HFLAV

Charm CPV and Oscillations

[Alan Schwartz, Marco Gersabeck, Jolanta Brodzicka]

Jolanta Brodzicka [INP Krakow]

CKM2023 Santiago de Compostela

HFLAV Report as of 2021

PHYSICAL REVIEW D **107,** 052008 (2023)

Editors' Suggestion | Featured in Physics

Averages of *b*-hadron, *c*-hadron, and τ-lepton properties as of 2021

About 50 pages on charm

Plan on 2023 Report

Inputs to global fit for D^0-$\bar{D}^0$ mixing

Decay mode	Observables
$D^0 \to K^+K^-,\ \pi^+\pi^-$	$y_{CP} - y_{CP}(K\pi),\ A_\Gamma$
$D^0 \to K_S K^+ K^-$	y_{CP}
$D^0 \to K_S \omega$	$y_{CP} - y_{CP}(K\pi)$
$D^0 \to K_S \pi^+\pi^-$	$x,\ y,\ \lvert q/p \rvert,\ \phi$
$D^0 \to K_S \pi^+\pi^-$	$x_{CP},\ y_{CP},\ \Delta x,\ \Delta y$
$D^0 \to K_S K^+ K^-$	$x,\ y$
$D^0 \to \pi^+\pi^-\pi^0$	$x,\ y$
$D^0 \to K^+\pi^-\pi^0$	$x'',\ y''$
$D^0 \to K^+\pi^-$	$x'^2,\ y',\ x'^{2\pm},\ y'^{\pm}$
$D^0 \to K^+\pi^-\pi^+\pi^-$	R_M
$D^0 \to K^+ l^- \bar{\nu}_l$	R_M
$D^0 \to K^+\pi^-,\ K^-\pi^+$	$R_D,\ R_D^{\pm},\ A_D$
$D^0 \to K^+K^-,\ \pi^+\pi^-$	$A_{CP},\ \Delta A_{CP}$
$\psi(3770) \to D\bar{D}$	$R_M,\ R_D,\ y,\ \sqrt{R_D}\cos\delta,\ A_{CP}(K\pi)$

Observables and underlying fit parameters (I)

- Allowing for CPV, there are 10 fitted parameters:

  x, y, |q/p|, ϕ, R_D, A_D, A_K, A_π, δ, $\delta_{K\pi\pi}$

$$2\,y_{CP} = \left(|q/p| + |p/q|\right) y\cos\phi - \left(|q/p| - |p/q|\right) x\sin\phi$$

$$2\,A_\Gamma = \left(|q/p| - |p/q|\right) y\cos\phi - \left(|q/p| + |p/q|\right) x\sin\phi$$

$$R_M = \frac{1}{2}(x^2 + y^2)$$

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix}_{K^+\pi^-\pi^0} = \begin{pmatrix} \cos\delta_{K\pi\pi} & \sin\delta_{K\pi\pi} \\ -\sin\delta_{K\pi\pi} & \cos\delta_{K\pi\pi} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix}_{K^+\pi^-} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x'^{\pm} = \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (x'\cos\phi \pm y'\sin\phi)$$

$$y'^{\pm} = \left(\frac{1 \pm A_M}{1 \mp A_M}\right)^{1/4} (y'\cos\phi \mp x'\sin\phi)$$

$$A_M = \frac{|q/p|^2 - |p/q|^2}{|q/p|^2 + |p/q|^2}$$

Observables and underlying fit parameters (II)

- Allowing for CPV, there are 10 fitted parameters:

  x, y, |q/p|, ϕ, R_D, A_D, A_K, A_π, δ, $\delta_{K\pi\pi}$

$$\frac{\Gamma(D^0\to K^+\pi^-)+\Gamma(\overline{D}^0\to K^-\pi^+)}{\Gamma(D^0\to K^-\pi^+)+\Gamma(\overline{D}^0\to K^+\pi^-)} = R_D \qquad \Gamma_{DCS}/\Gamma_{CF}$$

$$\frac{\Gamma(D^0\to K^+\pi^-)-\Gamma(\overline{D}^0\to K^-\pi^+)}{\Gamma(D^0\to K^+\pi^-)+\Gamma(\overline{D}^0\to K^-\pi^+)} = A_D$$

$$\frac{\Gamma(D^0\to K^+K^-)-\Gamma(\overline{D}^0\to K^+K^-)}{\Gamma(D^0\to K^+K^-)+\Gamma(\overline{D}^0\to K^+K^-)} = A_K + \frac{\langle t\rangle}{\tau_D}\mathcal{A}_{CP}^{\text{indirect}}$$

$$\frac{\Gamma(D^0\to \pi^+\pi^-)-\Gamma(\overline{D}^0\to \pi^+\pi^-)}{\Gamma(D^0\to \pi^+\pi^-)+\Gamma(\overline{D}^0\to \pi^+\pi^-)} = A_\pi + \frac{\langle t\rangle}{\tau_D}\mathcal{A}_{CP}^{\text{indirect}}$$

direct A_{CP}

$$2\mathcal{A}_{CP}^{\text{indirect}} = \left(|q/p|+|p/q|\right)x\sin\phi - \left(|q/p|-|p/q|\right)y\cos\phi \qquad -2A_\Gamma$$

Recent experimental additions

- A_Γ with $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$, LHCb Run2, (π-tag)

  PRD 104, 072010 (2021)

- x_{CP}, y_{CP}, Δx, Δy in $D^0 \to K_S\pi^+\pi^-$, LHCb Run2, (π-tag) + (μ-tag)

  PRL 127, 111801 (2021)   PRD108, 052005 (2023)

- y_{CP}-$y_{CP}(K\pi)$ with $D^0 \to K^+K^-$, $D^0 \to \pi^+\pi^-$, Run2, [π-tag)

  PRD 105, 092013 (2022)

- $A_{CP}(D^0 \to K^+K^-)$, $A_{CP}(D^0 \to \pi^+\pi^-)$, LHCb Run2, ($\pi$-tag)

  PRL 131, 091802 (2023)

- $A_{CP}[K\pi]$ [$\to \delta_{K\pi}$], BESIII

  Eur. Phys. J. C 82, 1009 (2022)

Included in global fit for CKM2023

[Thanks to Alan Schwartz]

A_Γ and y_{CP} averages [used in global mixing fit]

$$y_{CP} - y_{CP}^{K\pi} = \frac{\tau_{K^-\pi^+}}{\tau_{h^+h^-}} - 1$$

$$A_\Gamma = \frac{\tau_{\bar{D}^0\to h^+h^-} - \tau_{D^0\to h^+h^-}}{\tau_{\bar{D}^0\to h^+h^-} + \tau_{D^0\to h^+h^-}}$$

Index	Observable	Value	Source
1	$y_{CP} - y_{CP}(K\pi)$	$(0.697 \pm 0.028)\%$	World average of $D^0 \to K^+K^- / \pi^+\pi^- / K^+K^-K^0$ Our calculation of the $y_{CP}(K\pi)$ correction is from arXiv:2207.11867, Eq. (29). This correction was first pointed out by Pajero, Morello in JHEP 03 (2022) 162.
2	A_Γ	$(0.0089 \pm 0.0113)\%$	World average (COMBOS combination) of $D^0 \to K^+K^- / \pi^+\pi^-$ results

- $y_{CP}(K\pi)$: correction in decay-time distribution for Right-Sign and Wrong-Sign $D^0 \to K\pi$ decays

$$e^{-\Gamma t} \quad \to 2e^{-(1+y_{CP}^{K\pi})\Gamma t}$$

Pajero, Morello JHEP 03, 162 (2022)

$$y_{CP}^{K\pi} = \frac{y_{CP}^{RS} + y_{CP}^{WS}}{2} = -\frac{\sqrt{R_D}}{2}\left(\left|\frac{q}{p}\right|\sqrt{1-A_D} + \left|\frac{p}{q}\right|\sqrt{1+A_D}\right)(x\cos\phi\sin\delta + y\cos\phi\cos\delta)$$
$$+\frac{\sqrt{R_D}}{2}\left(\left|\frac{q}{p}\right|\sqrt{1-A_D} - \left|\frac{p}{q}\right|\sqrt{1+A_D}\right)(x\sin\phi\cos\delta - y\sin\phi\sin\delta)$$

Schwartz arXiv:2207.11867

Global mixing fit: versions with CPV

All CPV allowed

- Direct CPV at tree-level [A_D], indirect CPV [$|q/p|$, φ]
- Fit parameters: [x, y, $|q/p|$, φ]

Super-weak approximation

Kagan, Sokoloff PRD 80, 076008 (2009)

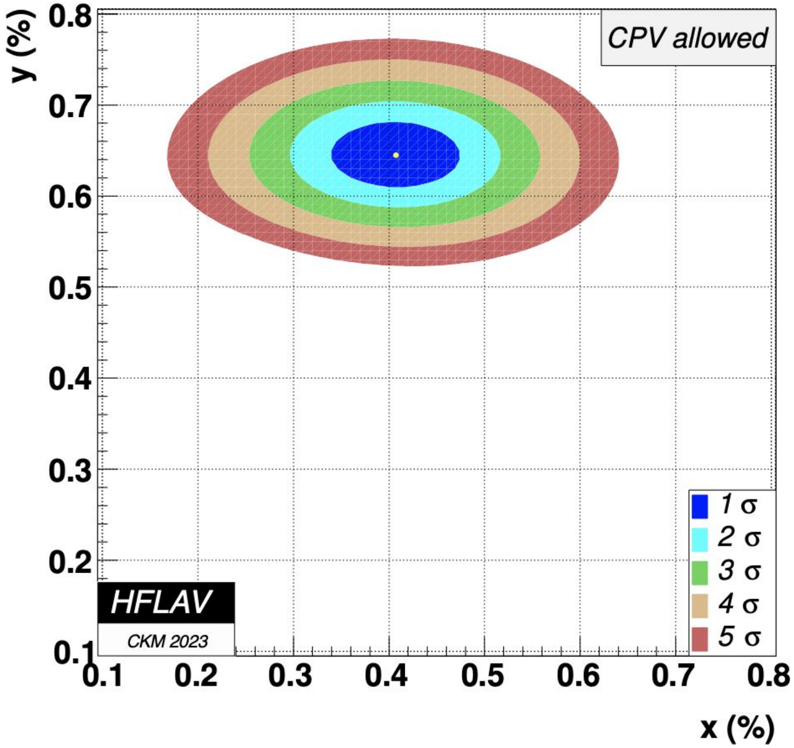
- No direct CPV at tree-level [$A_D=0$], no subleading phases in SCS [$\mathrm{Arg}(\Gamma_{12})=0$]
- One parameter for indirect CPV: $\varphi_{12}=\mathrm{Arg}[M_{12}/\Gamma_{12}]$
- Dispersive and absorptive mixing parameters: $x_{12}=2|M_{12}|/\Gamma$, $y_{12}=|\Gamma_{12}|/\Gamma$
- Fit parameters: [x, y, $|q/p|$, φ]$\longrightarrow$ [x_{12}, y_{12}, φ_{12}], [x, y, $|q/p|$], [x, y, φ]
- Additional constraint: $\tan\varphi=(1-|q/p|^2)/(1+|q/p|^2)(x/y)$

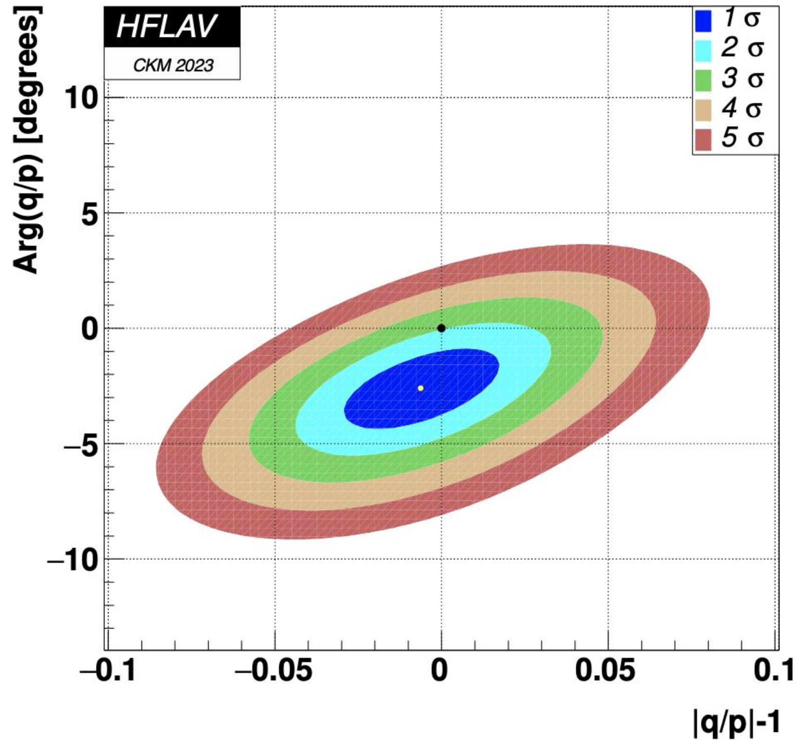
More general super-weak approximation

Kagan, Silvestrini PRD 103, 053008 (2021)

- No direct CPV at tree-level [$A_D=0$], subleading phases allowed in SCS [$\mathrm{Arg}(\Gamma_{12})\neq 0$]
- Two parameters for indirect CPV: $\varphi^M_2=\mathrm{Arg}[M_{12}]$, $\varphi^\Gamma_2=\mathrm{Arg}[\Gamma_{12}]$
- Fit parameters: [x, y, $|q/p|$, φ] $\longrightarrow$[x_{12}, y_{12}, φ^M_2, φ^Γ_2]

Mixing fit with all CPV allowed





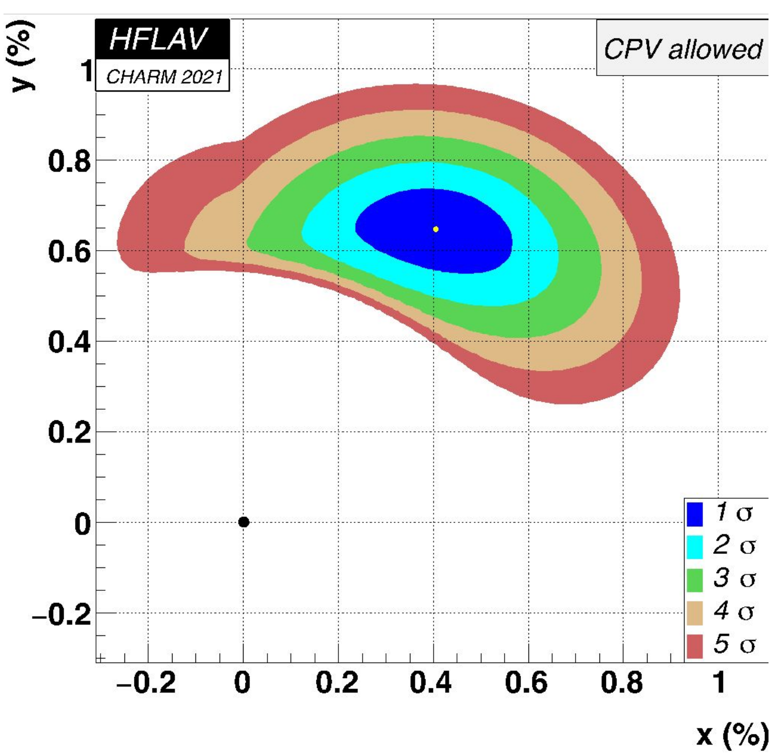
$$x = (0.407 \pm 0.044)\%$$
$$y = \left(0.645^{+0.024}_{-0.023}\right)\%$$

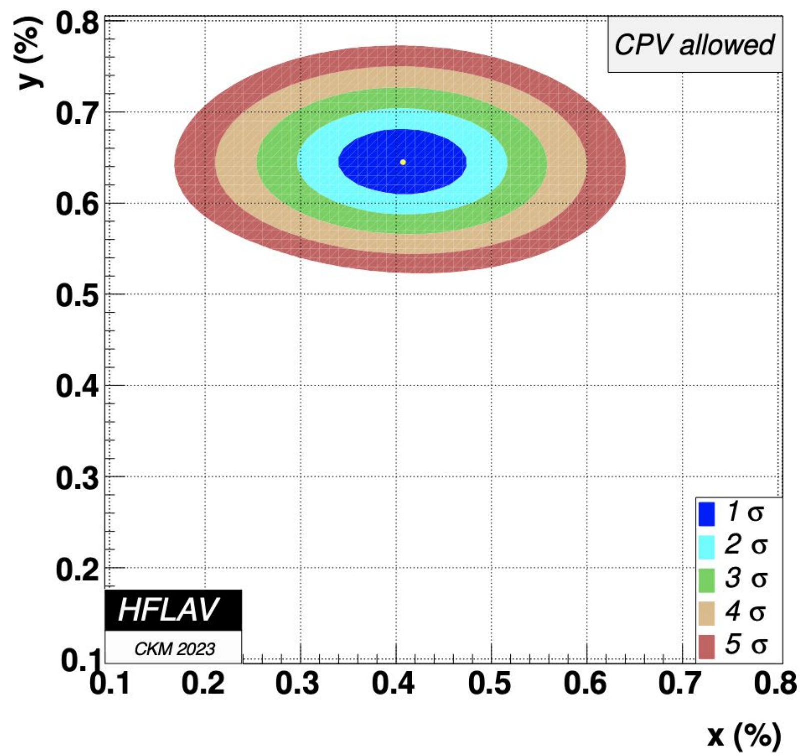
$$\phi = \left(-2.6^{+1.1}_{-1.2}\right)^{\circ}$$
$$|q/p| = 0.994^{+0.016}_{-0.015}$$

- Mixing well established, no indirect CPV, but ϕ non-zero at 2.2σ

Mixing fit all CPV allowed

- 2021 2023

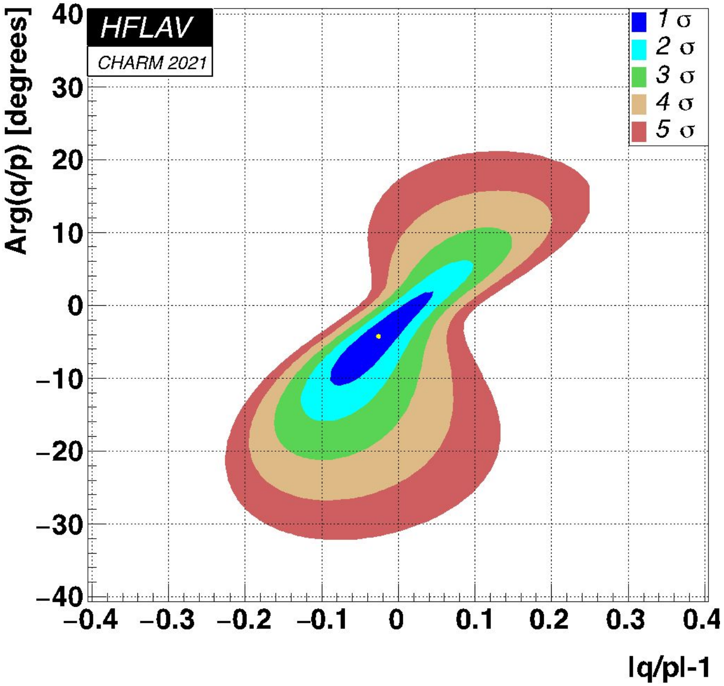


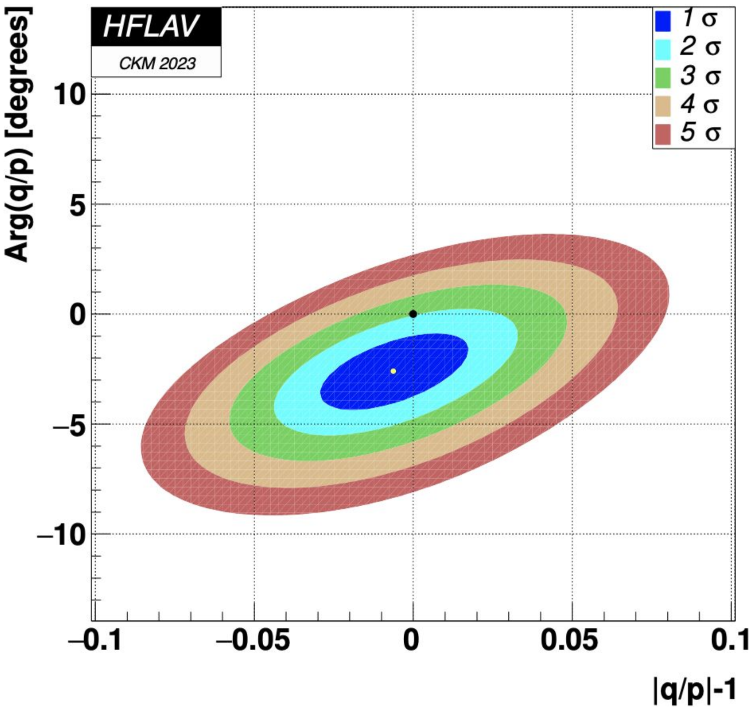


- Mostly impact of LHCb Run2 results from $D^0 \to K_S \pi^+ \pi^-$ and y_{CP} from LHCb Run2

Mixing fit all CPV allowed

- 2021 2023

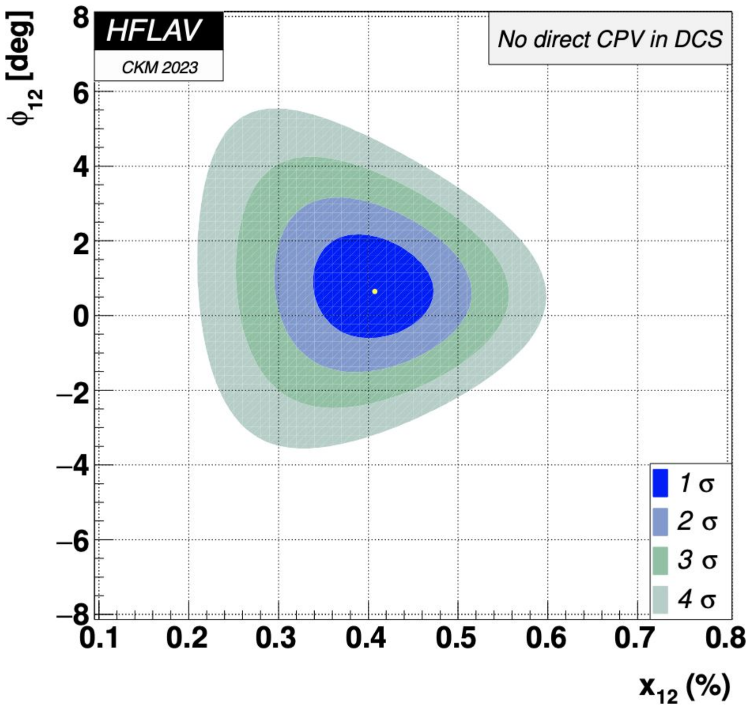


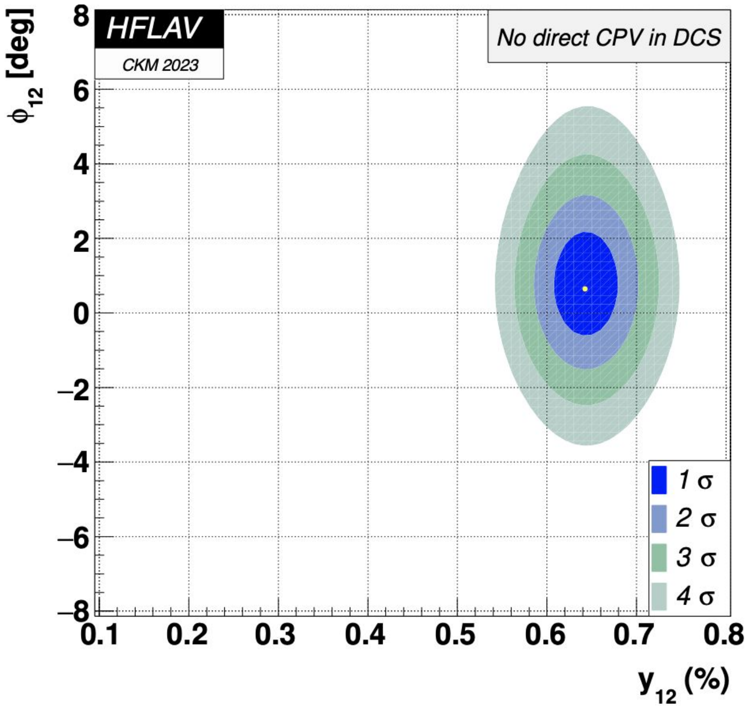


- Mostly impact of LHCb Run2 results from $D^0 \longrightarrow K_S \pi^+ \pi^-$
- A_Γ from LHCb Run2 included in 2021 fit

Mixing fit with super-weak approximation

- Dispersive and absorptive mixing, one parameter for indirect CPV





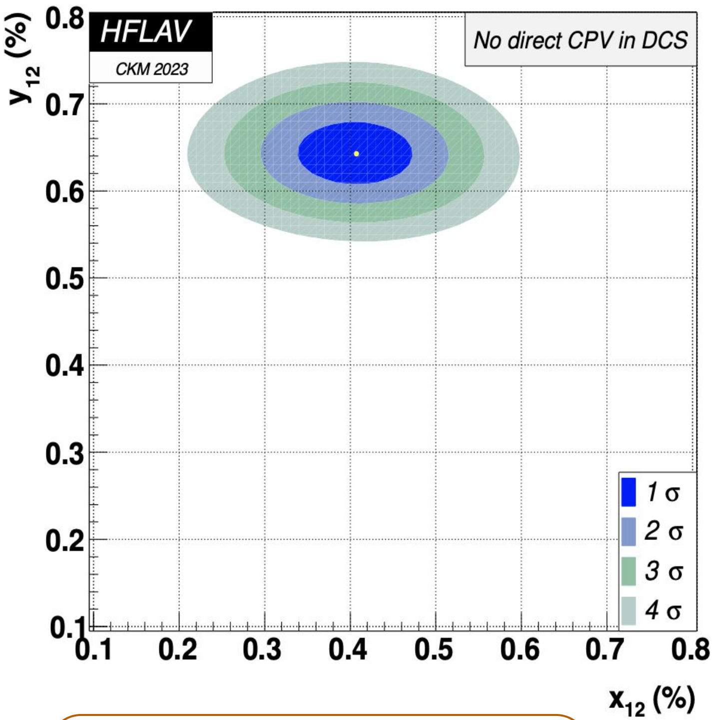
$$x_{12} = (0.407 \pm 0.044)\%$$

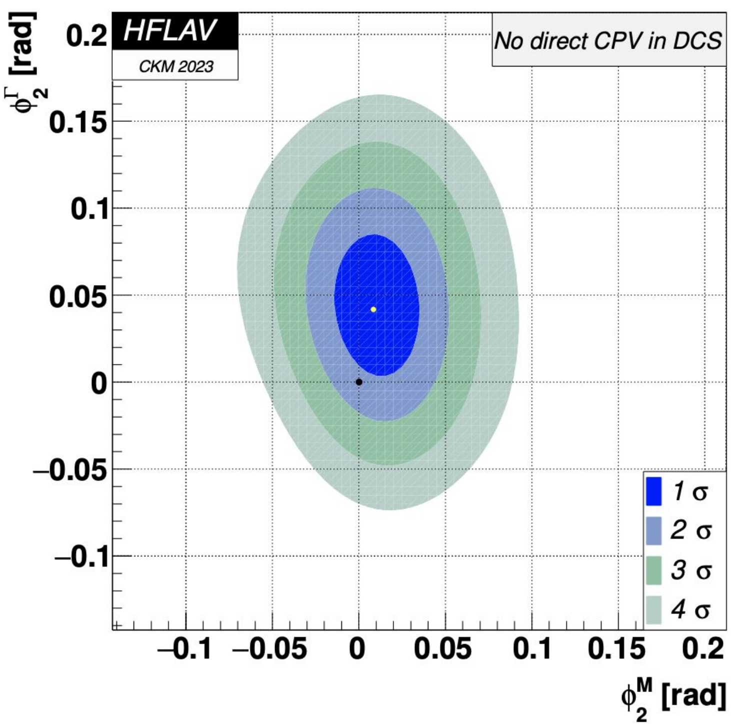
$$y_{12} = \left(0.641^{+0.024}_{-0.023}\right)\%$$

$$\phi_{12} = \left(+0.65^{+0.92}_{-0.90}\right)^{\circ}$$

Fit with more general super-weak limit

- Fit for dispersive and absorptive mixing and indirect CPV





$$x_{12} = (0.407 \pm 0.044)\%$$
$$y_{12} = 0.641^{+0.024}_{-0.023}\%$$

$$\phi_2^M = (+0.48 \pm 0.92)^\circ$$
$$\phi_2^\Gamma = \left(+2.40^{+1.55}_{-1.54}\right)^\circ$$

- Results on mixing parameters similar to those in super-weak limit

Results of global mixing fit

Parameter	No CPV (Fit #1)	No subleading ampl. in indirect CPV (Fit #2)	No subleading ampl. for CF/DCS decays (Fit #3)	all CPV allowed (Fit #4)	95% CL Interval (from Fit #2, Fit #3, Fit #4)
x (%)	$0.434^{+0.126}_{-0.139}$	0.407 ± 0.044	–	0.407 ± 0.044	[0.320, 0.493]
y (%)	0.646 ± 0.024	$0.643^{+0.024}_{-0.023}$	–	$0.645^{+0.024}_{-0.023}$	[0.600, 0.692]
$\delta_{K\pi}$ (°)	$11.5^{+3.6}_{-3.7}$	$11.3^{+3.6}_{-3.8}$	$10.9^{+3.6}_{-3.8}$	$11.4^{+3.5}_{-3.8}$	[3.7, 18.2]
R_D (%)	0.344 ± 0.002	0.344 ± 0.002	0.344 ± 0.002	0.344 ± 0.002	[0.340, 0.347]
A_D (%)	–	–	–	-0.77 ± 0.35	[−1.46, −0.08]
$\|q/p\|$	–	1.005 ± 0.007	–	$0.994^{+0.016}_{-0.015}$	[0.96, 1.02]
ϕ (°)	–	-0.19 ± 0.26	–	$-2.6^{+1.1}_{-1.2}$	[−4.88, −0.37]
$\delta_{K\pi\pi}$ (°)	$23.1^{+22.8}_{-23.2}$	$24.5^{+21.4}_{-22.5}$	$24.4^{+21.5}_{-22.5}$	$24.6^{+21.4}_{-22.5}$	[−20.1, 65.7]
A_π(%)	–	0.202 ± 0.059	0.200 ± 0.059	0.218 ± 0.060	[0.10, 0.34]
A_K(%)	–	0.045 ± 0.053	0.043 ± 0.053	0.062 ± 0.054	[−0.04, 0.17]
x_{12} (%)	–	0.407 ± 0.044	0.407 ± 0.044	–	[0.321, 0.494]
y_{12} (%)	–	$0.643^{+0.024}_{-0.023}$	$0.641^{+0.024}_{-0.023}$	–	[0.595, 0.687]
ϕ_{12}(°)	–	$0.65^{+0.92}_{-0.90}$	–	–	[−1.13, 2.49]
ϕ_2^M(°)	–	–	0.48 ± 0.92	–	[−1.35, 2.32]
ϕ_2^Γ(°)	–	–	$2.40^{+1.55}_{-1.54}$	–	[−0.61, 5.45]
χ^2/d.o.f.	$100.6/(63-5) = 1.74$	$68.9/(63-8) = 1.25$	$66.5/(63-9) = 1.23$	$65.4/(63-10) = 1.23$	

Averages of $A_{CP}(D \to \pi\pi)$

- $D^0 \to \pi^+\pi^-$

Year	Experiment	CP Asymmetry in the decay mode D0 to π+π-	[Γ(D0)-Γ(D0bar)]/[Γ(D0)+Γ(D0bar)]
2022	LHCb	R. Aaij et al. (LHCb Collab.), Phys. Rev. Lett. 131, 091802 (2023).	+0.00232 ± 0.00061(stat+syst)
2012	CDF	T. Aaltonen et al. (CDF Collab.), Phys. Rev. D 85, 012009 (2012).	+0.0022 ± 0.0024 ± 0.0011
2008	BABAR	B. Aubert et al. (BABAR Collab.), Phys. Rev. Lett. 100, 061803 (2008).	-0.0024 ± 0.0052 ± 0.0022
2008	BELLE	M. Staric et al. (BELLE Collab.), Phys. Lett.B 670, 190 (2008).	+0.0043 ± 0.0052 ± 0.0012
2002	CLEO	S.E. Csorna et al. (CLEO Collab.), Phys. Rev. D 65, 092001 (2002).	+0.019 ± 0.032 ± 0.008
2000	FOCUS	J.M. Link et al. (FOCUS Collab.), Phys. Lett. B 491, 232 (2000).	+0.048 ± 0.039 ± 0.025
1998	E791	E.M. Aitala et al. (E791 Collab.), Phys. Lett. B 421, 405 (1998).	-0.049 ± 0.078 ± 0.030
		HFLAV average	+0.00230 ± 0.00059

- $D^0 \to \pi^0\pi^0$

Year	Experiment	CP Asymmetry in the decay mode D0 to π0π0	[Γ(D0)-Γ(D0bar)]/[Γ(D0)+Γ(D0bar)]
2014	BELLE	N.K. Nisar et al. (BELLE Collab.), Phys. Rev. Lett. 112, 211601 (2014).	-0.0003 ± 0.0064 ± 0.0010
2001	CLEO	G. Bonvicini et al. (CLEO Collab.), Phys. Rev. D 63, 071101 (2001).	+0.001 ± 0.048
		HFLAV average	-0.0003 ± 0.0064

- $D^+ \to \pi^+\pi^0$

Year	Experiment	CP Asymmetry in the decay mode D+ to π+π0	[Γ(D+)-Γ(D-)]/[Γ(D+)+Γ(D-)]
2021	LHCb	R. Aaij et al. (LHCb Collab.), JHEP 06, 019 (2021).	-0.013 ± 0.009 ± 0.006
2018	BELLE	V. Babu et al. (BELLE Collab.), Phys. Rev. D 97, 011101 (2018).	+0.0231 ± 0.0124 ± 0.0023
2010	CLEO	H. Mendez et al. (CLEO Collab.), Phys. Rev. D 81, 052013 (2010).	+0.029 ± 0.029 ± 0.003
		HFLAV average	+0.004 ± 0.008

Averages of $A_{CP}(D \to \pi\pi)$

$$A_{CP}(D^0 \to \pi^+\pi^-) = (2.30 \pm 0.59) \times 10^{-3}$$

$$A_{CP}(D^0 \to \pi^0\pi^0) = (0.3 \pm 6.4) \times 10^{-3}$$

$$A_{CP}(D^+ \to \pi^+\pi^0) = (4.0 \pm 8.0) \times 10^{-3}$$

- To be used to test isospin sum rule

Isospin sum rule for D→$\pi\pi$

- Decompose D→$\pi\pi$ decay amplitudes into isospin amplitudes:

  $\mathcal{A}_1$, $\mathcal{A}_3$ isospin amplitudes of ΔI=1/2 and ΔI=3/2 transitions

$$A_{\pi^+\pi^-} = \sqrt{2}\mathcal{A}_3 + \sqrt{2}\mathcal{A}_1$$
$$A_{\pi^0\pi^0} = 2\mathcal{A}_3 - \mathcal{A}_1,$$
$$A_{\pi^+\pi^0} = 3\mathcal{A}_3,$$

⇨

$$\frac{1}{\sqrt{2}}A_{\pi^+\pi^-} + A_{\pi^0\pi^0} - A_{\pi^+\pi^0} = 0$$

- Differences of D and anti-D decay rates

$$|A_{\pi^+\pi^-}|^2 - |\overline{A}_{\pi^+\pi^-}|^2 + |A_{\pi^0\pi^0}|^2 - |\overline{A}_{\pi^0\pi^0}|^2 - \frac{2}{3}(|A_{\pi^+\pi^0}|^2 - |\overline{A}_{\pi^-\pi^0}|^2) \;=\; 3(|\mathcal{A}_1|^2 - |\overline{\mathcal{A}}_1|^2)$$

- Sums of D and anti-D decay rates

$$|A_{\pi^+\pi^-}|^2 + |\overline{A}_{\pi^+\pi^-}|^2 + |A_{\pi^0\pi^0}|^2 + |\overline{A}_{\pi^0\pi^0}|^2 - \frac{2}{3}(|A_{\pi^+\pi^0}|^2 + |\overline{A}_{\pi^-\pi^0}|^2) \;=\; 3(|\mathcal{A}_1|^2 + |\overline{\mathcal{A}}_1|^2)$$

Isospin sum rule for D→$\pi\pi$

- The ratio R of “differences” and “sums”

$$R \equiv \frac{|A_{\pi^+\pi^-}|^2 - |\overline{A}_{\pi^+\pi^-}|^2 + |A_{\pi^0\pi^0}|^2 - |\overline{A}_{\pi^0\pi^0}|^2 - \frac{2}{3}(|A_{\pi^+\pi^0}|^2 - |\overline{A}_{\pi^-\pi^0}|^2)}{|A_{\pi^+\pi^-}|^2 + |\overline{A}_{\pi^+\pi^-}|^2 + |A_{\pi^0\pi^0}|^2 + |\overline{A}_{\pi^0\pi^0}|^2 - \frac{2}{3}(|A_{\pi^+\pi^0}|^2 + |\overline{A}_{\pi^-\pi^0}|^2)}$$

- R corresponds to CP asymmetry in ΔI=1/2 amplitude
- if R≠0 ⇨ CPV in ΔI=1/2 amplitude (from SM penguin)
- if R=0 and individual A_{CP}'s are non-zero ⇨ CPV in ΔI=3/2 ⇨ New Physics

- Going from amplitudes to BFs and A_{CP}'s

$$R = \frac{A_{CP}(D^0 \to \pi^+\pi^-)}{1 + \frac{\tau_{D^0}}{\mathcal{B}_{+-}}\left(\frac{\mathcal{B}_{00}}{\tau_{D^0}} - \frac{2}{3}\frac{\mathcal{B}_{+0}}{\tau_{D^+}}\right)} + \frac{A_{CP}(D^0 \to \pi^0\pi^0)}{1 + \frac{\tau_{D^0}}{\mathcal{B}_{00}}\left(\frac{\mathcal{B}_{+-}}{\tau_{D^0}} - \frac{2}{3}\frac{\mathcal{B}_{+0}}{\tau_{D^+}}\right)} + \frac{A_{CP}(D^+ \to \pi^+\pi^0)}{1 - \frac{3}{2}\frac{\tau_{D^+}}{\mathcal{B}_{+0}}\left(\frac{\mathcal{B}_{00}}{\tau_{D^0}} + \frac{\mathcal{B}_{+-}}{\tau_{D^0}}\right)}$$

- Use HFLAV averages for A_{CP}'s and PDG for and BFs

$$R = (+0.9 \pm 3.1) \times 10^{-3}$$

- Consistent with zero, error dominated by A_{CP} uncertainties

More sum rules to be tested?

- Building sum rules for D⟶KK requires considering full SU(3)$_F$ multiplet
- U-spin sum rules to probe New Physics with ΔU=1 [Talk by Ulrich Nierste on Monday]
- Sum rules for charmed baryons [Talk by Fu-Sheng You on Tuesday]

$$A_{CP}(\Lambda_c^+ \to pK^-K^+) + A_{CP}(\Xi_c^+ \to \Sigma^+\pi^-\pi^+) = 0.$$
$$A_{CP}(\Lambda_c^+ \to p\pi^-\pi^+) + A_{CP}(\Xi_c^+ \to \Sigma^+K^-K^+) = 0.$$

- Hence ΔA$_{CP}$ *equivalent* for baryons would be:

$$\Delta\Delta A_{CP} \equiv \left(A_{CP}(\Lambda_c^+ \to pK^+K^-) - A_{CP}(\Lambda_c^+ \to p\pi^+\pi^-)\right) - \left(A_{CP}(\Xi_c^+ \to \Sigma^+K^+K^-) - A_{CP}(\Xi_c^+ \to \Sigma^+\pi^+\pi^-)\right)$$

- $\Delta\Delta$A$_{CP}$=0 in U-spin limit

Summary

- Mixing well established, no significant indirect CPV

- More HFLAV averages under:

  https://hflav.web.cern.ch/content/charm-cpv-and-oscillations

- 2023 HFLAV Report including results made public till end of CKM2023

HFLAV

Backup slides

Standard parameterization of mixing

$$|D_1\rangle = p|D^0\rangle + q|\overline{D}^0\rangle \qquad |D_1(t)\rangle = |D_1\rangle\, e^{-i\left(m_1 - \frac{i}{2}\Gamma_1\right)}$$
$$|D_2\rangle = p|D^0\rangle - q|\overline{D}^0\rangle \qquad |D_2(t)\rangle = |D_2\rangle\, e^{-i\left(m_2 - \frac{i}{2}\Gamma_2\right)}$$

$$x \equiv \frac{M_2 - M_1}{\Gamma} = \frac{\Delta M}{\Gamma} \qquad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma} = \frac{\Delta\Gamma}{2\Gamma}$$

$$\frac{q}{p} = \left|\frac{q}{p}\right| e^{i\phi} \qquad \frac{\mathcal{A}_{\overline{D}^0 \to f}}{\mathcal{A}_{D^0 \to f}} = \pm\sqrt{R_D}\, e^{i\delta}$$

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{matrix} x\cos\delta + y\sin\delta \\ y\cos\delta - x\sin\delta \end{matrix}$$

Standard parameterization of mixing and CPV

$$\frac{N(D^0 \to f)}{dt} \propto e^{-\overline{\Gamma}\, t} \left\{ R_D + \left|\frac{q}{p}\right| \sqrt{R_D}(y' \cos\phi - x' \sin\phi)(\overline{\Gamma} t) + \left|\frac{q}{p}\right|^2 \frac{(x'^2 + y'^2)}{4} (\overline{\Gamma}\, t)^2 \right\}$$

$$\frac{N(\overline{D}^0 \to \bar{f})}{dt} \propto e^{-\overline{\Gamma}\, t} \left\{ \overline{R}_D + \left|\frac{p}{q}\right| \sqrt{\overline{R}_D} y' \cos\phi + x' \sin\phi)(\overline{\Gamma} t) + \left|\frac{p}{q}\right|^2 \frac{(x'^2 + y'^2)}{4} (\overline{\Gamma}\, t)^2 \right\}$$

$A_D \equiv \frac{(R_D - \overline{R}_D)}{(R_D + \overline{R}_D)}$ CPV in the decay amplitude (direct CPV)

$\left|\frac{q}{p}\right|$ CPV in mixing

ϕ CPV in mixed/direct interference

Super-weak approximation

- No direct CPV at tree-level [A_D=0], no subleading phases in SCS [Arg(Γ_{12})=0]
- [x, y, |q/p|, φ]⟶[x_{12}, y_{12}, φ_{12}]
- Additional constraint:

$$\tan\phi = \frac{(1-|q/p|^2)}{(1+|q/p|^2)}\frac{x}{y}$$

- Relations between two parameter sets:

$$x = \left[\frac{x_{12}^2 - y_{12}^2 + \sqrt{(x_{12}^2+y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2\phi_{12}}}{2}\right]^{1/2}$$

$$y = \left[\frac{y_{12}^2 - x_{12}^2 + \sqrt{(x_{12}^2+y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2\phi_{12}}}{2}\right]^{1/2}$$

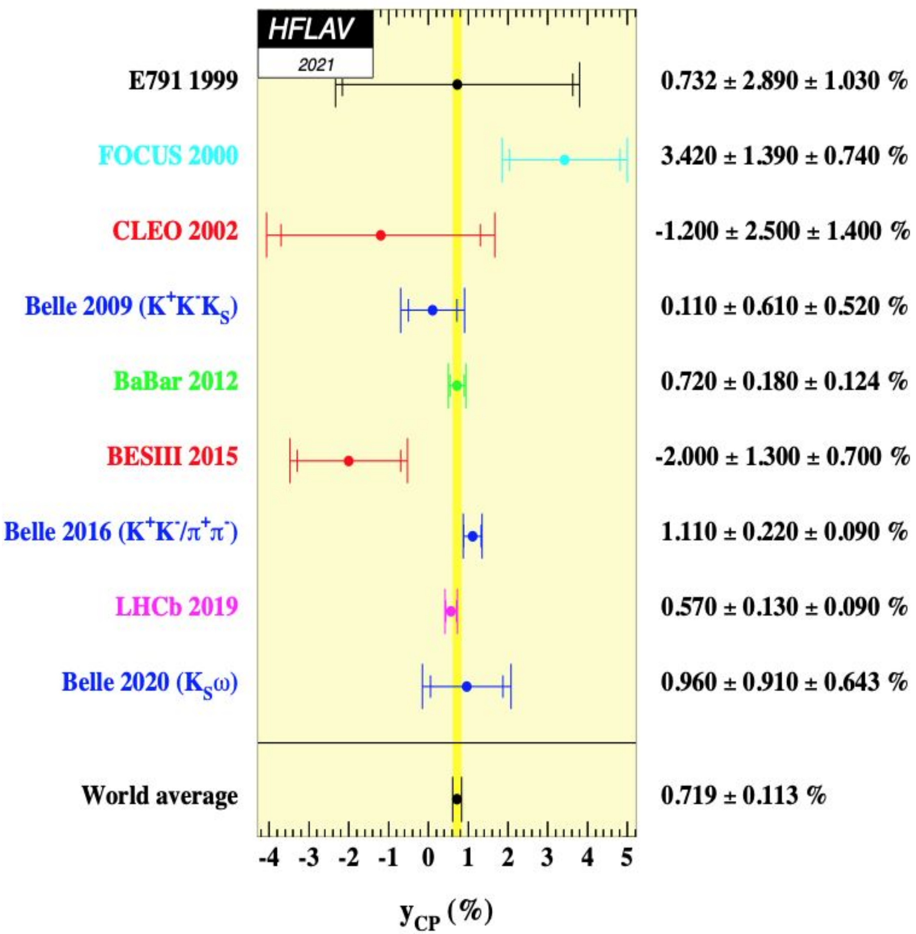
$$\left|\frac{q}{p}\right| = \left(\frac{x_{12}^2 + y_{12}^2 + 2x_{12}y_{12}\sin\phi_{12}}{x_{12}^2 + y_{12}^2 - 2x_{12}y_{12}\sin\phi_{12}}\right)^{1/4}$$

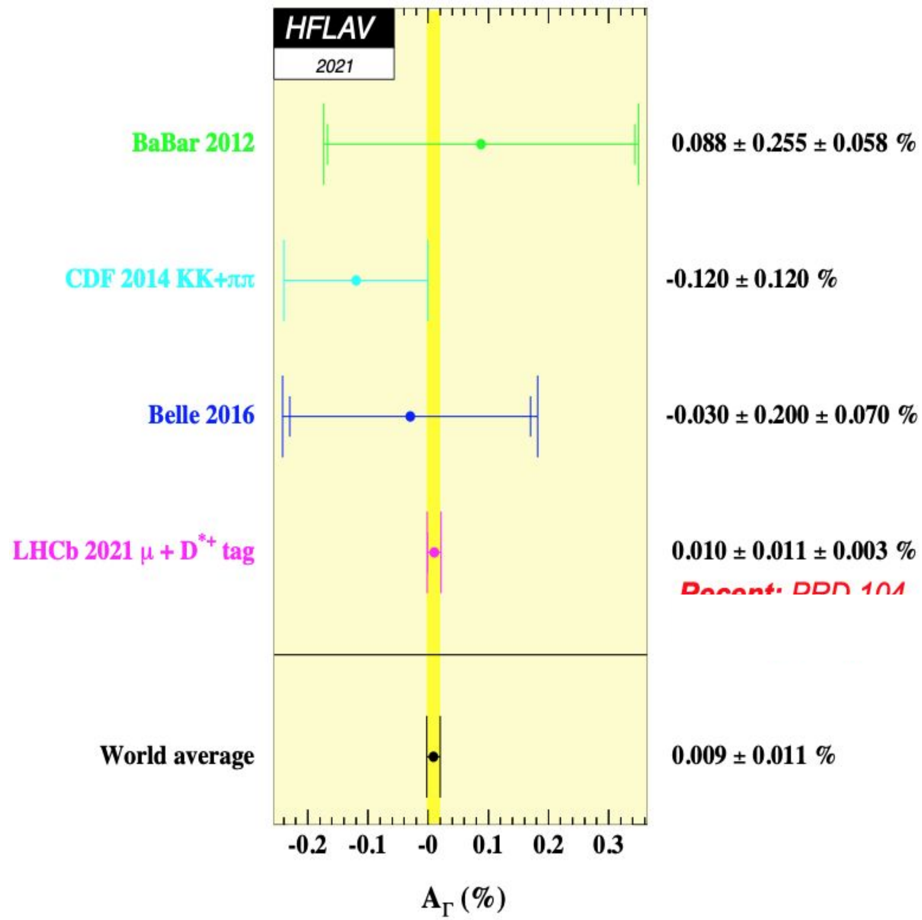
$$\tan 2\phi = -\frac{\sin 2\phi_{12}}{\cos 2\phi_{12} + (y_{12}/x_{12})^2}$$

A_Γ and y_{CP} averages [used in mixing fit]

$$y_{CP} = \frac{\tau_{K^-\pi^+}}{\tau_{h^+h^-}} - 1$$

$$A_\Gamma = \frac{\tau_{\bar{D}^0\to h^+h^-} - \tau_{D^0\to h^+h^-}}{\tau_{\bar{D}^0\to h^+h^-} + \tau_{D^0\to h^+h^-}}$$

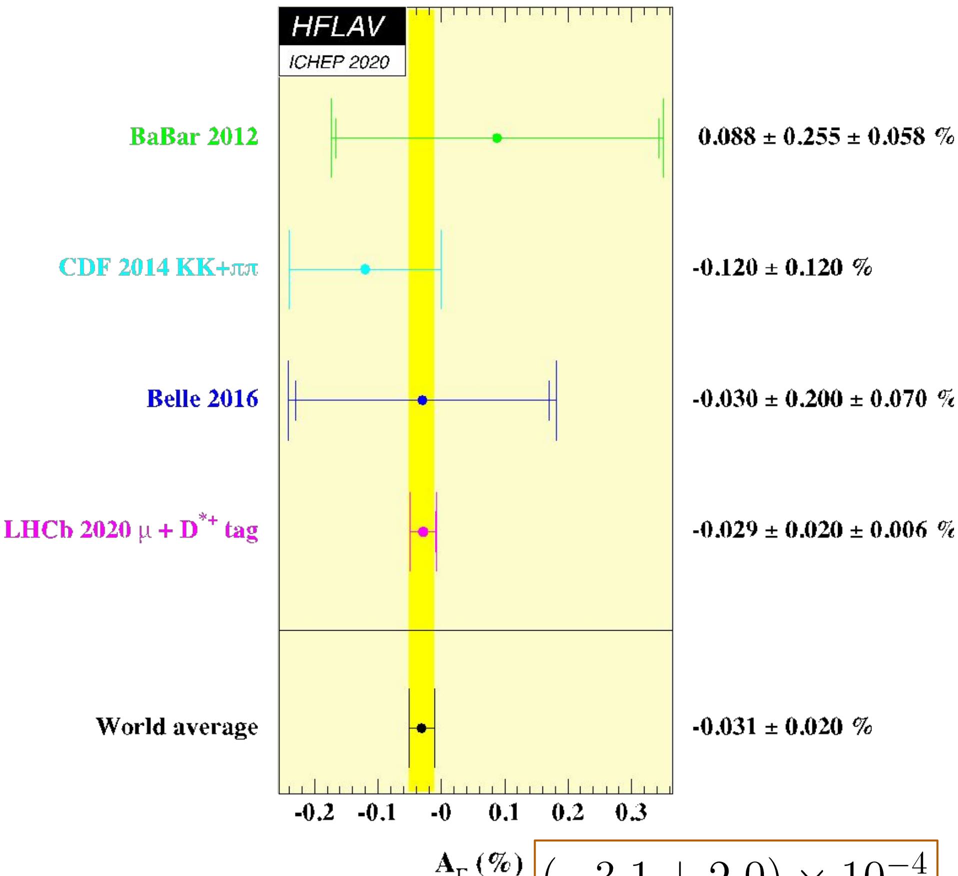




A_Γ world average

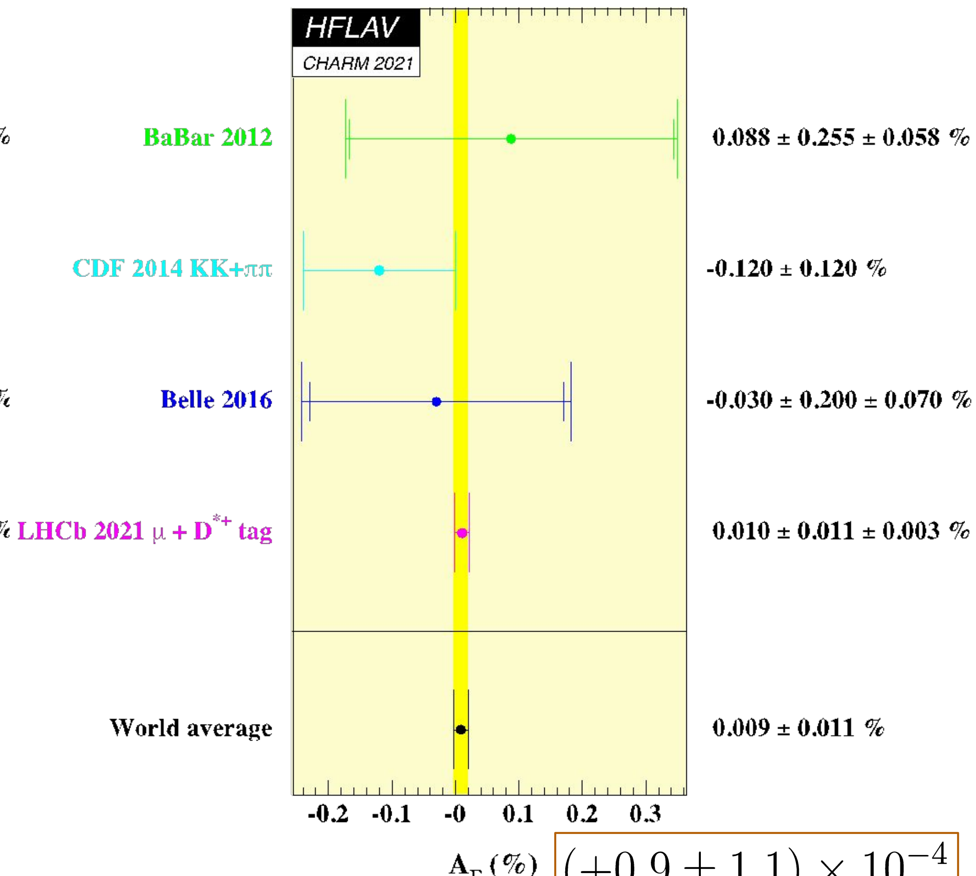
$$A_\Gamma = \frac{\tau_{\bar{D}^0\to h^+h^-} - \tau_{D^0\to h^+h^-}}{\tau_{\bar{D}^0\to h^+h^-} + \tau_{D^0\to h^+h^-}}$$

- 2020



$(-3.1 \pm 2.0) \times 10^{-4}$

2021



$(+0.9 \pm 1.1) \times 10^{-4}$

More general super-weak approximation

- No direct CPV at tree-level [A_D=0], subleading phases allowed in SCS [Arg(Γ_{12})≠0]
- Indirect CPV with two parameters φ^M_2=Arg[M_{12}], φ^Γ_2=Arg[Γ_{12}]
- [x, y, |q/p|, φ] ⟶[x_{12}, y_{12}, φ^M_2, φ^Γ_2]

- Relations between two parameter sets:

$$x = \left[x_{12}^2 - y_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2(\phi_2^M - \phi_2^\Gamma)}\right]^{1/2}$$

$$y = \left[y_{12}^2 - x_{12}^2 + \sqrt{(x_{12}^2 + y_{12}^2)^2 - 4x_{12}^2 y_{12}^2 \sin^2(\phi_2^M - \phi_2^\Gamma)}\right]^{1/2}$$

$$\left|\frac{q}{p}\right|^4 = \frac{x_{12}^2 + y_{12}^2 + 2x_{12}y_{12}\sin(\phi_2^M - \phi_2^\Gamma)}{x_{12}^2 + y_{12}^2 - 2x_{12}y_{12}\sin(\phi_2^M - \phi_2^\Gamma)}$$

$$\tan 2\phi = -\frac{x_{12}^2 \sin 2\phi_2^M + y_{12}^2 \sin 2\phi_2^\Gamma}{x_{12}^2 \cos 2\phi_2^M + y_{12}^2 \cos 2\phi_2^\Gamma}$$

Mixing parameterization

$$x_{CP} = -\,\mathrm{Im}\,(z_{CP}) = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\right]$$

$$\Delta x = -\,\mathrm{Im}\,(\Delta z) = \frac{1}{2}\left[x\cos\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) + y\sin\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right]$$

$$y_{CP} = -\,\mathrm{Re}\,(z_{CP}) = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right)\right]$$

$$\Delta y = -\,\mathrm{Re}\,(\Delta z) = \frac{1}{2}\left[y\cos\phi\left(\left|\frac{q}{p}\right| - \left|\frac{p}{q}\right|\right) - x\sin\phi\left(\left|\frac{q}{p}\right| + \left|\frac{p}{q}\right|\right)\right]$$