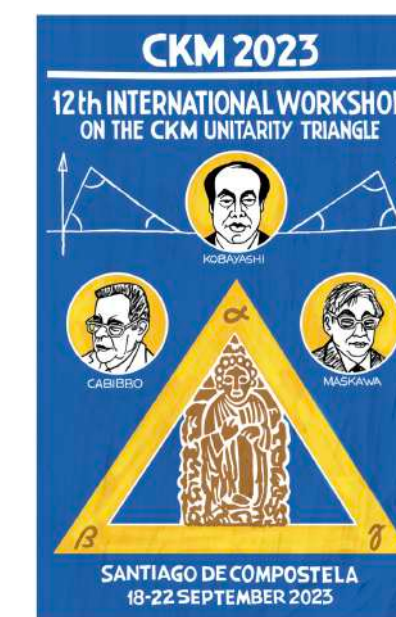


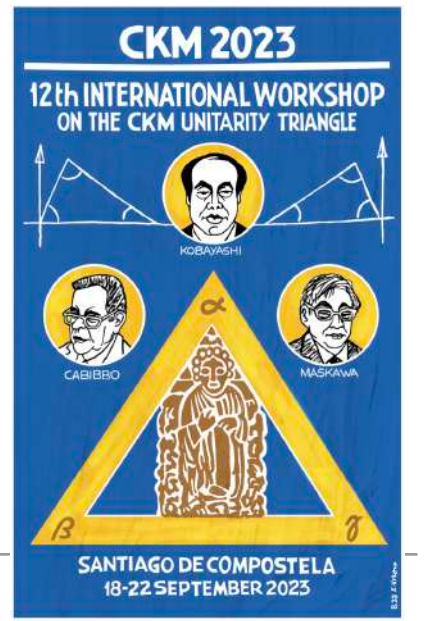
HQE, Mixing and lifetimes



21.9.2023

Alexander Lenz

CPV in Charm Decays



Alexander Lenz @alexlenz42 · Apr 1

Finally CPV in D mixing has been precisely predicted in the SM. The biggest breakthrough at [#charmingclues](#)! Quite a heroic effort!



3

62



Theoretical Peculiarities of Charm:

1. The strong coupling is strong

$$\alpha_s(m_c) = 0.33 \pm 0.01$$

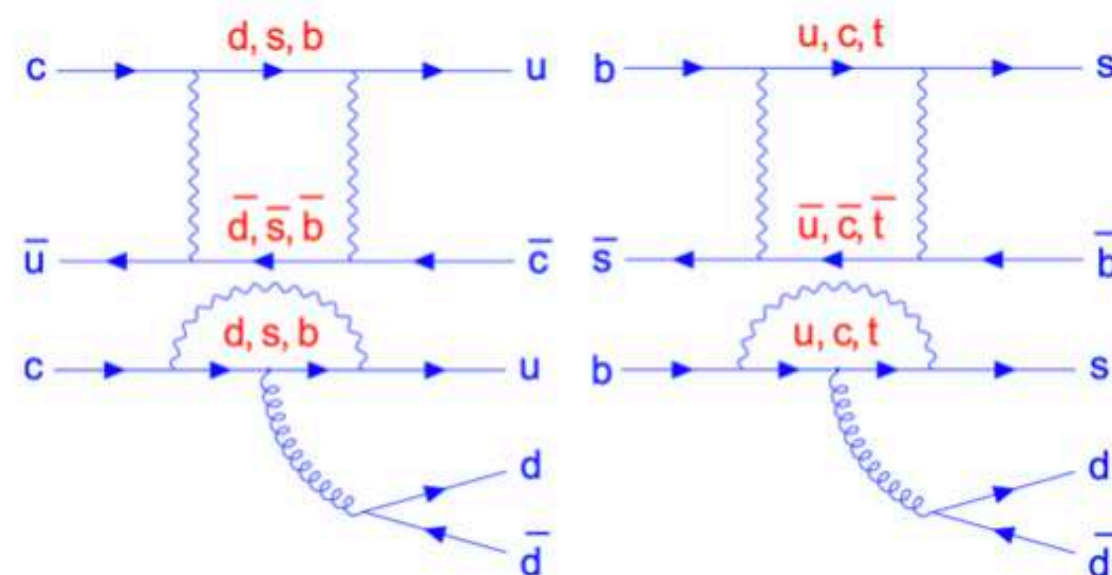
2. The charm quark is not really heavy

$$m_c^{\text{Pole}} = (1.67 \pm 0.07) \text{ GeV}, \quad \bar{m}_c(\bar{m}_c) = (1.27 \pm 0.02) \text{ GeV},$$

3. There is almost no CPV in charm

$$V_{cd} = -0.2247 - 1.4 \cdot 10^{-4}I, \quad V_{cs} = 0.97354 - 3.1 \cdot 10^{-5}I, \quad V_{cb} = 0.0416$$

4. There are extremely pronounced GIM cancellations in the charm sector

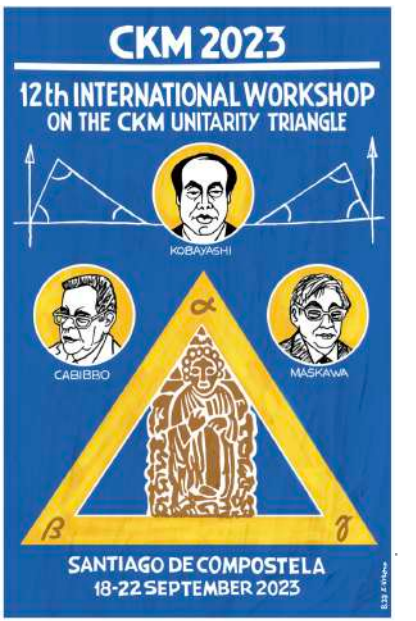


$$\begin{aligned} \left(\frac{m_d}{M_W}\right)^2 &\approx 0, & \left(\frac{m_u}{M_W}\right)^2 &\approx 0, \\ \left(\frac{m_s}{M_W}\right)^2 &\approx 1.3 \cdot 10^{-6}, & \left(\frac{m_c}{M_W}\right)^2 &\approx 2.5 \cdot 10^{-4}, \\ \left(\frac{m_b}{M_W}\right)^2 &\approx 2.8 \cdot 10^{-3}, & \left(\frac{m_t}{M_W}\right)^2 &\approx 4.5. \end{aligned}$$

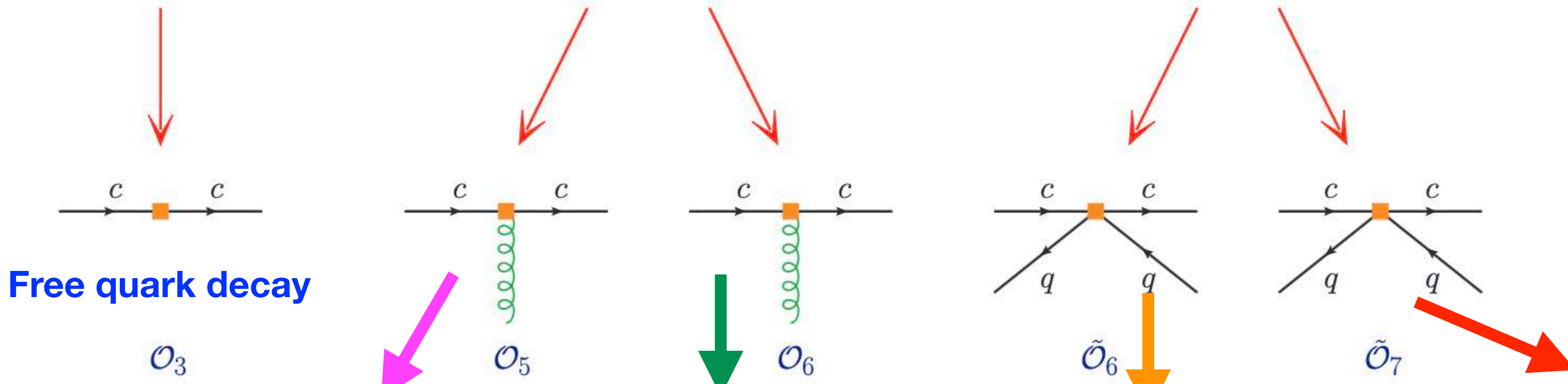
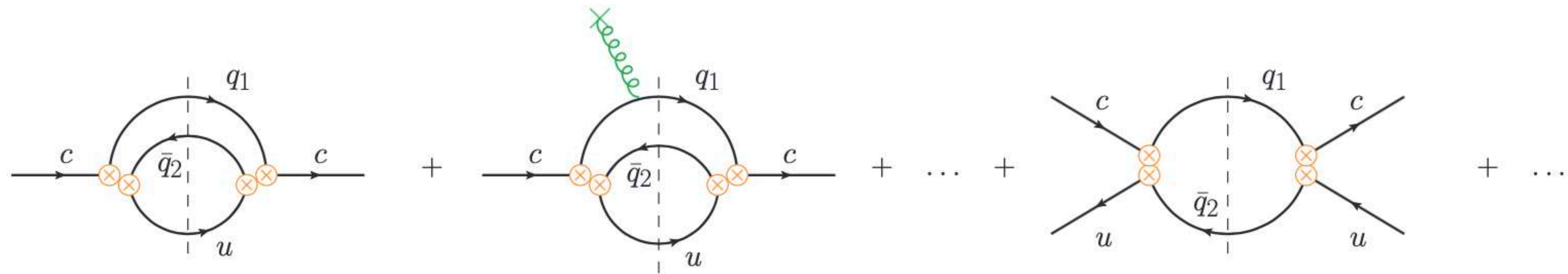
See e.g.
AL, G. Wilkinson
2011.04443



HEAVY QUARK EXPANSION



$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \dots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \dots \right), \quad \Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s(m_c)}{4\pi} \Gamma_i^{(1)} + \left[\frac{\alpha_s(m_c)}{4\pi} \right]^2 \Gamma_i^{(2)} + \dots$$



Free quark decay

\mathcal{O}_3

Kinetic operator μ_π^2

Chromomagnetic operator μ_G^2

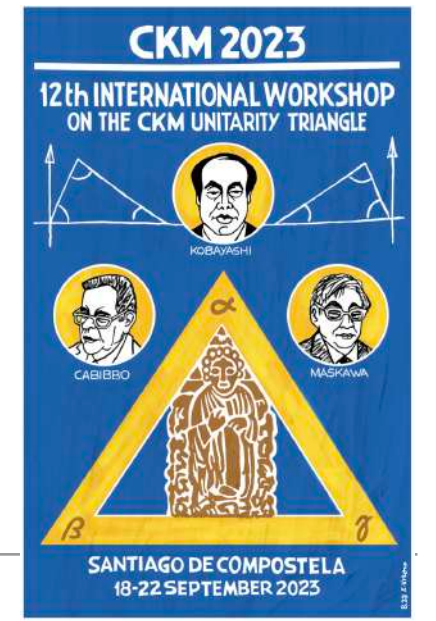
Darwin operator μ_π^2

4-quark operator B_i, ϵ_i

Eye contractions r

Dimension 7 operators
Vacuum insertion approximation

HQE: Status Quo



$$\Gamma(D) = \Gamma_3 + \Gamma_5 \frac{\langle \mathcal{O}_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle \mathcal{O}_6 \rangle}{m_c^3} + \dots + 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_c^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_c^4} + \dots \right), \quad \Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s(m_c)}{4\pi} \Gamma_i^{(1)} + \left[\frac{\alpha_s(m_c)}{4\pi} \right]^2 \Gamma_i^{(2)} + \dots$$

Perturbative semi-leptonic

$\Gamma_3^{(1)}$	1983	HoKim, Pham
$\Gamma_3^{(2)}$	1997 – 2013	Czarnecki, Melnikov, vanRitbergen, Pak, Dowling, Bonciani, Ferroglia, Biswas, Brucherseifer, Caola
$\Gamma_3^{(3)}$	2020 2021	Fael, Schoenwald, Steinhauser Czakon, Czarnecki, Dowling
$\Gamma_5^{(0)}$	1992	Bigi, Uraltsev, Vainshtein, Blok, Shifman
$\Gamma_5^{(1)}$	2013 – 2015	Alberti, Gambino, Nandi, Mannel, Pivovarov, Rosenthal
$\Gamma_6^{(0)}$	1996	Gremm, Kapustin
$\Gamma_6^{(1)}$	2019	Mannel, Pivovarov
$\Gamma_7^{(0)}$	2006	Dassinger, Mannel, Turczyk
$\Gamma_8^{(0)}$	2010	Mannel, Turczyk, Uraltsev

Perturbative non-leptonic

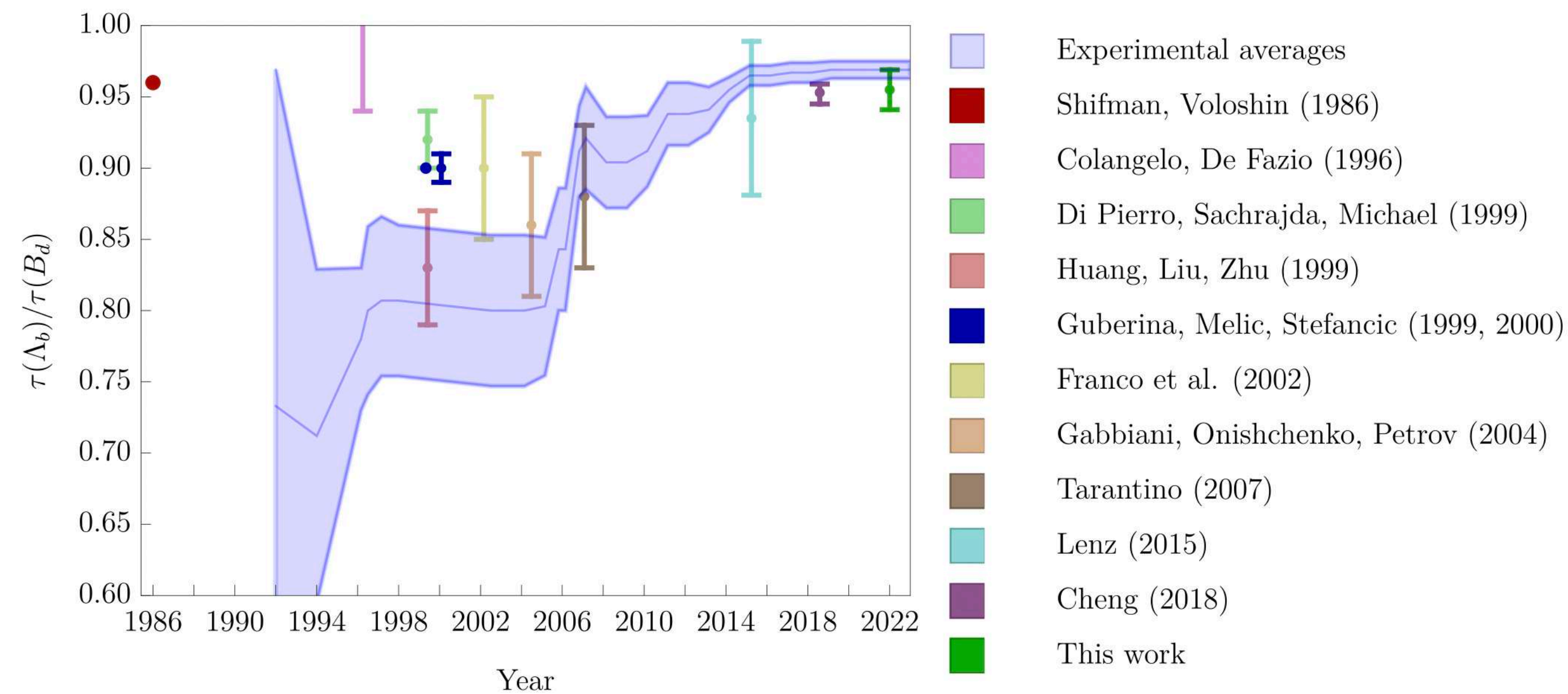
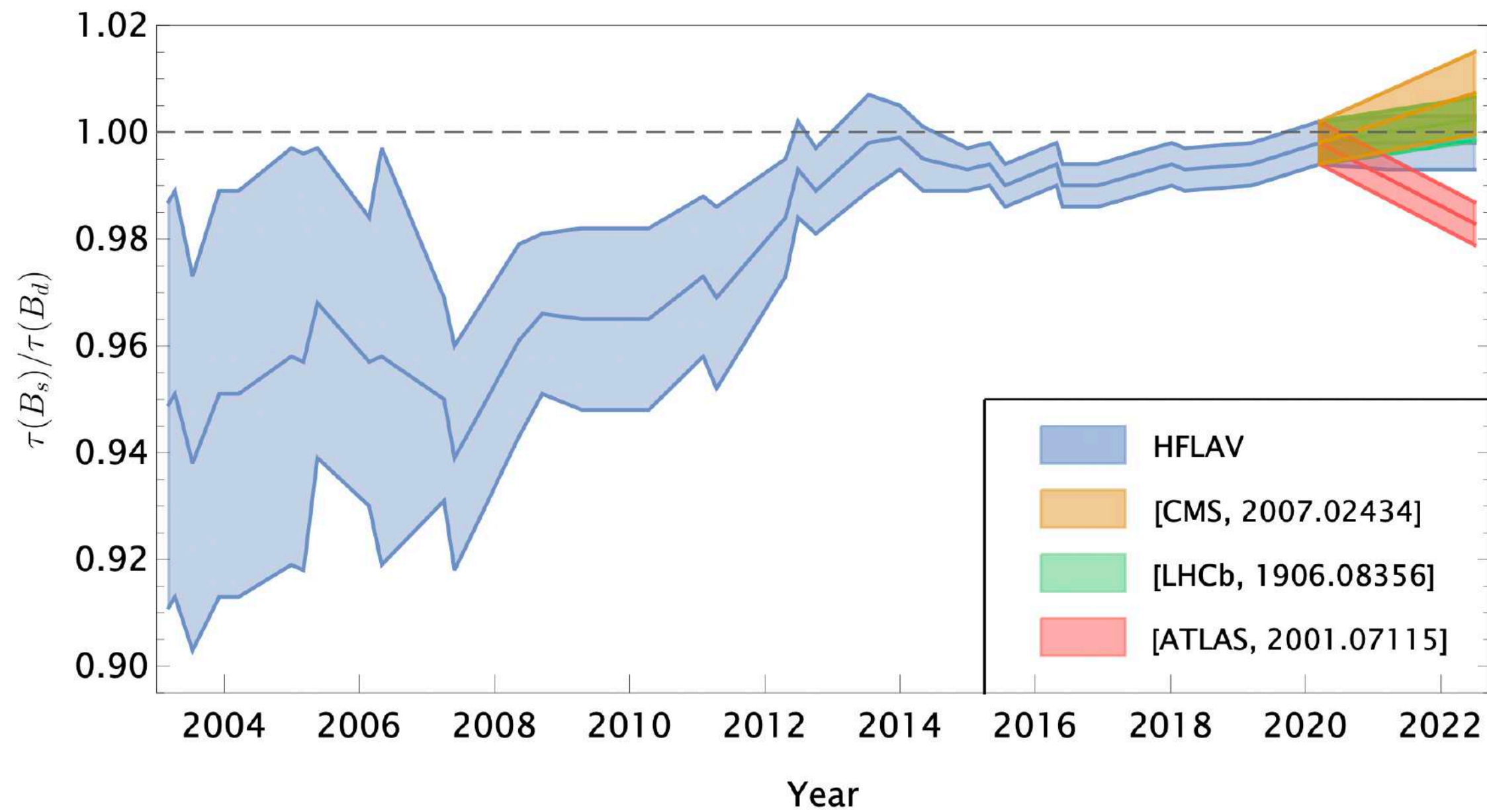
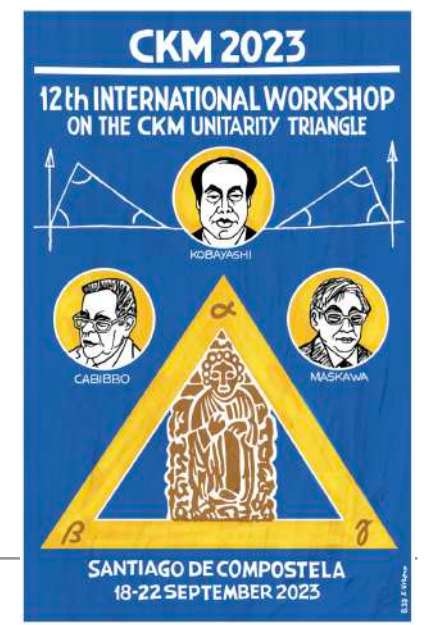
$\Gamma_3^{(1)}$	1983 – 2013	HoKim, Pham, Altarelli, Petrarca, Voloshin, Bagan, Ball, Braun, Goszinsky, Fiol, Lenz, Nierste, Ostermaier, Krinner, Rauh, Greub, Liniger
$\Gamma_3^{(2)}$	2005	partly by : Czarnecki, Slusarczyk
$\Gamma_5^{(0)}$	1992	Bigi, Uraltsev, Vainshtein, Blok, Shifman
$\Gamma_5^{(1)}$	2023	Mannel, Moreno, Pivovarov(m = 0)
$\Gamma_6^{(0)}$	2020	Lenz, Piscopo, Rusov, Mannel, Moreno, Pivovarov
$\tilde{\Gamma}_6^{(0)}$	1979 – 1996	Guberina, Nussinov, Peccei, Ruckl, Shifman, Voloshin, Uraltsev, Neubert, Sachrajda
$\tilde{\Gamma}_6^{(1)}$	2002	Beneke, Buchalla, Greub, Lenz, Nierste Franco, Lubicz, Mescia, Tarantino, Rauh
$\tilde{\Gamma}_7^{(0)}$	2004	Lenz, Nierste, Gabbiani, Onishchenko, Petrov

Non-perturbative matrix elements

	B_d, B^+	B_s	Λ_b	$\Xi_b^-, \Xi_b^0, \Omega_b^-$
$\langle \mathcal{O}_5 \rangle$	<i>Fits to SL data [26–29]</i> <i>HQET sum rules [30, 31]</i> <i>Lattice QCD [32, 33]</i>	<i>Spectroscopy [35]</i>	<i>Spectroscopy [37]</i>	<i>Spectroscopy [41]</i>
$\langle \mathcal{O}_6 \rangle$	<i>Fits to SL data [26–29]</i> <i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>	<i>Sum rules estimates [35]</i> <i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>	<i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>	<i>EOM relation to $\langle \tilde{\mathcal{O}}_6 \rangle$</i>
$\langle \tilde{\mathcal{O}}_6 \rangle$	<i>HQET sum rules [34]</i>	<i>HQET sum rules [36]</i>	<i>HQET SR [38]; NRCQM + spectroscopy [39, 40]</i>	<i>NRCQM + spectroscopy [39, 40]</i>
$\langle \tilde{\mathcal{O}}_7 \rangle$	<i>Vacuum insertion approximation</i>		—	—

No (state-of the art) lattice results for matrix elements of 4-quark operators

HQE: b hadrons



Disintegration of beauty: a precision study

Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Aleksey V. Rusov (Siegen U.) (Aug 4, 2022)

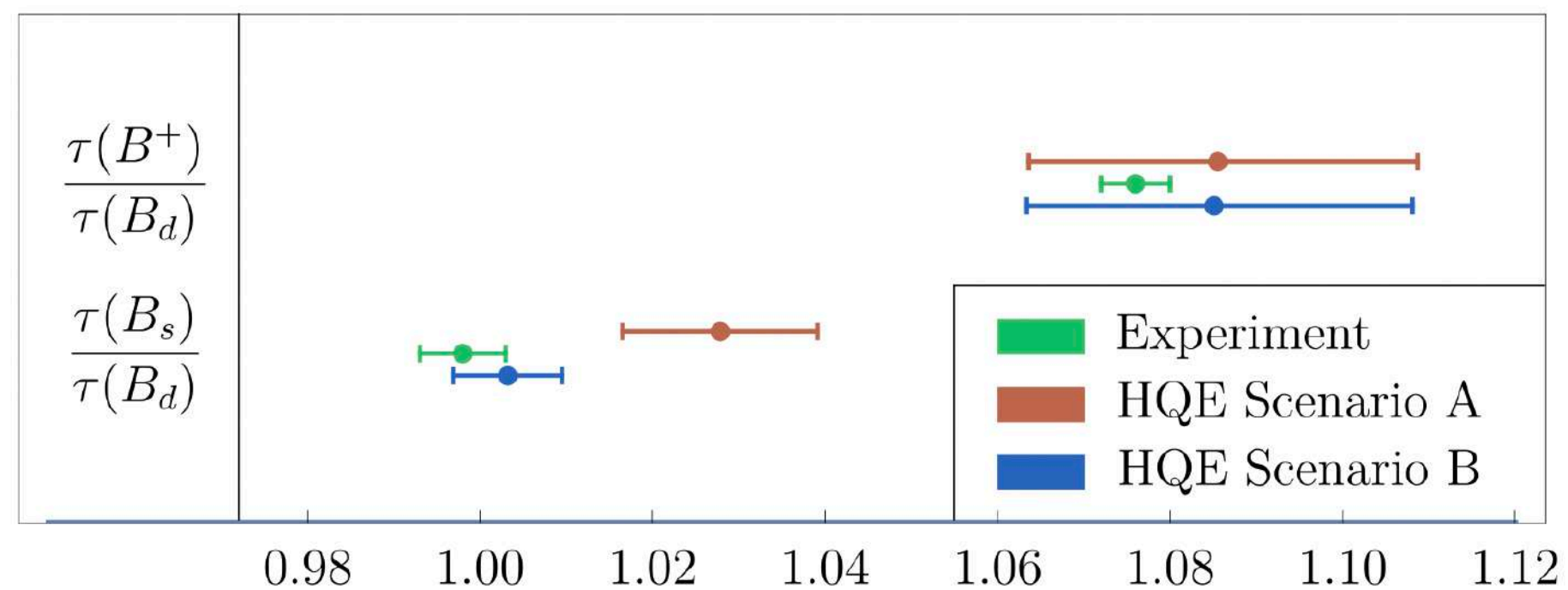
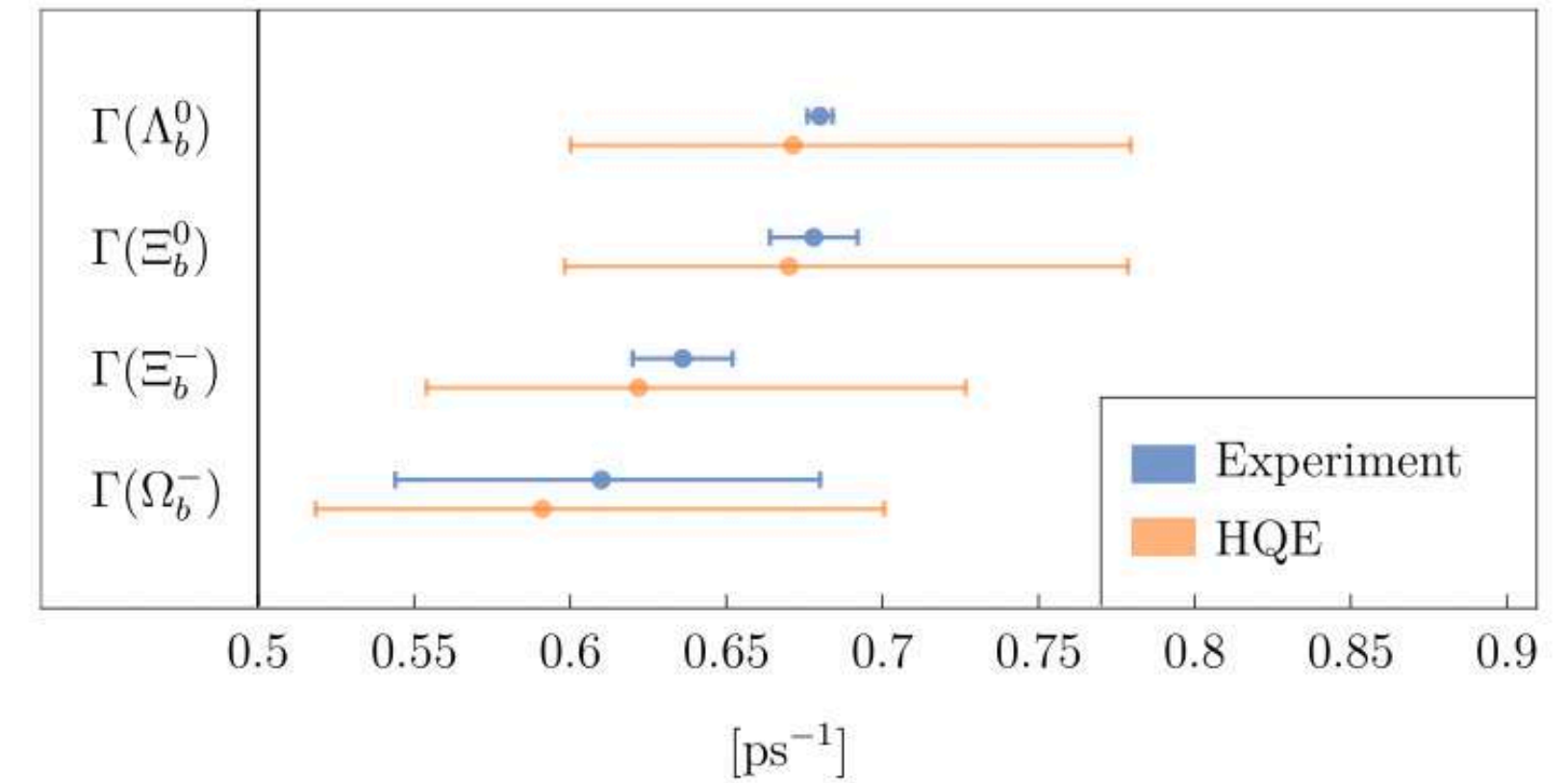
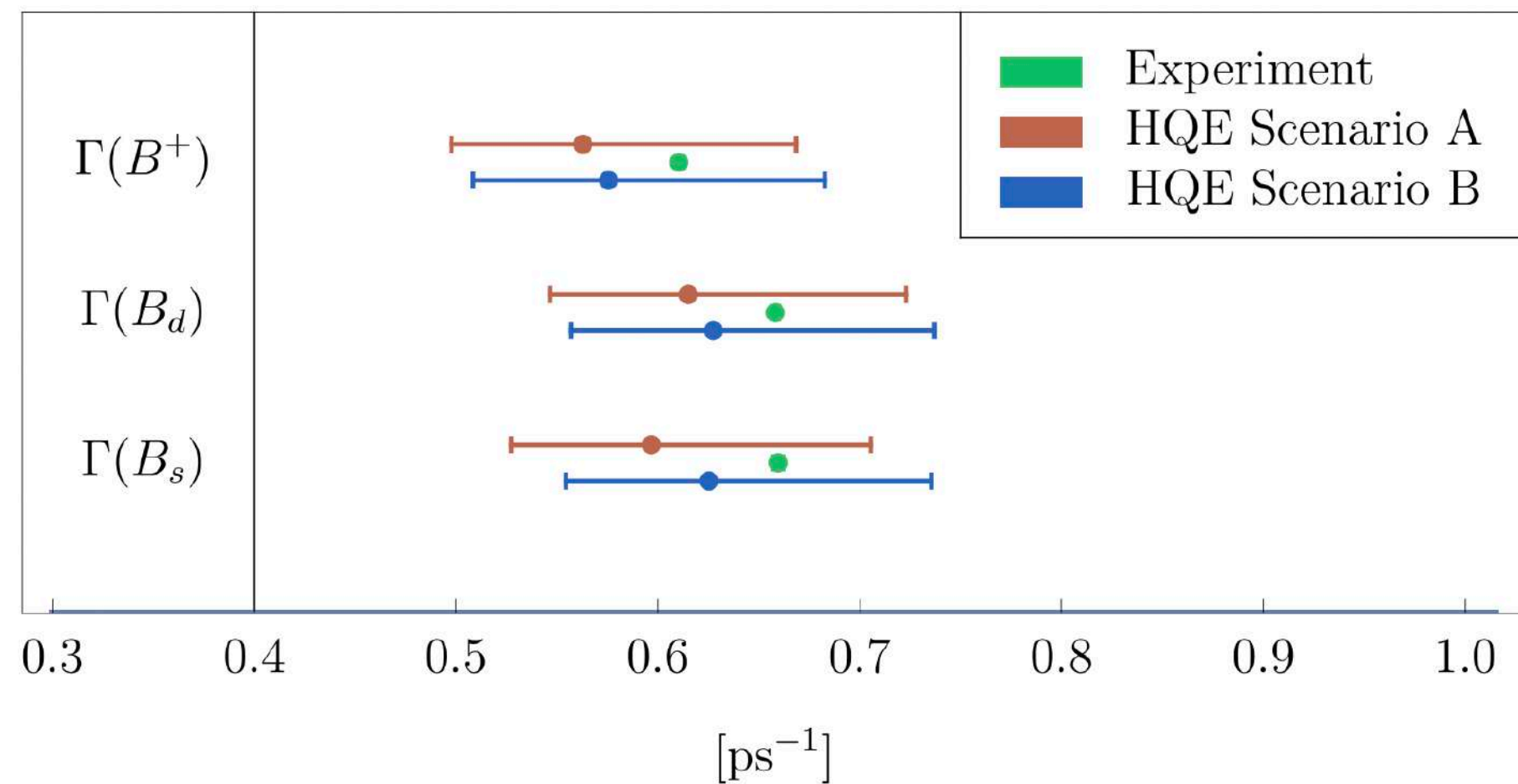
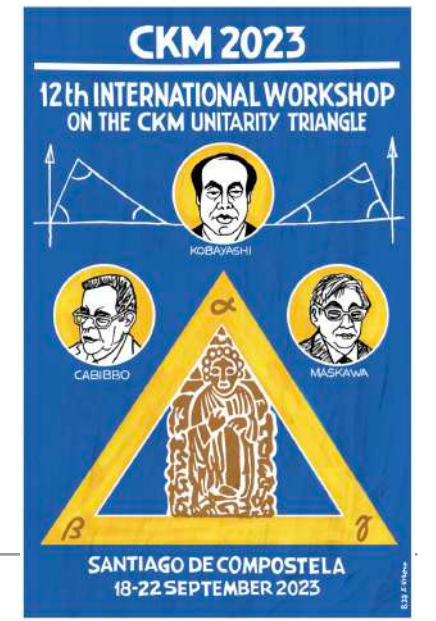
Published in: *JHEP* 01 (2023) 004 • e-Print: [2208.02643](https://arxiv.org/abs/2208.02643) [hep-ph]

Quark-hadron duality at work: lifetimes of bottom baryons

James Gratx (Boskovic Inst., Zagreb), Alexander Lenz (Siegen U.), Blaženka Melić (Boskovic Inst., Zagreb), Ivan Nišandžić (Boskovic Inst., Zagreb), Maria Laura Piscopo (Siegen U.) et al. (Jan 18, 2023)

Published in: *JHEP* 04 (2023) 034 • e-Print: [2301.07698](https://arxiv.org/abs/2301.07698) [hep-ph]

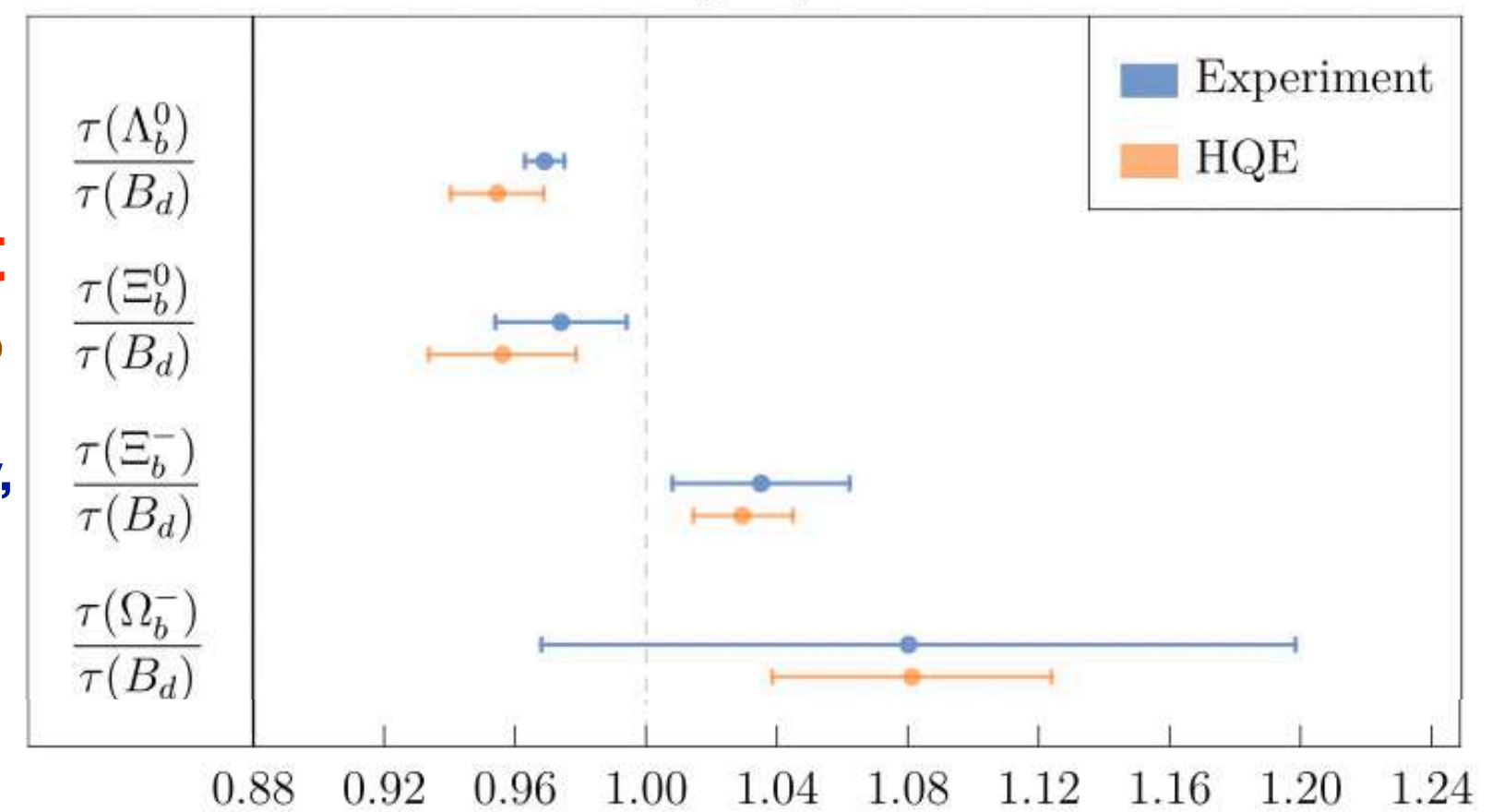
HQE: b hadrons



Size of Darwin term from inclusive V_{cb} fit

A) Bordone, Capdevila, Gambino 2021

B) Bernlochner, Fael Olschewsky, Persson, van Tonder, Vos, Welsch 2022



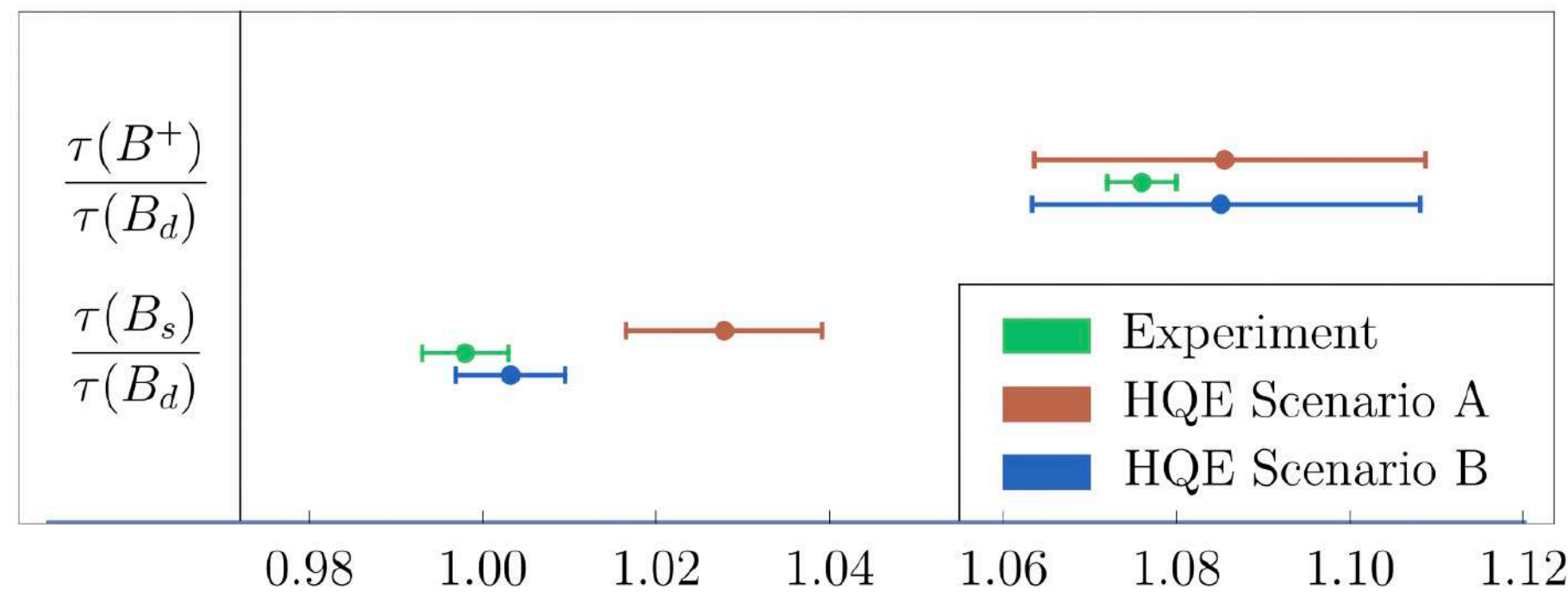
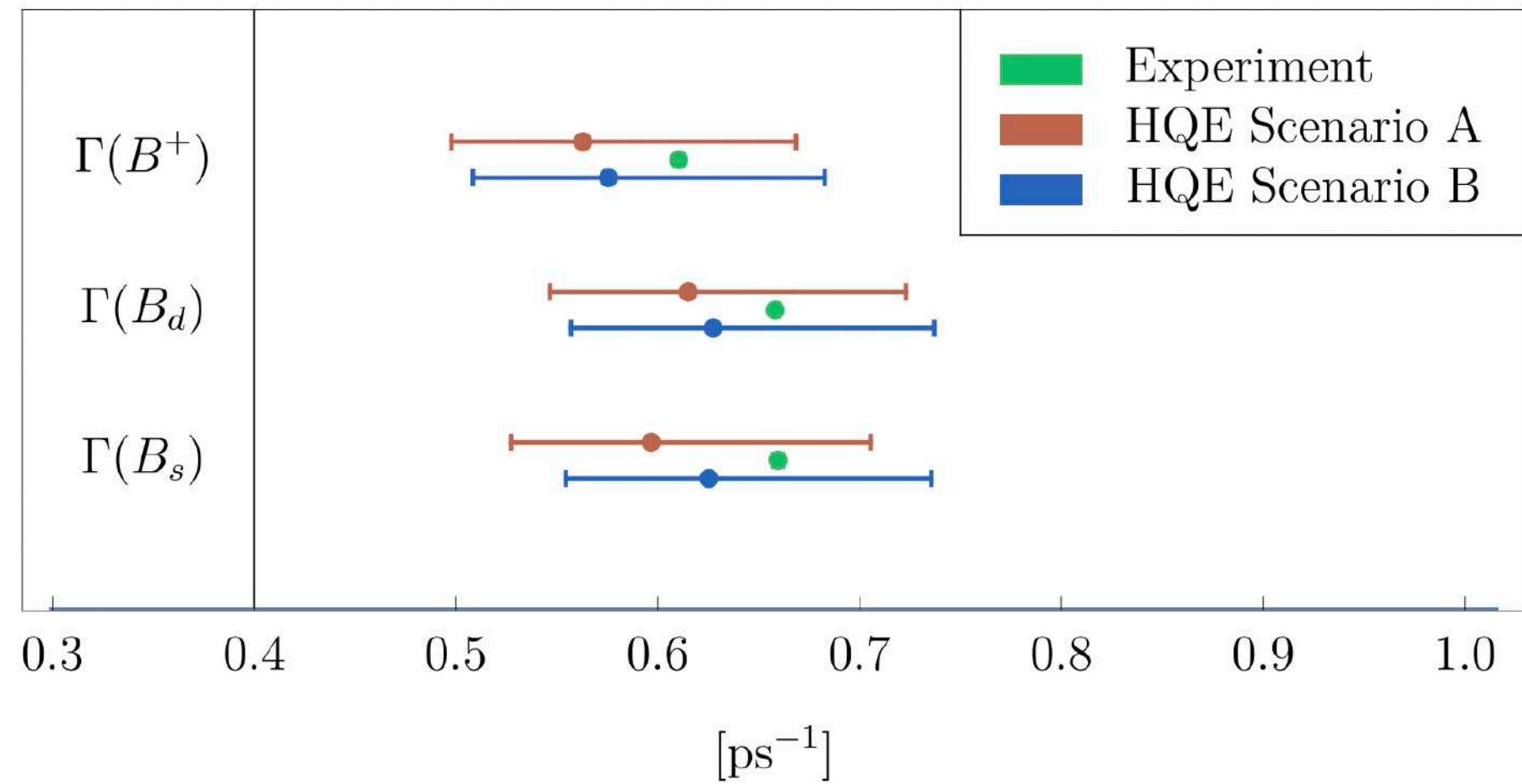
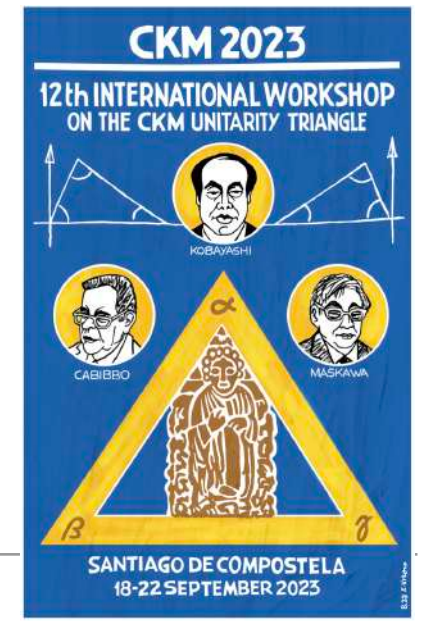
Disintegration of beauty: a precision study

Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Aleksey V. Rusov (Siegen U.) (Aug 4, 2022)
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Published in: *JHEP* 04 (2023) 034 • e-Print: 2301.07698 [hep-ph]

HQE: b hadrons



Size of Darwin term from inclusive V_{cb} fit

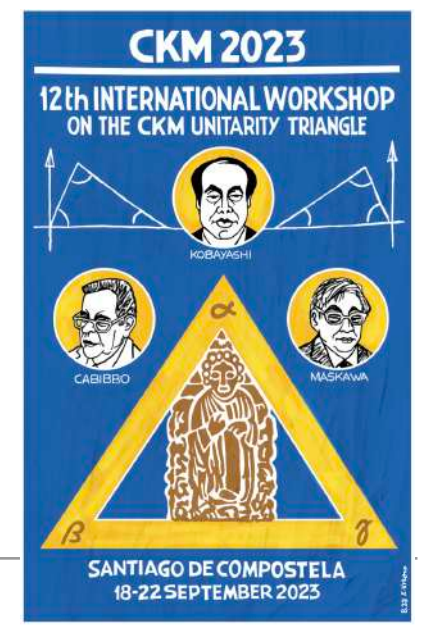
- A) Bordone, Capdevila, Gambino 2021
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 Published in: JHEP 04 (2023) 034 • e-Print: 2301.07698 [hep-ph]

HQE: b hadrons



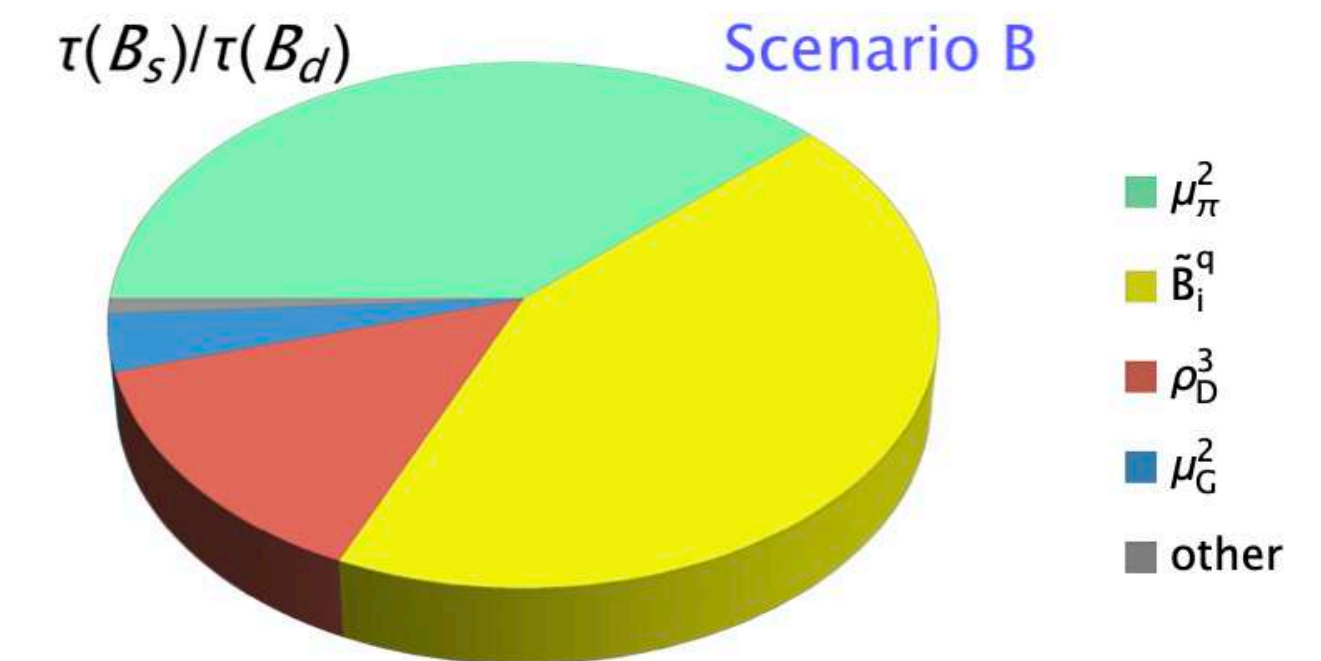
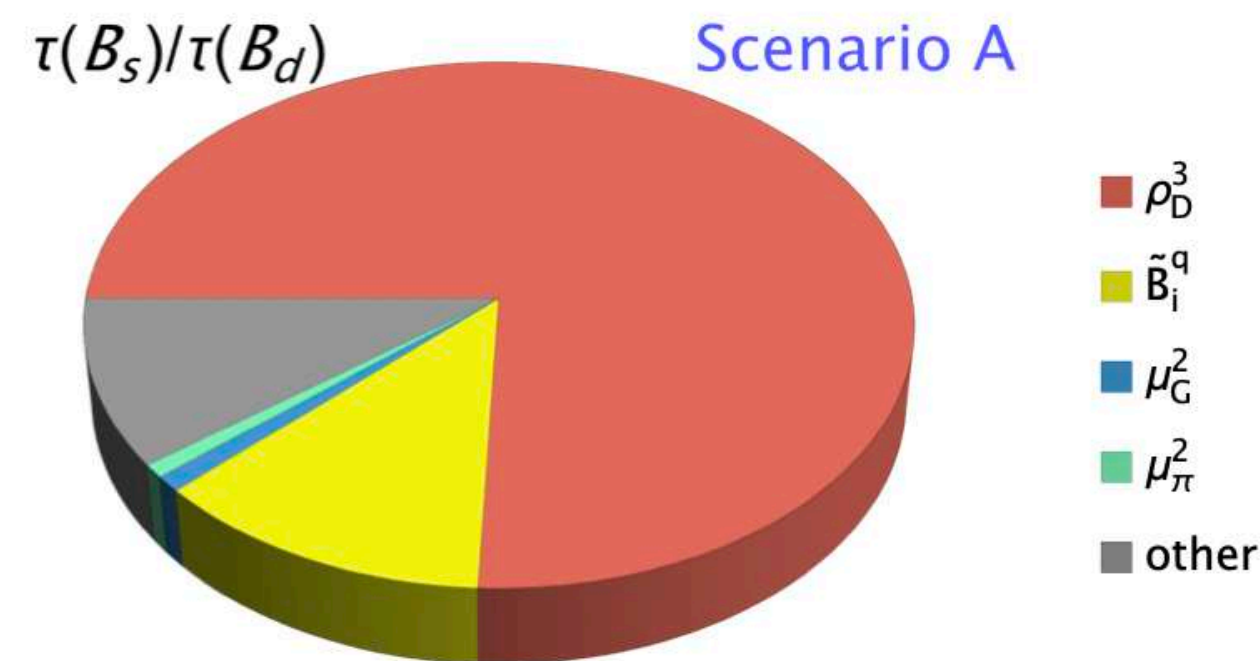
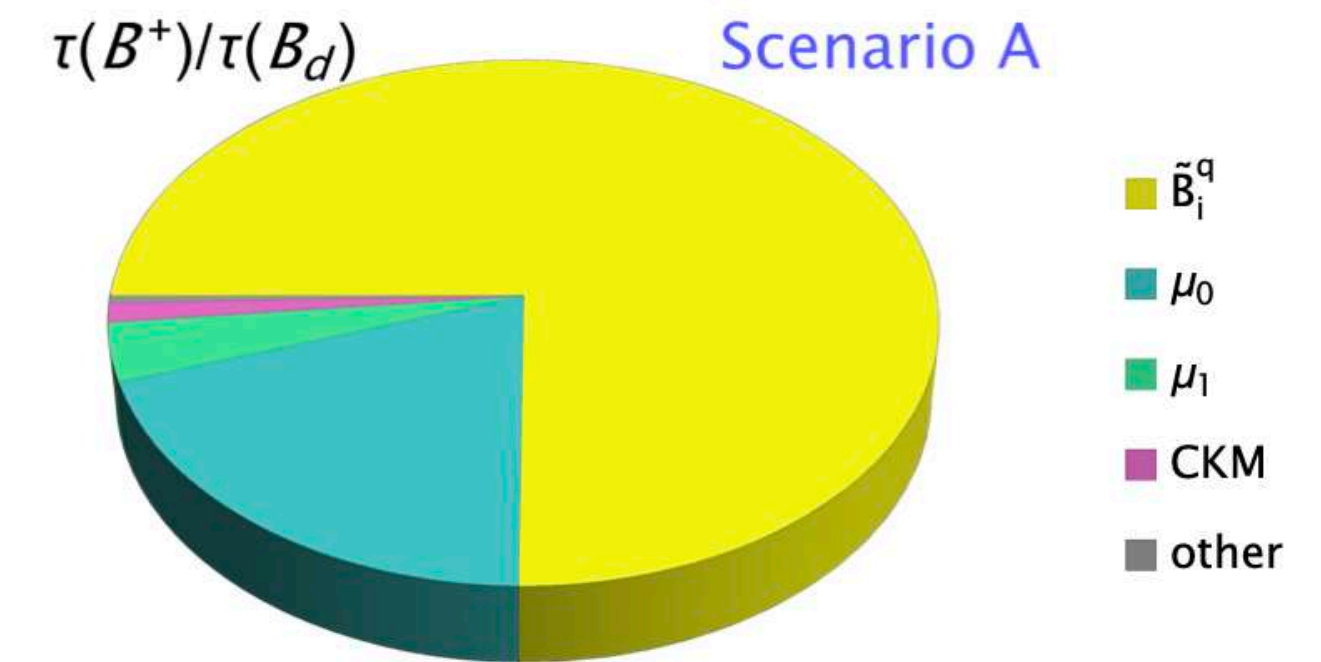
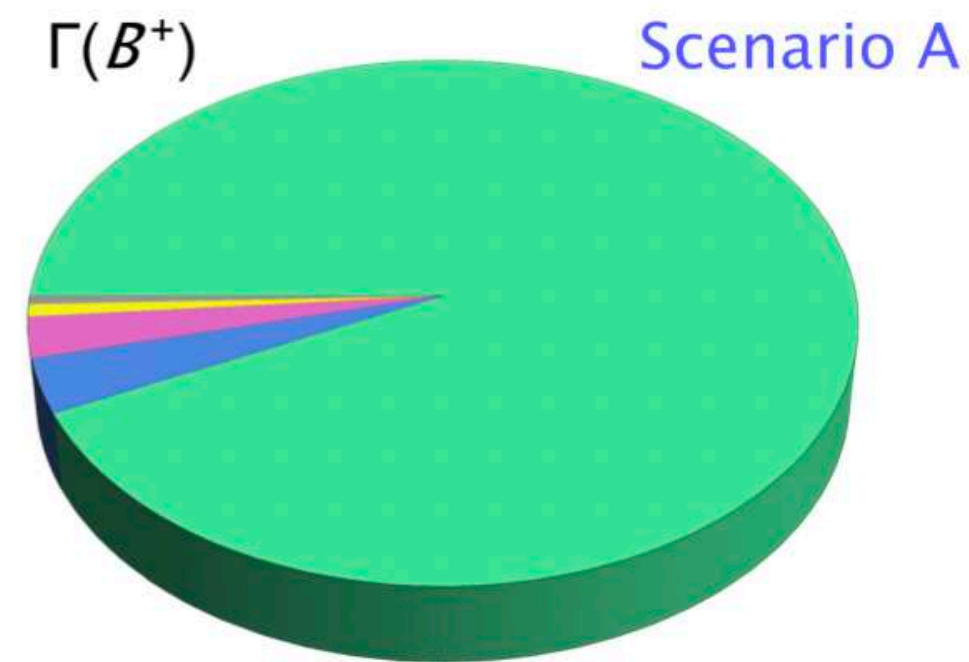
Total decay rate = free quark decay
+ hadron dependent contribution

$$\Gamma(H_b) = \Gamma_b + \delta\Gamma(H_b)$$

Free quark decay

$$\Gamma_b = \Gamma_0 [N_c (|V_{ud}|^2 f(x_c, x_u, x_d) + |V_{cs}|^2 f(x_c, x_c, x_s)) + f(x_c, x_e, x_{\nu_e}) + f(x_c, x_\mu, x_{\nu_\mu}) + f(x_c, x_\tau, x_{\nu_\tau}) + \dots \text{CKM suppressed decays}]$$

$$\Gamma_0 = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 \cdot \text{quark mass definition very sensitive to } \alpha_s \text{ corrections}$$



Free quark decay cancels in lifetime ratios!

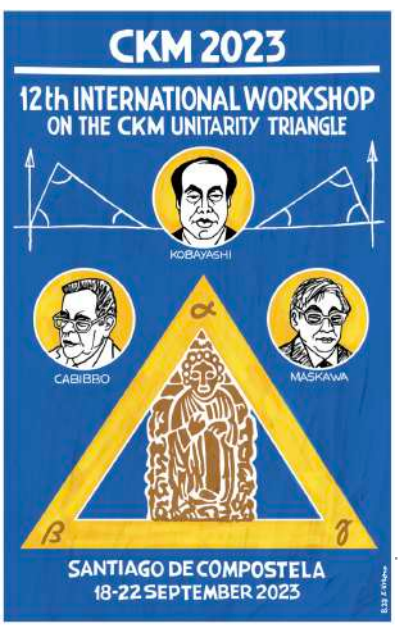
$$\frac{\tau(H_b)}{\tau(H'_b)} = 1 + [\delta\Gamma(H'_b) - \delta\Gamma(H_b)] \cdot \tau(H_b)$$

Disintegration of beauty: a precision study

Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Aleksey V. Rusov (Siegen U.) (Aug 4, 2022)

Published in: JHEP 01 (2023) 004 • e-Print: 2208.02643 [hep-ph]

Cancellations



The charm system is theoretically more difficult than the b system since

$$\alpha_s(m_c) \approx 0.33 \quad \text{and} \quad \frac{\Lambda_{QCD}}{m_c} \approx 3 \frac{\Lambda_{QCD}}{m_b}$$

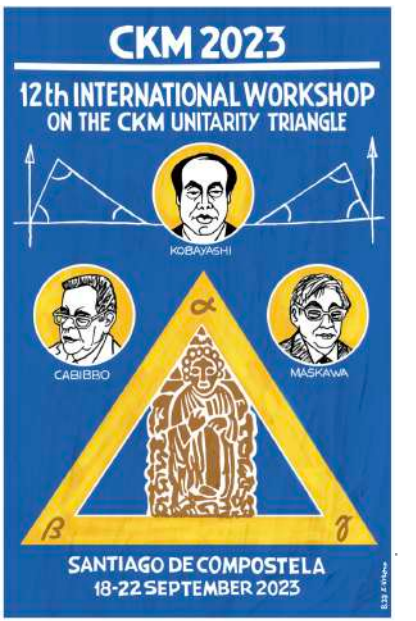
Nevertheless the **Heavy Quark Expansion** might still converge

But things will become very ugly, if in addition cancellations arising

- A. No cancellations, e.g. $\Gamma(D^0)$
- B. Strong cancellations, e.g. $\Gamma(D^+)$
- C. Crazy cancellations, e.g. D -mixing



A. No Cancellations



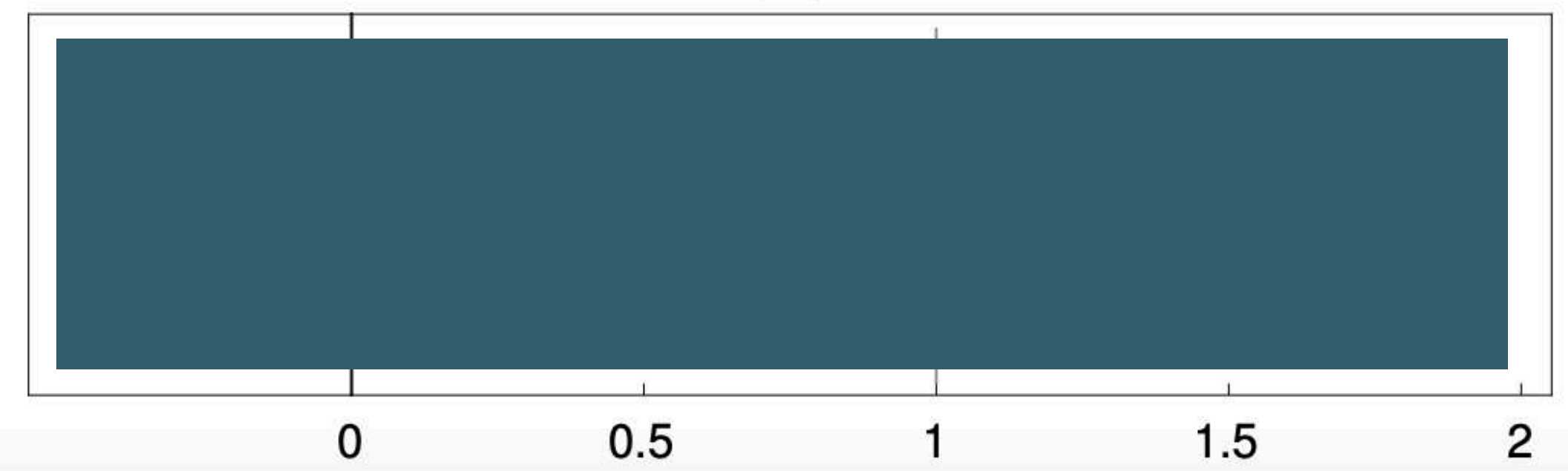
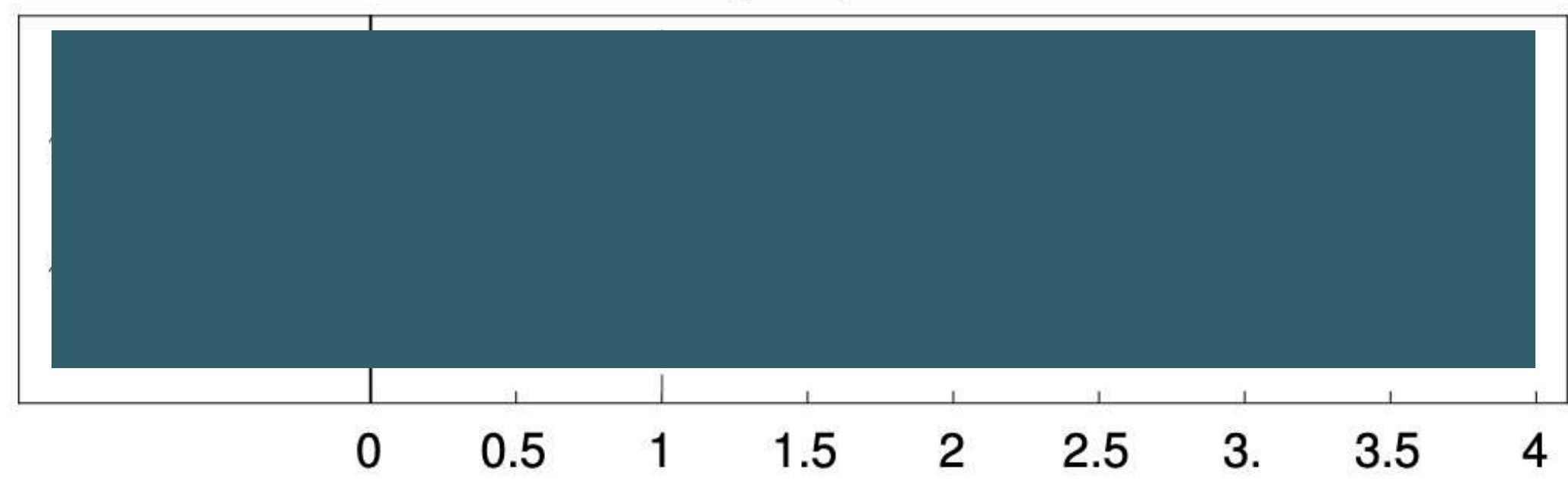
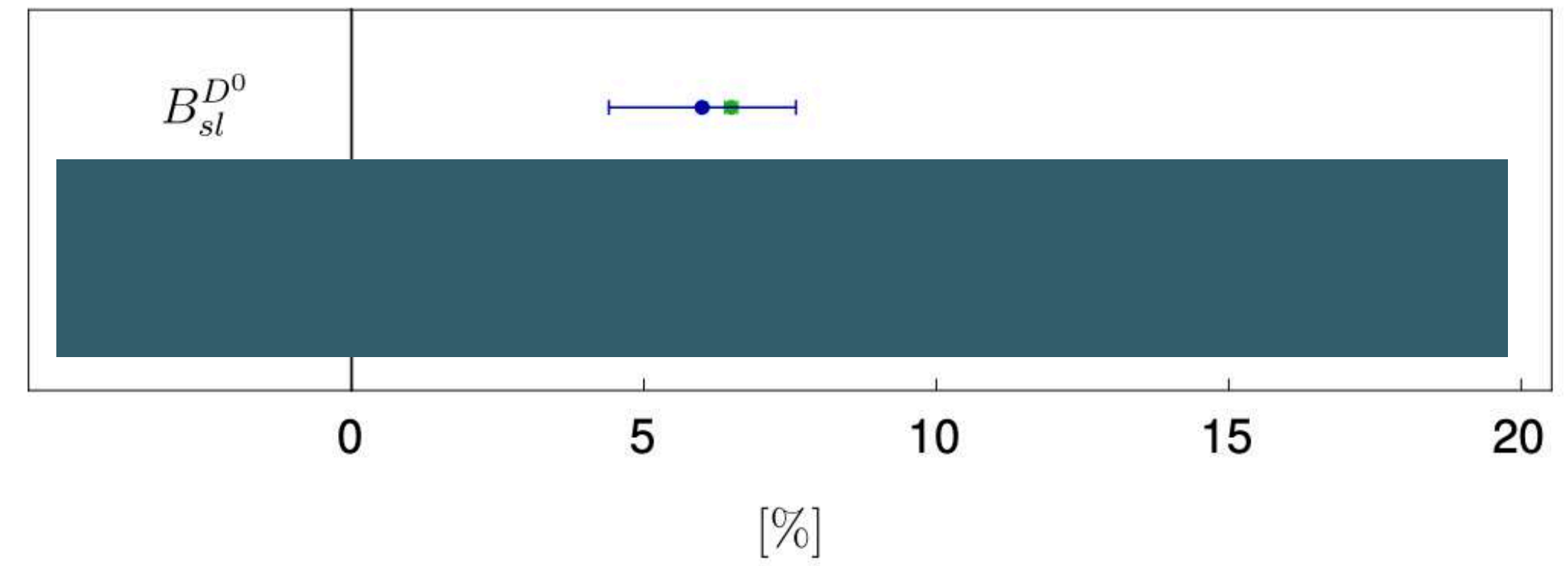
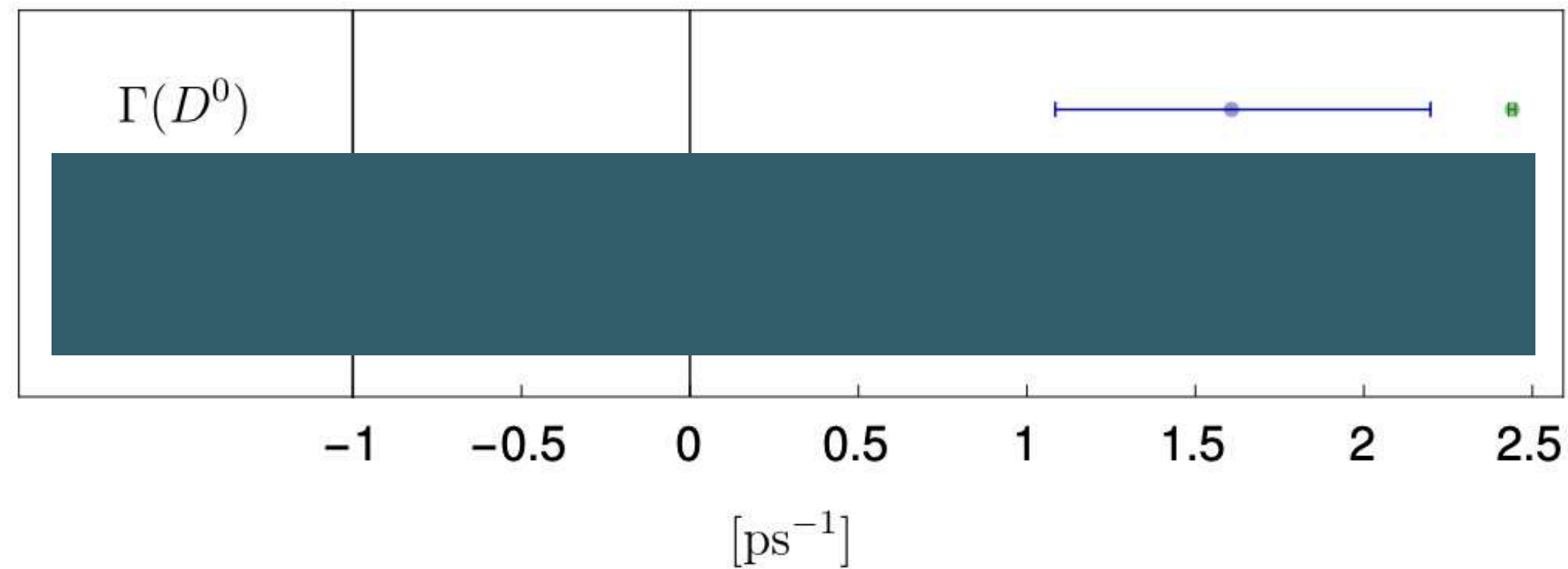
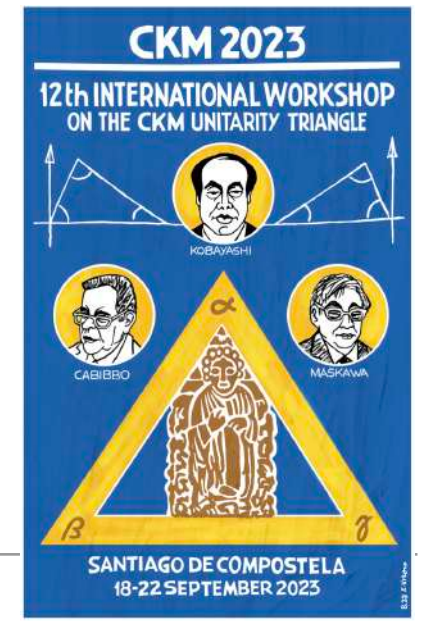
Huge NLO
Corrections

Size of Darwin term
from inclusive fit?

$$\Gamma(D^0) = 6.15 \Gamma_0 \left[1 + 0.48 - 0.13 \frac{\mu_\pi^2(D)}{0.48 \text{ GeV}^2} + 0.01 \frac{\mu_G^2(D)}{0.34 \text{ GeV}^2} + 0.31 \frac{\rho_D^3(D)}{0.082 \text{ GeV}^3} \right.$$

$$\begin{aligned} & - \underbrace{0.01}_{\text{dim-6, VIA}} - 0.005 \frac{\delta \tilde{B}_1^q}{0.02} + 0.005 \frac{\delta \tilde{B}_2^q}{0.02} + 0.137 \frac{\tilde{\epsilon}_1^q}{-0.04} - 0.125 \frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{0.00}_{\text{dim-7, VIA}} \\ & - 0.0045 r_1^{qq} - 0.0004 r_2^{qq} - 0.0035 r_3^{qq} + 0.0000 r_4^{qq} \\ & - 0.0109 r_1^{sq} - 0.0079 r_2^{sq} - 0.0000 r_3^{sq} + 0.0001 r_4^{sq} \end{aligned} \left. \right].$$

A. No Cancellations

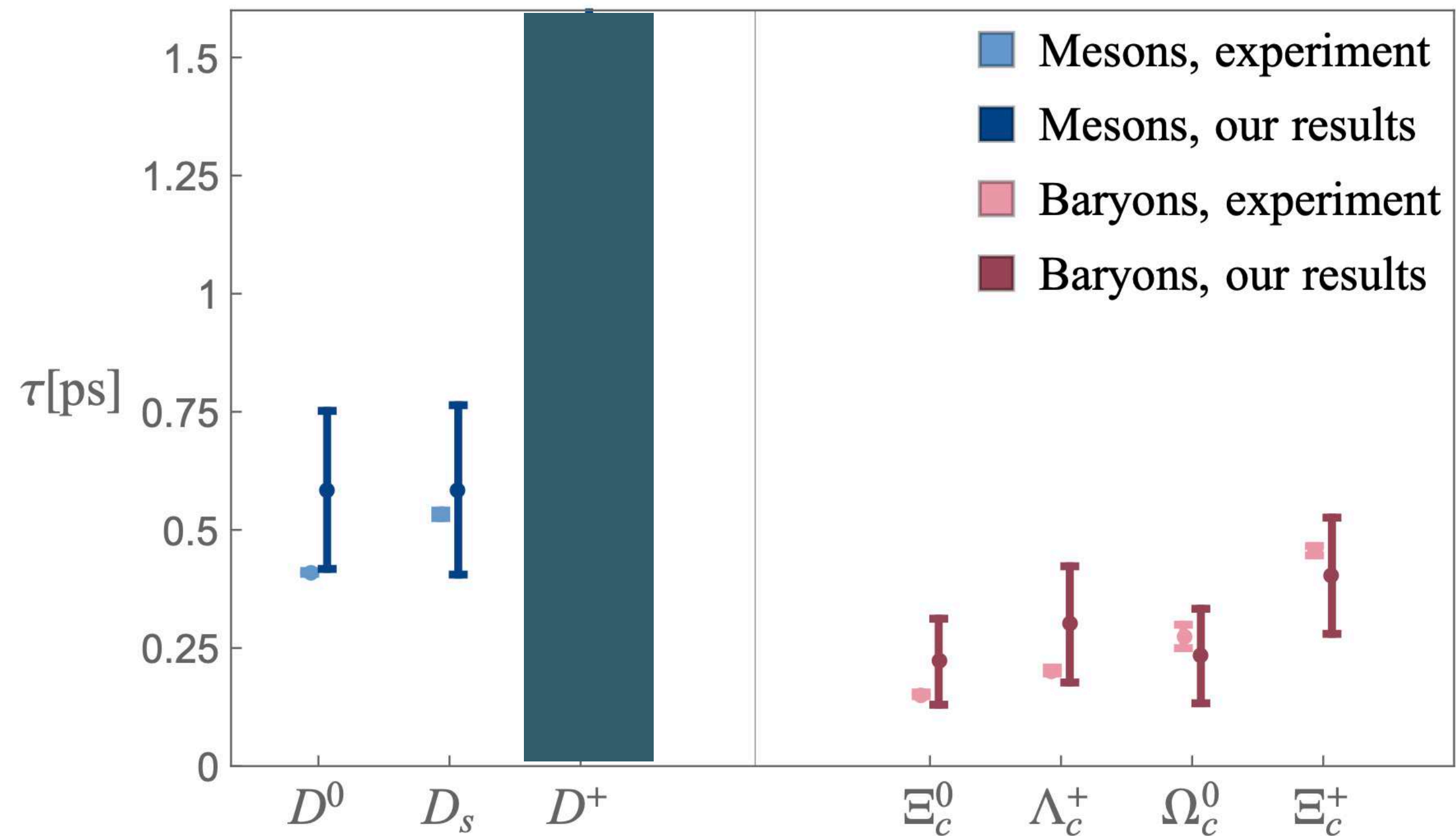
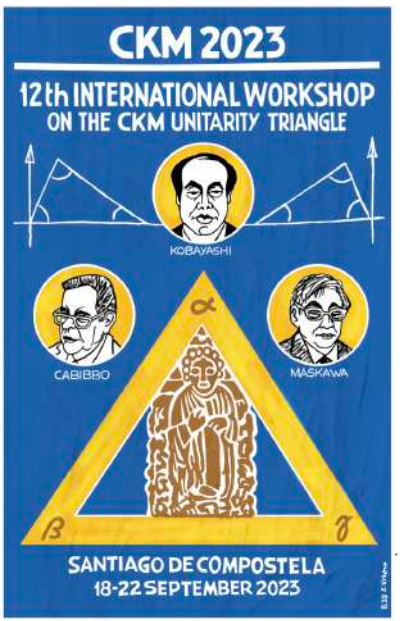


- Values of $\mu_\pi^2, \mu_G^2, \rho_D^3$ almost unknown: **incl. semileptonic fit**
- NNLO-QCD corrections to free quark decay in progress
Fael, Steinhauser,...
- NNLO-QCD corrections to spectator effects in progress
Nierste, Steinhauser,...



Revisiting Inclusive Decay Widths of Charmed Mesons #5
 Daniel King (Durham U., IPPP and Durham U.), Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Thomas Rauh (U. Bern, AEC), Aleksey V. Rusov (Siegen U.) et al. (Sep 27, 2021)
 e-Print: 2109.13219 [hep-ph]

A. No Cancellations



B_Q	BM ^a		NRQM		Experiment	
	$\mathcal{BF}_e^{\text{SL}}(\%)$	τ	$\mathcal{BF}_e^{\text{SL}}(\%)$	τ	$\mathcal{BF}_e^{\text{SL}}(\%)$	τ
Λ_c^+	4.57(54)	1.92(37)	$3.80_{-0.57}^{+0.58}$	$3.04_{-0.80}^{+1.06}$	3.95 ± 0.35	2.029(11)
Ξ_c^0	4.40(61)	1.66(32)	$4.31_{-0.84}^{+0.87}$	$2.31_{-0.59}^{+0.84}$	-	1.505(19)
Ξ_c^+	8.57(49)	3.27(76)	$12.74_{-2.45}^{+2.54}$	$4.25_{-1.00}^{+1.22}$	-	4.53(5)
Ω_c^0	1.88(1.69)	2.30(58)	$7.59_{-2.24}^{+2.49}$	$2.59_{-0.70}^{+1.03}$	-	2.73(12)
Λ_b	9.90(3)	1.48(22)	$11.0_{-0.5}^{+0.6}$	$1.490_{-0.207}^{+0.176}$	-	1.471(9)
Ξ_b^0	9.94(6)	1.49(22)	$11.1_{-0.6}^{+0.6}$	$1.493_{-0.207}^{+0.177}$	-	1.480(30)
Ξ_b^-	10.38(9)	1.55(23)	$11.7_{-0.6}^{+0.7}$	$1.608_{-0.230}^{+0.194}$	-	1.572(40)
Ω_b^-	10.76(14)	1.60(25)	$12.0_{-1.4}^{+1.4}$	$1.692_{-0.261}^{+0.231}$	-	$1.64_{-0.17}^{+0.18}$

Lifetimes of singly charmed hadrons

#17

James Gratx (Boskovic Inst., Zagreb), Blaženka Melić (Boskovic Inst., Zagreb), Ivan Nišandžić (Boskovic Inst., Zagreb)
(Apr 25, 2022)

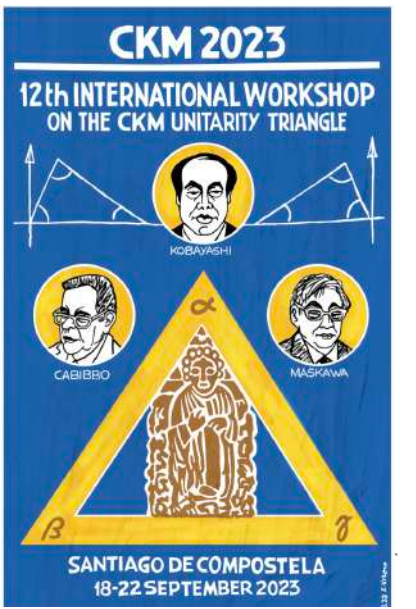
Published in: *JHEP* 07 (2022) 058, *JHEP* 07 (2022) 058 • e-Print: [2204.11935](https://arxiv.org/abs/2204.11935) [hep-ph]

Study of singly heavy baryon lifetimes

Hai-Yang Cheng (Taiwan, Inst. Phys.), Chia-Wei Liu (HIAS, UCAS, Hangzhou) (May 1, 2023)

Published in: *JHEP* 07 (2023) 114 • e-Print: [2305.00665](https://arxiv.org/abs/2305.00665) [hep-ph]

A. No Cancellations



Phenomenological Study of Heavy Hadron Lifetimes

Hai-Yang Cheng (Taiwan, Inst. Phys.) (Jul 2, 2018)

Published in: *JHEP* 11 (2018) 014 • e-Print: [1807.00916](https://arxiv.org/abs/1807.00916) [hep-ph]

- Huge α_s corrections to free quark decay neglected
- Huge α_s corrections to Pauli interference neglected
- 1 ad-hoc parameter α introduced to fit 1 lifetime
- Darwin term were not known
- Eye-contractions were not know

Many issues addressed in

Study of singly heavy baryon lifetimes

Hai-Yang Cheng (Taiwan, Inst. Phys.), Chia-Wei Liu (HIAS, UCAS, Hangzhou) (May 1, 2023)

Published in: *JHEP* 07 (2023) 114 • e-Print: [2305.00665](https://arxiv.org/abs/2305.00665) [hep-ph]

- $\frac{\Gamma(D_s \rightarrow X e \bar{\nu}_e)}{\Gamma(D^0 \rightarrow X e \bar{\nu}_e)} = 0.790 \pm 0.016 \pm 0.020$
in agreement with prediction from an effective quark model,
indicating non-spectator effects

9/19/2023

Markus Prim

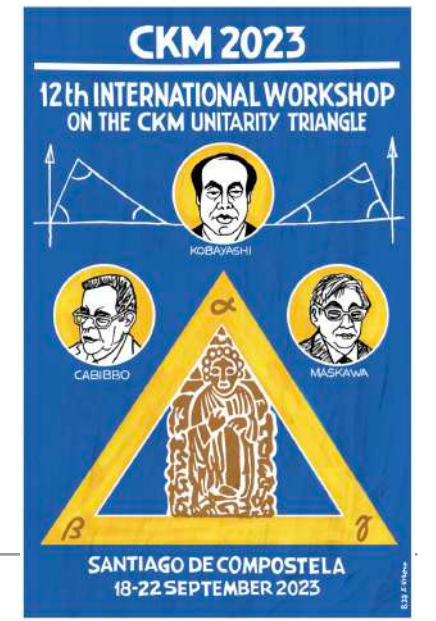
TECHNION-PH-2010-19
EFI 10-31
arXiv:1012.5098 [hep-ph]
December 2010

RATIOS OF HEAVY HADRON SEMILEPTONIC DECAY RATES

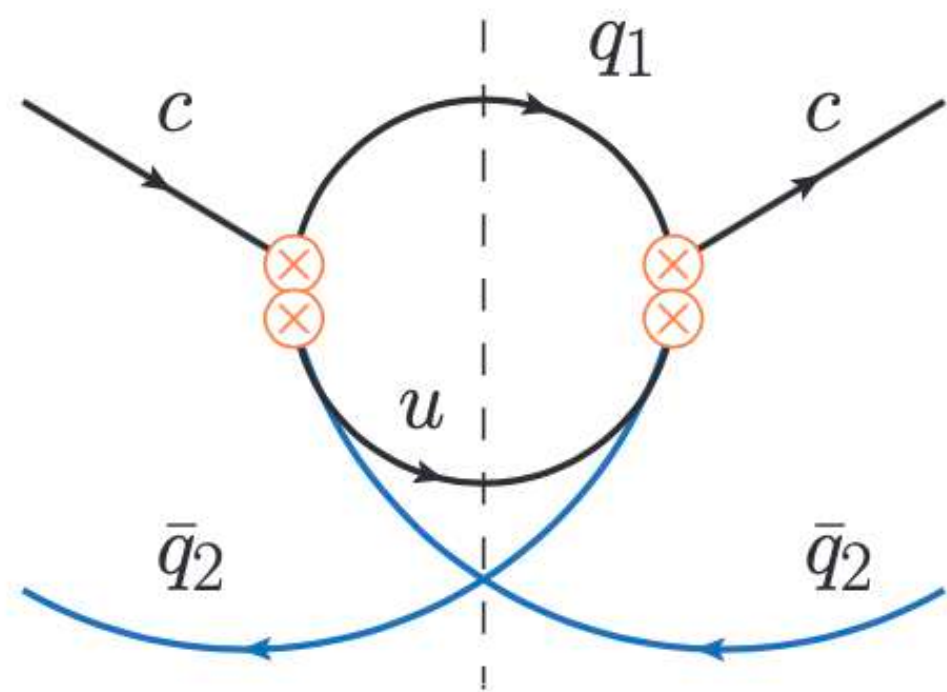
Michael Gronau
Physics Department, Technion, Haifa 32000, Israel.
Jonathan L. Rosner
Enrico Fermi Institute and Department of Physics, University of Chicago
Chicago, IL 60637, U.S.A.

with the physical masses of ground-state hadrons. This approach may be thought of as a cartoon version of local quark-hadron duality. It appears to reproduce known

B. Strong Cancellations



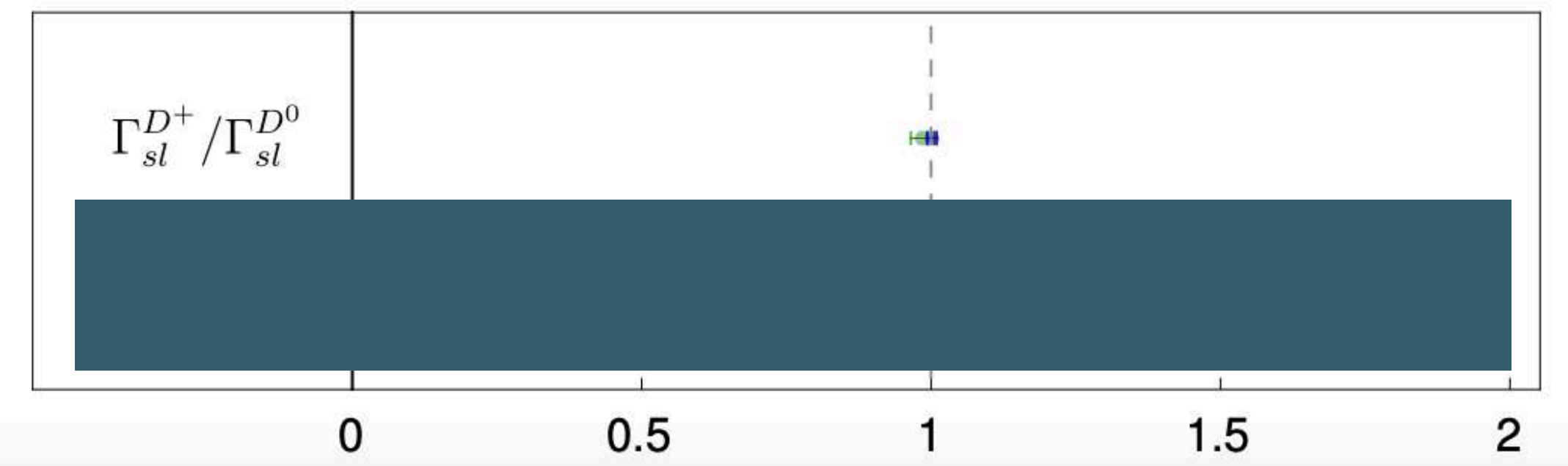
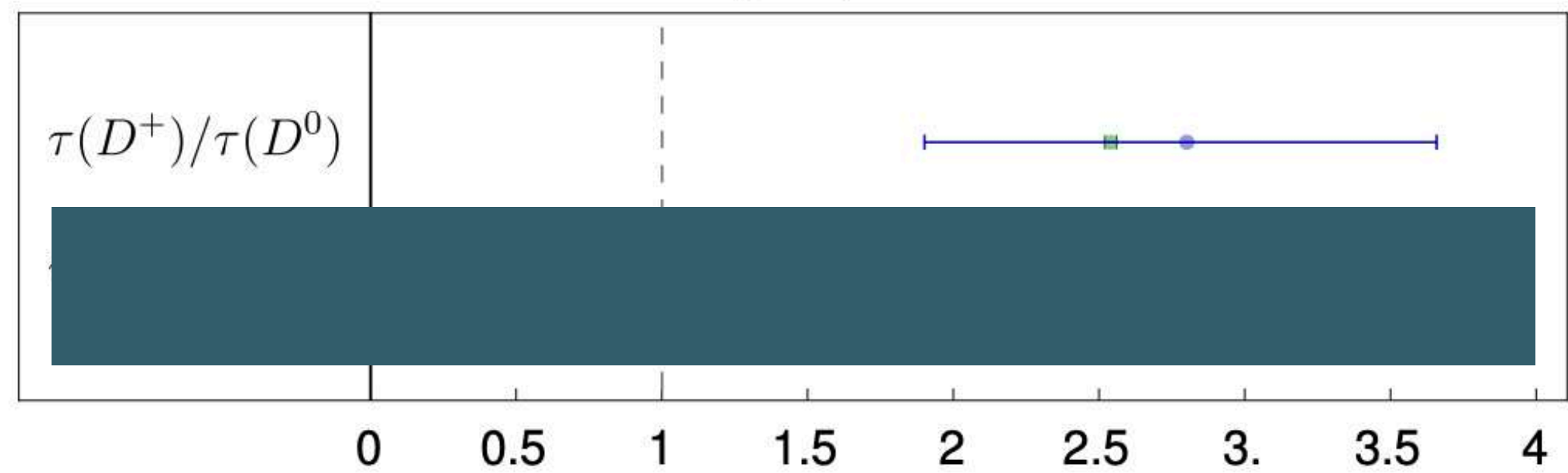
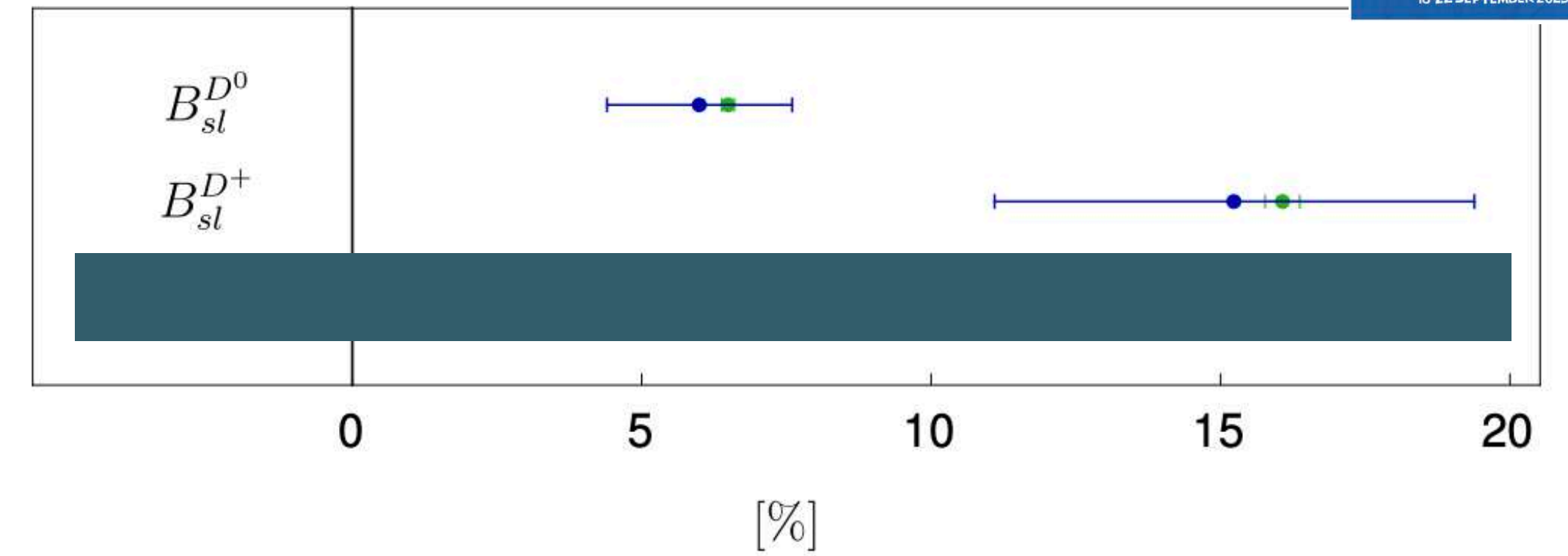
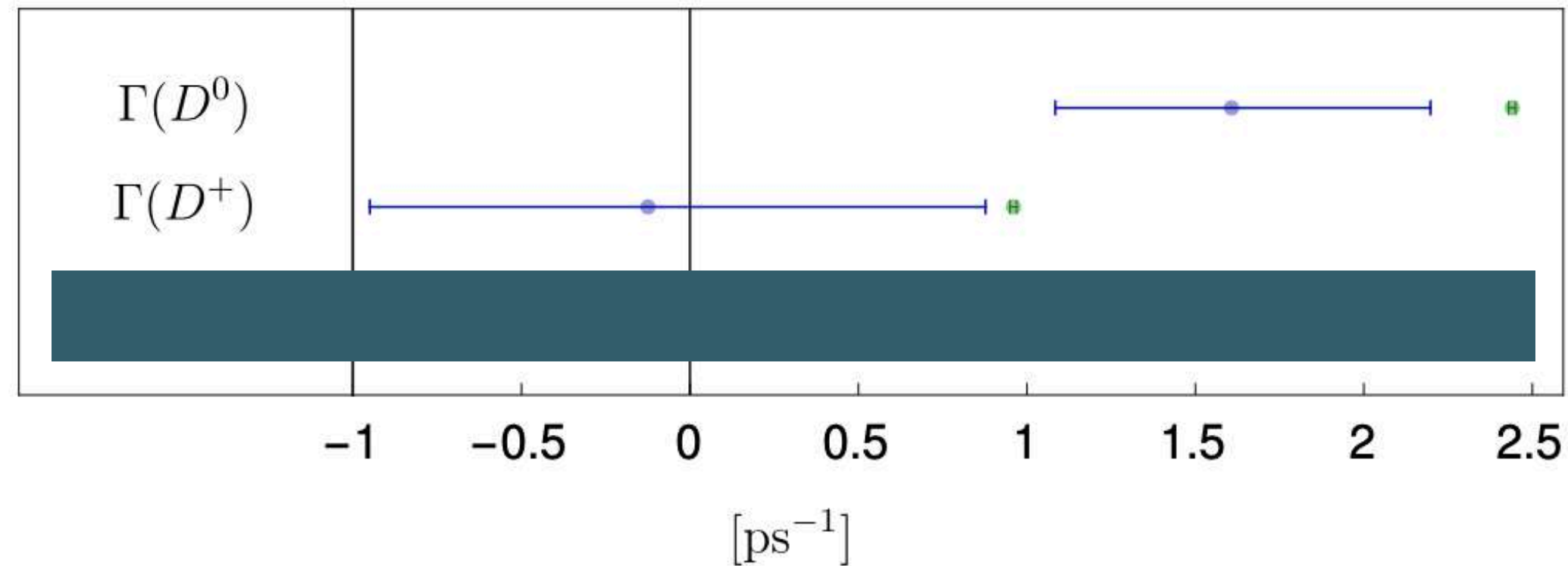
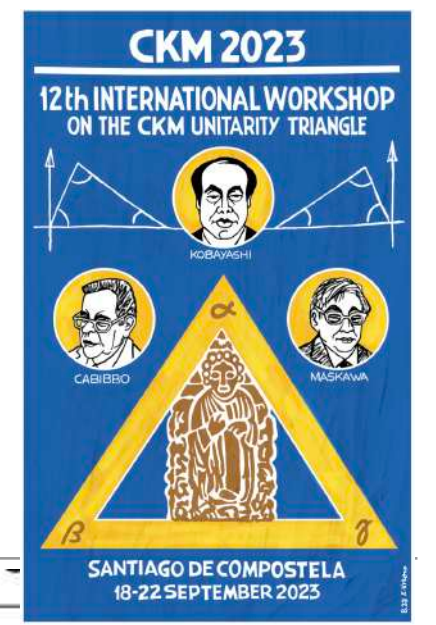
$$\Gamma(D^+) = 6.15 \Gamma_0 \left[1 + 0.48 - 0.13 \frac{\mu_\pi^2(D)}{0.48 \text{ GeV}^2} + 0.01 \frac{\mu_G^2(D)}{0.34 \text{ GeV}^2} + 0.31 \frac{\rho_D^3(D)}{0.082 \text{ GeV}^3} \right.$$



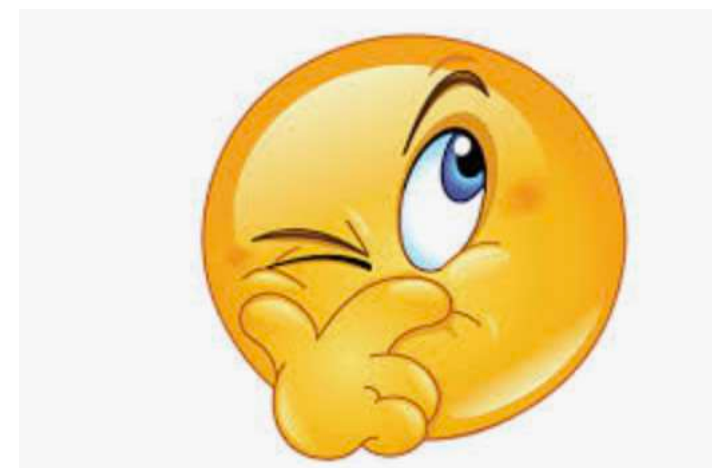
$$\begin{aligned} & - \underbrace{2.66}_{\text{dim-6, VIA}} - 0.055 \frac{\delta \tilde{B}_1^q}{0.02} + 0.002 \frac{\delta \tilde{B}_2^q}{0.02} - 0.546 \frac{\tilde{\epsilon}_1^q}{-0.04} + 0.009 \frac{\tilde{\epsilon}_2^q}{-0.04} + \underbrace{1.10}_{\text{dim-7, VIA}} \\ & - 0.0000 r_1^{qq} - 0.0000 r_2^{qq} + 0.0011 r_3^{qq} + 0.0008 r_4^{qq} \\ & - 0.0109 r_1^{sq} - 0.0080 r_2^{sq} - 0.0000 r_3^{sq} + 0.0001 r_4^{sq} \end{aligned} \Bigg],$$

Huge effects due to Pauli interference

B. Strong Cancellations



- Values of $\mu_\pi^2, \mu_G^2, \rho_D^3$ almost unknown
- NNLO-QCD corrections to free quark decay in progress
Fael, Steinhauser,...
- NNLO-QCD corrections to spectator effects in progress
Nierste, Steinhauser,...
- Check of HQET sum rule results with lattice
Black, Witzel, ... RBC-UK
- First non-perturbative determination of dimension 7

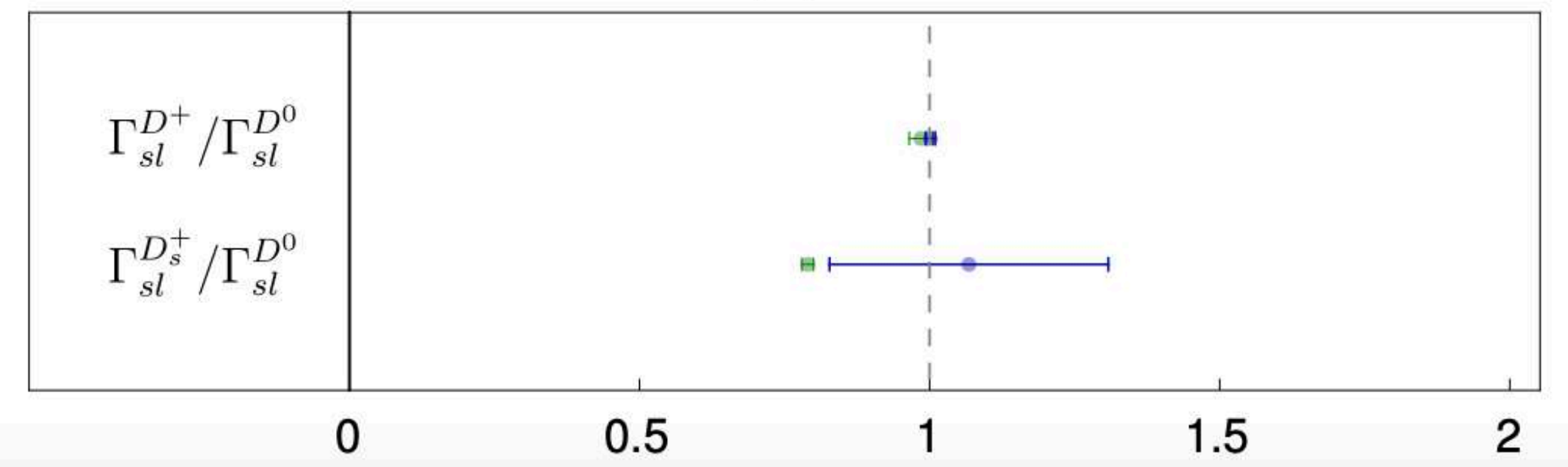
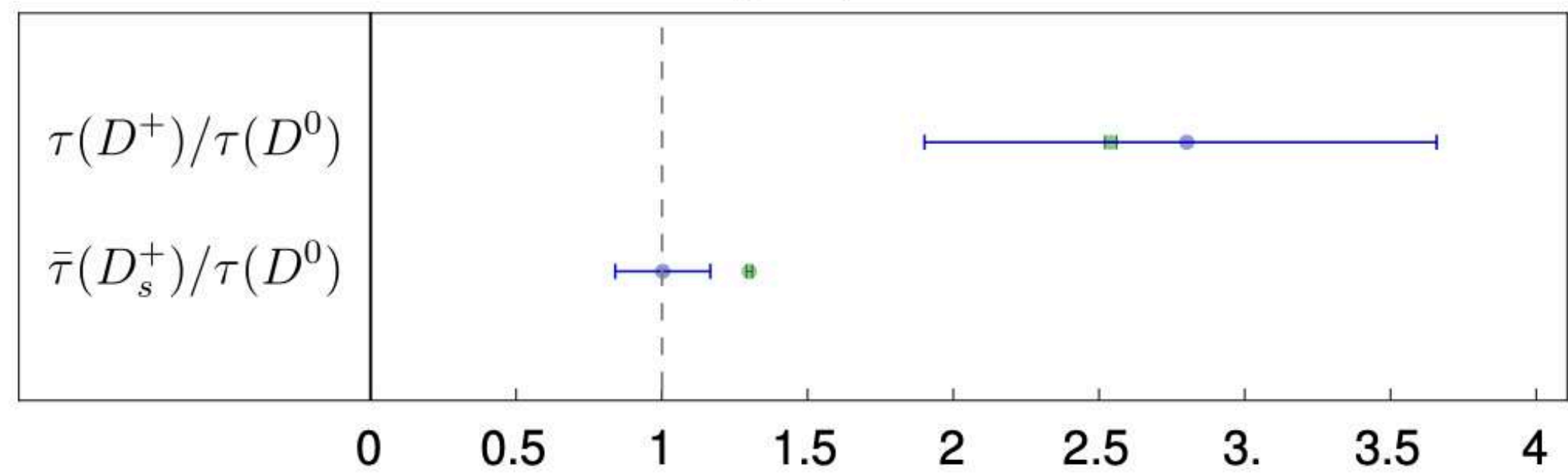
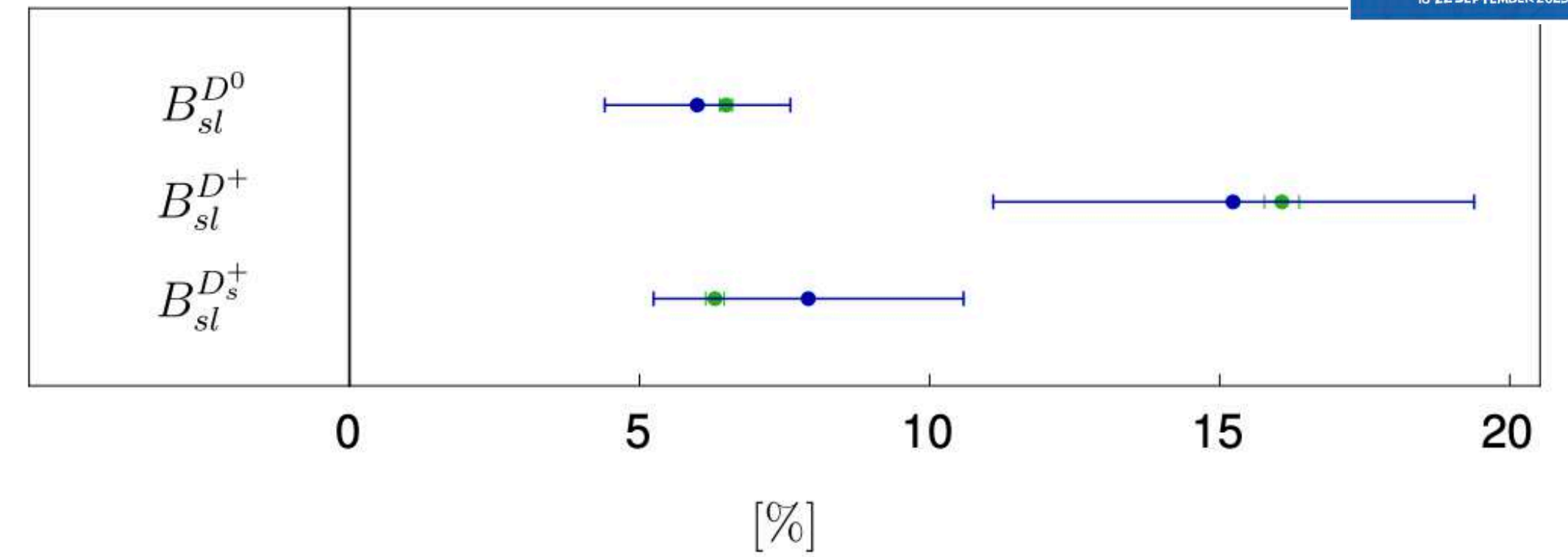
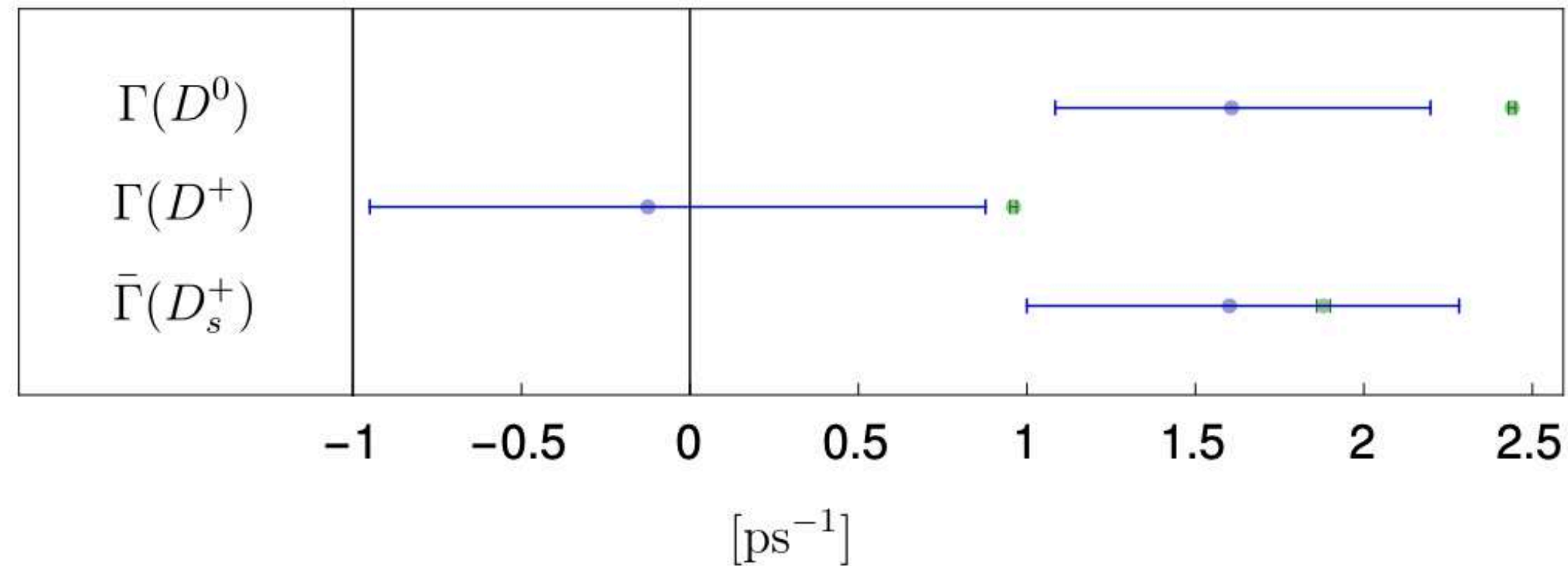
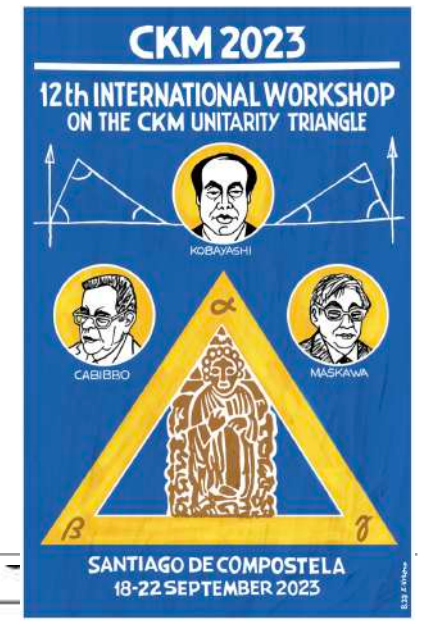


Revisiting Inclusive Decay Widths of Charmed Mesons #5

Daniel King (Durham U., IPPP and Durham U.), Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Thomas Rauh (U. Bern, AEC), Aleksey V. Rusov (Siegen U.) et al. (Sep 27, 2021)

e-Print: 2109.13219 [hep-ph]

B. Strong Cancellations



- Values of $\mu_\pi^2, \mu_G^2, \rho_D^3$ almost unknown — **inclusive semileptonic D_s^+ decays**

- NNLO-QCD corrections to free quark decay in progress

Fael, Steinhauser,...

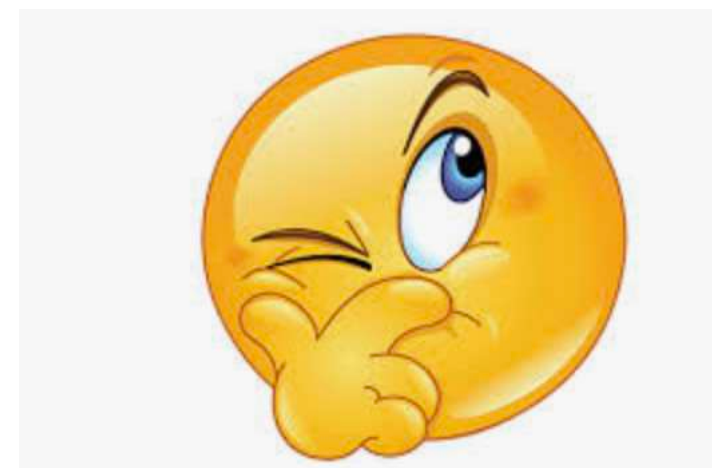
- NNLO-QCD corrections to spectator effects in progress

Nierste, Steinhauser,...

- Check of HQET sum rule results with lattice

Black, Witzel,...RBC-UK

- First non-perturbative determination of dimension 7

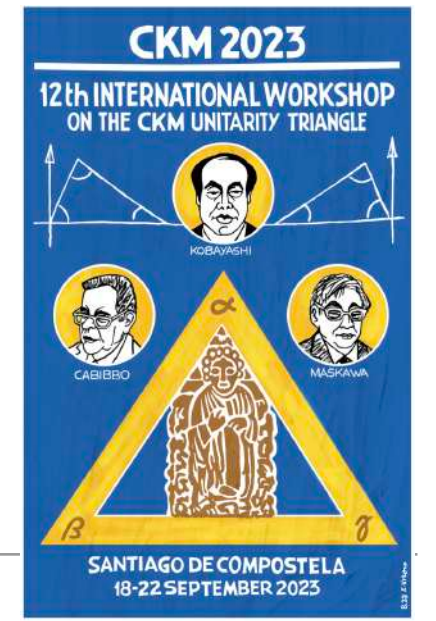


Revisiting Inclusive Decay Widths of Charmed Mesons #5

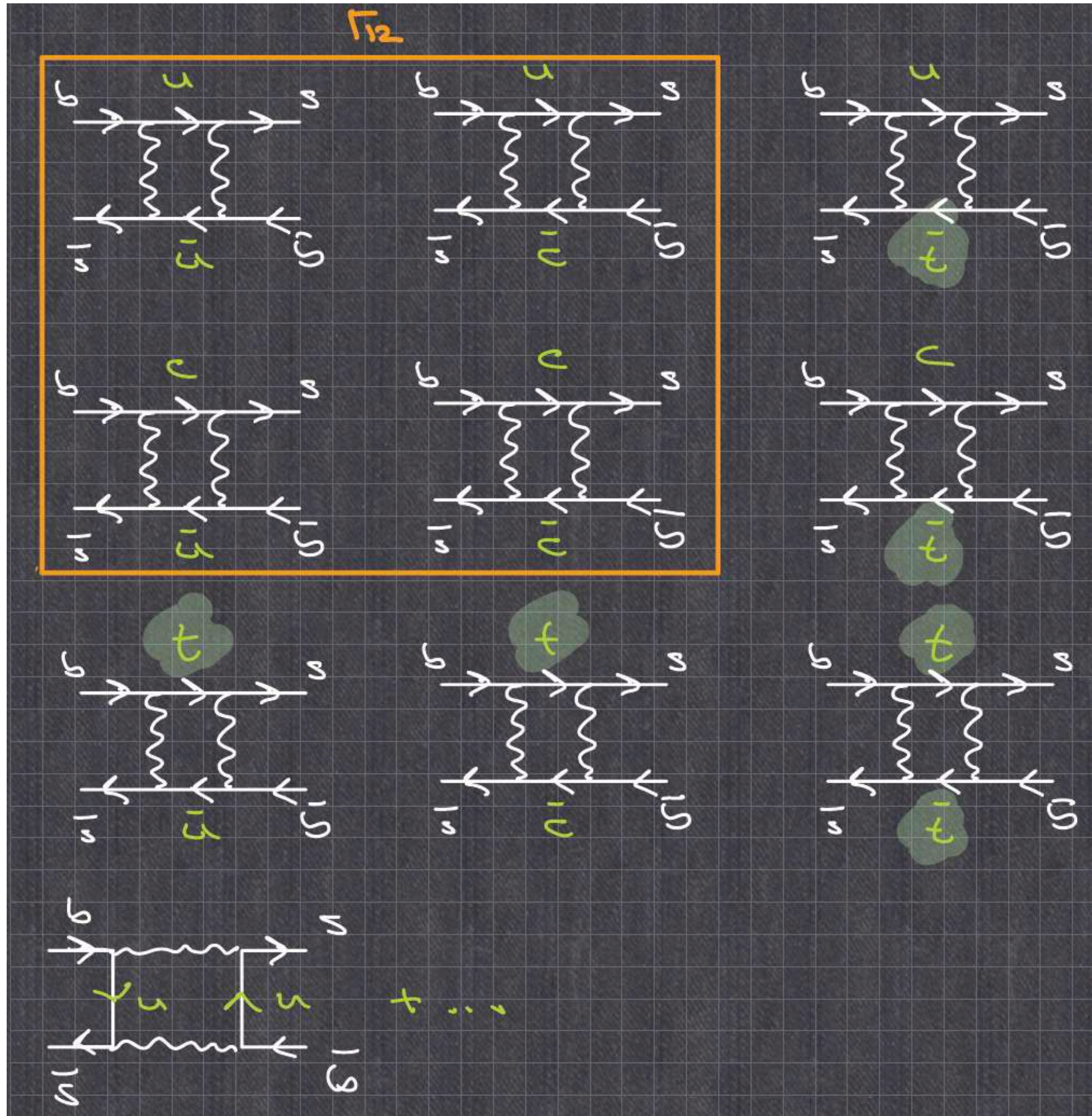
Daniel King (Durham U., IPPP and Durham U.), Alexander Lenz (Siegen U.), Maria Laura Piscopo (Siegen U.), Thomas Rauh (U. Bern, AEC), Aleksey V. Rusov (Siegen U.) et al. (Sep 27, 2021)

e-Print: 2109.13219 [hep-ph]

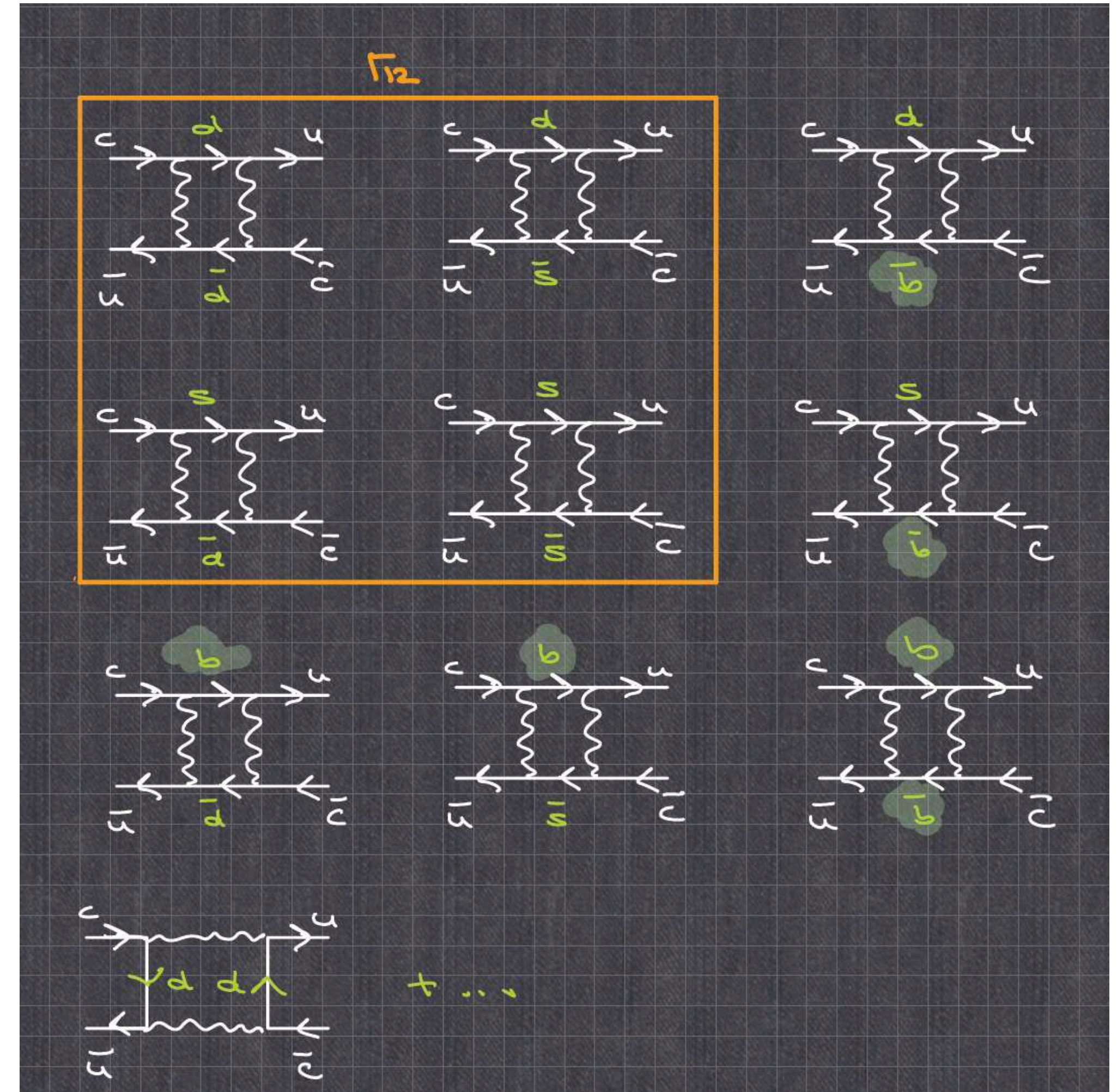
C: Crazy Cancellations



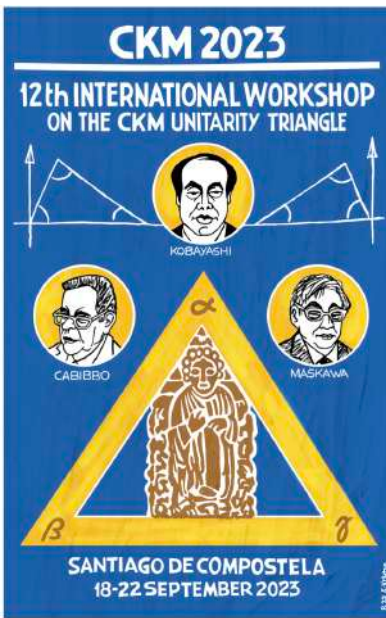
B-mixing



D-mixing



C: Crazy Cancellations



B-mixing



D-mixing



$$\begin{aligned}
 M_{12} &= \lambda_u^2 F(u,u) + \lambda_u \lambda_c F(u,c) + \lambda_u \lambda_t F(u,t) \\
 &+ \lambda_c \lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c \lambda_t F(c,t) \\
 &+ \lambda_t \lambda_u F(t,u) + \lambda_t \lambda_c F(t,c) + \lambda_t^2 F(t,t) \\
 \lambda_u + \lambda_c + \lambda_t &= 0 \\
 \downarrow \\
 &= \lambda_u^2 [F(c,c) - 2F(u,c) + F(u,u)] \\
 &+ 2\lambda_u \lambda_t [F(c,c) - F(u,c) + F(u,t) - F(c,t)] \\
 &+ \lambda_t^2 [F(c,c) - 2F(c,t) + F(t,t)]
 \end{aligned}$$

	Bd	Bs	
λ_u	$\lambda^{3.8}$	$\lambda^{4.8}$	$m_u^2/m_\tau^2 \approx 0$
λ_c	λ^3	λ^2	$m_c^2/m_\tau^2 \approx 2.5 \cdot 10^{-4}$
λ_t	λ^3	λ^2	$m_t^2/m_\tau^2 \approx 4.5$

$$\begin{aligned}
 M_{12} &= \lambda_d^2 F(d,d) + \lambda_d \lambda_s F(d,s) + \lambda_d \lambda_b F(d,b) \\
 &+ \lambda_s \lambda_d F(s,d) + \lambda_s^2 F(s,s) + \lambda_s \lambda_b F(s,b) \\
 &+ \lambda_b \lambda_d F(b,d) + \lambda_b \lambda_s F(b,s) + \lambda_b^2 F(b,b) \\
 \lambda_d + \lambda_s + \lambda_b &= 0 \\
 \downarrow \\
 &= \lambda_d^2 [F(d,d) - 2F(d,s) + F(s,s)] \\
 &+ 2\lambda_s \lambda_b [F(s,s) - F(d,s) + F(d,b) - F(s,b)] \\
 &+ \lambda_b^2 [F(s,s) - 2F(s,b) + F(b,b)]
 \end{aligned}$$

	D	
λ_d	λ^1	$m_d^2/m_\tau^2 \approx 0$
λ_s	λ^1	$m_s^2/m_\tau^2 \approx 1.3 \cdot 10^{-6}$
λ_b	$\lambda^{5.8}$	$m_b^2/m_\tau^2 \approx 2.8 \cdot 10^{-3}$

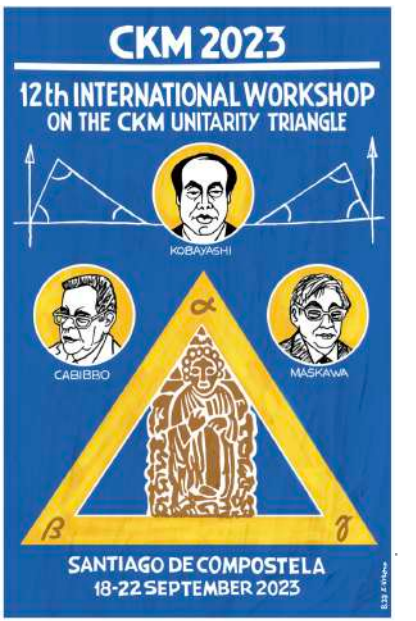
CKM dominant \equiv GIM dominant



CKM suppressed \equiv GIM suppressed

CKM suppressed \equiv GIM dominant

CKM dominant \equiv GIM suppressed

C: Crazy Cancellations



$x \equiv \frac{\Delta M_D}{\Gamma_D} = (4.07 \pm 0.44) \cdot 10^{-3}$ 
 $\frac{\Delta \Gamma_D}{2\Gamma_D} = (6.47 \pm 0.24) \cdot 10^{-3}$
 $\frac{\Delta \Gamma_D^{\text{naive HQE}}}{\Delta \Gamma_D^{\text{Exp.}}} \approx 10^{-5} \dots 10^{-4}$ 

HFLAV

AL, Piscopo, Vlahos
2007.03022

B^0: ARGUS <i>Observation of B^0 oscillations</i> Phys.Lett.B 192 (1987) 245	D^0: Belle & BaBar <i>Evidence of D^0 oscillations</i> Phys.Rev.Lett. 98 (2007) 211802 Phys.Rev.Lett. 98 (2007) 211803	D^0: LHCb <i>Observation of D^0 mass difference</i> LHCb-PAPER-2021-009
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K^0 <i>Behavior of neutral particles</i> e.g. Phys.Rev. 97 (1955) 1387	B_s^0: CDF <i>Observation of B_s^0 oscillations</i> Phys.Rev.Lett. 97 (2006) 242003	D^0: LHCb <i>Observation of D^0 oscillations</i> Phys.Rev.Lett. 110 (2013) 101802
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2012: $\Delta \Gamma_s$ by LHCb

8 June 2021: First observation of the mass difference between neutral charm mesons.

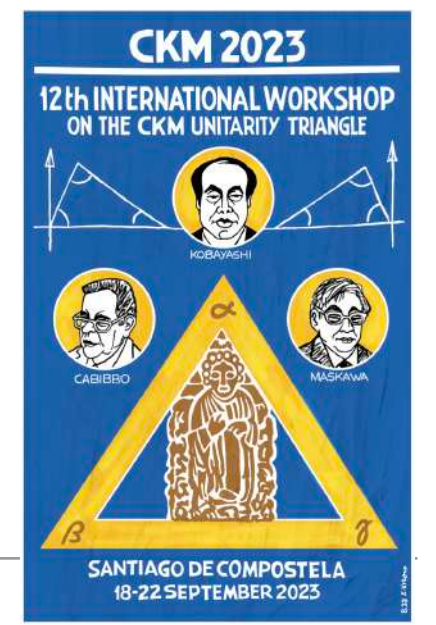
$x = (3.98^{+0.55}_{-0.55}) \times 10^{-3}$
 $m_1 - m_2 = 6.4 \times 10^{-6} \text{ eV} = 0.00000000000000000000000000000001 \text{ grams } (1 \times 10^{-38} \text{g})$
 $(m_1 - m_2) / (D^0 \text{mass}) = 3 \times 10^{-15}$

Today, the LHCb Collaboration submitted a paper for publication that reports the first observation of the mass difference between neutral charm mesons (or rather their mass eigenstates for experts). The result is also presented today at the CERN [seminar](#) and was reported last week at the [10th International Workshop on CHARM Physics](#). This mass difference determines the

Exclusive approaches

- Estimate phase space effects for y : Falk et al. 0110317
- assume pert. SU(3)F breaking $y \approx 1\%$ 
- neglect 3rd family
- neglect SU(3)F breaking in matrix elements - no QCD calculation
- Mass difference from a dispersion relation Falk et al. 0402204 $x \approx y$
- Exp. data Cheng, Chiang 1005.1106 $x \propto \mathcal{O}(0.1\%)$ $y \propto \mathcal{O}(\text{few } 0.1\%)$
- U-Spin sum rule Gronau, Rosner 2012
- Factorisation-assisted topological amplitude approach Jiang et al. 1705.07335 $y \approx 0.2\%$

C: Crazy Cancellations



The HQE is successful in the B system and for D meson lifetimes
=> apply it for D-mixing

$$y_D^{\text{HQE}} \approx \lambda_s^2 (\Gamma_{12}^{ss} - 2\Gamma_{12}^{sd} + \Gamma_{12}^{dd}) \approx 10^{-5} y_D^{\text{Exp.}}$$

How can this be?

Look only at a single diagram:

$$y_D^{\text{HQE}} \neq \lambda_s^2 \Gamma_{12}^{ss} \tau_D = 3.7 \cdot 10^{-2} \approx 5.6 y_D^{\text{Exp.}}$$

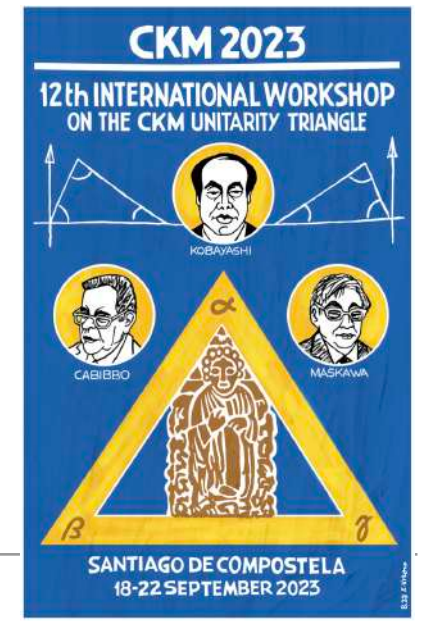
pert. calculation: **Bobrowski et al 1002.4794**

lattice input: **ETM 1403.7302; 1505.06639; FNAL/MILC 1706.04622**

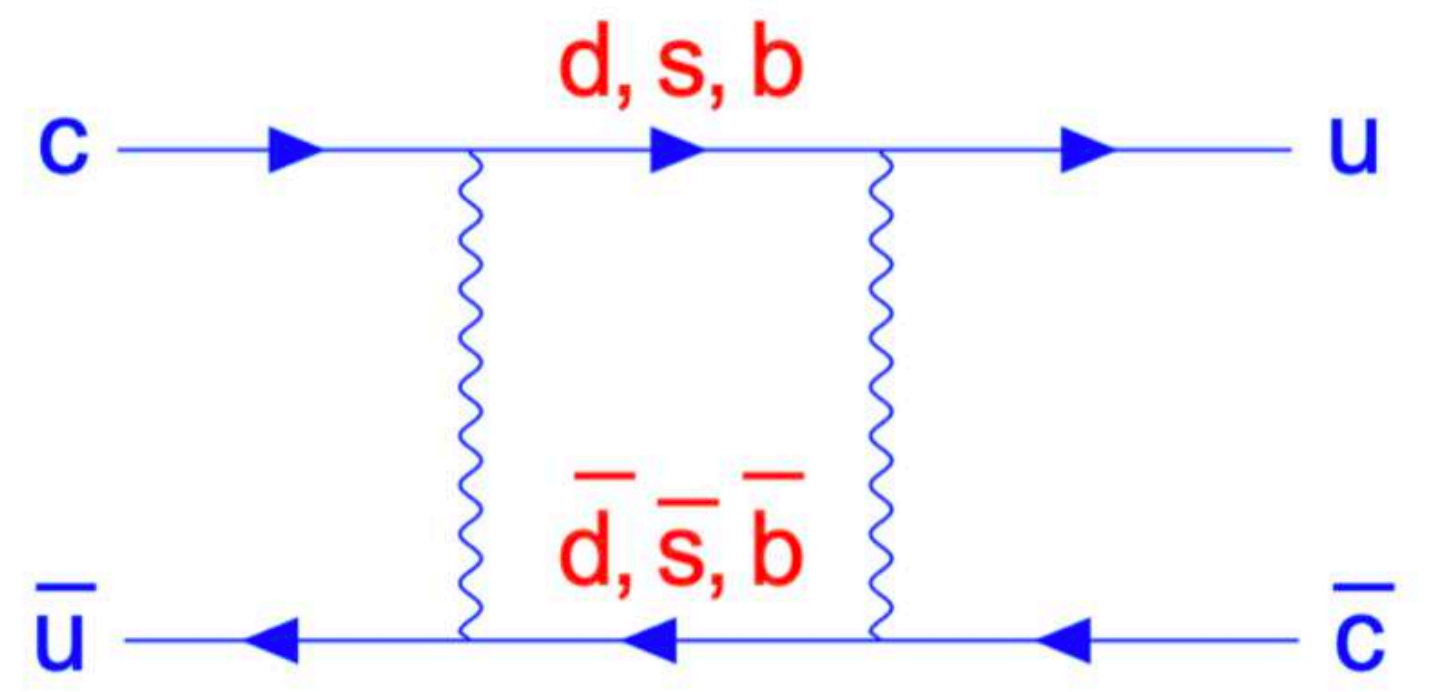
HQET sum rules: **Kirk, AL, Rauh 1711.02100**

The problem seems to originate in the extreme GIM cancellations

C: Crazy Cancellations



GIM cancellation vs CKM hierarchy: $|\lambda_b| \ll |\lambda_s|$, but complex!!!



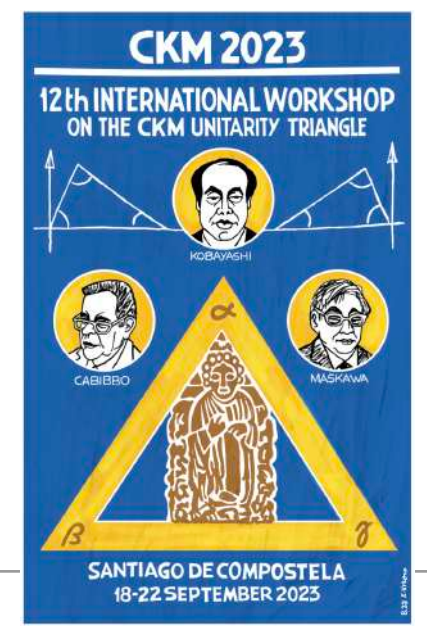
survives in
SU(3)F limit!

dominant for
B mixing

$$\Gamma_{12}^D = -\lambda_s^2 (\Gamma_{ss}^D - 2\Gamma_{sd}^D + \Gamma_{dd}^D) + 2\lambda_s\lambda_b (\Gamma_{sd}^D - \Gamma_{dd}^D) - \lambda_b^2 \Gamma_{dd}^D,$$

$$M_{12}^D = \lambda_s^2 [M_{ss}^D - 2M_{sd}^D + M_{dd}^D] + 2\lambda_s\lambda_b [M_{bs}^D - M_{bd}^D - M_{sd}^D + M_{dd}^D] + \lambda_b^2 [M_{bb}^D - 2M_{bd}^D + M_{dd}^D].$$

C: Crazy Cancellations



1. Duality violations - break down of HQE

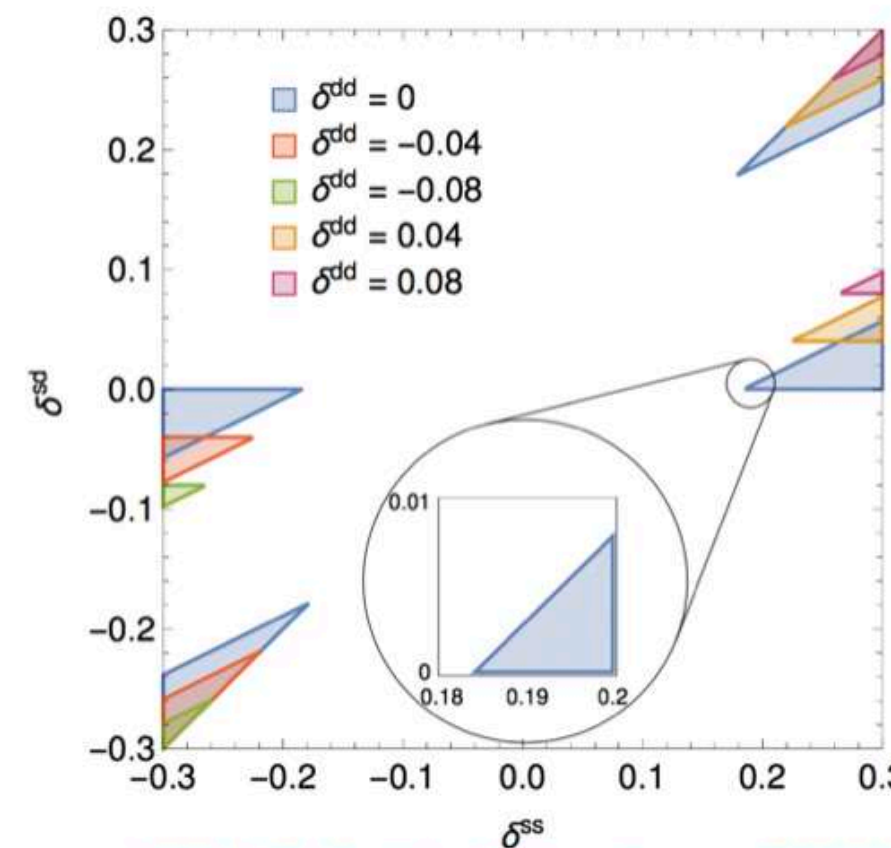
$$\Gamma_{12}^{ss} \rightarrow \Gamma_{12}^{ss}(1 + \delta^{ss}),$$

$$\Gamma_{12}^{sd} \rightarrow \Gamma_{12}^{sd}(1 + \delta^{sd}),$$

$$\Gamma_{12}^{dd} \rightarrow \Gamma_{12}^{dd}(1 + \delta^{dd}),$$

20% of duality violation is sufficient to explain experiment

Jubb, Kirk, AL, Tetlalmatzi-Xolocotzi 2016



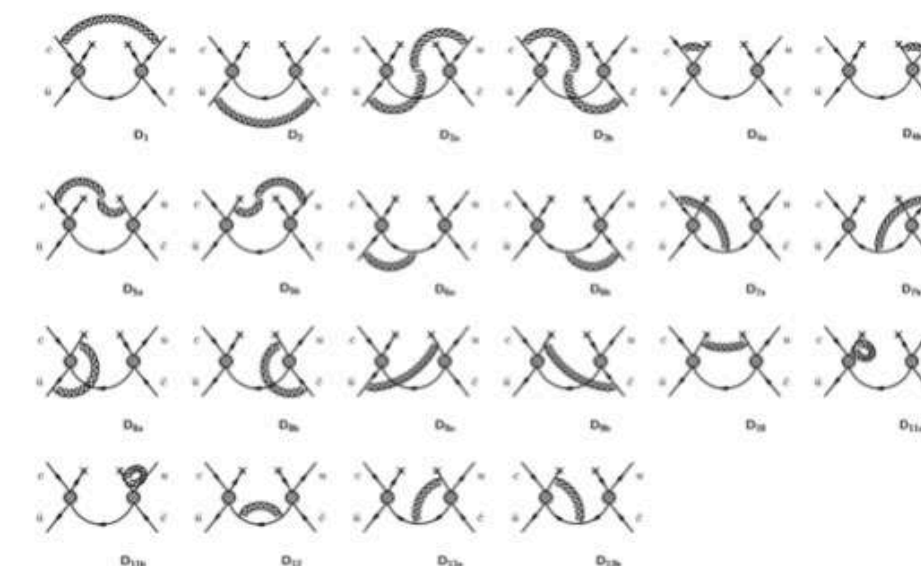
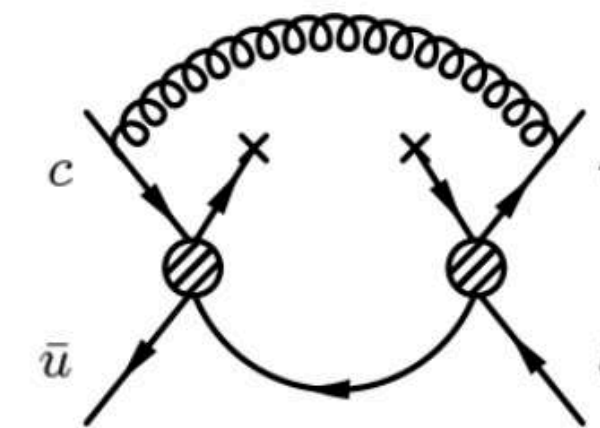
2. Higher dimensions

Georgi 9209291; Ohi, Ricciardi, Simmons 9301212; Bigi, Uraltsev 0005089

Idea: GIM cancellation is lifted by higher orders in the HQE - overcompensating the 1/mc suppression.

Partial calculation of D=9 yields an enhancement - but not to the experimental value

Bobrowski, AL, Rauh 2012



3. Renormalisation scale setting:

AL, Piscopo, Vlahos 2020

$$\mu_x^{ss} = \mu_x^{sd} = \mu_x^{dd}$$

Implicitly assumes a precision of 10^-5!

4. New Physics is present and we cannot prove it yet:-)

- 1) Vary $\mu^{ss,dd}$ and μ^{ds} independently between 1 GeV and $2 m_c$
 \Rightarrow uncertainty increases and exp. value is covered
- 2) Choose scales somehow phase space inspired as

$$\begin{aligned} \mu^{ss} &= m_c - 2\epsilon \\ \mu^{sd} &= m_c - \epsilon \\ \mu^{dd} &= m_c \end{aligned}$$

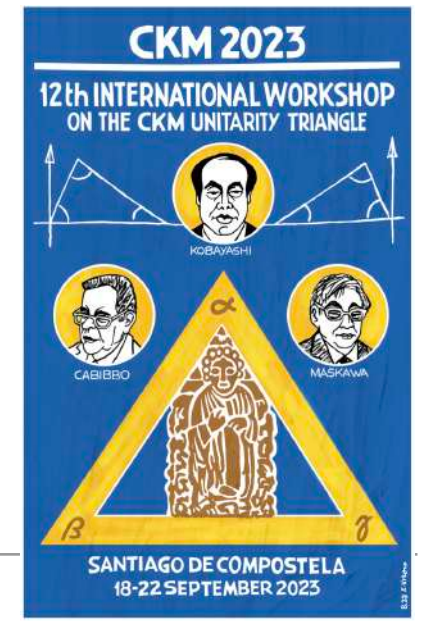
\Rightarrow exp. value is covered

Exclusive and inclusive approaches can cover the experimental regions



No precision determination possible

C: Crazy Cancellations



Exclusive approach

$$\Gamma_{12}^D = \sum_n \rho_n \langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle,$$

$$M_{12}^D = \sum_n \langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=2} | D^0 \rangle + P \sum_n \frac{\langle \bar{D}^0 | \mathcal{H}_{eff.}^{\Delta C=1} | n \rangle \langle n | \mathcal{H}_{eff.}^{\Delta C=1} | D^0 \rangle}{m_D^2 - E_n^2},$$

Cannot be calculated yet

Estimate phase space effects for y : [Falk et al. 0110317](#)

- assume pert. SU(3)F breaking $y \approx 1\%$
- neglect 3rd family
- neglect SU(3)F breaking in matrix elements - no QCD calculation

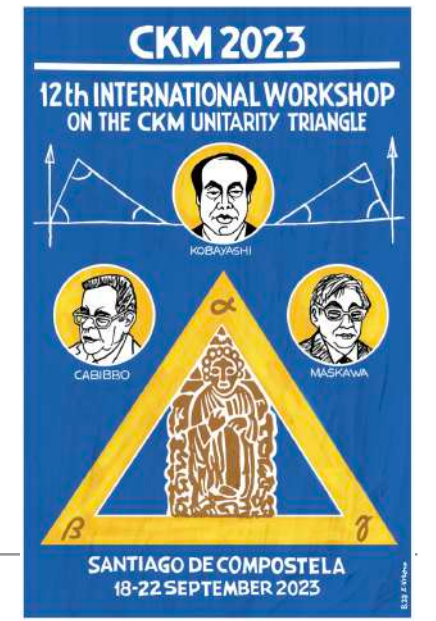
Mass difference from a dispersion relation [Falk et al. 0402204](#) $x \approx y$
 Exp. data [Cheng, Chiang 1005.1106](#) $x \propto \mathcal{O}(0.1\%)$ $y \propto \mathcal{O}(\text{few } 0.1\%)$

U-Spin sum rule [Gronau, Rosner 2012](#)

Factorisation-assisted topological amplitude approach

[Jiang et al. 1705.07335](#) $y \approx 0.2\%$

C: Crazy Cancellations



Direct lattice determination


**Still a very long way!
But not completely crazy
anymore!**



Multiple-channel generalization of Lellouch-Lüscher formula

#1




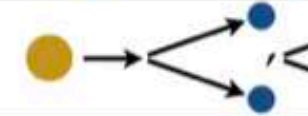
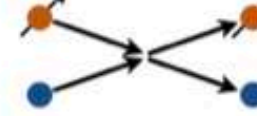

Maxwell T. Hansen (Washington U., Seattle), Stephen R. Sharpe (Washington U., Seattle) (Apr, 2012)

Published in: *Phys.Rev.D* 86 (2012) 016007 • e-Print: [1204.0826](https://arxiv.org/abs/1204.0826) [hep-lat]

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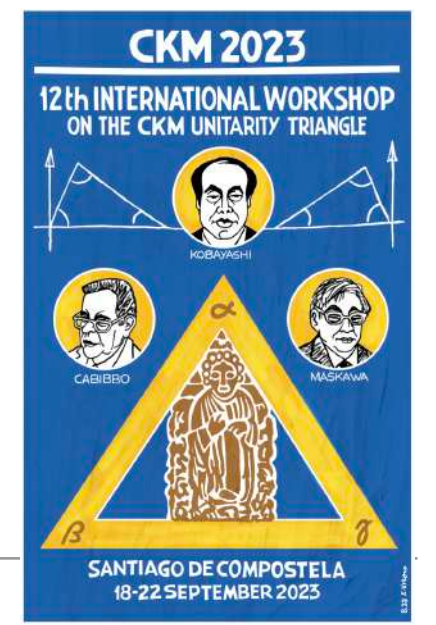
 reference search  314 citations

Status of multi-hadron matrix elements in LQCD...

physical system	Method to get it from LQCD
$\pi\pi \rightarrow \pi\pi$, $\sqrt{s} < 4M_\pi$ ($\mathbf{P} \neq 0$ in finite-volume frame)*	 Lüscher (1986, 1991) Rummukainen and Gottlieb (1995)*
$K \rightarrow \pi\pi$ (relies on $M_K < 4M_\pi$) ($\mathbf{P} \neq 0$ in finite-volume frame)*	 Lellouch and Lüscher (2001) Kim, Sachrajda and Sharpe (2005)*, Christ, Kim and Yamazaki (2005)*
$\pi\pi \rightarrow K\bar{K}$, $\sqrt{s} < 4M_\pi$ (not possible for physical masses)	 Bernard et al. (2011), Fu (2012), Briceño and Davoudi (2012)
$D \rightarrow \pi\pi, K\bar{K}$ (ignores four-particle states)	 MTH and Sharpe (2012)
$NN \rightarrow NN, N\pi \rightarrow N\pi$ (energies below three-particle production)	 Detmold and Savage (2004) Göckeler et al. (2012) Briceño (2014)
$\gamma^* \rightarrow \pi\pi, \pi\gamma^* \rightarrow \pi\pi,$ $N\gamma^* \rightarrow N\pi$ $B \rightarrow K^*(\rightarrow K\pi)\ell\ell$ (energies below three-particle production)	 Meyer (2011), Bernard et al. (2012), A. Agadjanov et al. (2014), Briceño, MTH and Walker-Loud (2014) Briceño and MTH (2015)

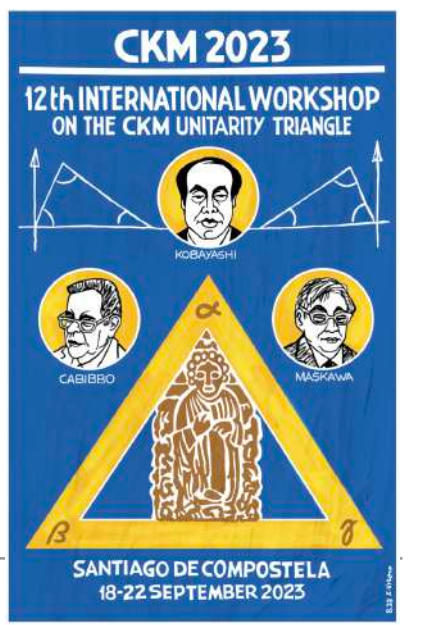
slide by Max Hansen

Theory for Charm



Theory for Charm Observable \neq Theory for Charm Observable

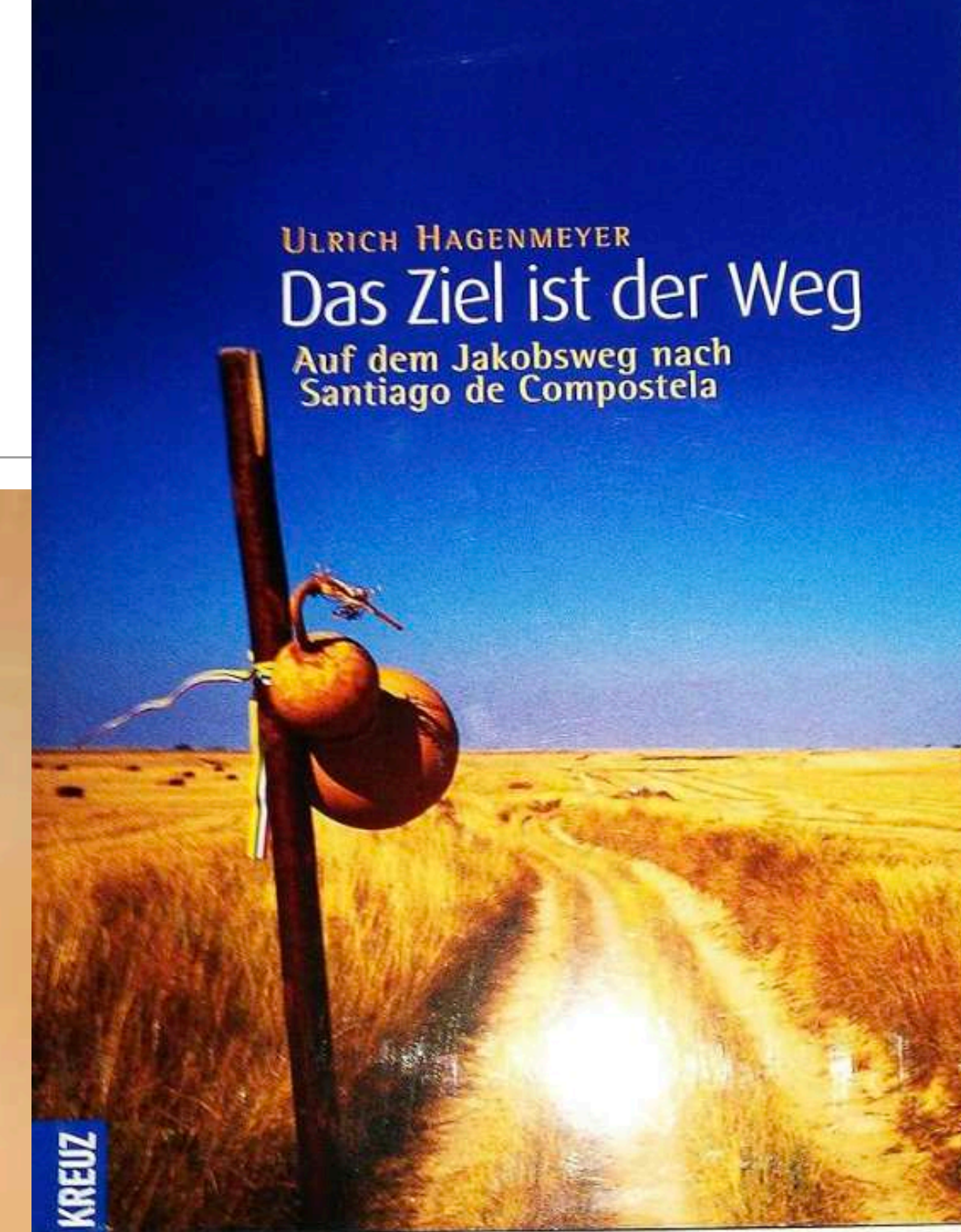
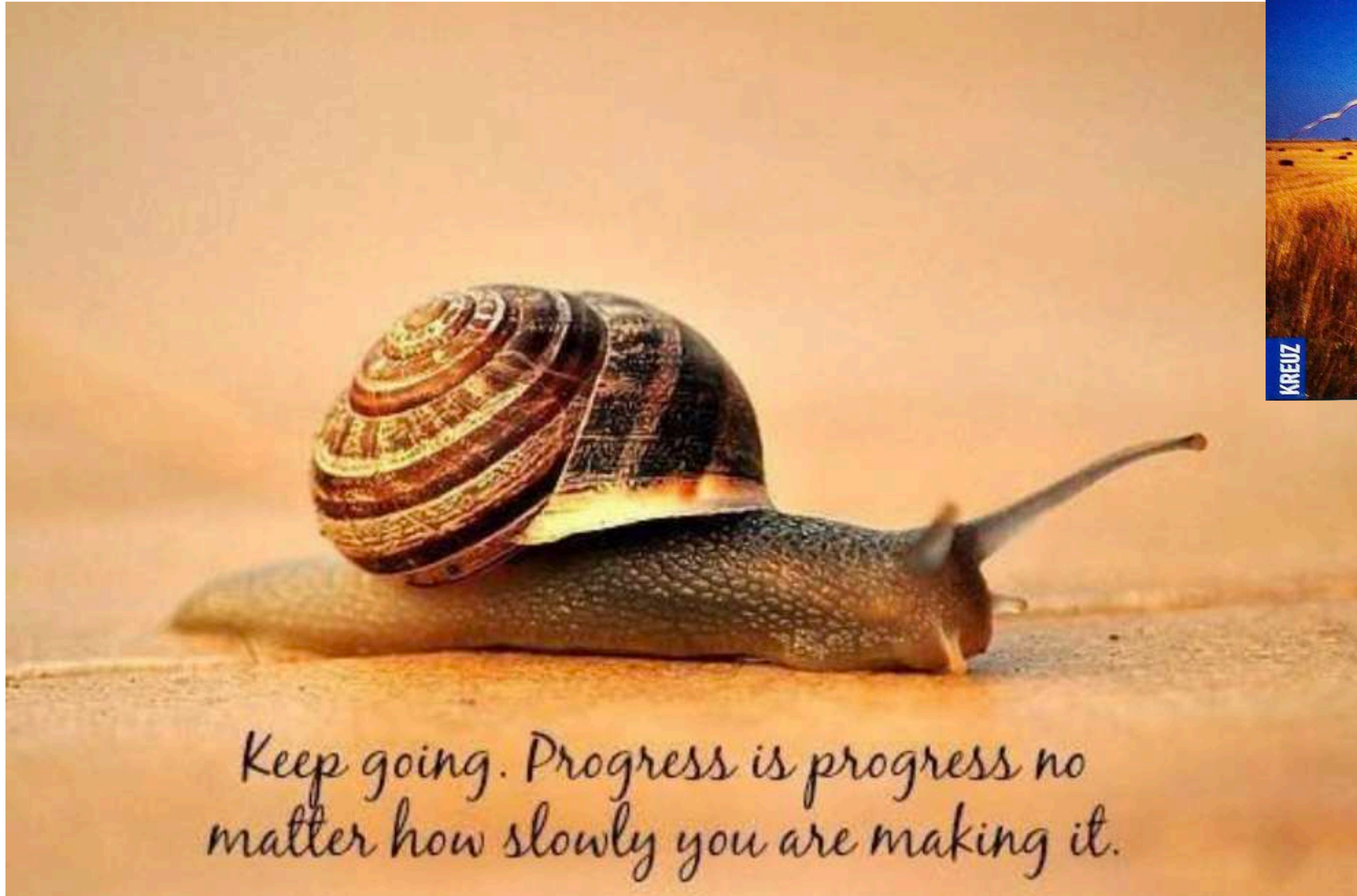
Theory for Charm



Theory for Charm Observable \neq Theory for Charm Observable

- No cancellations, e.g. Lifetime of D^0 can be predicted, **$1/m_c$ works**
- Cancellations, e.g. Lifetime of D^+ lies in the right ball park, **$1/m_c$ might work**
- Crazy cancellations, e.g. Mixing of D^0 HQE might overlap with exp., **$1/m_c$ not excluded**
- Hadronic decays: **we have to first understand the B-system!**

A BIG THANKS TO THE ORGANIZERS!!!!



Charm mixing - Theory


How Large Can the SM Contribution to CP Violation in $D^0 - \bar{D}^0$ Mixing Be?

#67

M. Bobrowski (Regensburg U.), A. Lenz (Dortmund U. and Regensburg U.), J. Riedl (Regensburg U.), J. Rohrwild (Regensburg U. and RWTH Aachen U.) (Feb, 2010)

Published in: *JHEP* 03 (2010) 009 • e-Print: [1002.4794](#) [hep-ph]

 pdf  DOI  cite  claim

 reference search  127 citations

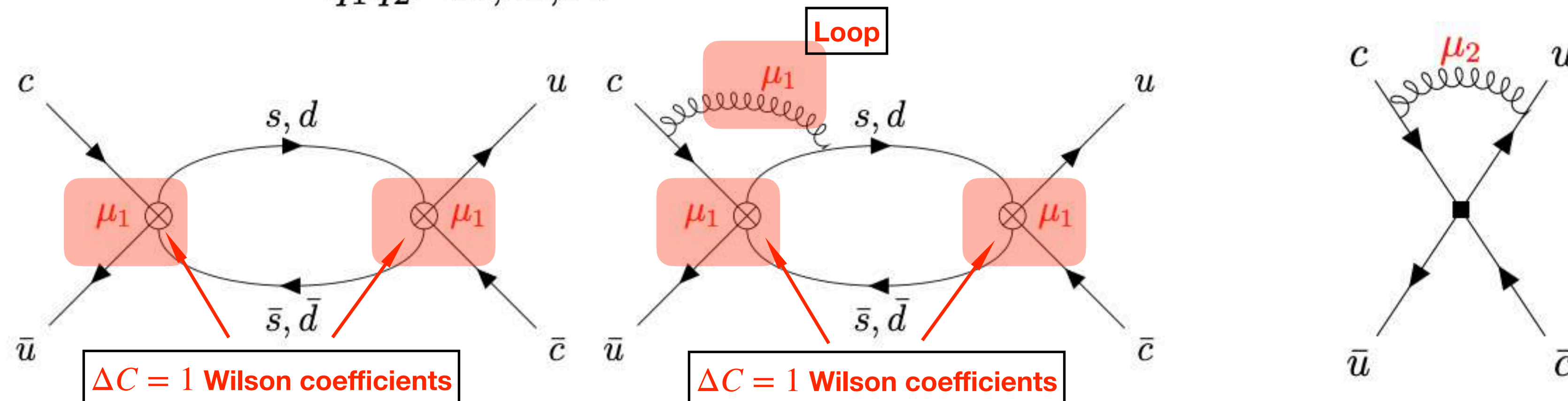
ABSTRACT: We investigate the maximum size of CP violating effects in D -mixing within the Standard Model (SM), using Heavy Quark Expansion (HQE) as theoretical working tool. For this purpose we determine the leading HQE contributions and also α_s corrections as well as subleading $1/m_c$ corrections to the absorptive part of the mixing amplitude of neutral D mesons. It turns out that these contributions to Γ_{12} do not vanish in the exact $SU(3)_F$ limit. Moreover, while the leading HQE terms give a result for Γ_{12} orders of magnitude lower than the current experimental value, we do find a sizeable phase. In the literature it was suggested that higher order terms in the HQE might be much less affected by the severe GIM cancellations of the leading terms; it is even not excluded that these higher order terms can reproduce the experimental value of y . If such an enhancement is realized in nature, the phase discovered in the leading HQE terms can have a sizeable effect. Therefore, we think that statements like: "*CP violating effects in D -mixing of the order of 10^{-3} to 10^{-2} are an unambiguous sign of new physics*"—given our limited knowledge of the SM prediction—are premature. Finally, we give an example of a new physics model that can enhance the leading HQE terms to Γ_{12} by one to two orders of magnitude.

Valid argument if D-mixing is described by higher dimension operators in the HQE

Charm mixing - Theory

Renormalisation scale setting?

$$\Gamma_{12} = \sum_{q_1 q_2 = ss, sd, dd} \Gamma_3^{q_1 q_2} (\mu_1^{q_1 q_2}, \mu_2^{q_1 q_2}) \langle Q \rangle (\mu_2^{q_1 q_2}) \frac{1}{m_c^3} + \dots$$



μ_1 and μ_2 cancel within the ss , sd and dd contributions independently

Is there any requirement to set exactly $\mu_1^{ss} = \mu_1^{sd} = \mu_1^{dd}$ (also during scale variation)?

ss and dd might be related via re-scattering, but sd is physically different from ss !

Charm mixing - Theory

Renormalisation scale setting?

$$\Delta\Gamma_D > 0.028\text{ps}^{-1} \Rightarrow \Omega \equiv \frac{2|\Gamma_{12}|^{\text{SM}}}{0.028\text{ps}^{-1}} \Rightarrow \Omega \approx 1 \text{ means HQE can describe Experiment}$$

Two scenarios:

1. Vary μ^{ss} , μ^{sd} , μ^{dd} independently around m_c between 1 GeV and $2 m_c$:

$$\Omega \in [4.6 \cdot 10^{-5}, 1.3]$$

2. Phase space inspired scale choice

$$\mu^{ss} = m_c - 2\epsilon$$

$$\mu^{sd} = m_c - \epsilon$$

$$\mu^{dd} = m_c$$

