The U-spin-CP anomaly in charm

Héctor Gisbert

U. Padova & INFN

Mainly based on 2210.16330.

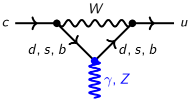
12th International Workshop on the CKM Unitarity Triangle, September 18-22, 2023, Santiago de Compostela, Spain

Charm physics is exceptional 2011.09478

1. **Window to test FCNCs in the up-sector!**

2. **Strong non-perturbative dynamics → "Null tests" $\mathcal{O} \pm \delta\,\mathcal{O}$**
   - Use SM symmetries: $\mathcal{O}_{\rm SM} = \mathbf{0}$,
   - Small uncertainties: $\mathcal{O}_{\rm SM} \gg \delta\,\mathcal{O}_{\rm SM}$,
   - Use large hadronic effects to enhance NP contributions, ...

3. **Very efficient GIM mechanism:** $\sum_i \lambda_i = 0$ with $\lambda_i \equiv V_{ci}^* V_{ui}$.



$$= \sum_{i=d,s,b} \lambda_i\, f_i \;=\; \lambda_s \left[(f_s - f_d) + \frac{\lambda_b}{\lambda_s}\,(f_b - f_d) \right]$$

$f_i \sim \frac{m_i^2}{(4\pi)^2\, M_W^2}$, $\mathrm{Im}(\lambda_b/\lambda_s) \sim 10^{-3}$

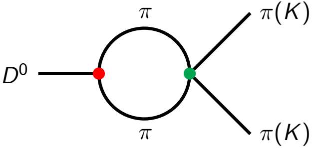
BRs ($A_{\rm CP}$) are loop-(CKM-) suppressed!

Excellent place to search for BSM physics!

ΔA_{CP} predictions in the SM Talk of U. Nierste

$$\Delta A_{CP}^{SM} \approx r \sin\phi_{CKM} \sin\delta_{QCD}$$

$$r = r_{CKM}\, r_{QCD}$$



- $\sin\phi_{CKM} \sim \mathrm{Im}(\lambda_b/\lambda_s) \sim 10^{-3}$.
- $\sin\delta_{QCD} \sim \mathcal{O}(1)$, large strong phases.
- $r_{CKM} = \left|\frac{\lambda_d}{\lambda_s}\right| = 1$, ratio of CKM factors.
- What is the ratio of rescattering r_{QCD}?

Light Cone Sum Rules (LCSR)

$$r_{QCD} \sim \mathcal{O}\left(\frac{\alpha_s}{\pi}\right) \sim 10^{-1}$$

$$\Delta A_{CP}^{SM} \sim 10^{-4}$$

Not explains exp. value

Low energy QCD

$$r_{QCD} \sim 1$$

$$\Delta A_{CP}^{SM} \sim 10^{-3}$$

Compatible with exp. value

$$\Delta A_{CP}^{LHCb} = (-15.4 \pm 2.9) \cdot 10^{-4}$$ 1903.08726

SM prediction of ΔA_{CP} is not well established!

Final results for $A_{CP}(K^-K^+)$ and combination

- Final results for $A_{CP}(K^-K^+)$ are:

$$C_{D^+}: \mathcal{A}_{CP}(K^-K^+) = [13.6 \pm 8.8\,(\text{stat}) \pm 1.6\,(\text{syst})] \times 10^{-4},$$
$$C_{D_s^+}: \mathcal{A}_{CP}(K^-K^+) = [\ 2.8 \pm 6.7\,(\text{stat}) \pm 2.0\,(\text{syst})] \times 10^{-4}.$$

with an overall correlation coefficient $\rho = 0.06$ and are found to be compatible within 1 standard deviation.

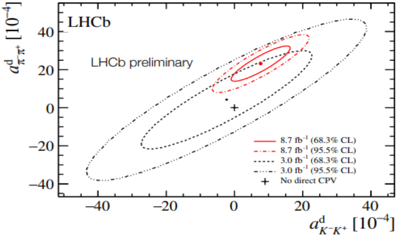
- The combination yields

$$\mathcal{A}_{CP}(K^-K^+) = [6.8 \pm 5.4\,(\text{stat}) \pm 1.6\,(\text{syst})] \times 10^{-4},$$

Serena Maccolini   Direct CPV in charm   7 July 2022   12

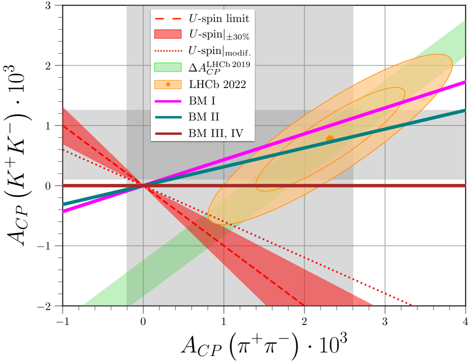
First evidence for direct *CP* violation

$$a^d_{K^-K^+} = (\ 7.7 \pm 5.7) \times 10^{-4}$$
$$a^d_{\pi^-\pi^+} = (23.2 \pm 6.1) \times 10^{-4}$$
with $\rho(a^d_{KK}, a^d_{\pi\pi}) = 0.88$



- They report the first evidence for **direct *CP* violation** in ***D*⁰→π⁻π⁺** decays at the level of **3.8**σ.
- *U-spin* breaking in CP asymmetries: $a^d_{KK} + a^d_{\pi\pi} \neq 0$ at the level of 2.7σ.

What tells the new LHCb result? 2207.08539, 2210.16330



1. $a^d_{\pi^-\pi^+}$ is larger than $|\Delta A_{\rm CP}|$. SM needs even more enhancement!

$$a^d_{\pi^-\pi^+}|_{\rm SM} \sim 2\,{\rm Im}(\lambda_b/\lambda_s)\left(\frac{h}{t}\right) \sim 1.2\cdot 10^{-3}\left(\frac{h}{t}\right) \longrightarrow \boxed{\frac{h}{t}\sim 2}$$

2. Violation of U-spin, $a^d_{K^-K^+} + a^d_{\pi^-\pi^+} \neq 0$, at the level of $2.7\,\sigma$!

BSM CP & U-spin violation?

Flavorful Z' models

- **Gauge symmetry:** $SU(3)_C \times SU(2)_L \times U(1)_Y \times \underbrace{U(1)'}_{g_4}$

- **Representations:**

$$Q_i = (3, 2, 1/6, F_{Q_i}) , \quad u_i = (3, 1, 2/3, F_{u_i}) , \quad d_i = (3, 1, -1/3, F_{d_i}) ,$$
$$L_i = (1, 2, -1/2, F_{L_i}) , \quad e_i = (1, 1, -1, F_{e_i}) , \quad \nu_i = (1, 1, 0, F_{\nu_i}) .$$

- **Z' Lagrangian (in the gauge basis):**

$$\mathcal{L}_{Z'} = g_4 \sum_i \sum_{\psi_i} F_{\psi_i} (\bar{\psi}_i \gamma^\mu \psi_i) Z'_\mu , \quad \psi = Q, L, u, d, e, \nu .$$

- **Charge F_ψ assignment: guarantee anomaly-cancellation.**

Z' effects in $\Delta A_{\rm CP}$ 2004.01206

$$\Delta A_{\rm CP}^{\rm NP} = A_{\rm CP}^{\rm NP}\left(K^+K^-\right) - A_{\rm CP}^{\rm NP}\left(\pi^+\pi^-\right)$$

with (assuming maximal strong phases $\sin\delta_{\pi,K} \sim 1$)

$$A_{\rm CP}^{\rm NP}\left(K^+K^-\right) \sim \left(\frac{g_4}{M_{Z'}}\right)^2 \sin\phi_R\, \Delta\widetilde{F}_R \left[c_K\, F_{Q_2} + d_K\, F_{d_2}\right]$$

$$A_{\rm CP}^{\rm NP}\left(\pi^+\pi^-\right) \sim \left(\frac{g_4}{M_{Z'}}\right)^2 \sin\phi_R\, \Delta\widetilde{F}_R \left[c_\pi\, F_{Q_1} + d_\pi\, F_{d_1}\right]$$

with $\Delta\widetilde{F}_R = \sin\theta_u \cos\theta_u \Delta F_R$ and

$$c_K = \frac{\chi_K}{a_K}\, r_1 \sim +\mathcal{O}(1)\,, \quad c_\pi = -\frac{\chi_\pi}{a_\pi}\, r_1 \sim -\mathcal{O}(1)\,,$$
$$d_K = \frac{1}{a_K}\, r_2 \sim -\mathcal{O}(0.1)\,, \quad d_\pi = -\frac{1}{a_\pi}\, r_2 \sim +\mathcal{O}(0.1)\,.$$

a_P is tree-level amplitude fixed by $\mathcal{B}(D^0 \to P^+P^-)_{\rm exp}$ and $r_{1,2}$ encode RGE effects.

CP asymmetries constraints 2210.16330

$$a^d_{K^-K^+} = \frac{g_4^2}{M_{Z'}^2} \Delta \widetilde{F}_R \left[c_K F_{Q_2} + d_K F_{d_2} \right],$$

$$a^d_{\pi^-\pi^+} = \frac{g_4^2}{M_{Z'}^2} \Delta \widetilde{F}_R \left[c_\pi F_{Q_1} + d_\pi F_{d_1} \right],$$

with $F_{Q_{1,2}} = 0$ (D-mixing constraints), the ratio F_{d_2}/F_{d_1} is fixed:

$$\boxed{\frac{F_{d_2}}{F_{d_1}} = \frac{d_\pi \, a^d_{K^-K^+}}{d_K \, a^d_{\pi^-\pi^+}} \simeq -0.42^{+0.83}_{-0.13}}$$

resulting in a large hierarchy

$$\boxed{|F_{d_2}| \ll |F_{d_1}|}$$

D-mixing constraints 2004.01206, 2210.16330

D-meson mixing constraints from HFLAV average

$$\frac{g_4\,\Delta\widetilde{F}_R}{M_{Z'}} < 7.1\cdot 10^{-4}\,\text{TeV}^{-1}\ \ (95\%\,\text{C.L.})$$

which includes new data from LHCb 2106.03744

Remainder:

$\Delta\widetilde{F}_R$ contains the mixing angle θ_u which can be freely adjusted:

$$\Delta\widetilde{F}_R = \sin\theta_u\cos\theta_u\,(F_{u_2} - F_{u_1}) \approx \theta_u\,(F_{u_2} - F_{u_1})$$

θ_u small is instrumental to build our models

$$\theta_u \ll 1$$

Rare charm decays & Drell-Yan constraints 2210.16330

- BRs of rare D-decays (2011.09478):

$$g_4^2|\Delta\widetilde{F}_R|\sqrt{F_{L_2}^2+F_{e_2}^2}\lesssim 0.02\left(\frac{M_{Z'}}{\text{TeV}}\right)^2,$$

$$g_4^2|\Delta\widetilde{F}_R(F_{L_2}-F_{e_2})|\lesssim 0.02\left(\frac{M_{Z'}}{\text{TeV}}\right)^2.$$

- Drell-Yan data for $\ell = e, \tau$ (2003.12421):

$$g_4^2|\Delta\widetilde{F}_R|\sqrt{F_{L_{1\,(3)}}^2+F_{e_{1\,(3)}}^2}\lesssim 0.06\,(0.12)\left(\frac{M_{Z'}}{\text{TeV}}\right)^2.$$

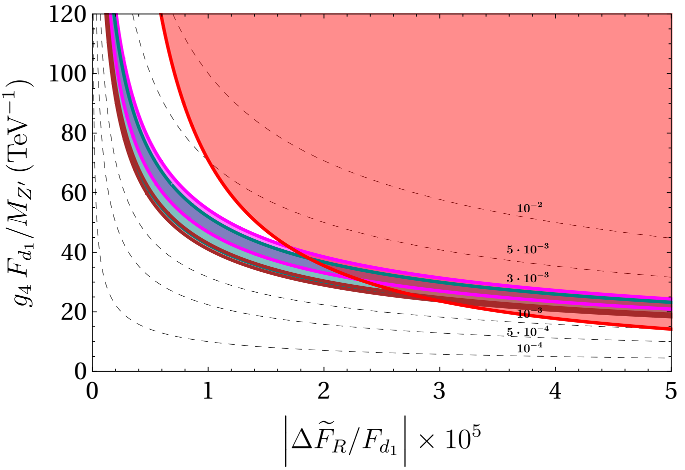
- Combined with the D-mixing constraint:

$$|F_{L_2}-F_{e_2}|,\ \sqrt{F_{L_2}^2+F_{e_2}^2}\lesssim 0.8\,|F_{d_1}|$$

$$\sqrt{F_{L_1}^2+F_{e_1}^2}\lesssim 2.3\,|F_{d_1}|$$

$$\sqrt{F_{L_3}^2+F_{e_3}^2}\lesssim 4.7\,|F_{d_1}|$$

Charm constraints synopsis 2210.16330



$$\frac{g_4 F_{d_1}}{M_{Z'}} \sim \frac{1}{0.025\ \text{TeV}} \times \frac{|a^d_{\pi^-\pi^+}|}{0.002} \rightarrow$$ **light Z' mass or large $g_4\, F_{d_1}$**

Anomaly-free $U(1)'$ models 2210.16330

$$
\begin{aligned}
&2\langle \mathcal{F}_Q \rangle - \langle \mathcal{F}_u \rangle - \langle \mathcal{F}_d \rangle = 0\,,\\
&3\langle \mathcal{F}_Q \rangle + \langle \mathcal{F}_L \rangle = 0\,,\\
&\langle \mathcal{F}_Q \rangle + 3\langle \mathcal{F}_L \rangle - 8\langle \mathcal{F}_u \rangle - 2\langle \mathcal{F}_d \rangle - 6\langle \mathcal{F}_e \rangle = 0\,,\\
&6\langle \mathcal{F}_Q \rangle + 2\langle \mathcal{F}_L \rangle - 3\langle \mathcal{F}_u \rangle - 3\langle \mathcal{F}_d \rangle - \langle \mathcal{F}_e \rangle - \langle \mathcal{F}_\nu \rangle = 0\,,\\
&\langle \mathcal{F}_Q^2 \rangle - \langle \mathcal{F}_L^2 \rangle - 2\langle \mathcal{F}_u^2 \rangle + \langle \mathcal{F}_d^2 \rangle + \langle \mathcal{F}_e^2 \rangle = 0\,,\\
&6\langle \mathcal{F}_Q^3 \rangle + 2\langle \mathcal{F}_L^3 \rangle - 3\langle \mathcal{F}_u^3 \rangle - 3\langle \mathcal{F}_d^3 \rangle - \langle \mathcal{F}_e^3 \rangle - \langle \mathcal{F}_\nu^3 \rangle = 0\,.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_X &= \mathrm{diag}(F_{X_1}, F_{X_2}, F_{X_3})\\
\langle \mathcal{F}_X \rangle &= \mathrm{Tr}(\mathcal{F}_X)
\end{aligned}
$$

Model	F_{Q_i}			F_{u_i}			F_{d_i}			F_{L_i}			F_{e_i}			F_{ν_i}		
BM I	0	0	0	9	-16	7	20	-11	-9	15	-6	-9	-16	0	16	6	12	-18
BM II	0	0	0	-19	9	10	20	-8	-12	4	1	-5	15	2	-17	8	2	-10
BM III	0	0	0	0	-1	0	1	0	0	0	0	0	0	0	1	0	0	-1
BM IV	0	0	0	$-\frac{985}{1393}$	$\frac{985}{1393}$	0	1	0	-1	0	0	0	$\frac{1}{1393}$	0	$-\frac{1}{1393}$	F_ν	$-F_\nu$	0

- **All these BMs survive the previous constraints!**
- **BMs feature U-spin and IB with signals in $\pi^+\pi^0$, $\pi^0\pi^0$:**

$$
A_{\rm CP}(\pi^0\pi^0) \simeq A_{\rm CP}(\pi^+\pi^0) \simeq \left(1 - \frac{F_{u_1}}{F_{d_1}}\right) |\Delta A_{\rm CP}| \simeq (1-2)\, 10^{-3}
$$

A flavorful Z' of $\mathcal{O}(10)$ GeV? 2210.16330

⋆ **Constraints from $Z' \to \ell^+\ell^-$ searches:**

- Severe constraints in 1-100 GeV range.
- Exp. constraints from e & μ (1801.04847 & 1910.06926)

$$\boxed{g_4\, F_{e_{1,2},L_{1,2}} \lesssim 4 \cdot 10^{-4}}$$

- Combined with previous bounds, leads to

$$\boxed{\frac{F_{e_{1,2},L_{1,2}}}{F_{d_1}} \lesssim \frac{1}{750}}$$

directly excluding BM I and II and dictating a strong quark and lepton charge hierarchy in BM III and IV.

A flavorful Z' of $\mathcal{O}(10)$ GeV? 2210.16330

⋆ **Constraints from dijets searches:**

- For $10\,\text{GeV} \lesssim M_{Z'} \lesssim 50\,\text{GeV}$, the strongest constraints are from CMS (1905.10331), and their dijet plus initial state radiation (2112.05392). Using their results, approximately $g_4 F_{d_1} \lesssim 0.5$, together with the previous constraints:

$$\boxed{10\,\text{GeV} \lesssim M_{Z'} \lesssim 20\,\text{GeV}}$$

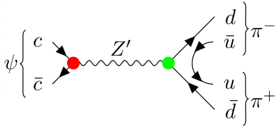
- $\Upsilon(\bar{b}b) \to j\,j$ constraints around 10 GeV (1404.3947).

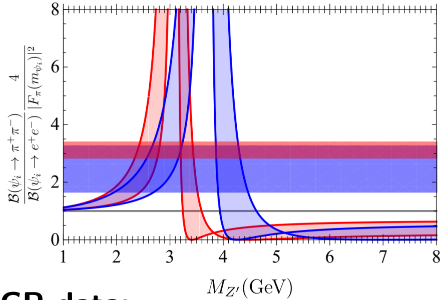
$$\boxed{M_{Z'} \lesssim 7\,\text{GeV} \text{ or } M_{Z'} \gtrsim 15\,\text{GeV} \text{ (BM IV)}}$$

- BM III does not couple to b's, we can evade the Υ-limits.

A flavorful Z' of $\mathcal{O}(10)$ GeV? 2210.16330

⋆ **Constraints from charmonium decays:**





Charmonia + Υ + charm CP data:

$M_{Z'} \sim [2.3, 2.8]$ **GeV** or $M_{Z'} \sim [3.2, 3.3]$ **GeV (BM III)**

$M_{Z'} \sim [4.6, 7]$ **GeV (BM IV)**

Resolving the tension between F_π extracted from J/ψ-decays assuming the leading photon exchange hep-ph/0409080

A flavorful Z' of $\mathcal{O}(10)$ GeV? 2210.16330

$\star$ Constraints in BM III from $\psi(2s) \to \tau^+\tau^-$:

Similarly as $\psi_i \to \pi^+\pi^-$, using $\mathcal{B}(\psi(2s) \to \tau^+\tau^-)$ from PDG:

$$M_{Z'} \lesssim 2.2\,\text{GeV or } [4.0, 4.8]\,\text{GeV}$$

very close to the windows implied by F_π.

$\star$ Constraints from $J/\psi(1s) \to$ nothing:

Using $\mathcal{B}(J/\psi(1s) \to \text{nothing})$ from PDG:

$$M_{Z'} \lesssim 0.7\,\text{GeV}$$

in conflict with F_π windows, or the BSM neutrino mass $m_\nu > m_{J/\psi(1s)}/2$ to forbid the decay kinematically.

Summary

- **The data from LHCb require a huge amount of U-spin breaking.**
- **Explaining the data poses a challenge to model building, given the low NP scale and the severe constraints.**
- **A $e\,\&\,\mu-$phobic light Z' can explain the U-spin-CP anomaly.**
- **Signatures & search channels:**
  - CP asymmetries in $D^0 \to \pi^0\pi^0$ and $D^+ \to \pi^0\pi^+$.
  - Low mass dijets $Z' \to q\bar{q}$.
  - Enhanced $D\bar{D}$, $\pi\pi$, $\tau\tau$ production.
  - Dark photon searches
  - Invisible & hadronic D decays
  - J/ψ, $\psi(2s)$, Υ decays

Thank you for your attention!

BACKUP

From gauge to mass basis via rotations

- **Rotations:** 4 unitary matrices, $V_u^\dagger V_u = V_d^\dagger V_d = U_u^\dagger U_u = U_d^\dagger U_d = I$

$$(u'_L)_i = (V_u)_{ij}\,(u_L)_j \,, \qquad (u'_R)_i = (U_u)_{ij}\,(u_R)_j \,,$$

$$(d'_L)_i = (V_d)_{ij}\,(d_L)_j \,, \qquad (d'_R)_i = (U_d)_{ij}\,(d_R)_j \,. \qquad \boxed{V_{\rm CKM} = V_u^\dagger V_d}$$

- **Z' Lagrangian for charm FCNCs (in the mass basis):**

$$\mathcal{L}_{Z'} \supset \left(g_L^{uc}\,\bar{u}_L\gamma^\mu c_L Z'_\mu + g_R^{uc}\,\bar{u}_R\gamma^\mu c_R Z'_\mu + \text{h.c.}\right) + g_L^d\,\bar{d}_L\gamma^\mu d_L Z'_\mu + g_R^d\,\bar{d}_R\gamma^\mu d_R Z'_\mu + g_L^s\,\bar{s}_L\gamma^\mu s_L Z'_\mu + g_R^s\,\bar{s}_R\gamma^\mu s_R Z'_\mu + \sum_{\ell=e,\mu,\tau}\left(g_L^{\ell\ell}\bar{\ell}_L\gamma^\mu\ell_L + g_R^{\ell\ell}\bar{\ell}_R\gamma^\mu\ell_R\right) Z'_\mu$$

$$\boxed{g_L^{d,s} = g_4\,F_{Q_{1,2}}, \quad g_R^{d,s} = g_4\,F_{d_{1,2}}, \quad g_L^{\ell\ell} = g_4\,F_{L_\ell}, \quad g_R^{\ell\ell} = g_4\,F_{e_\ell}}$$

- **Avoid strong constraints in the kaon sector** $\rightarrow$ $\boxed{V_d = U_d = I}$

$$\boxed{g_L^{uc} = g_4\,\Delta F_L\,\lambda} \qquad \boxed{g_R^{uc} = g_4\,\Delta F_R\,\sin\theta_u\,\cos\theta_u\,e^{i\,\phi_R}}$$

with $\Delta F_L = F_{Q_2} - F_{Q_1}$ and $\Delta F_R = F_{u_2} - F_{u_1}$.

$|\Delta c| = |\Delta u| = 1$ FCNC couplings $g^{uc}_{L,R}$

- Avoid strong constraints in the kaon sector $\rightarrow$ $V_d = U_d = I$

$$V_{CKM} = V_u^\dagger \rightarrow (V_{\rm CKM})_{2\times 2} = \begin{pmatrix} \cos\Phi_u & \sin\Phi_u \\ -\sin\Phi_u & \cos\Phi_u \end{pmatrix}, \quad \sin\Phi_u = \lambda \approx 0.2\,.$$

$$(U_u)_{2\times 2} = \begin{pmatrix} \cos\theta_u & \sin\theta_u e^{-i\,\phi_R} \\ -\sin\theta_u e^{i\,\phi_R} & \cos\theta_u \end{pmatrix} \rightarrow \boxed{\text{1 CP-phase in RH up sector}}$$

- After rotation:

$$g_L^{uc} = g_4\,(V_{\rm CKM}\,F_Q\,V_{\rm CKM}^\dagger)_{12} = g_4\,(F_{Q_2} - F_{Q_1})\,\sin\Phi_u\,\cos\Phi_u\,,$$

$$g_R^{uc} = g_4\,(U_u^\dagger\,F_u\,U_u)_{12} = g_4\,(F_{u_2} - F_{u_1})\,\sin\theta_u\,\cos\theta_u\,e^{i\,\phi_R}\,,$$

- CP violation BSM generated by RH up rotation in g_R^{uc},

$$\boxed{g_L^{uc} = g_4\,\Delta F_L\,\lambda} \qquad \boxed{g_R^{uc} = g_4\,\Delta F_R\,\sin\theta_u\,\cos\theta_u\,e^{i\,\phi_R}}$$

with $\Delta F_L = F_{Q_2} - F_{Q_1}$ and $\Delta F_R = F_{u_2} - F_{u_1}$.

Four-fermion operators

New $U(1)'$ charges require new operators, like EW penguins.

High-energy scales

8 additional operators:

$$\widetilde{Q}_7 = (\bar{u}c)_{V-A} \sum_q F_{u_i,d_i} (\bar{q}q)_{V+A} \ , \qquad \widetilde{Q}'_7 = (\bar{u}c)_{V+A} \sum_q F_{Q_i} (\bar{q}q)_{V-A} \ ,$$

$$\widetilde{Q}_8 = (\bar{u}_\alpha c_\beta)_{V-A} \sum_q F_{u_i,d_i} (\bar{q}_\beta q_\alpha)_{V+A} \ , \quad \widetilde{Q}'_8 = (\bar{u}_\alpha c_\beta)_{V+A} \sum_q F_{Q_i} (\bar{q}_\beta q_\alpha)_{V-A},$$

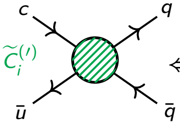
$$\widetilde{Q}_9 = (\bar{u}c)_{V-A} \sum_q F_{Q_i} (\bar{q}q)_{V-A} \ , \qquad \widetilde{Q}'_9 = (\bar{u}c)_{V+A} \sum_q F_{u_i,d_i} (\bar{q}q)_{V+A} \ ,$$

$$\widetilde{Q}_{10} = (\bar{u}_\alpha c_\beta)_{V-A} \sum_q F_{Q_i} (\bar{q}_\beta q_\alpha)_{V-A} \ , \quad \widetilde{Q}'_{10} = (\bar{u}_\alpha c_\beta)_{V+A} \sum_q F_{u_i,d_i} (\bar{q}_\beta q_\alpha)_{V+A},$$

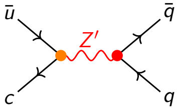
with $q = u, c, d, s, b$ and α, β are color indices.

Matching and RGEs

Matching condition at high-energy scales:



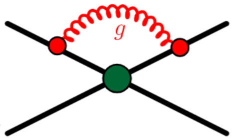
$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \sum_i \widetilde{C}_i^{(\prime)}\, \widetilde{Q}_i^{(\prime)} = \mathcal{L}_{Z'}$$



$$\widetilde{C}_{7,9}(M_{Z'}) = \frac{\sqrt{2}}{G_F}\,\frac{g_4\, g_L^{uc}}{4\, M_{Z'}^2}\,, \quad \widetilde{C}'_{7,9}(M_{Z'}) = \frac{\sqrt{2}}{G_F}\,\frac{g_4\, g_R^{uc}}{4\, M_{Z'}^2}\,, \quad \widetilde{C}^{(\prime)}_{8,10}(M_{Z'}) = 0\,.$$

QCD plays a role at low-energy: RGEs mix different operators

$$\left(\frac{\lambda^a}{2}\right)_{\alpha\beta}\left(\frac{\lambda^a}{2}\right)_{\gamma\delta} = \delta_{\alpha\delta}\,\delta_{\beta\gamma} - \frac{1}{N_C}\,\delta_{\alpha\beta}\,\delta_{\gamma\delta}$$



Anomalous dimension

$$\Rightarrow \quad \gamma_F^0 \quad \Rightarrow$$

$$\begin{aligned}
\widetilde{C}_7^{(\prime)}(m_c) &= 0.829\, \widetilde{C}_7^{(\prime)}(M_{Z'})\,,\\
\widetilde{C}_8^{(\prime)}(m_c) &= 1.224\, \widetilde{C}_7^{(\prime)}(M_{Z'})\,,\\
\widetilde{C}_9^{(\prime)}(m_c) &= 1.404\, \widetilde{C}_9^{(\prime)}(M_{Z'})\,,\\
\widetilde{C}_{10}^{(\prime)}(m_c) &= -0.718\, \widetilde{C}_9^{(\prime)}(M_{Z'})\,.
\end{aligned}$$

Estimation of hadronic matrix elements (HME)

Factorization of currents: $Q_i = (\bar{q}_1\,\Gamma_1\,q_2)\,(\bar{q}_3\,\Gamma_2\,q_4)$

$$\left\langle P^+\,P^- \middle|\, Q_i \,\middle| D^0 \right\rangle = \left\langle P^+ \middle| \,(\bar{q}_1\,\Gamma_1\,q_2)\, \middle| 0 \right\rangle \left\langle P^- \middle| \,(\bar{q}_3\,\Gamma_2\,q_4)\, \middle| D^0 \right\rangle B_i^{P^+P^-}$$

where $B_i^{P^+P^-}$ parametrizes the deviation of the true HME from $B_i^{P^+P^-}|_{\text{naïve}} = 1$.

After Fierz identities in the flavor and color space:

$$\left\langle P^+P^- \middle|\, Q_i \,\middle| D^0 \right\rangle_{\text{Penguin}} = (\text{factor}) \times (\text{HME}_{\text{Tree}})$$

then it cancels in the CP-asymmetry: $A_{CP} \propto \frac{\text{HME}_{\text{Penguin}}}{\text{HME}_{\text{Tree}}}$.

What does the "factor" contain?

- **Chiral factor (Hadronization):**
  - Non-enhanced: $\widetilde{Q}_{9,10}$
  - Enhanced: $\widetilde{Q}_{7,8}$

$$\left\langle P^+P^- \middle|\, Q_i^{(V-A)\times(V+A)} \,\middle| D^0 \right\rangle \propto \frac{2\,M_P^2}{m_c\,(m_{q_1}+m_{q_2})}$$

- **Color factor (Fierz):**
  - Non-suppressed: $\widetilde{Q}_{8,10}$
  - Suppressed: $\widetilde{Q}_{7,9}$

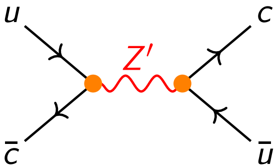
$$\left(\frac{\lambda^a}{2}\right)_{\alpha\beta}\left(\frac{\lambda^a}{2}\right)_{\gamma\delta} = \delta_{\alpha\delta}\,\delta_{\beta\gamma} - \frac{1}{N_C}\,\delta_{\alpha\beta}\,\delta_{\gamma\delta}$$

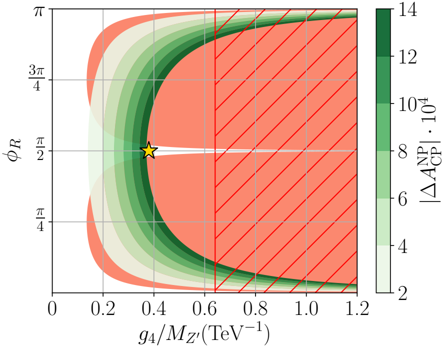
$D^0 - \overline{D}^0$ mixing constraints

- **Amplitude:** $\langle D^0|\mathcal{H}_{\rm eff}^{\Delta c=2}|\overline{D}^0\rangle = M_{12} - \frac{i}{2}\Gamma_{12}$
- **3 physical quantities:**
  $x_{12} = 2\frac{|M_{12}|}{\Gamma}\,,\; y_{12} = \frac{|\Gamma_{12}|}{\Gamma}\,,\; \phi_{12} = \arg\left(\frac{M_{12}}{\Gamma_{12}}\right)\,.$
- **Require NP contributions to saturate the current world averages (HFLAV):**
  $x_{12}^{\rm NP} \leq x_{12}\,,\quad x_{12}^{\rm NP}\sin\phi_{12}^{\rm NP} \leq x_{12}\sin\phi_{12}$
- **Constraint from x_{12}:**
  $\left|(g_L^{uc})^2 + (g_R^{uc})^2 - X\, g_L^{uc}\, g_R^{uc}\right| \lesssim 6\cdot 10^{-7}\left(\frac{M_{Z'}}{\rm TeV}\right)^2$
- **Avoided via aligment:** $g_L^{uc} \sim X\, g_R^{uc}$
- **Implies:** $\mathrm{Arg}(g_L^{uc}) \sim \mathrm{Arg}(g_R^{uc})$
- **BUT kaon constraints kill $\mathrm{Arg}(g_L^{uc})$!**
- $g_L^{uc} = 0 \;\rightarrow\; \Delta F_L = 0 \;\rightarrow\; F_{Q_1} = F_{Q_2}$!
  ⋆: **Model 2 with $\Delta A_{\rm CP}^{\rm NP} \sim 10^{-3}$**

$\Delta F_R = 12,\; \phi_R \sim \pi/2\,,\; g_4/M_{Z'} \sim 0.38/\text{TeV}\,,\; \theta_u \sim 1\cdot 10^{-4}\,.$

Same couplings as rare $|\Delta c| = |\Delta u| = 1$ decays!

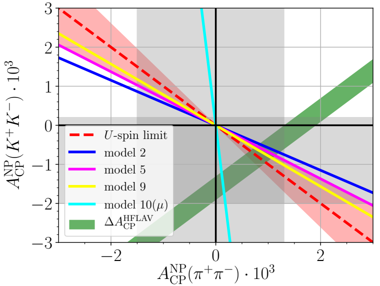




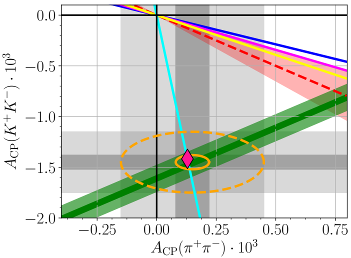
U-spin patterns in $D^0 \to \pi^+\pi^-$, K^+K^-

- **U-spin symmetry:** invariant under $d \Longleftrightarrow s$.
- **Obviously is broken** (by M_P and f_P, $\pi^+ = u\,\bar{d}$ and $K^+ = u\,\bar{s}$).
- **Z' model:** U–spin breaking arises for $F_{Q_1} \neq F_{Q_2}$ or $F_{d_1} \neq F_{d_2}$!
- **U-spin sum rule** (broken $\delta U_{\text{break}} \lesssim 30\,\%$ 1308.4143):

$$A_{\text{CP}}(D^0 \to K^+K^-) + A_{\text{CP}}(D^0 \to \pi^+\pi^-) = 0 + \delta U_{\text{break}}$$



Green and gray bands are the $1\,\sigma$ experimental world averages (HFLAV).



Future experimental projections over model 10(μ). Ligher (darker) bands correspond to LHCb Run 1-3 (1-5).

Isospin breaking patterns in $D^+ \to \pi^+\pi^0$

- **Isospin symmetry:** invariant under $u \Longleftrightarrow d$.
- **Softly broken** (10% by $m_u \neq m_d$ and QED corrections).
- Z' **model:** Isospin breaking arises for $F_{u_1} \neq F_{d_1}$!

$$A_{\rm CP}^{\rm NP}(\pi^+\pi^0) \sim \frac{g_4^2}{M_{Z'}^2}\,\Delta\widetilde{F}_R\, d_{\pi'}\,(F_{d_1} - F_{u_1})$$

Models 9 and 10(μ):

$$A_{\rm CP}^{\rm NP}(\pi^+\pi^0) \sim (1-2)\cdot \Delta A_{\rm CP}^{\rm NP}$$

for $\Delta A_{\rm CP}^{\rm NP} \sim 10^{-3}$ is within the projected sensitivity of Belle II,

$$\sigma(A_{\rm CP}(\pi^+\pi^0))_{\text{Belle II}} = 1.7\cdot 10^{-3} \text{ for } 50\text{ab}^{-1}\,.$$

Constraints from invisibles data

- Missing energy can stem from ν_R and/or vector-like dark fermions χ charged under the $U(1)'$ only.
- $\mathcal{B}(D^0 \to \pi^0 + \text{inv.})$ is constrained by BES III (2112.14236)

$$\mathcal{B}(D^0 \to \pi^0\ \text{inv.}) < 2.1 \cdot 10^{-4}\ (90\%\ \text{C.L.})\,.$$

Neglecting finite m_χ corrections

$$\mathcal{B}(D^0 \to \pi^0 \nu\bar{\nu}, \chi\bar{\chi}) \approx \frac{2\pi^2 A_+}{G_F^2 \alpha_e^2} \left(\frac{g_4^2 \Delta \widetilde{F}_R F_{\nu,\chi}}{M_{Z'}^2} \right)^2$$

- Following a similar procedure as the dilepton constraints

$$\boxed{|F_{\nu,\chi}| \lesssim 110\, |F_{d_1}|}$$

for $m_{\nu_R,\chi} < m_D/2 \approx 0.9$ GeV.

BM III: Can we avoid them? Setting them to zero?

- If the Z' does not couple directly to e and μ, one can still induce a small coupling ε from $Z' - \gamma$ gauge-kinetic mixing

$$\mathcal{L} \supset -\tfrac{1}{4} F^{\mu\nu} F_{\mu\nu} - \tfrac{1}{4} Z'^{\mu\nu} Z'_{\mu\nu} - \tfrac{\eta}{2} F^{\mu\nu} Z'_{\mu\nu}\,, \quad \varepsilon = -\frac{\eta}{\sqrt{1-\eta^2}}\,.$$

- ε is also related to the parameter $\rho = M_W/(M_Z \cos\theta_W)$ which from global fit of electroweak precision parameters leads to

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{NP}} = (3.8 \pm 2.0) \cdot 10^{-4} \rightarrow \boxed{|\varepsilon(M_Z)| \lesssim 4 \cdot 10^{-1}}$$

- η cannot be switched off at more than one scale, so it has to be compatible with $|\varepsilon(M_{Z'})| \lesssim 10^{-3}$. Using RG evolution, the U-spin-CP anomaly requires

$$|\varepsilon(M_Z) - \varepsilon(M_{Z'})| \gtrsim \frac{10\,e}{\pi^2} \frac{M_{Z'}}{\text{TeV}} \ln\left(\frac{M_Z}{M_{Z'}}\right) \gtrsim 10^{-3}$$

- ρ & $Z' \to \ell\ell$ constraints can be avoided if $|\varepsilon(M_Z)| \sim \mathcal{O}(10^{-2})$.

BM IV: Fun with Diophantine equations!

If Z' directly couples to e or μ (like BM IV)? Can we find solutions anomaly-free such as

$$\frac{F_{e_{1,2},L_{1,2}}}{F_{d_1}} \lesssim \frac{1}{750} \ ?$$

The gauge anomaly cancellation conditions (ACCs) read

$$2\langle\mathcal{F}_Q\rangle - \langle\mathcal{F}_u\rangle - \langle\mathcal{F}_d\rangle = 0\,, \tag{1}$$
$$3\langle\mathcal{F}_Q\rangle + \langle\mathcal{F}_L\rangle = 0\,, \tag{2}$$
$$\langle\mathcal{F}_Q\rangle + 3\langle\mathcal{F}_L\rangle - 8\langle\mathcal{F}_u\rangle - 2\langle\mathcal{F}_d\rangle - 6\langle\mathcal{F}_e\rangle = 0\,, \tag{3}$$
$$6\langle\mathcal{F}_Q\rangle + 2\langle\mathcal{F}_L\rangle - 3\langle\mathcal{F}_u\rangle - 3\langle\mathcal{F}_d\rangle - \langle\mathcal{F}_e\rangle - \langle\mathcal{F}_\nu\rangle = 0\,, \tag{4}$$
$$\langle\mathcal{F}_Q^2\rangle - \langle\mathcal{F}_L^2\rangle - 2\langle\mathcal{F}_u^2\rangle + \langle\mathcal{F}_d^2\rangle + \langle\mathcal{F}_e^2\rangle = 0\,, \tag{5}$$
$$6\langle\mathcal{F}_Q^3\rangle + 2\langle\mathcal{F}_L^3\rangle - 3\langle\mathcal{F}_u^3\rangle - 3\langle\mathcal{F}_d^3\rangle - \langle\mathcal{F}_e^3\rangle - \langle\mathcal{F}_\nu^3\rangle = 0\,. \tag{6}$$

In addition, avoiding $Z' - Z$ kinetic mixing at one loop requires

$$\langle\mathcal{F}_Q\rangle - \langle\mathcal{F}_L\rangle + 2\langle\mathcal{F}_u\rangle - \langle\mathcal{F}_d\rangle - \langle\mathcal{F}_e\rangle = 0\,. \tag{7}$$

First, we focus on those equations that are linear with trace charge matrices, that are Eqs. (1),(2),(3),(4), and (7). Solving them,

$$\langle\mathcal{F}_Q\rangle = -\langle\mathcal{F}_u\rangle = \frac{1}{3}\langle\mathcal{F}_d\rangle = -\frac{1}{3}\langle\mathcal{F}_L\rangle = -\langle\mathcal{F}_e\rangle = -\frac{1}{5}\langle\mathcal{F}_\nu\rangle$$

BM IV: Fun with Diophantine equations!

Setting $F_{Q_{1,2,3}} = 0$ to avoid kaon constraints, makes all charge matrices traceless, $\langle \mathcal{F}_A \rangle = 0$.

Let us work on the remaining Eqs (5) and (6). To solve the problem mathematical relations between $\langle \mathcal{F}_A \rangle$, $\langle \mathcal{F}_A^2 \rangle$ and $\langle \mathcal{F}_A^3 \rangle$ would be helpful. For 3×3 matrices holds (Cayley-Hamilton relation)

$$\langle \mathcal{F}_A^3 \rangle - \frac{3}{2} \langle \mathcal{F}_A^2 \rangle \langle \mathcal{F}_A \rangle + \frac{1}{2} \langle \mathcal{F}_A \rangle^3 = 3 \det(\mathcal{F}_A) \;=\; 3\, F_{A_1} F_{A_2} F_{A_3}$$

It follows from $\langle \mathcal{F}_A \rangle = 0$ that $\langle \mathcal{F}_A^3 \rangle$ vanishes if one charge vanishes, then Eq. (6) is trivially fulfilled. In general,

$$\mathcal{F}_A = F_A \text{diag}(+1, -1, \quad 0), \quad A = u, d, L, e, \nu$$

where F_A are integers. The ordering $(+1, -1, 0)$ can be changed.

BM IV: Fun with Diophantine equations!

It remains to solve Eq. (5), which now simply reduces to

$$F_d^2 + F_e^2 = F_L^2 + 2\,F_u^2$$

Setting $F_e = F_L = 0$? Only the trivial solution $F_u = F_d = 0$. Let us simplify by setting just $F_L = 0$:

$$F_d^2 + F_e^2 = 2F_u^2 \;\rightarrow\; \langle F|\mathcal{J}|F\rangle = F_e^2$$

with $|F\rangle = (F_u, F_d)$ and $\mathcal{J} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}$. The first non-trivial solution, $(F_d, F_e, F_u) = (1, 1, 1)$, is

$$\langle F_0|\mathcal{J}|F_0\rangle = 1\,, \quad |F_0\rangle = (1, 1)\,. \tag{8}$$

Are there more solutions such as $F_e \ll F_u, F_d$? One possibility is if a transformation $\mathcal{J} \longrightarrow \mathcal{J}' = U^T\,\mathcal{J}\,U = \mathcal{J}$ by a 2×2 matrix with integer entries U, leads invariant Eq. (8) so that we can generate recursively solutions

$$|F_i\rangle = (U)^i\,|F_0\rangle$$

which could get enlarged while keeping F_e fixed.

BM IV: Fun with Diophantine equations!

This matrix needs to satisfy

$$\begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix}^T \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} .$$

The smallest integer solution reads

$$U = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} .$$

We then find the solutions

$$|F_1\rangle = (5, 7)\,,\ |F_2\rangle = (29, 41)\,,\ |F_3\rangle = (169, 239)\,,$$
$$|F_4\rangle = (985, 1393)\,,\ |F_5\rangle = (5741, 8119)\,,\ \ldots$$

To avoid e & μ constraints, we need solutions $|F_i\rangle$ with $i \geq 4$. In our analysis, we considered the solution $i = 4$ for BM IV,

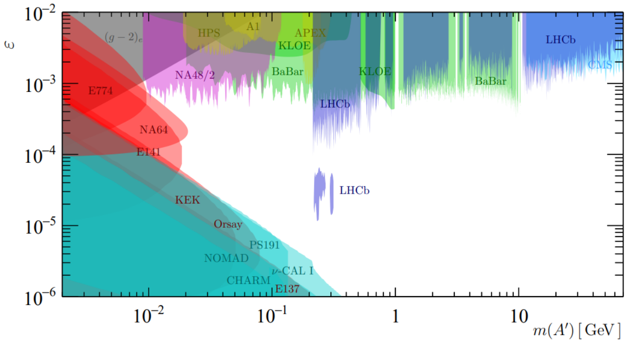
$$\boxed{(F_d, F_e, F_u) = (1393, 1, 985)}$$

with zero neutrino charges $F_\nu = 0$.

A flavorful Z' of $\mathcal{O}(10)$ GeV?

- The scale required points to a light Z' (large couplings get in trouble with perturbativity).
- Long-lived $Z' \to e^- e^+, \mu^- \mu^+$ searches provide severe constraints in the 1-100 GeV range (1801.04847 & 1910.06926).

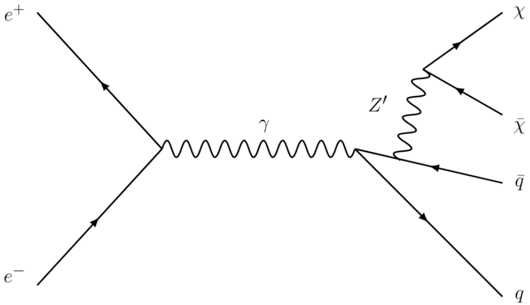
  $\mathcal{L}_\varepsilon = -\varepsilon\, e\, J^\mu A'_\mu$



- Small e & μ couplings (stronger than rare decays & DY).

Branching ratios of the light Z' boson

Model	light quarks	b	c	e	μ	τ	ν_R
BM III$\|_{M_{Z'}=2.5\,\text{GeV}}$	75	0	0	0	0	0	25
BM III$\|_{M_{Z'}=15\,\text{GeV}}$	38	0	37	0	0	12	13
BM III-s$\|_{M_{Z'}=2.5\,\text{GeV}}$	86	0	0	0	0	0	14
BM III-s$\|_{M_{Z'}=15\,\text{GeV}}$	75	0	0	0	0	12	13
BM IV$\|_{M_{Z'}=5\,\text{GeV}}$	79	0	21	0	0	0	0
BM IV$\|_{M_{Z'}=15\,\text{GeV}}$	54	28	18	0	0	0	0



Smoking gun signature for e^+e^- machines.

High energy behaviour

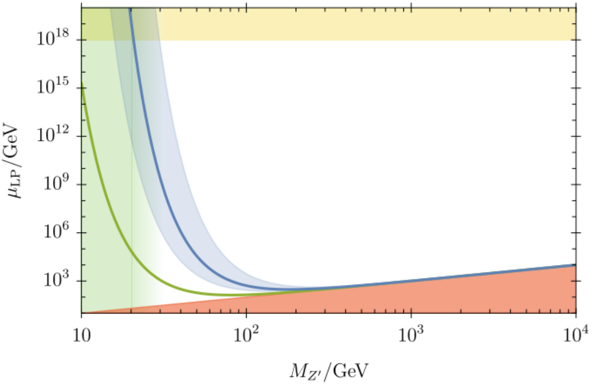


FIGURE 1: Scale of the Landau pole for BM III (blue), depending on the $M_{Z'}$ mass. An uncertainty of 30% is considered for $a^d_{\pi^+\pi^-}$ (blue shaded area). The shifted central value is also shown if one dark fermion is included (solid green line). The red shaded area is excluded, as $\mu_{\mathrm{LP}} \leq M_{Z'}$. The preferred range by CP-data is shaded in green. The yellow band indicates the μ_{LP} regime an order of magnitude around the Planck scale.