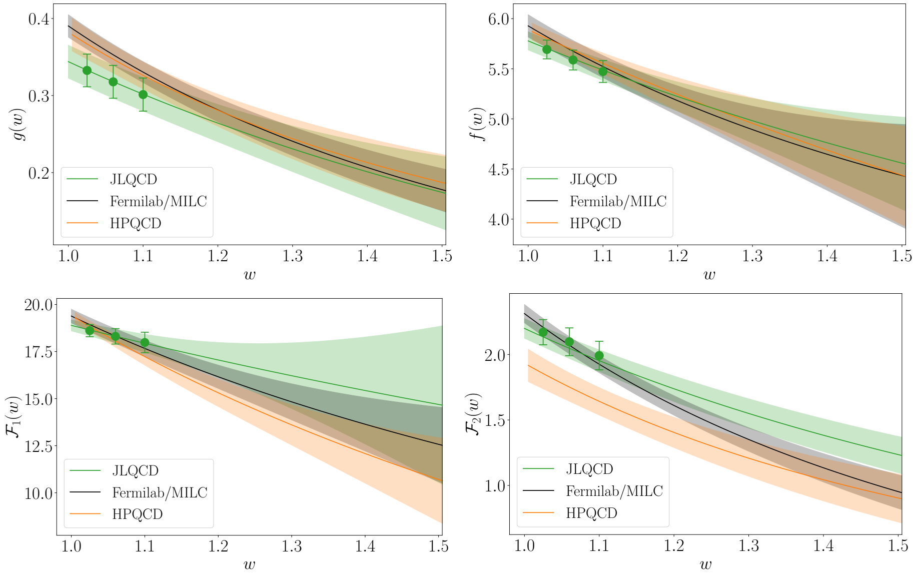
- Continuum fits of $h_{V,A_{1,2,3}} \to f, g, \mathcal{F}_{1,2}$
- Generate synthetic data at reference w ($w_{\mathrm{ref}} = 1.025, 1.060, 1.100$)
- Fit synthetic data to BGL form to gives FFs across w range

BGL fit form:

$$F(Z) = \frac{1}{P_F(z)\phi_f(z)} \sum_{k=0}^{N_F} a_{F,k} z^k \qquad (F = f, g, \mathcal{F}_1, \mathcal{F}_2)$$

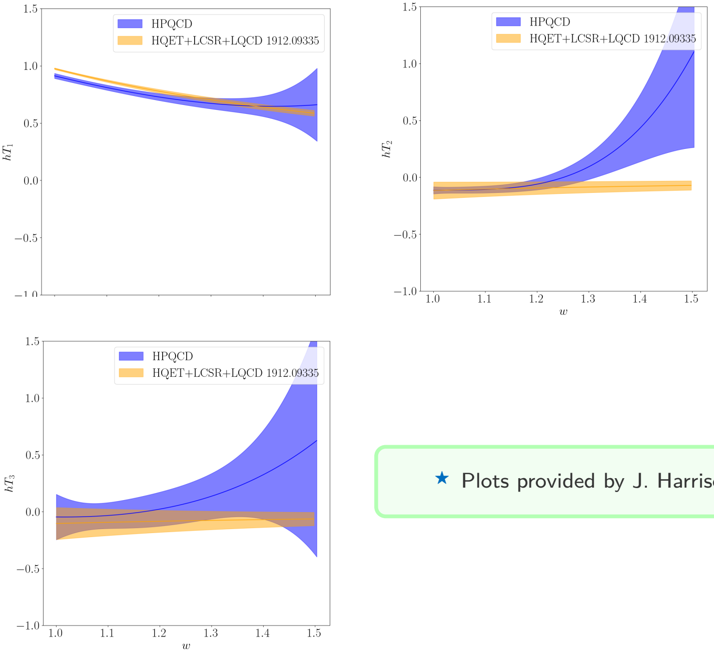
$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}} \qquad (0 \leq z \leq 0.06)$$

Form factors in HQET convention



HPQCD bands updated since [2304.03137]

Tensor form factors by HPQCD



★ Plots provided by J. Harrison

Differential decay rate and $R(D^*)$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{16\pi^3}|V_{cb}|^2|\eta_{\mathrm{EW}}|^2 M_B r^2 \sqrt{w^2-1}\left(1-\frac{m_\ell^2}{q^2}\right)^2$$
$$\times\left[\frac{q^3}{3}\left(1+\frac{m_\ell^2}{2q^2}\right)\left\{|H_+(w)|^2+|H_-(w)|^2+|H_0(w)|^2\right\}+\frac{m_\ell^2}{2}|H_S(w)|^2\right]$$

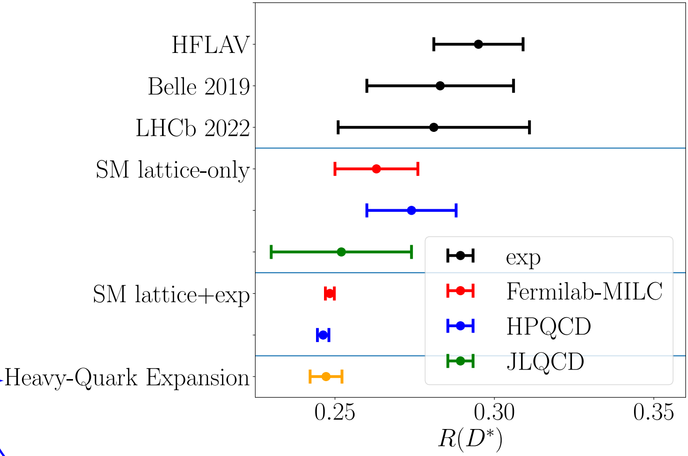
★ Form factors from BGL $\to$ helicity basis: $H_\pm(w)$, $H_0(w)$, $H_S(w)$

Fitting this expression alongside experimental data gives access to $|V_{cb}|$. Can also integrate over w using lattice BGL fit, which gives:

$$R(D^*) = 0.252(22)$$

$R(D^*)$ Summary

Thanks to J. Harrison for this summary plot

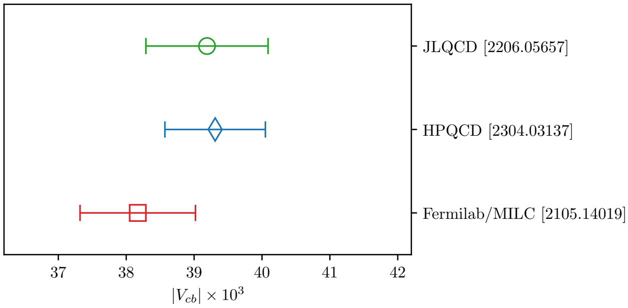


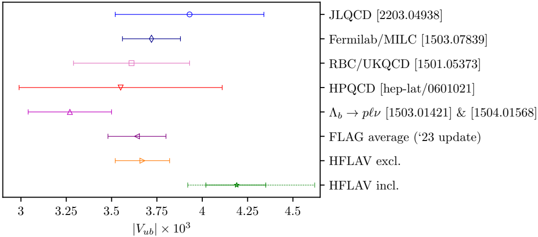
M. Bordone *et al* [1912.09335]

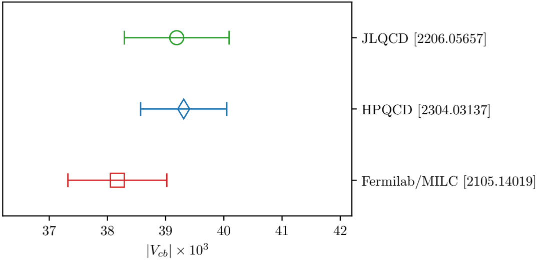
$$\frac{d\Gamma}{dw} = \frac{G_F^2}{16\pi^3}|V_{cb}|^2|\eta_{\mathrm{EW}}|^2 M_B r^2 \sqrt{w^2-1}\left(1-\frac{m_\ell^2}{q^2}\right)^2 \times \left[\frac{q^3}{3}\left(1+\frac{m_\ell^2}{2q^2}\right)\left\{|H_+(w)|^2+|H_-(w)|^2+|H_0(w)|^2\right\}+\frac{m_\ell^2}{2}|H_S(w)|^2\right]$$

$$|V_{cb}| = 39.19(90)\times 10^{-3}$$

- ★ good agreement with previous determinations of $|V_{cb}|$ from exclusive decay
  - ▶ (but tension with inclusive result)







Recent results

★ JLQCD calculated CKM matrix elements $|V_{ub}|$ and $|V_{cb}|$, finding:

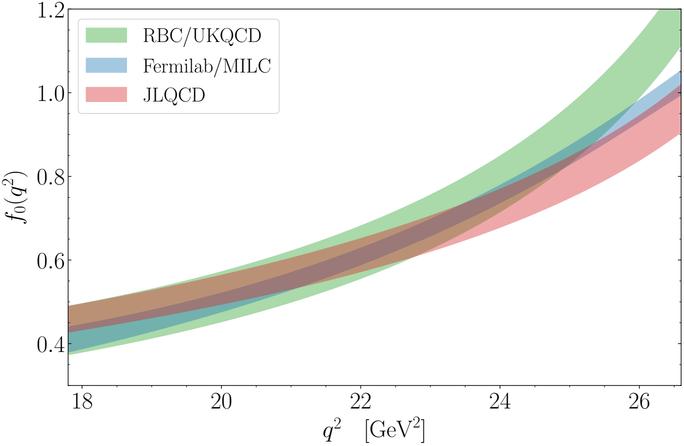
$$|V_{ub}| = 3.93(41) \times 10^{-3}$$
$$|V_{cb}| = 39.19(90) \times 10^{-3}$$

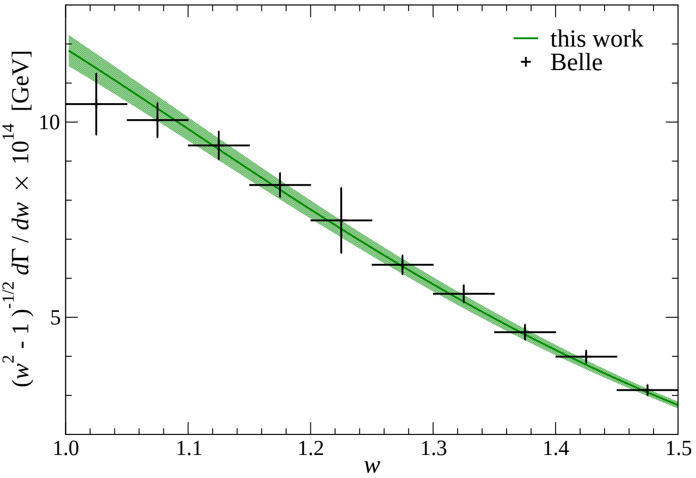
- Both consistent with exclusive decays.
- $B \to \pi$ also consistent with inclusive result

Thank you!

Extra stuff

Comparison with other lattice calculations: f_0




this work
Belle
10
5
1.0
1.1
1.2
1.3
1.4
1.5
w
(w^2 - 1)^-1/2 dΓ / dw × 10^14 [GeV]