Three-body non-leptonic B decays in QCD factorization

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Collaborative Research Center TRR 257
P H
Particle Physics Phenomenology after the Higgs Discovery

Mainly based on	J. Virto, K. K. Vos, TH	2007.08881 (JHEP),
See also	Kränkl, Mannel, Virto	1505.04111 (NPB),
	Klein, Mannel, Virto, Vos	1708.02047 (JHEP)

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Outline

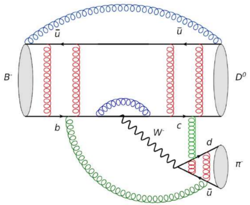
- Introduction / motivation
- Three-body nonleptonic B decays into heavy-light final states
- Three-body nonleptonic B decays into charmless final states
- Conclusion and outlook

Introduction/motivation

- Nonleptonic B decays provide a lot of useful information
  - CP violation and UT angles
  - QCD with heavy quarks and energetic light particles
  - BSM physics in quark sector
- Extraction of interesting physics requires precision in theory and experiment
- Generic structure of decay amplitude for B decays

$$\mathcal{A}(\bar{B} \to f) = \lambda_{\rm CKM} \times C \times \langle f|\mathcal{O}|\bar{B}\rangle_{\rm QCD+QED}$$

- Computation of hadronic matrix elements highly non-trivial
- QCD effects from many different scales
- QCD effects could overshadow the interesting fundamental dynamics



[courtesy of Alex Lenz]

State of the art

- Two-body nonleptonic decays extensively studied [Too many to mention...]
- Three-body decay events populate two-dim. Dalitz-plot, also local CP violation possible

- Theory approaches
  - Flavour SU(3) or one of its SU(2) subgroups
    [Gronau,Rosner,Bhattacharya,Imbeault,Bertholet,Ben-Haim, London]
    [Engelhard, Nir, Raz, Charles, Descotes-Genon, Ocariz, Pérez Pérez]
    [Fines-Neuschild, Houck, Jean, F.S. Yu, . . .]
  - Final-state interaction effects
    [Guo,Danilkin,Szczepaniak'14; Bediaga,Frederico,Magalhães,'15+]
  - Via quasi-two body decays
    $B \to R(\to M_1 M_2) M_3$ [e.g. Cheng,Chua,Soni'07]
    [Li,Ma,Wang,Xiao'16'18; Wang,Chai'18; Cheng,Chua,Zhang'13,'16]
    [Boito,Dedonder,El-Bennich,Escribano,Kaminski,Lesniak,Loiseau'17]
    [Zou,Fang,Liu,Li'21'22]
  - FAT, pQCD [e.g. Li,Yan,Rui,Liu,Zhang,Xiao'20; Cheng,Chua'21]
    [Zhou,Li,Wei,Lu'21; Zhou,Hai,Li,C.D.Lu'23]
  - Models [e.g. Bediaga,Miranda et al.; Mannel,Olschewsky,Vos'20]

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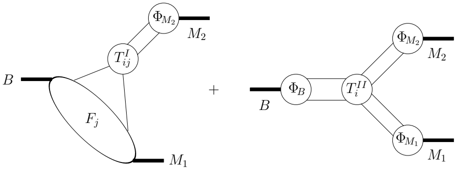
- Theory approaches cont'd
  - QCD factorization
    - Well-established for two-body decays
      [Beneke,Buchalla,Neubert,Sachrajda'99-'04]
    - Recent progress also for three-body decays
      [Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15]
      [Klein,Mannel,Virto,Vos'17; Virto,Vos,TH'20]
    - Separate factorization formulas for different regions of phase space
- New non-perturbative objects
  - $B \to M_1 M_2$ form factors
    [Faller,Feldmann,Khodjamirian,Mannel,van Dyk'13; Hambrock,Khodjamirian'15]
    [Böer,Feldmann,van Dyk'16; Cheng,Khodjamirian,Virto'17'17]
    [Descotes-Genon,Khodjamirian,Virto'19(+Vos'23)]
  - Di-meson light-cone distribution amplitudes
    [Polyakov'98]

QCD factorization for two-body nonleptonic decays



- Amplitude in the limit $m_b \gg \Lambda_{\rm QCD}$ [Beneke,Buchalla,Neubert,Sachrajda'99-'04]

$$\langle M_1 M_2 | Q_i | \bar{B} \rangle \simeq m_B^2 \, F_+^{B\to M_1}(0) \, f_{M_2} \int_0^1 du \; T_i^I(u) \, \phi_{M_2}(u) \;+\; (M_1 \leftrightarrow M_2)$$
$$+ f_B \, f_{M_1} \, f_{M_2} \int_0^\infty d\omega \int_0^1 dv \, du \; T_i^{II}(\omega, v, u) \, \phi_B(\omega) \, \phi_{M_1}(v) \, \phi_{M_2}(u)$$

- $T^{I,II}$: Hard scattering kernels, perturbatively calculable

- F_+ : $B \to M$ form factor
  f_i : decay constants
  ϕ_i : light-cone distribution amplitudes

  } Nonperturbative, but universal. From sum rules, lattice, dispersion relation, analyticity, data, ...

- Strong phases are $\mathcal{O}(\alpha_s)$ and/or $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$

QCD factorization for two-body heavy-light final states

- Particularly simple and clean for heavy-light final states such as in $B \to D\pi$ [Beneke,Buchalla,Neubert,Sachrajda'99-'04]

$$\langle D_q^{(*)+} L^- | \, \mathcal{Q}_i \, | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \int_0^1 du \, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

- Only vertex-terms of colour-allowed tree-amplitude a_1 contribute
- No colour-suppressed tree amplitude, no penguins
- Spectator scattering and weak annihilation power suppressed
- Weak annihilation absent if all final-state flavours distinct
  - as in $\bar{B}_s^0 \to D_s^+ \pi^-$ and $\bar{B}^0 \to D^+ K^-$ but not in $\bar{B}^0 \to D^+ \pi^-$ [cf. Bordone,Gubernari,Jung,v.Dyk,TH'20]

QCDF for three-body $B \to D(K,\pi)\pi$ decays

Idea / goal

Establish factorization for $\bar{B}^0 \to D^+ M^- \pi^0$ decays ($M = K, \pi$) in phase space region of small $M^- \pi^0$ invariant mass, i.e. $(M\pi)$-system recoils against heavy D meson.

- Kinematics: Consider $\bar{B}^0(p) \to D^+(q)\, M^-(k_1)\pi^0(k_2)$
- Describe phase space in terms of two variables
  - Invariant mass of light di-meson system $(M\pi)$: $k^2 = (k_1 + k_2)^2$
  - Angle $\theta_\pi = \angle(\vec{k}_2, \vec{p})$ in $(M\pi)$ rest frame (where $\vec{k} = 0$ holds)
- Decay amplitude $\mathcal{A} = \mathcal{A}(k^2, \theta_\pi)$ can be factorized by expanding in partial waves,

$$\mathcal{A}(k^2, \theta_\pi) = \sum_{\ell=0}^{\infty} \mathcal{A}^{(\ell)}(k^2) P_\ell(\cos\theta_\pi)$$

$\uparrow$ Legendre polynomials

Factorization formula for three-body $B \to D(K,\pi)\pi$ decays

- Lagrangian and operators in CMM basis ($x = d, s$)

$$\mathcal{L}_{\text{eff}}^{(6)} = -\frac{4G_F}{\sqrt{2}} V_{ux}^* V_{cb} \, (C_1 Q_1 + C_2 Q_2) + h.c. \, , \qquad Q_{1,2} = (\bar{c}\gamma^\mu P_L \{T^a, \mathbb{1}\} b) \, (\bar{x}\gamma_\mu P_L \{T^a, \mathbb{1}\} u)$$

Factorization formula at leading power $(L = M^- \pi^0)$

$$\mathcal{A}(\bar{B} \to D^+ L^-) = \frac{4G_F}{\sqrt{2}} V_{ux}^* V_{cb} \, k^- \, F_n^{B\to D} \int_0^1 du \, \Big[C_1 T_1(u) + C_2 T_2(u)\Big] \Phi_L(u,k)$$

$$a_1(D^+ L^-) = \int_0^1 du \; [C_1(\mu) T_1(u,\mu) + C_2(\mu) T_2(u,\mu)] \; \Phi_L(u,k)$$

- Hard kernels $T_i(u)$ are identical to two-body decay and known to NNLO [Kränkl,Li,TH'16]
- $\Phi_L(u,k)$: Di-meson distribution amplitude [Polyakov'98]

Di-meson LCDA

- Not so much is known about the di-meson distribution amplitude

$$k^- \int_0^1 du\, e^{iutk^-}\, \Phi_L(k,u) = -2\sqrt{2}\,\langle L^-(k)|\bar{q}^{(x)}(t\bar{n})\frac{\not{\bar{n}}}{2}P_L q^{(u)}(0)|0\rangle$$

- Non-local, non-perturbative object

- Local limit given by pion and $K\pi$ timelike form factors
[Shekhovtsova,Przedzinski,Roig,Was'12; Hanhart'12]
[Celis,Cirigliano,Passemar'13; Daub,Hanhart,Kubis'15]
[Belle'07'08; Cheng,Khodjamirian,Virto'17]
[Descotes-Genon,Khodjamirian,Virto'19]

$$\langle \pi^-(k_1)\pi^0(k_2)|\bar{d}\gamma_\mu u|0\rangle = -\sqrt{2}\,F_\pi(k^2)\,\bar{k}_\mu\,,$$

$$\langle K^-(k_1)\pi^0(k_2)|\bar{s}\gamma_\mu u|0\rangle = -\frac{f_+^{K\pi}(k^2)}{\sqrt{2}}\,\bar{k}_\mu - \frac{\Delta m^2_{K\pi}}{\sqrt{2}k^2}\,f_0^{K\pi}(k^2)\,k_\mu$$

- Normalization

$$\int_0^1 du\,\Phi_{\pi\pi}(u,k^2,\theta_\pi) = \cos\theta_\pi\,\beta_\pi(k^2)F_\pi(k^2)$$

$$\int_0^1 du\,\Phi_{K\pi}(u,k^2,\theta_\pi) = \cos\theta_\pi\,\frac{\sqrt{\lambda_{K\pi}(k^2)}}{2k^2}f_+^{K\pi}(k^2) + \frac{\Delta m^2_{K\pi}}{2k^2}f_0^{K\pi}(k^2)$$

- Expand in Gegenbauer coefficients:

$$\Phi_L(u,k) = 6u\bar{u}\sum_{n=0}^{\infty}\alpha_n^L(k^2,\theta_\pi)\,C_n^{3/2}(u-\bar{u})$$

Di-meson LCDA

- Expansion of Gegenbauer coefficients in partial waves
- In $\bar{B}^0 \to D^+\pi^-\pi^0$ case

$$\alpha_n^{\pi\pi}(k^2,\theta_\pi) = \sum_{\ell=1,3,\cdots}^{n+1} B_{n\ell}^{\pi\pi}(k^2)\, P_\ell(\cos\theta_\pi) \qquad (n \text{ even})$$

  - Normalization fixes $B_{01}^{\pi\pi}(k^2) = \beta_\pi(k^2) F_\pi(k^2)$

- In $\bar{B}^0 \to D^+K^-\pi^0$ case

$$\alpha_n^{K\pi}(k^2,\theta_\pi) = \sum_{\ell=0}^{n+1} B_{n\ell}^{K\pi}(k^2)\, P_\ell(\cos\theta_\pi) \qquad (\text{all } n)$$

  - Normalization fixes $B_{00}^{K\pi}(k^2) = \frac{\Delta m_{K\pi}^2}{2k^2} f_0^{K\pi}(k^2)\,, \qquad B_{01}^{K\pi}(k^2) = \frac{\sqrt{\lambda_{K\pi}(k^2)}}{2k^2} f_+^{K\pi}(k^2)$

- The $B_{n\ell}^L(k^2)$ determine the k^2 spectrum of each partial wave

Numerical size of Gegenbauer terms at NLO and NNLO

- Gegenbauer expansion of amplitude. Define $\mathcal{V}_{in} = \int_0^1 du\, T_i(u)\, 6u\bar{u}\, C_n^{3/2}(u-\bar{u})$. Then,

$$a_1(D^+L^-) = \sum_{n\geq 0} \alpha_n^L(k^2,\theta_\pi)\left[C_1(\mu)\mathcal{V}_{1n}(\mu) + C_2(\mu)\mathcal{V}_{2n}(\mu)\right] \equiv \sum_{n\geq 0} \alpha_n^L(k^2,\theta_\pi)\,\mathcal{G}_n(\mu)$$

- Numerical size of Gegenbauer expansion coefficients $\mathcal{G}_n$ at various loop orders (in the CMM basis)

$$\begin{aligned}
\mathcal{G}_0(\mu_b) &= 1.034_{\rm LO} + (0.026 + i\,0.020)_{\rm NLO} + (0.013 + i\,0.027)_{\rm NNLO} &&= 1.07 + i\,0.047 \\
\mathcal{G}_1(\mu_b) &= (-0.013 + i\,0.030)_{\rm NLO} + (-0.044 + i\,0.018)_{\rm NNLO} &&= -0.057 + i\,0.048 \\
\mathcal{G}_2(\mu_b) &= (0.0023 - i\,0.0017)_{\rm NLO} + (0.0017 - i\,0.0054)_{\rm NNLO} &&= 0.0040 - i\,0.0071
\end{aligned}$$

- Features
  - Only $\mathcal{G}_0$ non-vanishing at LO
  - NLO corrections small due to vanishing color-factor of $\mathcal{V}_{2n}$ at one loop and small WC C_1
  - NNLO corrections large relative to NLO

Numerical size of amplitude through to NNLO

- Gegenbauer expansion of amplitude

$$a_1(D^+L^-) = \sum_{n\geq 0} \alpha_n^L \left[C_1(\mu)\mathcal{V}_{1n}(\mu) + C_2(\mu)\mathcal{V}_{2n}(\mu)\right] \equiv \sum_{n\geq 0} \alpha_n^L \, \mathcal{G}_n(\mu)$$

- Numerical size of squared amplitude $|a_1(D^+L^-)|^2$, use $\hat{\alpha}_i^L \equiv \alpha_i^L/\alpha_0^L$

$$|a_1(D^+L^-)|^2 = |\alpha_0^L|^2 \Big\{1.07_{\rm LO} + \big[0.053 - 0.026\,{\rm Re}\,\hat{\alpha}_1^L - 0.062\,{\rm Im}\,\hat{\alpha}_1^L + 0.0047\,{\rm Re}\,\hat{\alpha}_2^L + 0.0034\,{\rm Im}\,\hat{\alpha}_2^L\big]_{\rm NLO}$$

$$+ \big[0.029 - 0.091\,{\rm Re}\,\hat{\alpha}_1^L - 0.040\,{\rm Im}\,\hat{\alpha}_1^L + 0.0036\,{\rm Re}\,\hat{\alpha}_2^L + 0.011\,{\rm Im}\,\hat{\alpha}_2^L\big]_{\rm NNLO}\Big\}$$

$$= 1.15|\alpha_0^L|^2 \Big\{1 - 0.10\,{\rm Re}\,\hat{\alpha}_1^L - 0.09\,{\rm Im}\,\hat{\alpha}_1^L + 0.007\,{\rm Re}\,\hat{\alpha}_2^L + 0.014\,{\rm Im}\,\hat{\alpha}_2^L\Big\}$$

- Features
  - $n=1$ ($n=2$) corrections are of order 10% (1%) of leading $n=0$ terms
  - In each case, NNLO corrections are essential, ${\rm Re}\,\hat{\alpha}_1^L$ is even dominated by NNLO
  - Important for $L = K\pi$ where $\alpha_1^{K\pi} \neq 0$

Modeling the dimeson system

- The $B^{L}_{n\ell}(k^2)$ can be extracted from data or modeled (or both)
- Take model as sum over resonances R

[see also Descotes-Genon,Khodjamirian,Virto'19]

$$B^{M\pi}_{n0}(s) = \sum_{R_0} \frac{m_{R_0}\, f_{R_0}\, g_{R_0 M\pi}\, e^{i\varphi_{R_0}}}{\sqrt{2}[m^2_{R_0} - s - i\sqrt{s}\,\Gamma_{R_0}(s)]}\, \alpha^{R_0}_n$$

$$B^{M\pi}_{n1}(s) = \frac{\sqrt{\lambda_{M\pi}(s)}}{s} \sum_{R} \frac{m_R\, f_R\, g_{RM\pi}\, e^{i\varphi_R}}{\sqrt{2}[m^2_R - s - i\sqrt{s}\,\Gamma_R(s)]}$$

- Yields for S- and P-wave coefficients

$$B^{K\pi}_{00}(s) = \frac{\Delta m^2_{K\pi}}{2k^2} f^{K\pi}_0(s) = \sum_{R_0} \frac{m_{R_0}\, f_{R_0}\, g_{R_0 K\pi}\, e^{i\varphi_{R_0}}}{\sqrt{2}[m^2_{R_0} - s - i\sqrt{s}\,\Gamma_{R_0}(s)]}$$

$$B^{\pi\pi}_{01}(s) = \beta_\pi(s) F_\pi(s) = \beta_\pi(s) \sum_{R} \frac{m_R\, f_R\, g_{R\pi\pi}\, e^{i\varphi_R}}{\sqrt{2}[m^2_R - s - i\sqrt{s}\,\Gamma_R(s)]}$$

$$B^{K\pi}_{01}(s) = \frac{\sqrt{\lambda_{K\pi}(s)}}{2s} f^{K\pi}_+(s) = \frac{\sqrt{\lambda_{K\pi}(s)}}{s} \sum_{R} \frac{m_R\, f_R\, g_{RK\pi}\, e^{i\varphi_R}}{\sqrt{2}[m^2_R - s - i\sqrt{s}\,\Gamma_R(s)]}$$

- Satisfies narrow-width limit in case of stable vector resonance (ρ or K^*)

Corrections to narrow-width limit

- Leading corrections to the narrow-width limit (NWL)

$$\Gamma_{[R]} \equiv \int_{(m_R-\delta)^2}^{(m_R+\delta)^2} ds\, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{ds}$$

- Already integrated over θ_π $\Longrightarrow$ no interference among different partial waves. Therefore:

$$\Gamma_{[R]} = \sum_\ell \frac{2}{2\ell+1} \int_{(m_R-\delta)^2}^{(m_R+\delta)^2} ds\, \frac{\sqrt{\lambda_{BD}(s)\,\lambda_{M\pi}(s)}}{64(2\pi)^3 s m_B^3} |\mathcal{A}^{(\ell)}(s)|^2 = \sum_\ell \Gamma_{[R]}^{(\ell)}$$

- Define the ratio $$\mathcal{W}_R^{(\ell)} = \frac{\Gamma_{[R]}^{(\ell)}}{\Gamma_{[R]\,,\mathrm{NWL}}^{(\ell)}}$$

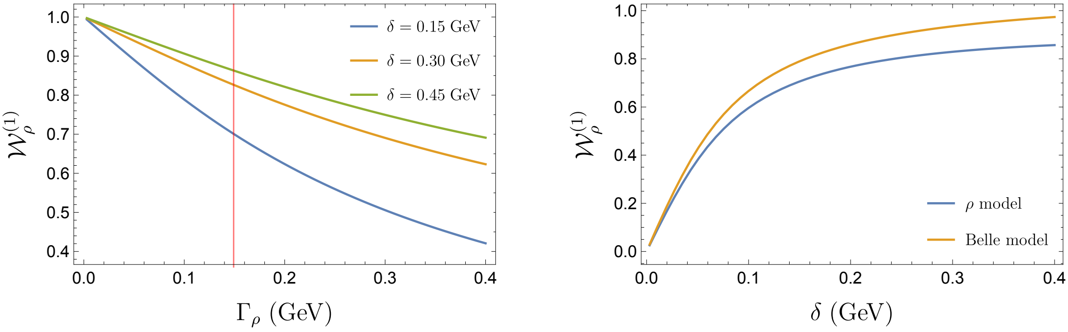
- $\Gamma_{[R]\,,\mathrm{NWL}}^{(\ell)}$ is $\Gamma_{[R]}^{(\ell)}$ in the narrow-width limit,

$$\Gamma_{[R]\,,\mathrm{NWL}}^{(\ell)} = \Gamma(\bar{B} \to D^+ R^-)\, \mathcal{B}(R \to M\pi)$$

Corrections to narrow-width limit

- Example: ρ contribution to $B \to D\pi\pi$ (neglect B_{n1} for $n \geq 2$ and use $B_{01}(k^2) = \beta_\pi(k^2) F_\pi(k^2)$)

$$\mathcal{W}_\rho^{(1)} = \int_{(m_\rho-\delta)^2}^{(m_\rho+\delta)^2} ds \, \frac{\lambda_{BD}^{1/2}(s)}{\lambda_{BD}^{1/2}(m_\rho^2)} \, \frac{[\beta_\pi(s)]^3 \, |F_\pi(s)|^2}{24\pi^2 f_\rho^2 \, \mathcal{B}(\rho \to \pi\pi)}$$

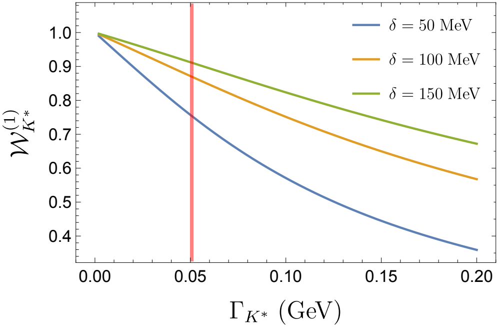


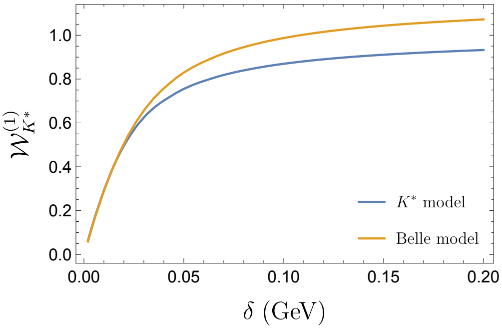
- Belle model: Include also higher ρ resonances [Belle'08]

Corrections to narrow-width limit

- Example: K^* contribution to $B \to DK\pi$

$$\mathcal{W}_{K^*}^{(1)} = \int_{(m_{K^*}-\delta)^2}^{(m_{K^*}+\delta)^2} ds \, \frac{\lambda_{BD}^{1/2}(s)}{\lambda_{BD}^{1/2}(m_{K^*}^2)} \frac{[\lambda_{K\pi}(s)]^{3/2} \, |f_+^{K\pi}(s)|^2}{96\pi^2 \, s^3 f_{K^*}^2 \, \mathcal{B}(K^* \to \pi K)}$$





- Expect somewhat smaller finite width-effects since K^* is much narrower than ρ.

Phenomenological aspects

- Probing higher-order QCD effects, higher Gegenbauer moments, higher partial waves.
- Define ratios of angular-integrated decay widths ($z = \cos\theta_\pi$)

$$\mathcal{R}_{MM}[z_1, z_2; z_1', z_2'](k^2) \equiv \frac{\displaystyle\int_{z_1}^{z_2} dz \, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{dk^2\, dz}}{\displaystyle\int_{z_1'}^{z_2'} dz \, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{dk^2\, dz}} = \frac{\displaystyle\int_{z_1}^{z_2} dz \, \left|a_1(D^+ M^- \pi^0)\right|^2}{\displaystyle\int_{z_1'}^{z_2'} dz \, \left|a_1(D^+ M^- \pi^0)\right|^2}$$

- Examples from $D\pi\pi$ final state
  - Forward-backward asymmetry vanishes (only odd partial waves), tests isospin-violating corrections

$$A_{\rm FB}^{\pi\pi}(k^2) = \mathcal{R}_{\pi\pi}[0,1;-1,1](k^2) - \mathcal{R}_{\pi\pi}[-1,0;-1,1](k^2) = 0$$

  - Assuming P-wave dominance and small $\alpha_2^{\pi\pi}$, angular dependence factorises at each order

$$\mathcal{R}_{\pi\pi}[z_1, z_2, z_1', z_2'](k^2) = \frac{I[z_1, z_2, P_1^2]}{I[z_1', z_2', P_1^2]} + \frac{I[z_1, z_2, P_1\, P_3]\, I[z_1', z_2', P_1^2] - I[z_1', z_2', P_1\, P_3]\, I[z_1, z_2, P_1^2]}{(I[z_1', z_2', P_1^2])^2}$$
$$\times \frac{2\,\mathrm{Re}\left(B_{01}^{\pi\pi}(k^2)\, B_{23}^{\pi\pi\,*}(k^2)\, \mathcal{G}_0(\mu_b)\, \mathcal{G}_2^*(\mu_b)\right) + 2\,\mathrm{Re}\left(B_{21}^{\pi\pi}(k^2)\, B_{23}^{\pi\pi\,*}(k^2)\right) |\mathcal{G}_2(\mu_b)|^2}{\left|B_{01}^{\pi\pi}(k^2)\, \mathcal{G}_0(\mu_b) + B_{21}^{\pi\pi}(k^2)\, \mathcal{G}_2(\mu_b)\right|^2}$$

$$I[z_1, z_2, f] \equiv \int_{z_1}^{z_2} dz\, f(z)$$

P_i : Legendre polynomials

Phenomenological aspects

$$\mathcal{R}_{MM}[z_1, z_2; z_1', z_2'](k^2) \equiv \frac{\displaystyle\int_{z_1}^{z_2} dz\, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{dk^2\, dz}}{\displaystyle\int_{z_1'}^{z_2'} dz\, \frac{d\Gamma(\bar{B} \to D^+ M^- \pi^0)}{dk^2\, dz}} = \frac{\displaystyle\int_{z_1}^{z_2} dz\, \left|a_1(D^+ M^- \pi^0)\right|^2}{\displaystyle\int_{z_1'}^{z_2'} dz\, \left|a_1(D^+ M^- \pi^0)\right|^2}$$

- In $DK\pi$ final state, also first Gegenbauer moment $\alpha_1^{K\pi}$ and partial waves of even ℓ appear
  - Forward-backward asymmetry does not vanish

$$A_{\mathrm{FB}}^{K\pi}(k^2) = \mathcal{R}_{K\pi}[0, 1; -1, 1](k^2) - \mathcal{R}_{K\pi}[-1, 0; -1, 1](k^2) \neq 0$$

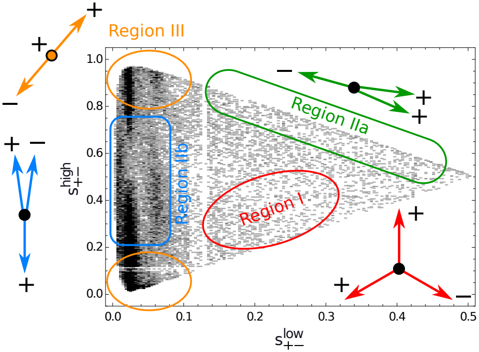
  - Assume S and P-wave dominance and expand in small $\mathcal{G}_{1,2}$

$$A_{\mathrm{FB}}^{K\pi}(k^2) \simeq \frac{2\mathrm{Re}(B_{00}B_{01}^*)}{2|B_{00}|^2 + 2/3|B_{01}|^2} + \frac{2\mathrm{Re}\left[\left(2(B_{00}^*)^2 - 2/3(B_{01}^*)^2\right)\mathcal{G}_0^*\left(B_{00}(B_{11}\mathcal{G}_1 + B_{21}\mathcal{G}_2) - B_{01}(B_{10}\mathcal{G}_1 + B_{20}\mathcal{G}_2)\right)\right]}{|\mathcal{G}_0|^2\left(2|B_{00}|^2 + 2/3|B_{01}|^2\right)^2}$$

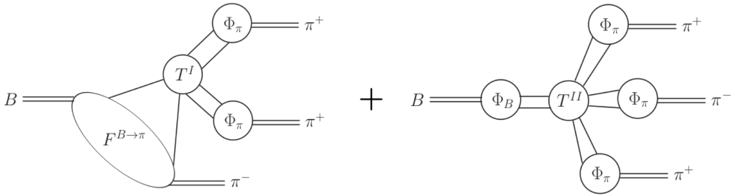
  - To leading order in α_s, $A_{\mathrm{FB}}^{K\pi}$ vanishes in the limit $m_K = m_\pi$
  - From NLO one starts probing the higher Gegenbauer coefficients B_{11}, B_{20} and B_{21}

Three-body charmless nonleptonic B-decays in QCDF

- Focus on $B^+ \to \pi^+ \pi^- \pi^+$ in a factorisation approach [Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15]
  - Identify different regions in Dalitz plot
  - Each region obeys its own factorization formula



[Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15]



- Factorization formula

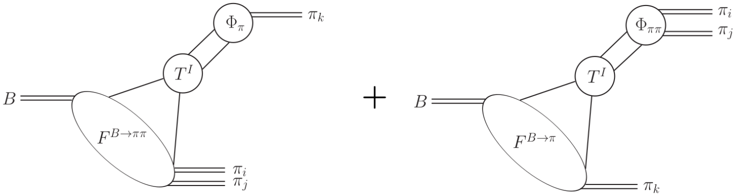
$$\langle \pi^+\pi^+\pi^-|Q_i|B^+\rangle_c = T_i^I \otimes F^{B\to\pi} \otimes \Phi_\pi \otimes \Phi_\pi + T_i^{II} \otimes \Phi_B \otimes \Phi_\pi \otimes \Phi_\pi \otimes \Phi_\pi$$

- At tree-level all convolutions are finite
- $1/m_b^2$ and α_s suppressed compared to two-body case

Edges of Dalitz plot

[Beneke'06; Stewart '06; Kränkl,Mannel,Virto'15; Klein,Mannel,Virto,Vos'17]

- Features of the edges
  - Three-body decays resemble two-body ones
  - Resonances close to the edges



- Factorisation formula

$$\langle \pi^+\pi^+\pi^- | \mathcal{Q}_i | B \rangle_e = T_i^I \otimes F^{B\to\pi^+} \otimes \Phi_{\pi^+\pi^-} + T_i^I \otimes F^{B\to\pi^+\pi^-} \otimes \Phi_{\pi^+}$$

- New nonperturbative input: 2πLCDA, $B \to \pi\pi$ form factor

New nonperturbative input

- New nonperturbative input from data or model [Klein,Mannel,Virto,Vos'17]
- 2πLCDA [Polyakov'98]

$$\phi^q_{\pi\pi}(u,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iu(k^+_{12}x^-)} \left\langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\right\rangle$$

$$s = (k_1+k_2)^2,\, \zeta = k_1/s$$

- Both isoscalar $(I=0)$ and isovector $(I=1)$ contribute
- At leading order only normalization needed

$$\int du\, \phi^{I=1}_{\pi\pi}(u,\zeta,s) = (2\zeta-1)F_\pi(s) \qquad \int du\, \phi^{I=0}_{\pi\pi}(u,\zeta,s) = 0$$

- $B \to \pi\pi$ form factor [Klein,Mannel,Virto,Vos'17]
  - Was studied in $B \to \pi\pi\,\ell\,\nu$ decays [Faller,Feldmann,Khodjamirian,Mannel,van Dyk'13; Böer,Feldmann,van Dyk'16]
- For $B^+ \to \pi^+\pi^-\pi^+$ only vector form factor relevant

$$k_{3\mu} \left\langle \pi^+(k_1)\pi^-(k_2)|\bar{b}\gamma^\mu\gamma^5 u|B^+(p)\right\rangle = i\, m_\pi\, F_t(s,\zeta)$$

Conclusion

Conclusion

- Nonleptonic two- and three body decay offer an interesting testing groud for conceptual and phenomenological applications
- Three-body decay $\bar{B}^0 \to D^+(K^-/\pi^-)\pi^0$ in recoil region is the simplest from QCDF point of view. Important for studying di-meson LCDAs.
  - Can probe expansion in α_s, partial waves, Gegenbauer polynomials

Outlook / wishlist

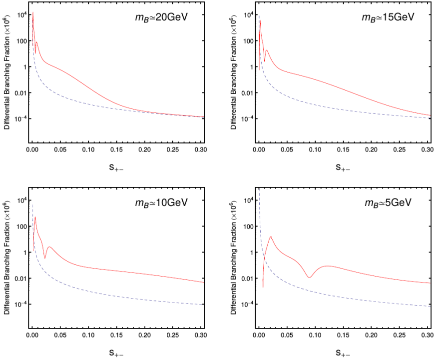
- Work out three-body charmless decays to final states with K's and π's in QCDF. Work out connection between SU(3), topological, QCDF approach. Study phenonomenological implications [Malami,Mannel,TH w.i.p.]

Backup slides

Matching central and edge region of Dalitz plot

- For large enough m_B the two regions should be well separated

[Kränkl,Mannel,Virto'15]



Full 2πLCDA (red) and perturbative contribution (dashed)

- For realistic values of m_B
  - not enough phase space to reach a perturbative regime in the center
  - Dalitz plot completely dominated by the edges
  - No part of the Dalitz plot is really center-like