University of CINCINNATI

Earth Mover‘s Distance as a meassure of CP violation

CKM 2023, Santiago de Compostela, Spain

Based on [J. High Energ. Phys. 2023, 98 (2023)]

Ahmed Youssef
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Sept 19th, 2023

In collaboration with:
Adam Davis, Tony Menzo, and Jure Zupan

Outline

Current state of the art: Energy Test

Earth Mover' Distance (EMD) as test statistic
→ B decay

Modified EMD for large samples
→ D decay

Conclusion and Outlook

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Direct CPV

How do we quantify direct CP violation in 3 body decay?

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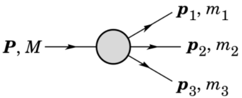
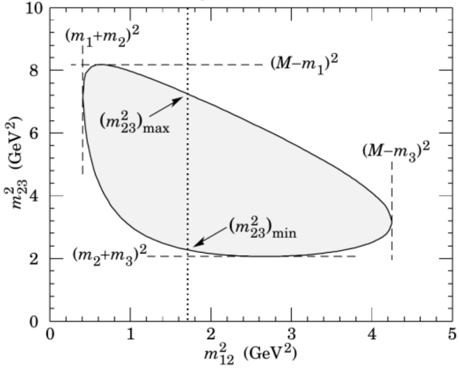


$$\frac{d\Gamma(B \to f)}{d\,\Omega} \neq \frac{d\Gamma(\bar{B} \to \bar{f})}{d\,\Omega}$$ ➡ **Visualize using Dalitz plots!**

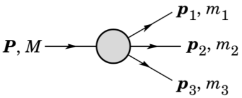
$$m_{ij}^2 = (p_i + p_j)^2$$



m_{ij} - invariant mas of a final state particle

Visualizes the differential decay rate across the phase space of the three-body decay

How do we quantify direct CP violation in 3 body decay?



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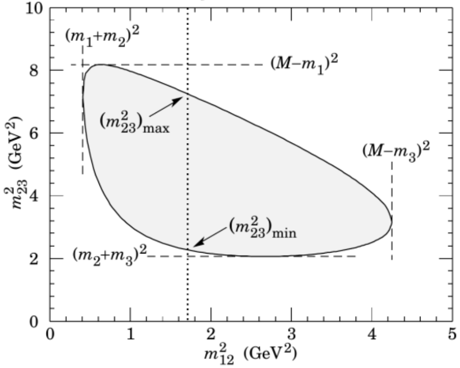

m_{ij} - **invariant mas of a final state particle**

Visualizes the differential decay rate across the phase space of the three-body decay

Compare particle and its antiparticle distribution

→ **Hints to CP violation**

Direct CPV

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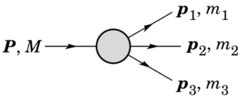

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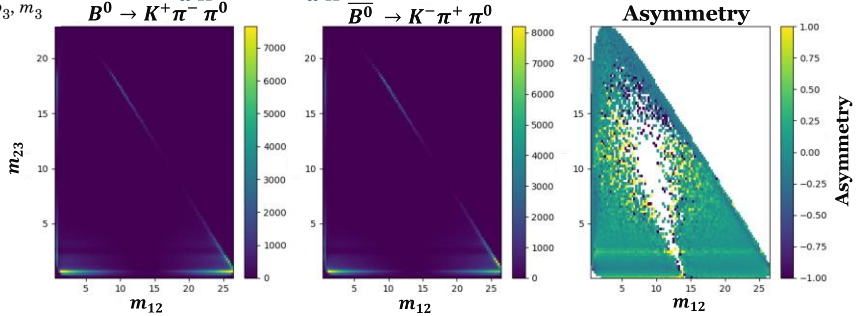

→ **Direct CPV manifests as local density asymmetries between conjugate Dalitz plots!**

Current State of the art

Model the amplitude → **Very Complicated!**

Use a Test Statistic

- **Binned Test Statistic**
  - → **e.g. Miranda Test** → **Washes out Asymmetry!**
- **Unbinned Test Statistic**
  - → **e.g. Energy Test** → **Hard to interpret!**

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We need an alternative approach!

Motivation

What requirements do we need?

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Is it highly sensitive to CP violation?

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Earth Mover's Distance (EMD) as test statistic

→ **Comparable sensitivity to established method! (Comparison with the Energy Test)**

→ **Tells us which part of the Dalitz plot the CPV originated from!**

Energy Test

The energy test has already been successfully applied to search for CPV in multibody decays LHCB Collaboration, Phys. Lett. B 740 (2015) 158

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Unbinned two-sample test utilizing a test statistic:

$$T = \sum_{i,j>i}^{N} \frac{\psi_{ij}}{N(N-1)} + \sum_{i,j>i}^{\bar{N}} \frac{\psi_{ij}}{\bar{N}(\bar{N}-1)} - \sum_{i,j}^{N,\bar{N}} \frac{\psi_{ij}}{N\bar{N}},$$

Sum over index i — **Sum over index j** — **Sum over indices i,j**

Weighting distance function:

$$\psi_{ij} \equiv \psi(d_{ij};\sigma) = e^{-d_{ij}^2/2\sigma^2}$$

$$B^0(\bar{B}^0) \to f\ (\bar{f})$$

i – B^0 sample
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Energy Test is not interpretable!

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$(\bar{f})$

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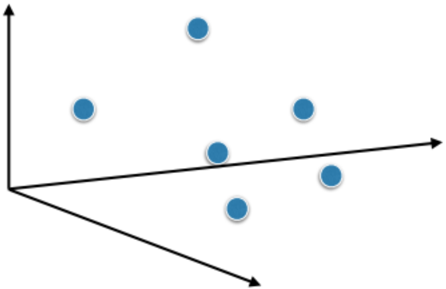
Can we come up with a more interpretable test statistic?

Events from two identical distribution → T close to zero

Events from two dissimilar distribution → T is non zero

Perform a hypothesis test to obtain a p-value!

Optimal Transport (OT)



Example from: Marco Cuturi, MLSS summer school presentaation

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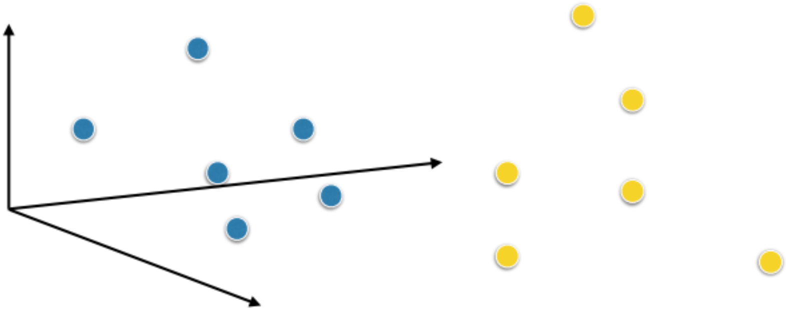

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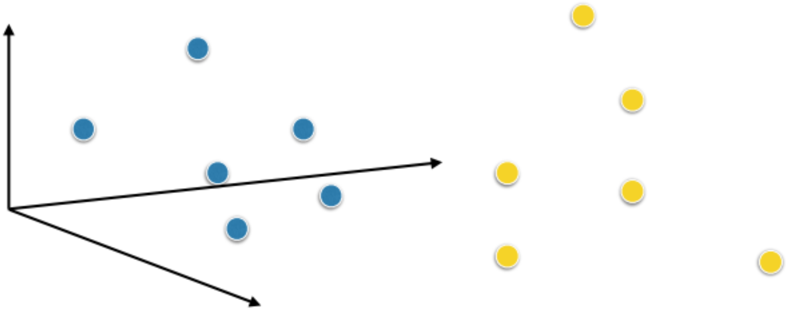
Optimal Transport (OT)

What is the most optimal way to move one sample to another?



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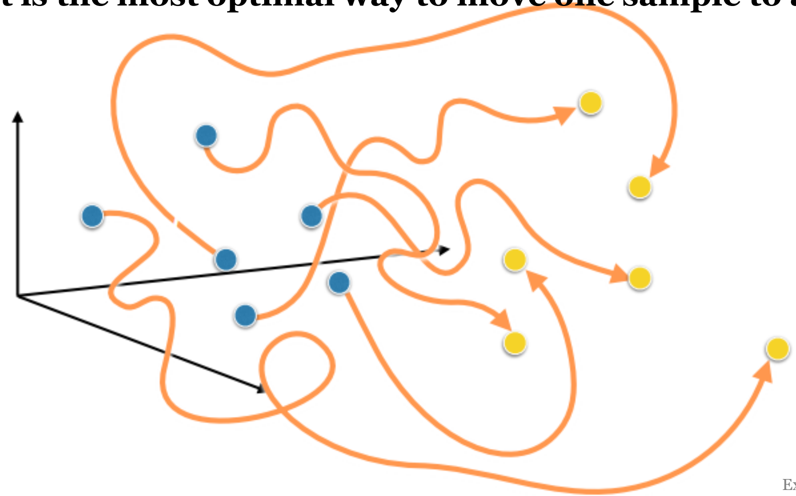

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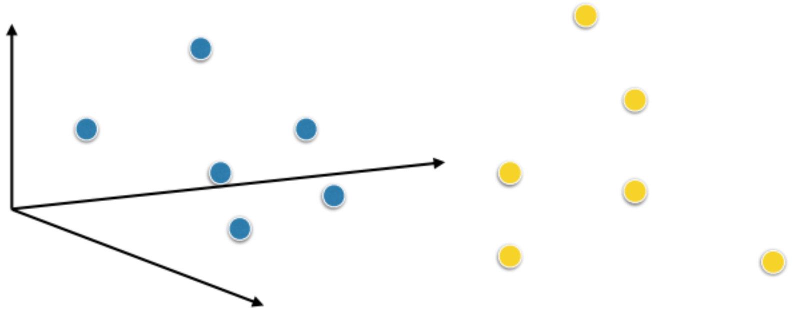

Goal of OT: Find the most "natural" way to move points

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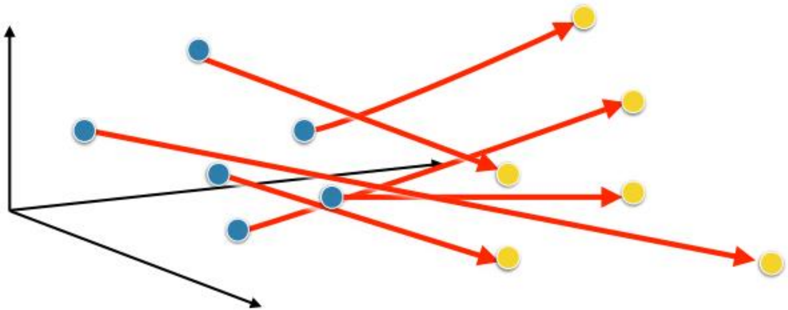

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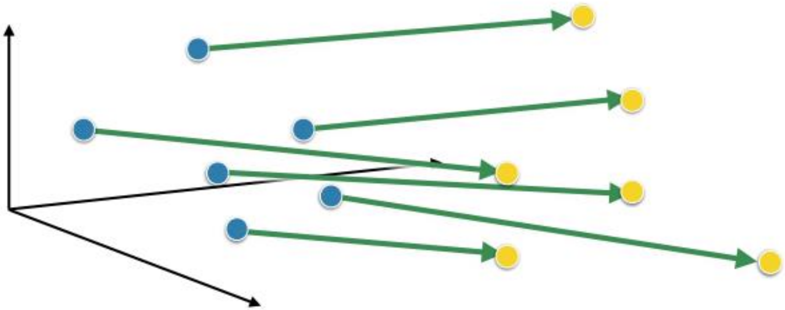

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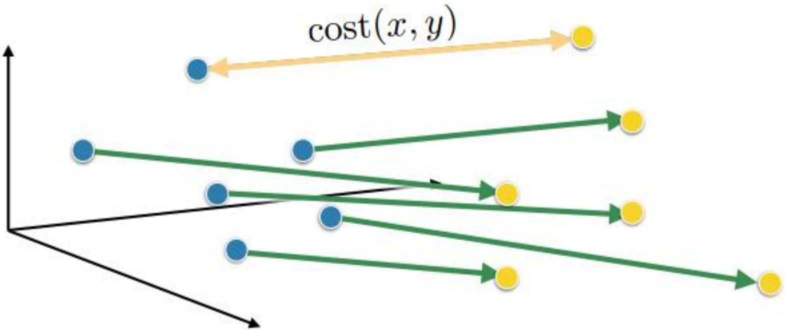

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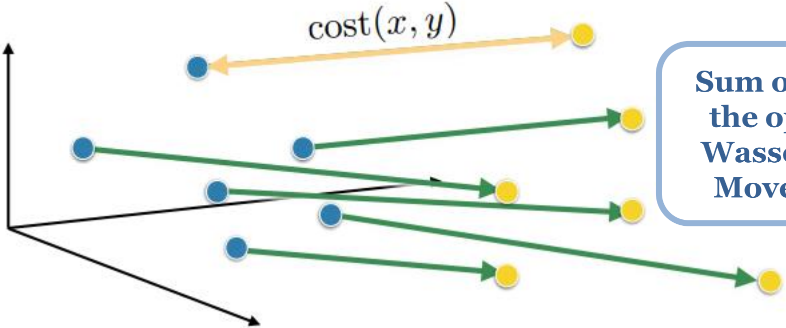

Sum over the cost of the optimal path = Wasserstein (Earth Mover's) distance

Goal of OT: Find the most "natural" way to move points

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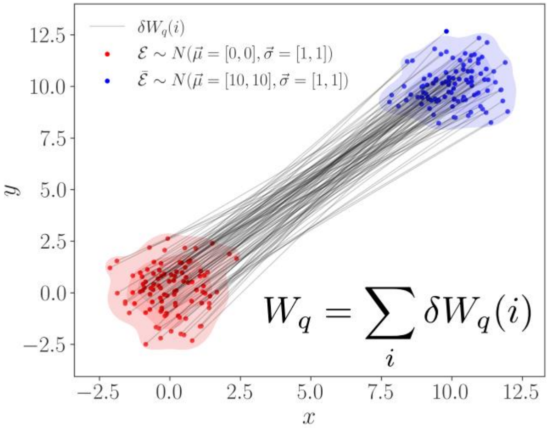
Wasserstein Distance

Wasserstein distance (WD)

$$W_q(\mathcal{E},\bar{\mathcal{E}}) = \left[\min_{\{f_{ij}\geq 0\}} \sum_{i=1}^{N}\sum_{j=1}^{\bar{N}} f_{ij}\left(\hat{d}_{ij}\right)^q\right]^{1/q}$$



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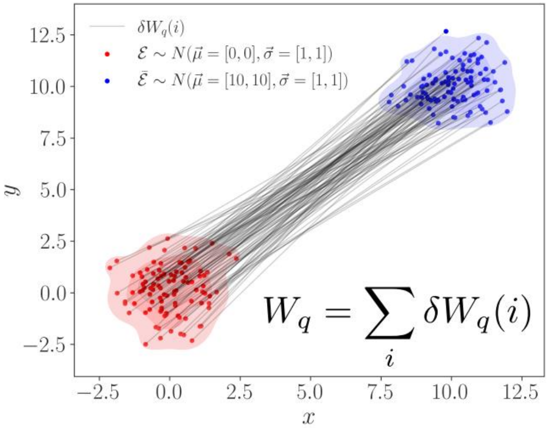

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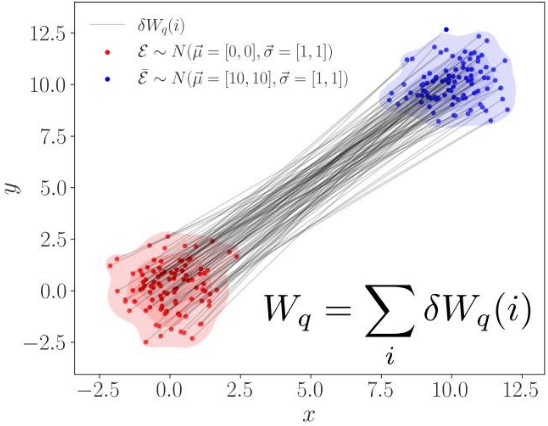
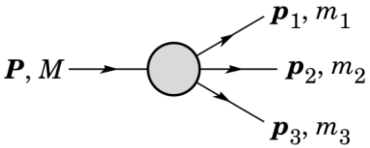


Application to 3 Body decay



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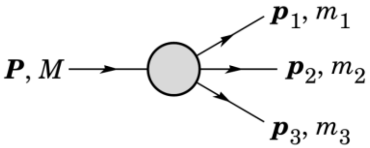

Sample Size = $\sim 10^3$

$$B^0 \to K^+\pi^-\pi^0$$
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EMD as a test statistic

Sample Size = $\sim 10^5 - 10^6$

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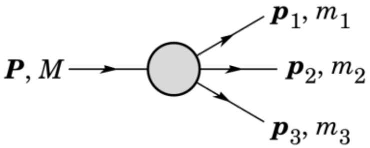

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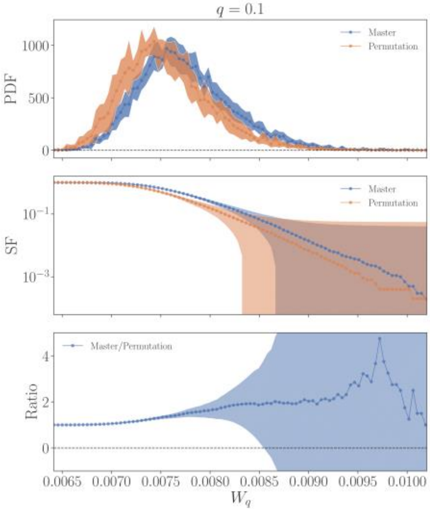

Permutation Method

- Permuting the original B^0 and $\bar{B}^0$ samples
- Calculate the test statistic for each permutation

Master Method

- Generate an ensemble of B^0 and $\bar{B}^0$ decay event samples, using the B^0 decay model for both
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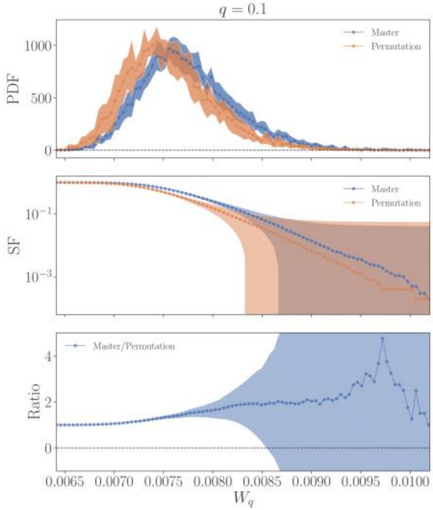

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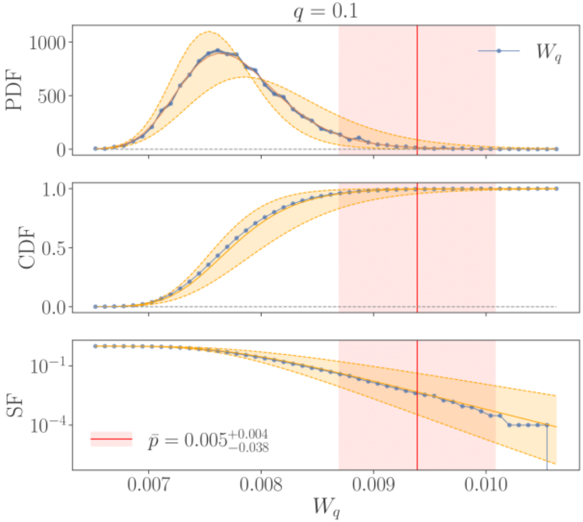
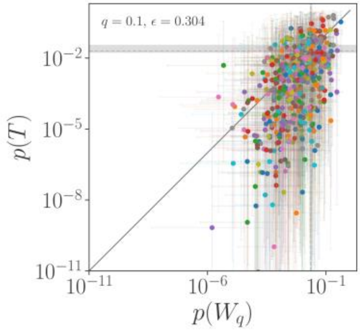
Compare the sensitivity of W_q and Energy test using the master method

Results for B decay



$$\epsilon \equiv \frac{1}{N_e} \sum_{i=1}^{N_e} \begin{cases} +1 & p_i(W_q) < p_i(T), \\ 0 & \text{otherwise,} \end{cases}$$



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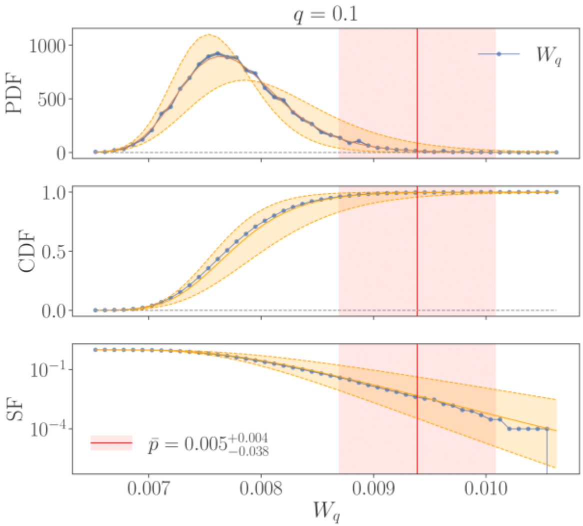

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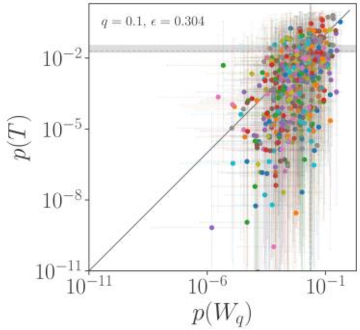

We have a similar sensitivity as the ET

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Can we locate the CP violation?

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EMD traces the variation of the CP asymmetry across the Dalitz plot!

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CP asymmetry:

$B^0(\bar{B}^0) \to K^+\pi^-\pi^0\ (K^-\pi^-\pi^0)$

$$\mathcal{A}_{\mathrm{CP}}(s_{12}, s_{13}) = \frac{d\bar{\Gamma}(\bar{s}_{12}, \bar{s}_{13}) - d\Gamma(s_{12}, s_{13})}{d\bar{\Gamma}(\bar{s}_{12}, \bar{s}_{13}) + d\Gamma(s_{12}, s_{13})}$$

BaBar amplitude model

BarBar Collaboration, Phys. Rev. D 83 (2011) 112010

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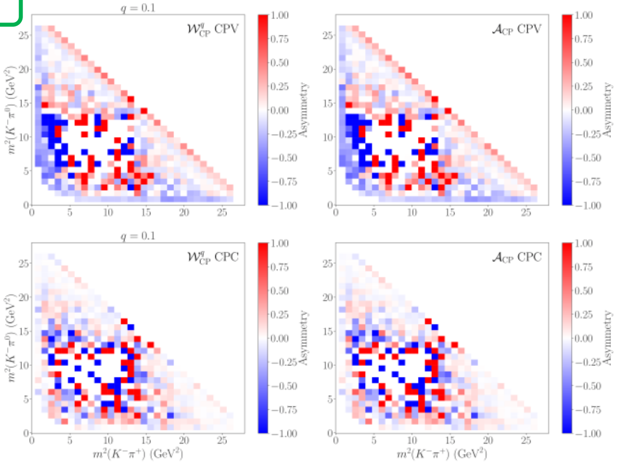

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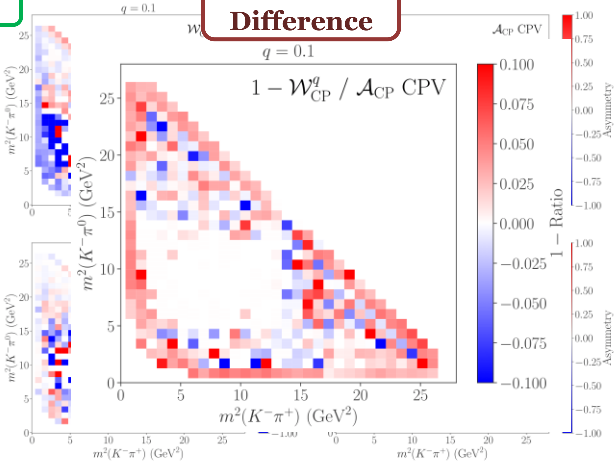
Relative Difference



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What about larger Data sets?

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$$\overline{D}^0 \to \pi^-\pi^+\pi^0$$

→ Very small non-zero CP violation

→ Studied at the LHCb using the ET

Can we still use the EMD?

Yes, but ...

→ Computationally expensive

→ Very memory intensive

What about larger Data sets?

$D^0 \to \pi^+\pi^-\pi^0$
$\bar{D}^0 \to \pi^-\pi^+\pi^0$

➡ Very small no… o CP violation

➡ Stud… using the ET

We need a more efficient solution for larger data sets!

Ca… the EMD?

Yes, but …

➡ Computationally expensive

➡ Very memory intensive

Alternative Solution

We propose two solutions

Binned Wasserstein distance

Not part of this talk

Sliced Wasserstein distance

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Sliced Wasserstein distance

Use Sliced Wasserstein Distance as test statistic!

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Sliced Wasserstein distance

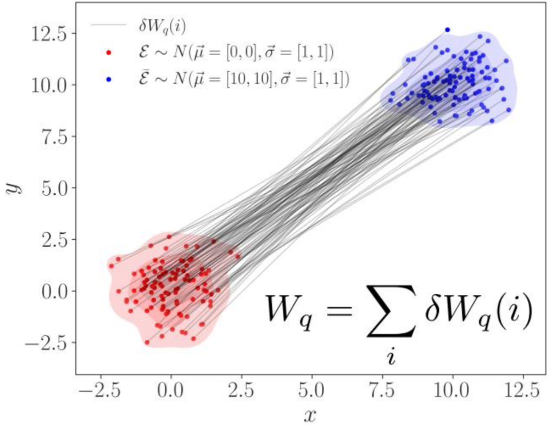
Use Sliced Wasserstein Distance as test statistic!

Wasserstein distance (WD)

$$W_q(\mathcal{E},\bar{\mathcal{E}}) = \left[\min_{\{f_{ij}\geq 0\}} \sum_{i=1}^{N}\sum_{j=1}^{\bar{N}} f_{ij}\left(\hat{d}_{ij}\right)^q\right]^{1/q}$$

Sliced Wasserstein distance

- Projects high dimensional data into one dimensional “slices”
- WD in 1D has a closed form solution
  - Sorted Difference of the two samples



Sliced Wasserstein distance

How many slices do we need to converge?

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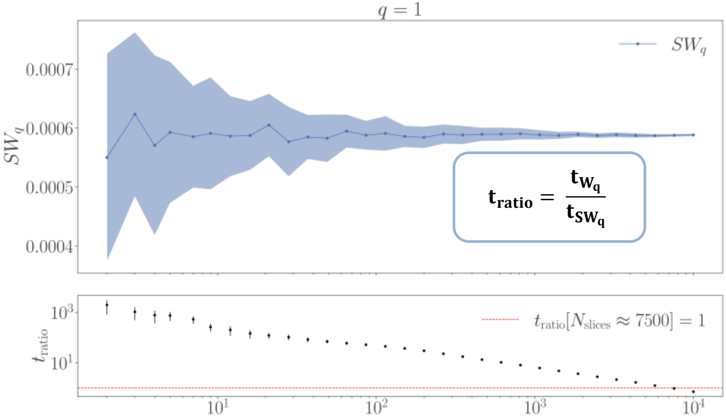
Sliced Wasserstein distance

How many slices do we need to converge?

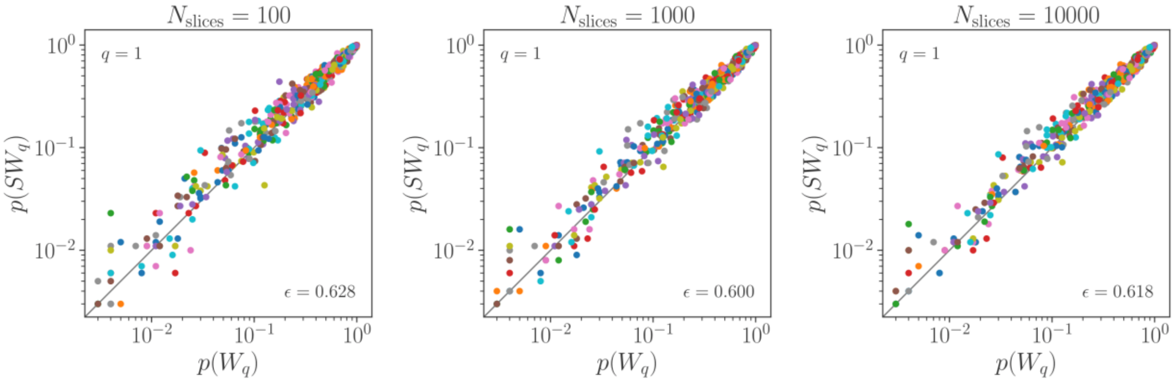


$$t_{ratio} = \frac{t_{W_q}}{t_{SW_q}}$$

- **Starts converging at 1000 slices**
- **1000 slices: speed up of a factor ~7 over W_q**
- **Comp of N_{Slices} can be done in parallel!**
- **Does not require large memory resources!**

Sliced Wasserstein distance

Comparison with W_q



Conclusion and Outlook

EMD is a robust, model independent, and unbinned test statistic for CPV!

highly sensitive to CPV

Interpretable

Future work

- Improving the test further
- Time-dependent CPV
- Flavor Violation

Public code:

https://github.com/adamdddave/EMD4CPV

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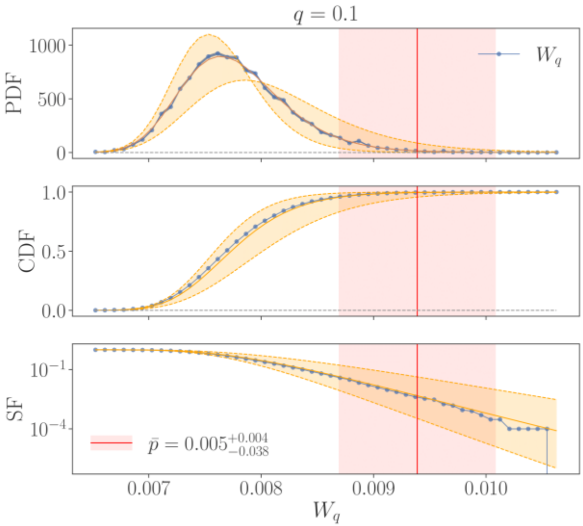
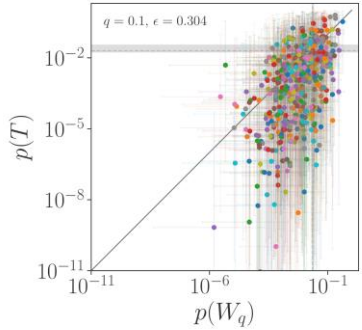
Back up

Results for B decay



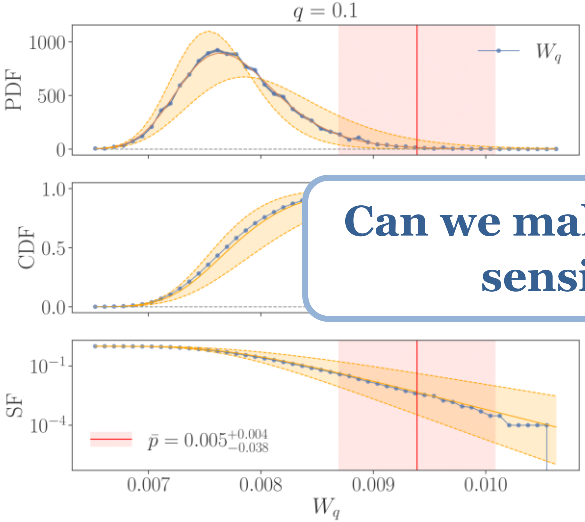
$$\epsilon \equiv \frac{1}{N_e} \sum_{i=1}^{N_e} \begin{cases} +1 & p_i(W_q) < p_i(T), \\ 0 & \text{otherwise,} \end{cases}$$



$\delta W_q > 0$: W_q receives contributions from non CPV areas

→ **Slightly smaller sensitivity than ET**

Results for B decay



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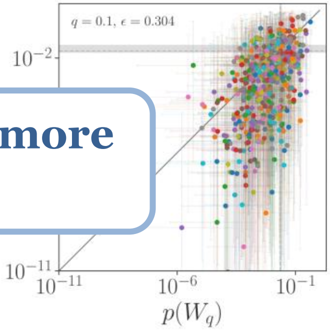
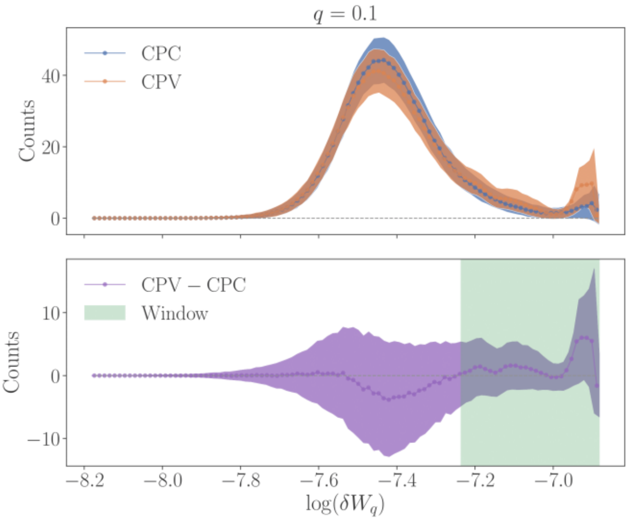


Can we make W_q more sensitive?

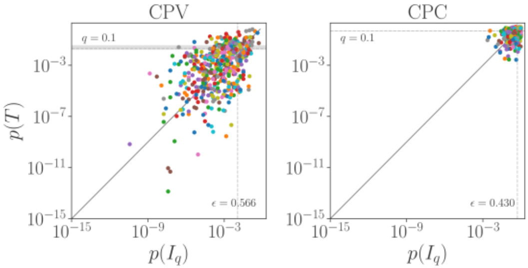
$\delta W_q > 0$: W_q receives contributions from non CPV areas

→ Slightly smaller sensitivity than ET

Windowed Wasserstein distance



$$w(x) = \begin{cases} +1 & x \in [\delta W^{\text{win}}_{\text{min}}, \delta W^{\text{win}}_{\text{max}}], \\ 0 & \text{otherwise.} \end{cases}$$



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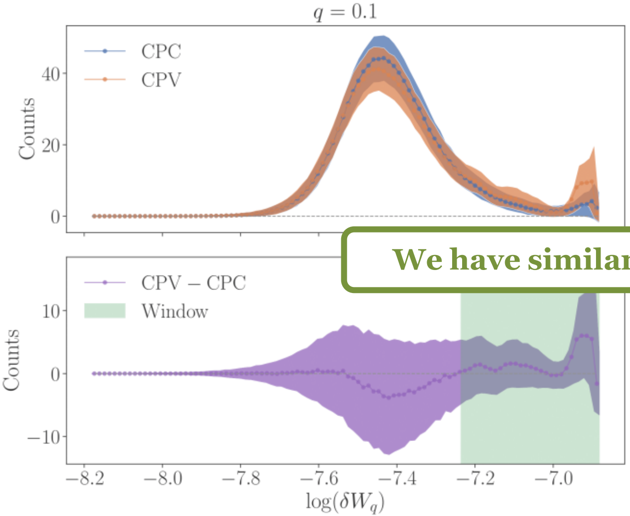
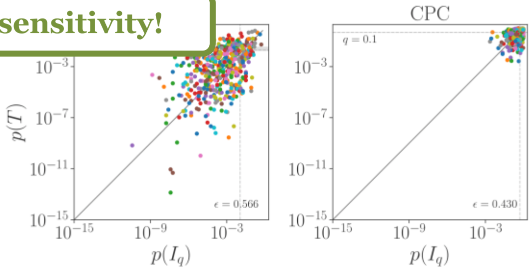
Windowed Wasserstein distance



$$w(x) = \begin{cases} +1 & x \in [\delta W^{\text{win}}_{\text{min}}, \delta W^{\text{win}}_{\text{max}}], \\ 0 & \text{otherwise.} \end{cases}$$

We have similar sensitivity!



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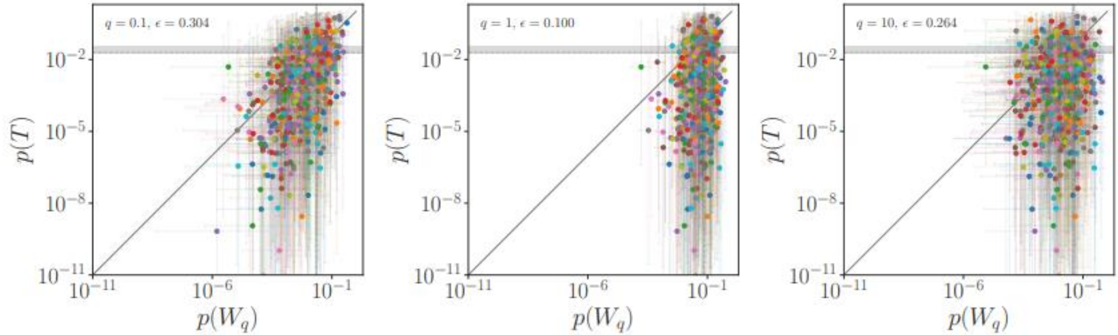
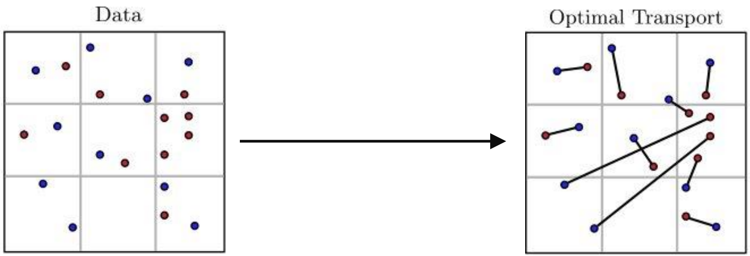
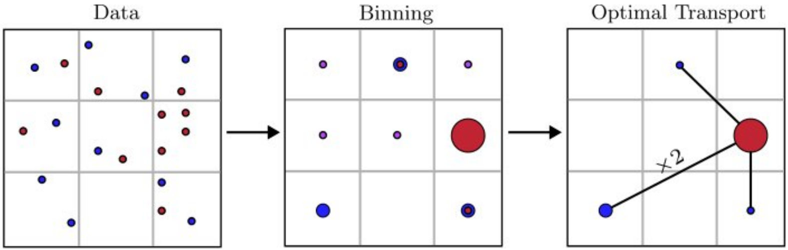
Results for B decay



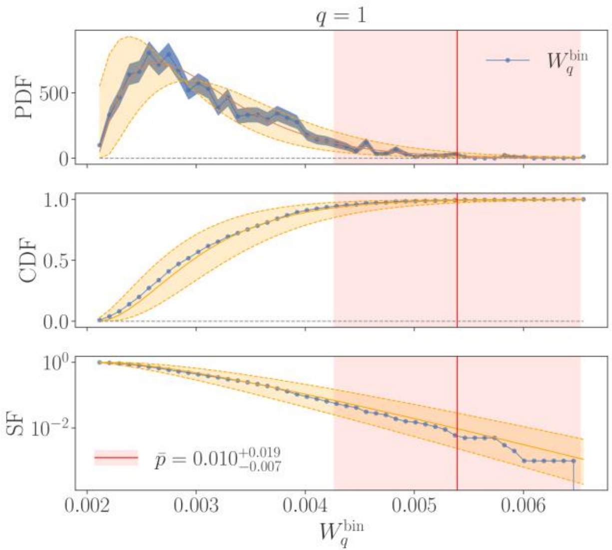
“Normal” Wasserstein distance

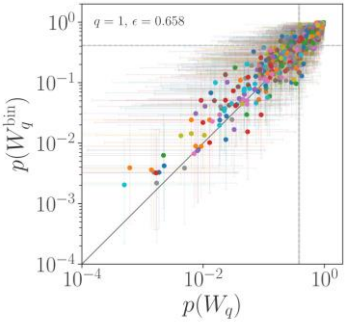


Binned Wasserstein distance

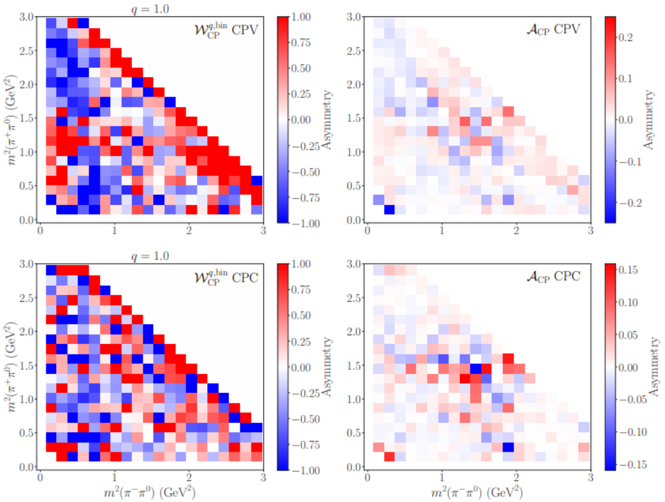


Binned Wasserstein distance





Binned EMD



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