Overview of D-Hadron Form Factors & Decay Constants: Lattice QCD

~~Elvira Gamiz~~

William I. Jay — MIT

CKM 2023

12th International Workshop on the CKM Unitarity Triangle

Santiago de Compostela, Spain

18-22 September 2023

Outline

- Connections to program at CKM 2023
- Motivation & Review of Lattice QCD
- Leptonic decays of D-mesons
- Semi-leptonic Decays of D-mesons
- Summary

Enormous lattice literature on D-hadrons weak decays.

Impossible to be entirely comprehensive.

Talk is unavoidably selective, focusing attention on published results from the past few years

Apologies for any omissions

Related talks at CKM 2023

Topically Adjacent

- Takashi Kaneko, M 13:05 - *Status and Progress of Lattice QCD*
- Alan Schwartz, T 9:00 - *Charmed meson lifetimes and prospects for future determinations of V_{cs}, V_{cd} at Belle/Belle II*
- Tengjiao Wang T 9:30 - *Experimental Status of V_{cs} and V_{cd}*

Methodologically Adjacent

- Tuesday 16:00 - Alejandro Vaquero - $B_{(s)} \to D^{(\star)}_{(s)}$ from Fermilab-MILC
- Thursday 18:20 - Chris Bouchard - Rare $B \to \pi$, $B \to K$ on the lattice
- Thursday 18:50 - Brian Colquhoun - $B \to \pi$, $B \to D^{(\star)}$ from JLQCD

Context & Motivation

Quark Flavor and Lattice QCD

Accessing the CKM Matrix

“Gold-plated processes” $\Longleftrightarrow$

Single-hadron initial state.

Zero- or one-hadron final state.

All hadrons stable under QCD.

***Nota bene*: Different formalism for exclusive multi-hadron or inclusive final states**

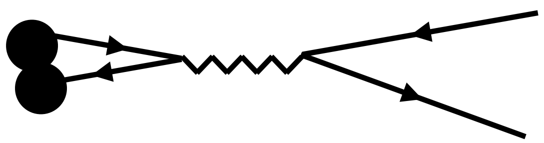
Luka Leskovec, Th 17:50
Lattice outlook on $B \to \rho, B \to K^{\star}$

Ryan Kellerman, T 12:00
Updates on inclusive charmed and bottomed meson decays from the lattice

Quark Flavor and Lattice QCD

Accessing the CKM Matrix

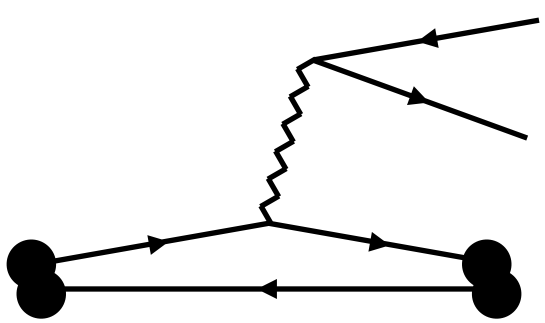
Leptonic decays



(Decay constants)

$$\langle 0|\, A^\mu \,|H(P)\rangle = i f_H\, p^\mu$$

Semi-leptonic decays

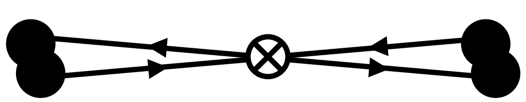


(Form factors)

$$f_J(p) \propto \langle \text{final}|\, J(p) \,|\text{initial}\rangle$$

$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$ $K \to \pi\ell\nu$	$B \to \ell\nu$ $B \to \pi\ell\nu$ $\Lambda_b \to p\ell\nu$
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$ $D \to \pi\ell\nu$ $D_s \to K\ell\nu$ $\Lambda_c \to N\ell\nu$	$D_s \to \ell\nu$ $D \to K\ell\nu$ $\Lambda_c \to \Lambda\ell\nu$ $\Xi_c \to \Xi\ell\nu$	$B \to D\ell\nu$ $B \to D^\star\ell\nu$ $\Lambda_b \to \Lambda_c\ell\nu$
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$\langle B_d \| \bar{B}_d\rangle)$	$\langle B_s \| \bar{B}_s\rangle)$	

Neutral-meson mixing



(Matrix elements)

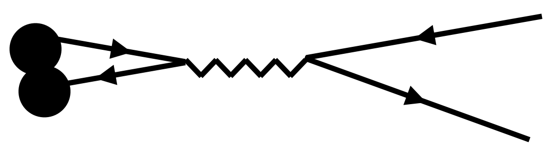
$$\langle \bar{B}^0 | \mathcal{H}_{\text{eff}} | B^0 \rangle$$

Felix Erben, Th 12:30
Update on SU(3)-breaking ratios and bag parameters for $B_{(s)}$ mesons

Quark Flavor and Lattice QCD

Accessing the CKM Matrix

Leptonic decays



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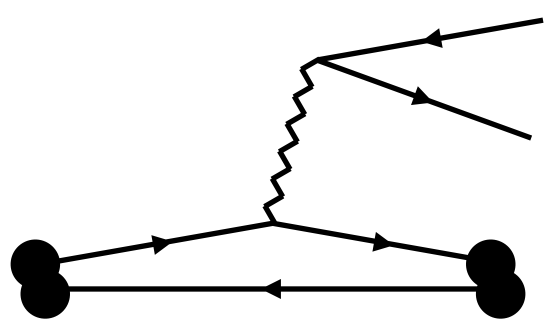
Semi-leptonic decays



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$D \to \ell\nu$	$D_s \to \ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$
$D_s \to K\ell\nu$	$\Lambda_c \to \Lambda\ell\nu$
$\Lambda_c \to N\ell\nu$	$\Xi_c \to \Xi\ell\nu$

Lattice QCD with Heavy Quarks

Lattice QCD

- Lattice QCD gives complete non-perturbative definition to the strong interactions
- This framework gives:

$$\mathcal{Z} = \int \mathcal{D}[\text{fields}] e^{-S_E[\text{fields}]}$$

- **Fundamental approximations:**
  - UV cutoff: lattice spacing *a* [target: a ≪ physical scales]
  - IR cutoff: finite spacetime volume $V = L^3 \times T$ [target: $1 \ll m_\pi L$]

- **Approximations of convenience:**
  - Often: Heavier-than-physical pions: $(m_\pi)^{\text{lattice}} > (m_\pi)^{\text{PDG}}$
  - Often: Isospin limit $m_u = m_d$
  - Often: QCD interactions only, no QED
  - Often: lighter-than-physical or static heavy quarks

Lattice QCD is systematically improvable

- All approximations admit theoretical descriptions via EFT
  - Cutoff dependence $\Longleftrightarrow$ Symanzik effective theory
  - Finite-volume dependence $\Longleftrightarrow$ Finite-volume χPT
  - Chiral extrapolation / interpolation $\Longleftrightarrow$ χPT
  - Heavy quark extrapolation / interpolation $\Longleftrightarrow$ HQET, NRQCD, etc…
  - QED, isospin breaking $\Longleftrightarrow$ perturbative expansion of path integral
- Careful treatment of all systematic effects is key to modern high-precision lattice QCD
- Technical advances in controlling these systematics have been drivers of progress in lattice QCD, especially in charm physics

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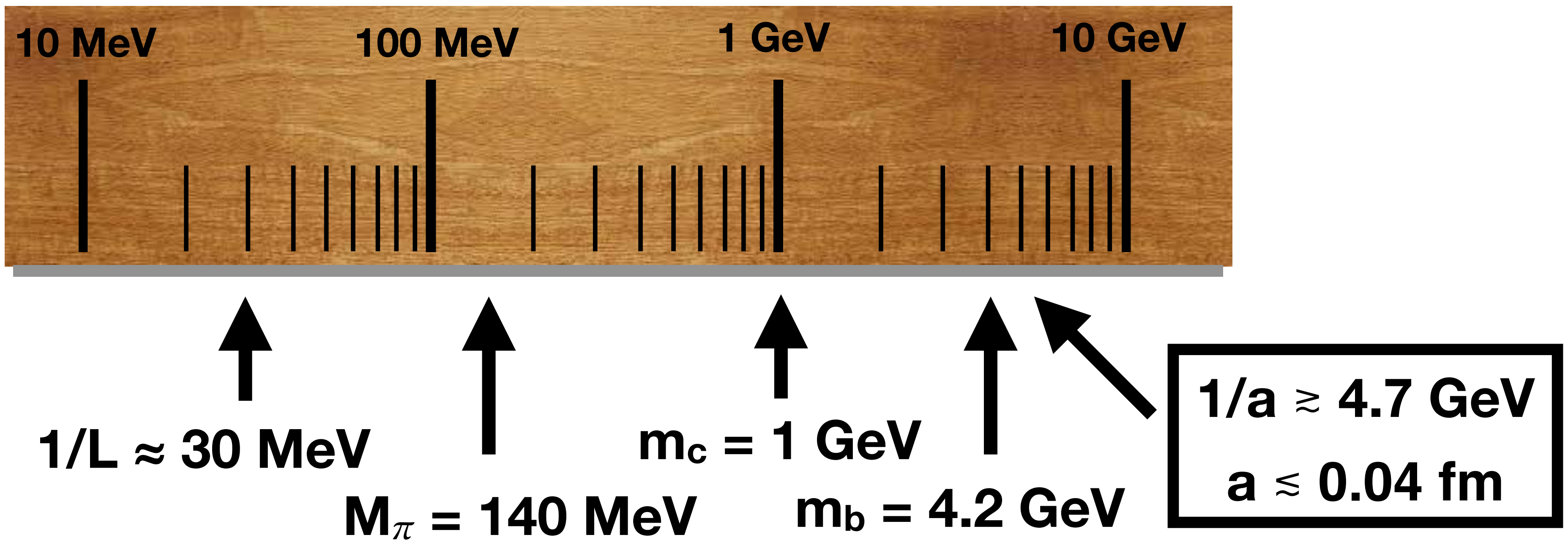
Lattice QCD with Heavy Quarks

A challenging multi-scale problem



Heavy quarks are hard: lattice artifacts grow like powers $(am_h)^n$ — especially tricky for masses near or above the cutoff

$$\frac{1}{L} \ll M_\pi \ll m_h \ll \frac{1}{a}$$

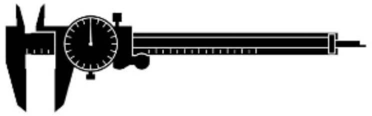
Lattice QCD with Heavy Quarks

A challenging multi-scale problem

Solutions to the cutoff challenge?

1. Use an "effective theory" for heavy quarks (b, sometimes c)
   - "FNAL interpretation," NRQCD, RHQ, Oktay-Kronfeld
   - Good: Solves problem with artifacts (am_h)
   - No free lunch: EFTs require matching and/or parameter tuning, which introduces systematic effects
   - (1-3)% total errors

2. Use highly-improved relativistic light-quark action on fine lattices
   - Good: advantageous renormalization, continuum limit
   - No free lunch: simulations still need $am_h < 1$ and often an extrapolation to the physical bottom mass
   - (< 1)% total errors possible



Lattice QCD with Heavy Quarks

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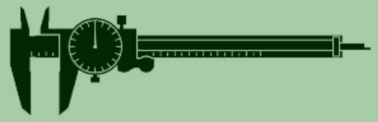


Semileptonic decays: $H \to L\ell\nu$

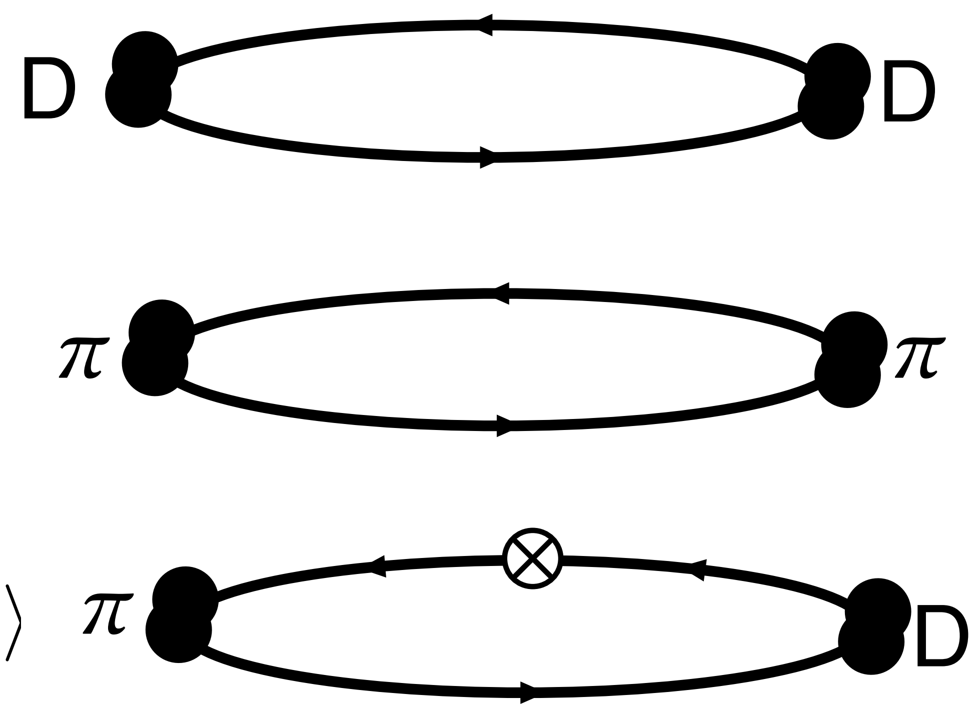
Anatomy of a calculation: correlation functions

- Hadron masses $\Longleftrightarrow$ QCD 2pt functions
- Matrix elements $\Longleftrightarrow$ QCD 3pt functions
- For concreteness: consider $D \to \pi\ell\nu$

$$C_D(t) = \sum_{\boldsymbol{x}} \langle \mathcal{O}_D(0,\mathbf{0}) \mathcal{O}_D(t,\boldsymbol{x}) \rangle$$

$$C_\pi(t,\boldsymbol{p}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\boldsymbol{x}} \langle \mathcal{O}_\pi(0,\mathbf{0}) \mathcal{O}_\pi(t,\boldsymbol{x}) \rangle$$

$$C_3(t,T,\boldsymbol{p}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{p}\cdot\boldsymbol{y}} \langle \mathcal{O}_\pi(0,\mathbf{0}) J(t,\boldsymbol{y}) \mathcal{O}_D(T,\boldsymbol{x}) \rangle$$



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$$\longrightarrow \langle 0|\,\mathcal{O}_\pi\,|\pi\rangle \langle \pi|\,J\,|D\rangle \langle D|\,\mathcal{O}_D\,|0\rangle\, e^{-E_\pi t} e^{M_D(T-t)}$$

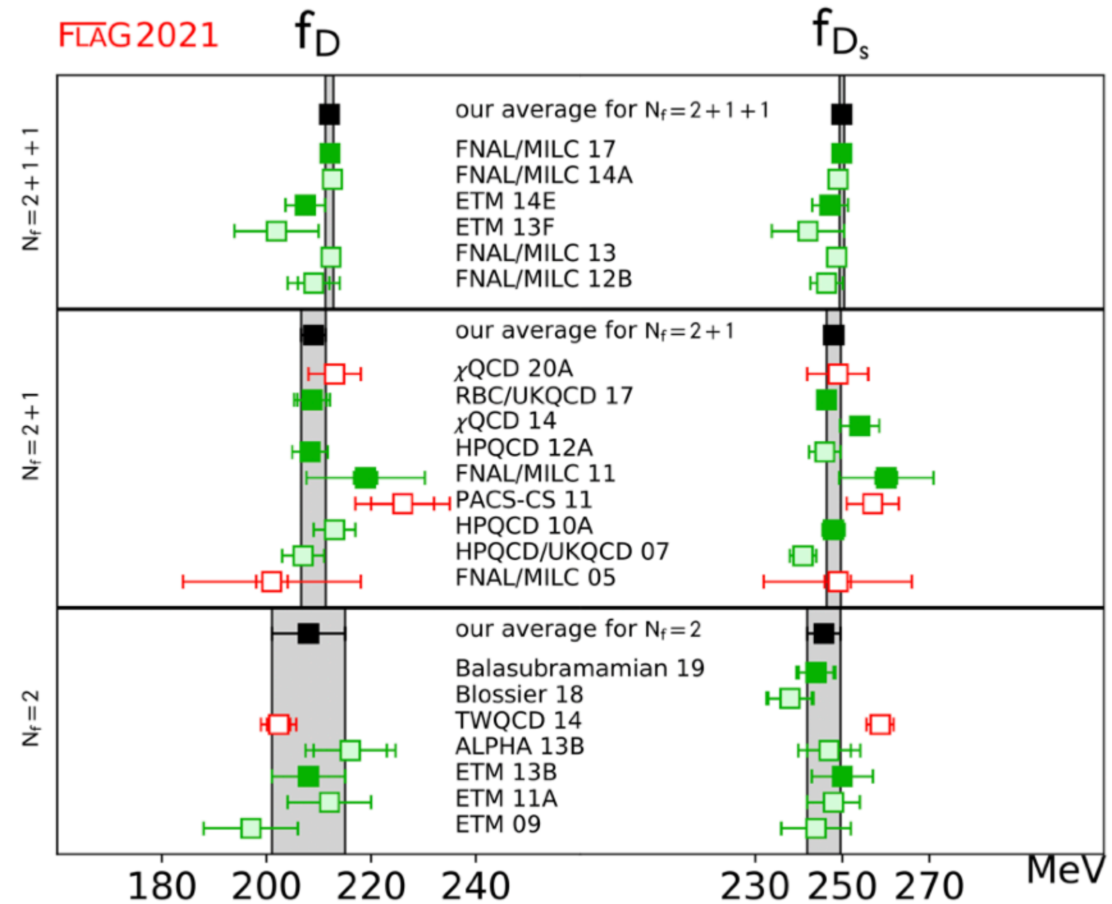
Matrix elements ⇒ Form factors

Leptonic Decays

An invitation to precision in lattice QCD

FLAG Review 21
Y. Aoki et al.
EPJC 82 (2022) 10, 869
arXiv: 2111.09849

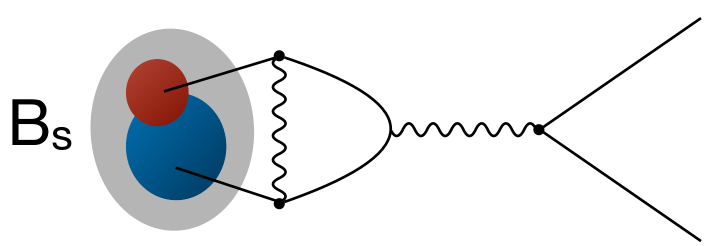
- Sub-percent precision for $f_{D_{(s)}}$ and $f_{B_{(s)}}$
  - LQCD precision is below existing/expected experimental uncertainties
  - Complementary calculations and discretizations bolster confidence in results
- "Pure QCD problem is solved"
  - Further improvement: systematic inclusion of QED, isospin breaking



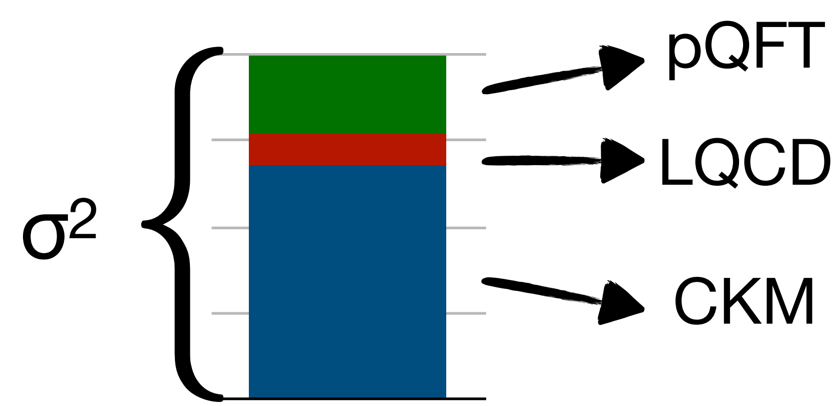
SM prediction for rare leptonic decay rate

Beneke et al, arXiv:1908.07011, JHEP 2019

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.660(38) \times 10^{-9}$$



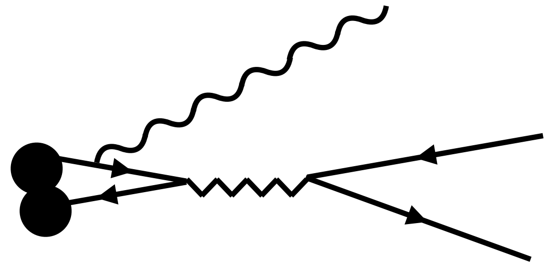
Lattice QCD value for f_{B_s} is now a sub-dominant source of uncertainty



Leptonic Decays

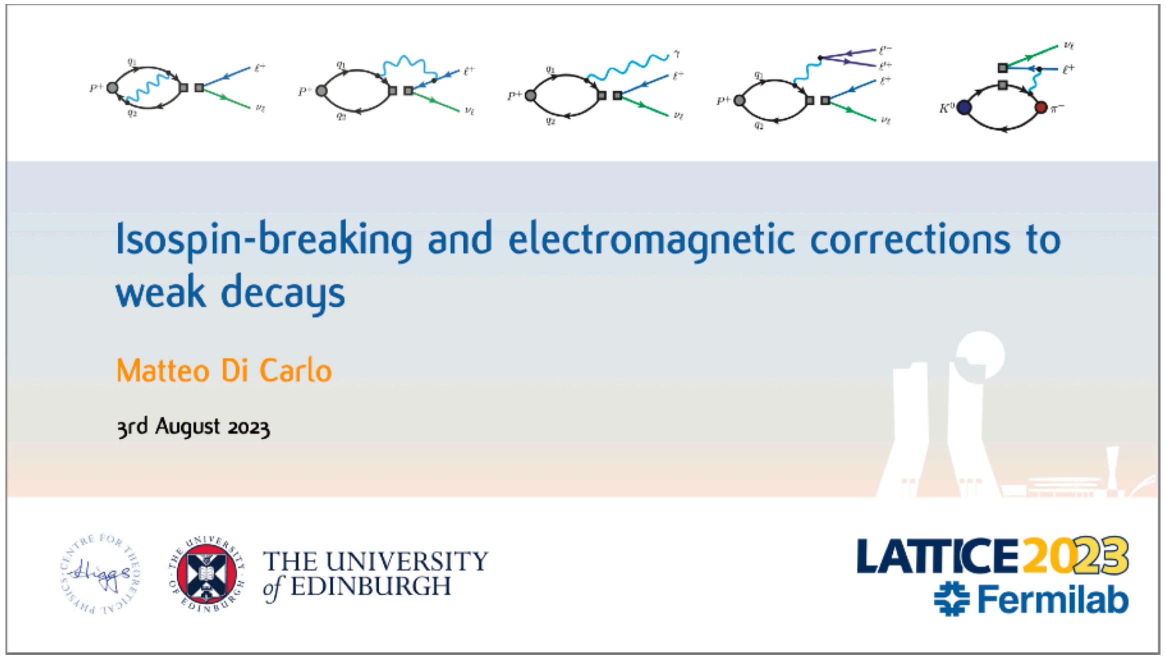
An invitation to precision in lattice QCD

- Isospin / QED corrections to weak decays have been considered by the lattice community since ≈ 2015



$$\epsilon_\mu(k)\int d^4y\, e^{iky}\mathscr{T}\langle\varnothing\,|\,j_W^\alpha(0)j_{EM}^\mu(y)\,|\,P(\mathbf{p})\rangle$$

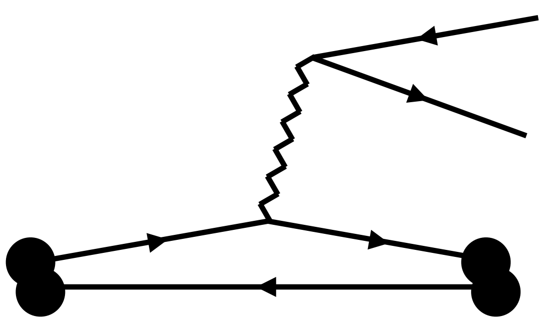
Structure-dependent form factors: qualitatively new element for leptonic decays.



Matteo Di Carlo
<u>Plenary Review</u> @ Lattice 2023
Includes discussion and references to literature, recent work reported at Lattice 2023

Semileptonic Decays

of D-mesons



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Semileptonic Decays

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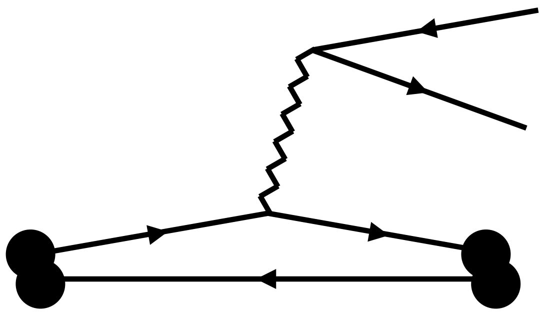
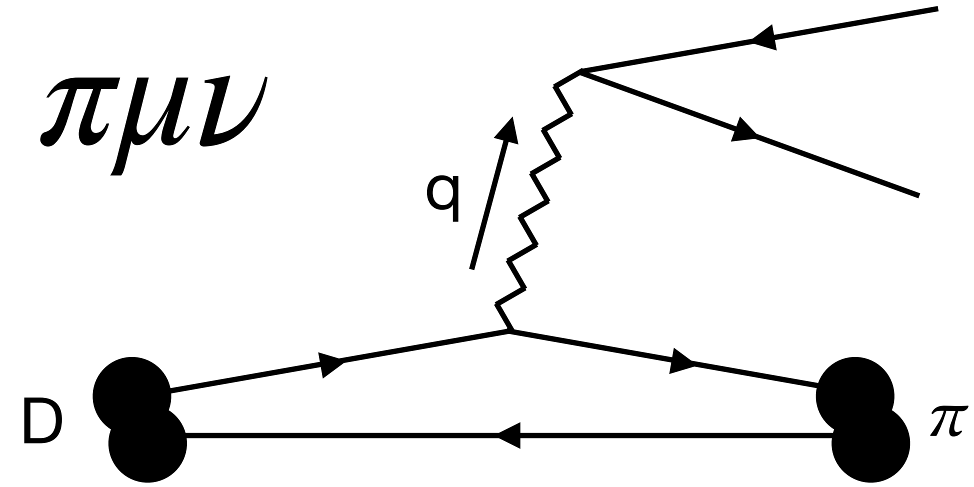


$$\langle \pi | \bar{d}\gamma^{\mu} c | D \rangle$$

Vector form factors: $f_{+,0}$

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



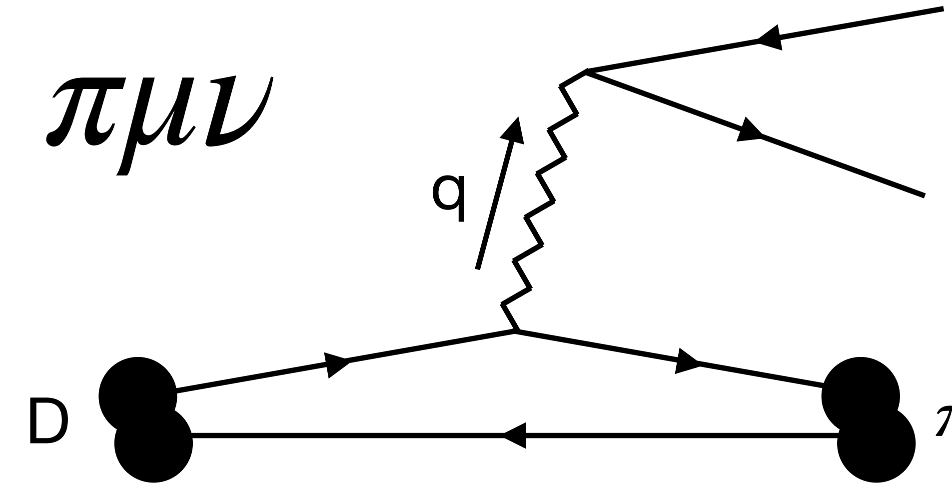
$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \eta_{EW}^2}{24\pi^3} \left|V_{cd}\right|^2 (1-\epsilon)^2 (1+\delta_{\mathrm{EM}}) \times$$

$$\left[\left|\mathbf{p}\right|^3 \left(1+\frac{\epsilon}{2}\right) \left|f_+(q^2)\right|^2 + \left|\mathbf{p}\right| M_D^2 \left(1-\frac{M_\pi^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} \left|f_0(q^2)\right|^2 \right]$$

: measured decay rate

$$\epsilon = m_\mu^2/q^2 \ll 1$$

Semileptonic decays: $D \to \pi\mu\nu$



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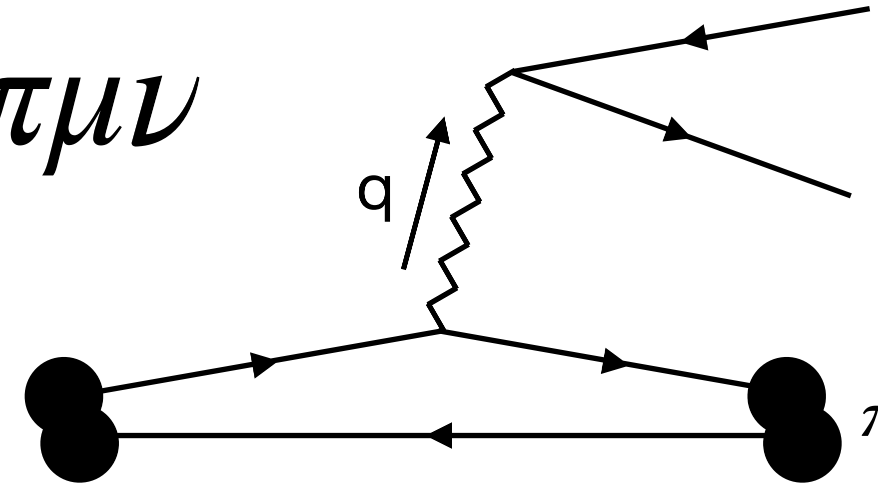
: measured decay rate (blue box)

: (non-perturbative) hadronic form factors (green box)

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



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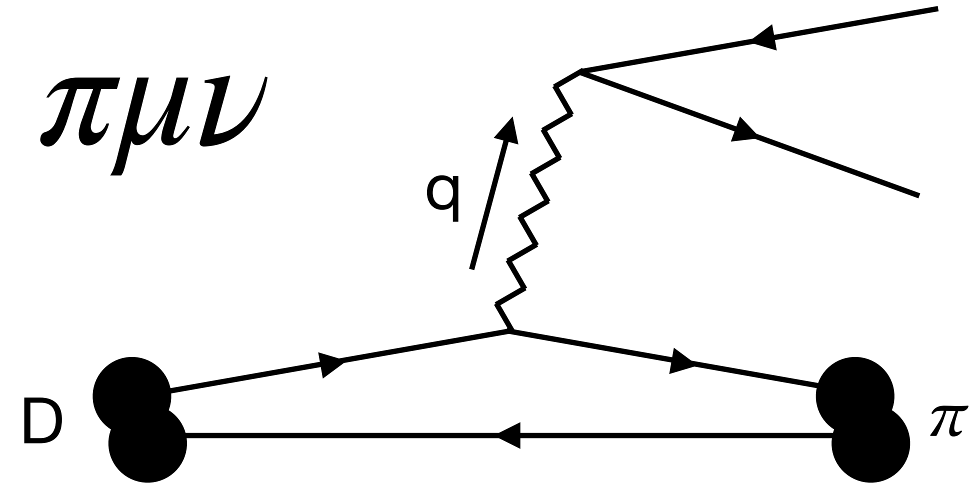
- (blue): **measured decay rate**
- (green): **(non-perturbative) hadronic form factors**
- (yellow): **kinematic factors**

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



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: measured decay rate

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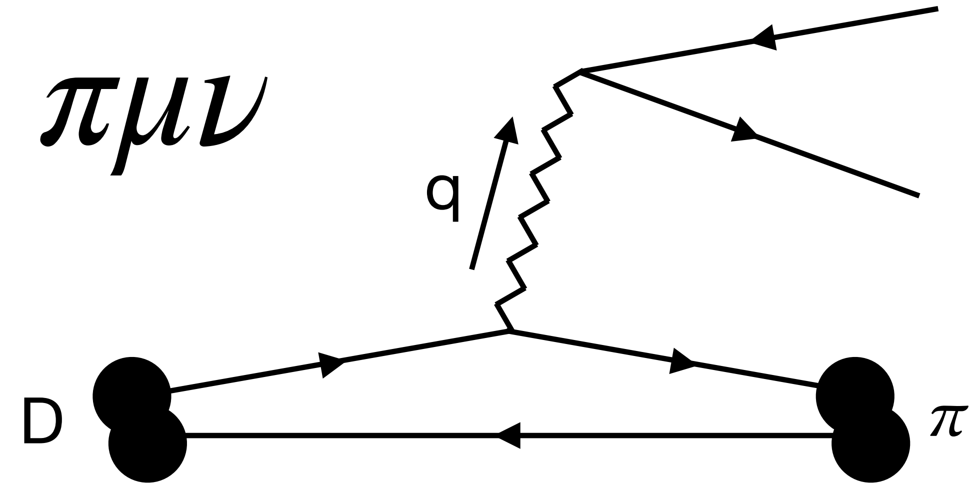
: perturbative corrections

Semileptonic decays: $D \to \pi\mu\nu$

Theoretical preliminaries



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$$\epsilon = m_\mu^2 / q^2 \ll 1$$

: measured decay rate (blue)

: (non-perturbative) hadronic form factors (green)

: kinematic factors (yellow)

: perturbative corrections (red)

At O(1%) precision, all sectors of SM become important: QCD, QED, EW

D-meson Semileptonic Decays

Measure: Expt.

Calculate: LQCD

Pseudoscalar final state: $D_{(s)} \to \pi/K\ell\nu$

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{24\pi^3}\eta_{\mathrm{EW}}^2|V_{cx}|^2(1-\epsilon)^2(1+\delta_{\mathrm{EM}})\times \left[|\boldsymbol{p}|^3\left(1+\frac{\epsilon}{2}\right)|f_+(q^2)|^2 + |\boldsymbol{p}|\,M_H^2\left(1-\frac{M_L^2}{M_H^2}\right)^2\frac{3\epsilon}{8}|f_0(q^2)|^2\right]$$

+tensor-current form factors for FCNC, BSM

$$|V_{cd}^{\mathrm{excl.}}| = 0.2330(0.0029)^{\mathrm{Expt}}(0.0133)^{\mathrm{QCD}}$$

- Status as of PDG 2022
- Combined precision for $D \to \pi$ ~ 6%
- Theory errors dominated
- Today: recent significant improvement

$$|V_{cs}^{\mathrm{excl.}}| = 0.972(0.007)$$

- Combined precision for $D \to K \lesssim 1\%$
- Theory errors dominant
- Percent-level total errors now possible, with QCD subdominant

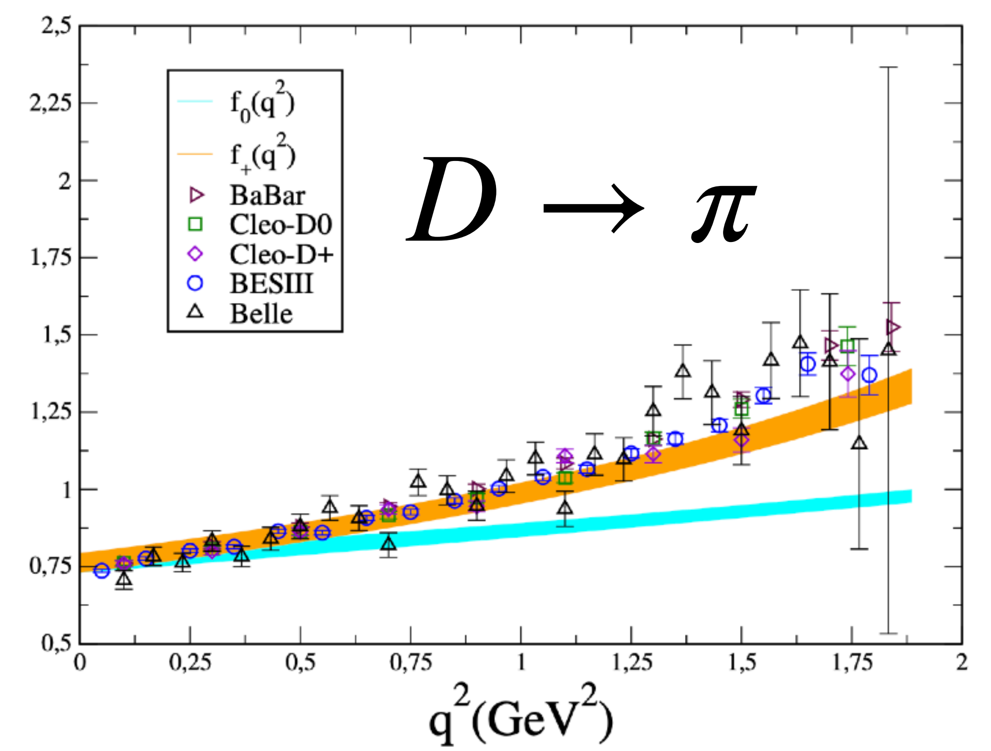
A. Ceccucci, Z. Ligeti, Y. Saka
PDG 2022: 12. CKM Quark-Mixing Matrix

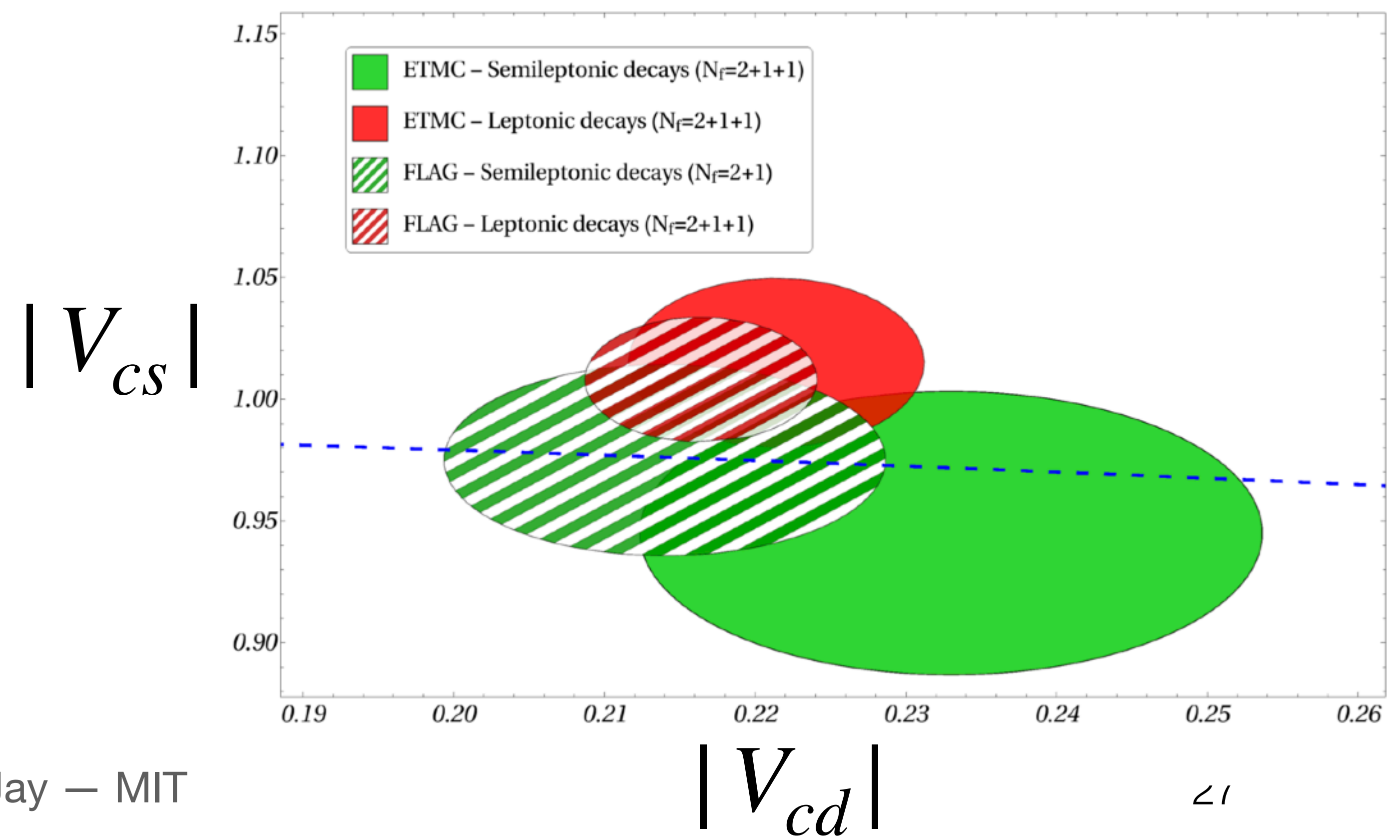
D-meson Semileptonic Decays

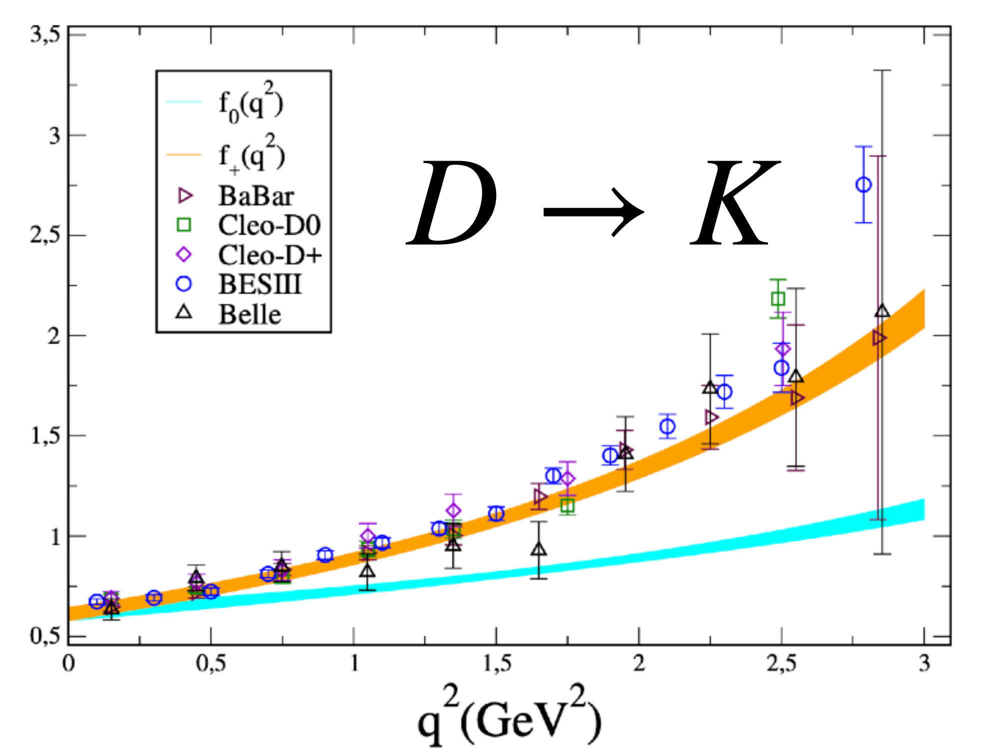
ETMC
PRD 96 (2017) 5, 054514
arXiv:1706.03017

$D \to K/\pi \ell \nu$ and $|V_{cd}|$, $|V_{cs}|$

- (N_f=2+1+1)ETMC Wilson twisted mass ensembles
- Lattice spacings: $a \in \{0.09, 0.08, 0.06\}$ fm
- $M_\pi \simeq 210 - 450$ MeV
- $\approx 4-6\,\%$ precision for $f_{+/0}(0)$
- $|V_{cd}| = 0.2330(133)^{\mathrm{LQCD}}(31)^{\mathrm{EXP}}$ [≈6%]
- $|V_{cs}| = 0.945(38)^{\mathrm{LQCD}}(4)^{\mathrm{EXP}}$ [≈4%]





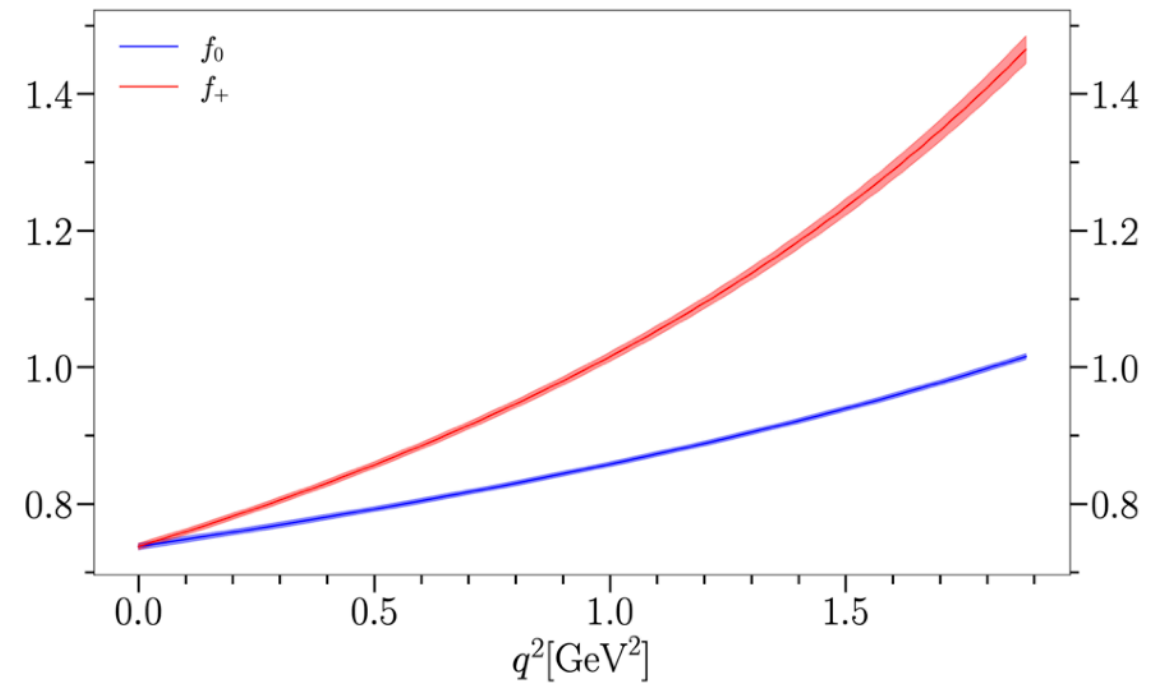


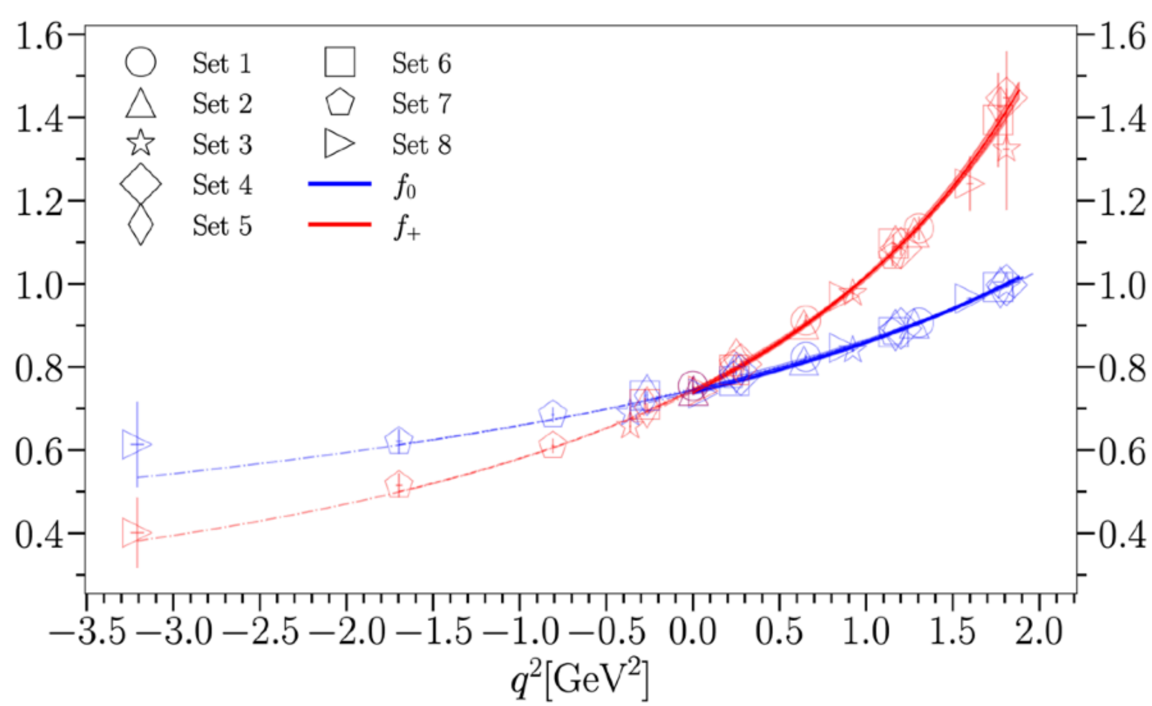
D-meson Semileptonic Decays

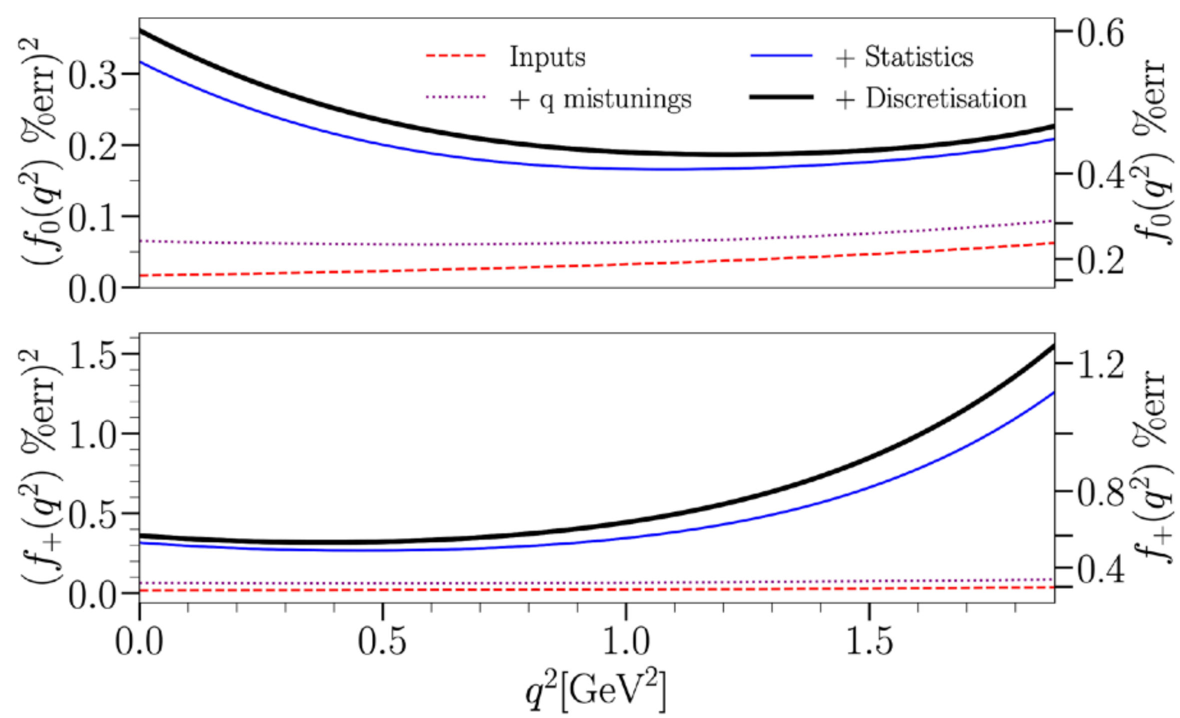
HPQCD
PRD 104 (2021) 3, 034505
arXiv:2104.09883

$D \to K\ell\nu$ and $|V_{cs}|$

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: $a \in \{0.045 - 0.15\}$ fm
- $M_\pi \simeq 135 - 320$ MeV
- Valence: heavy HISQ
- Chiral-continuum analysis via "modified z-expansion"
- $\lesssim 1\,\%$ precision for $f_{+/0}(0)$
- $|V_{cs}| = 0.9663(53)^{\mathrm{LQCD}}(39)^{\mathrm{EXP}}(19)^{\mathrm{EW}}(40)^{\mathrm{EM}}$ [≈1%]





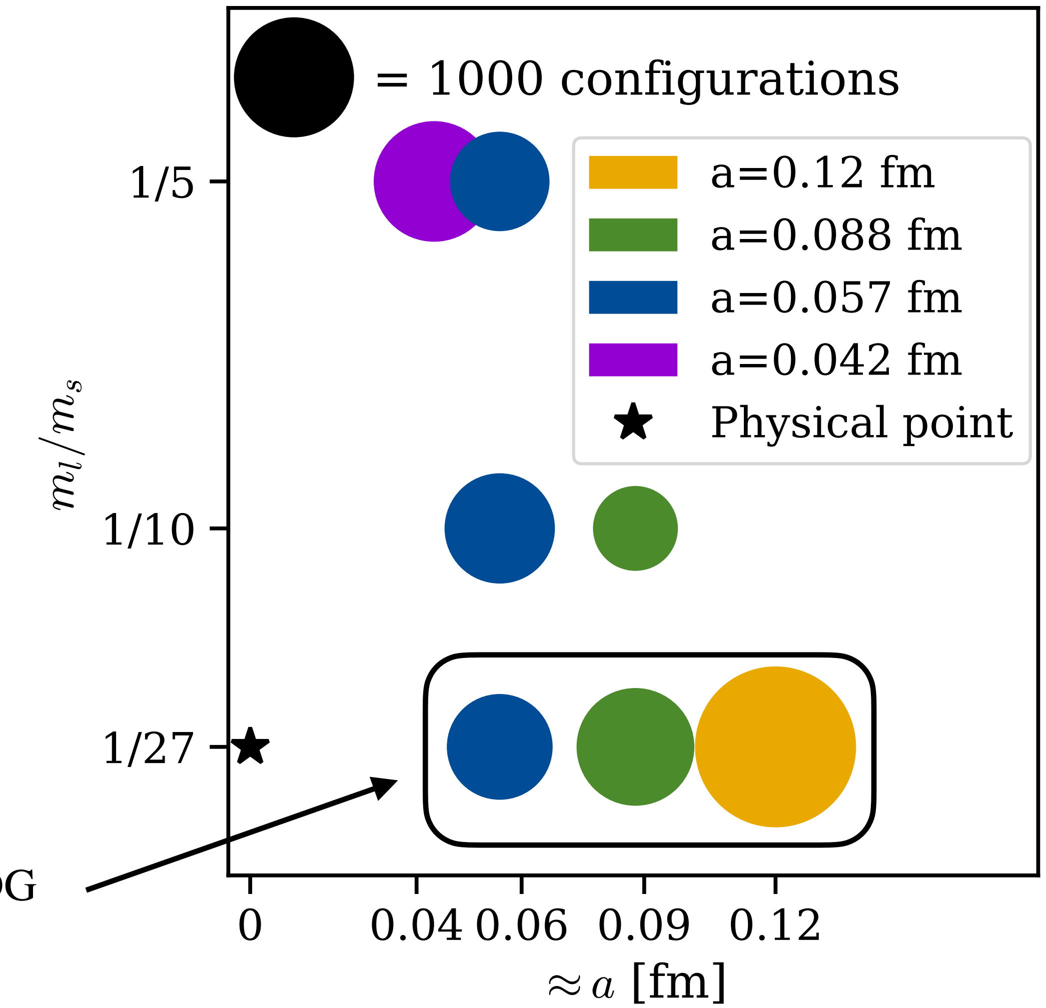


D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

$D_{(s)} \to K/\pi \ell \nu$ and $|V_{cd}|, |V_{cs}|$

- (N_f=2+1+1) MILC HISQ ensembles
- Lattice spacings: [0.045 - 0.12] fm
- Valence: heavy HISQ
- Percent-level determinations of $|V_{cd}|$, $|V_{cs}|$
  - Consistent with $|V_{cs}|$ from HPQCD 2021
- First-ever $|V_{cd}|$ from $D_s \to K \ell \nu$ when combined with recent first measurements from BESIII
- First time that LQCD and experimental errors are commensurate for $D \to \pi \ell \nu$
- All results from a **blinded analysis**



$$|V_{cd}|^{D\to\pi} = 0.2338(11)^{\mathrm{Expt}}(15)^{\mathrm{LQCD}}[22]^{\mathrm{EW/QED/SIB}}$$

$$|V_{cs}|^{D\to K} = 0.9589(23)^{\mathrm{Expt}}(40)^{\mathrm{LQCD}}[96]^{\mathrm{EW/QED/SIB}}$$

Measure: Expt.

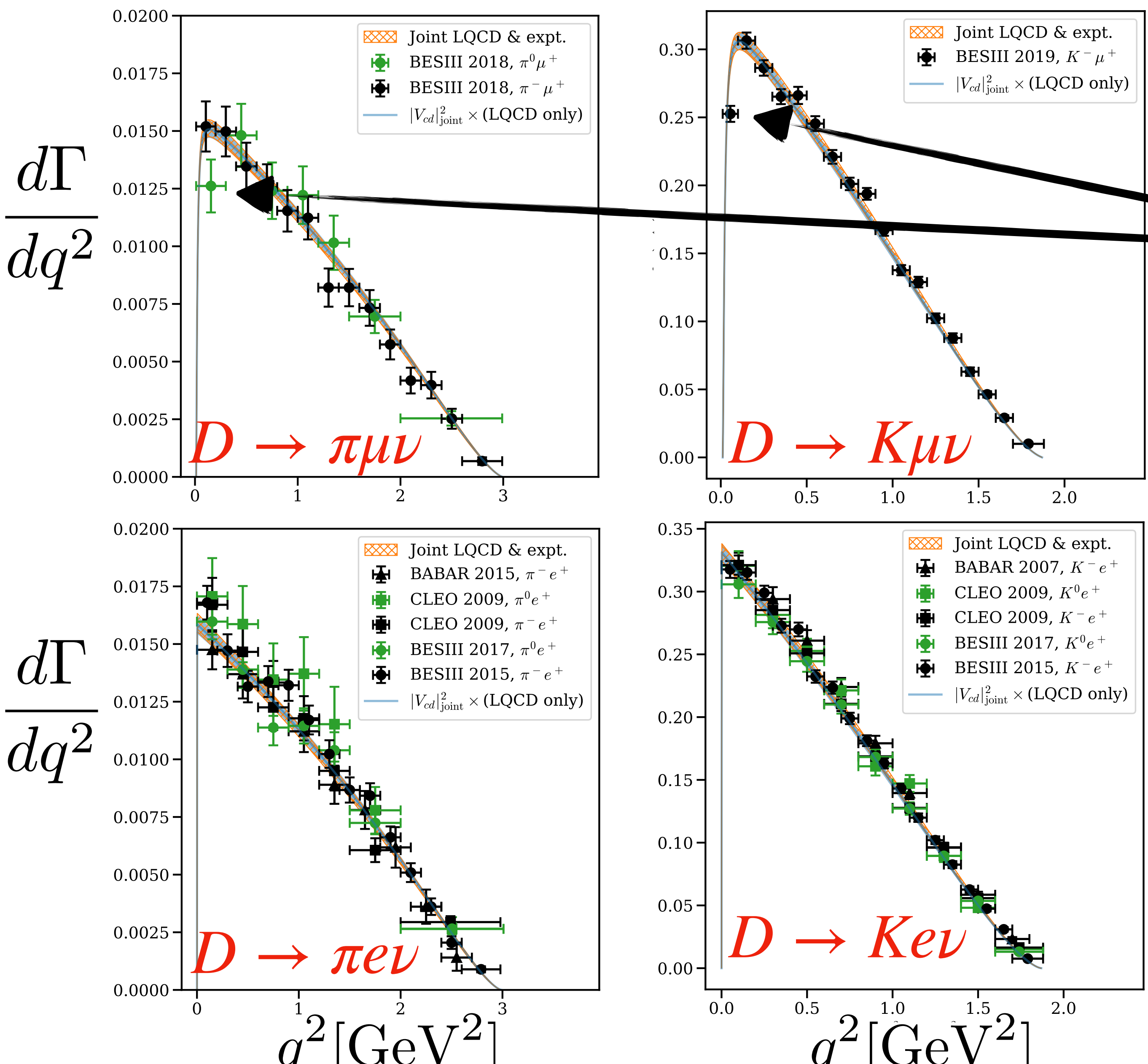
Calculate: LQCD

D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

Comparison to experimental data

$D \to \pi$ $\qquad$ $D \to K$



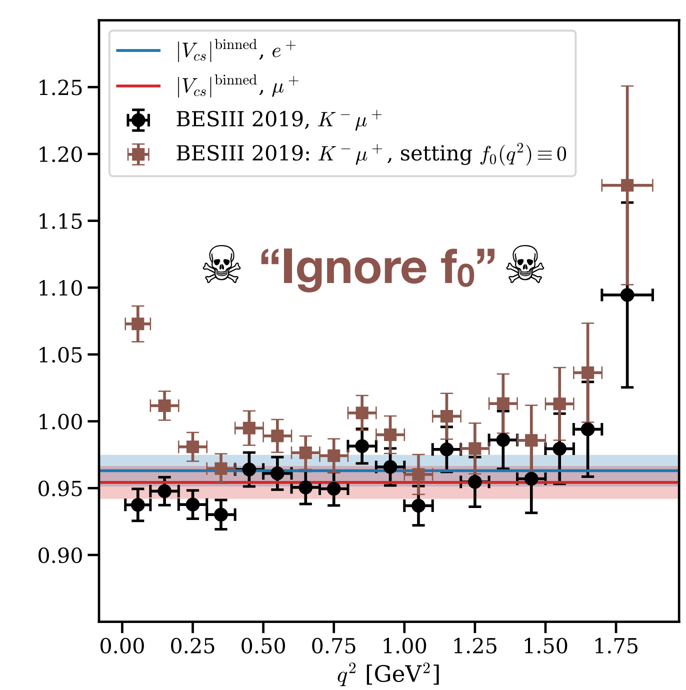
Suppressed scalar-form-factor contributions

$$\propto (m_\ell^2/q^2)\,|f_0|^2$$

These effects were first statistically relevant in the extraction of $|V_{cs}|$ by **HPQCD 2021**

How relevant is f_0?

See, e.g., binwise estimates of $|V_{cs}|$



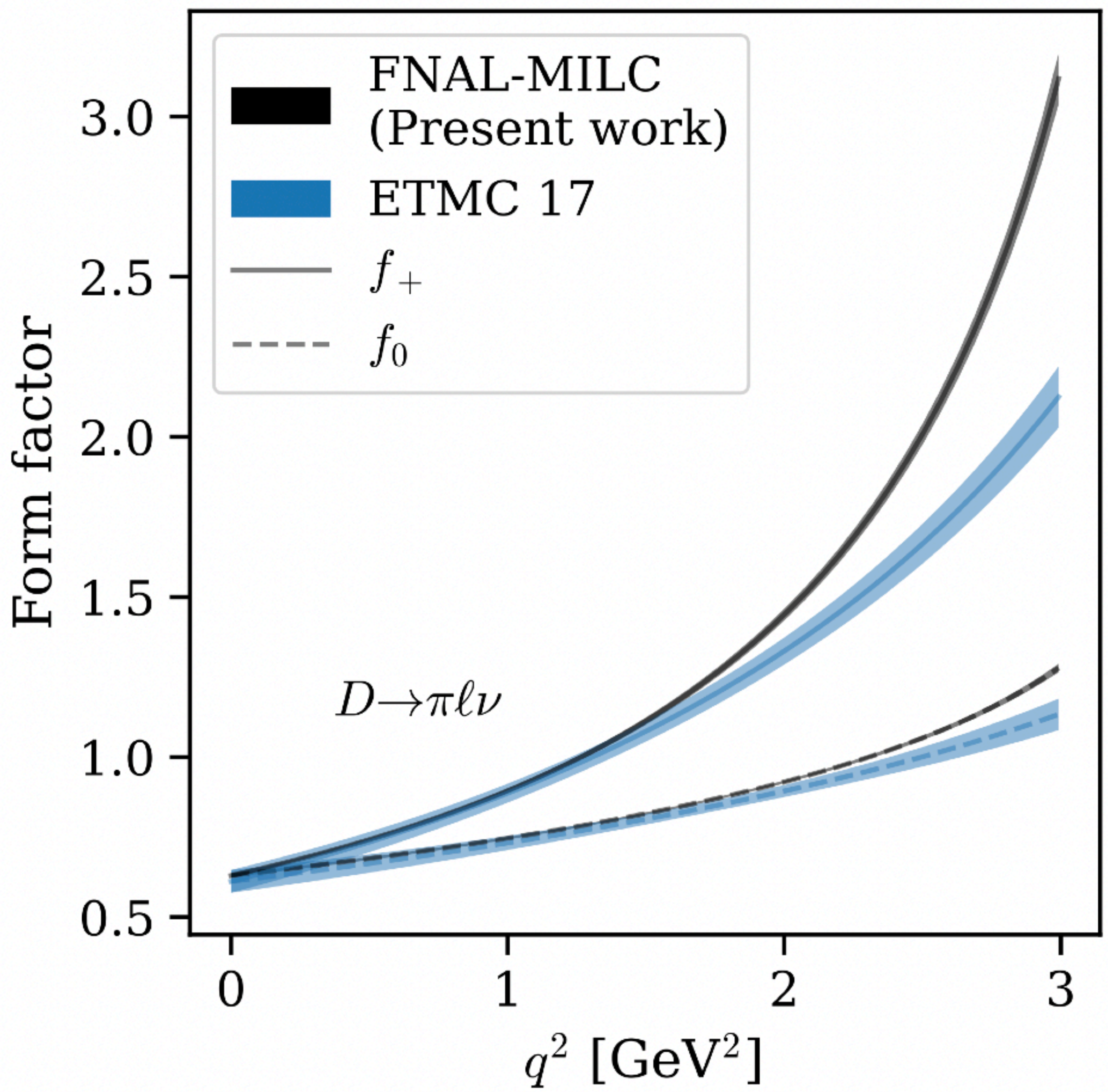
D-meson Semileptonic Decays

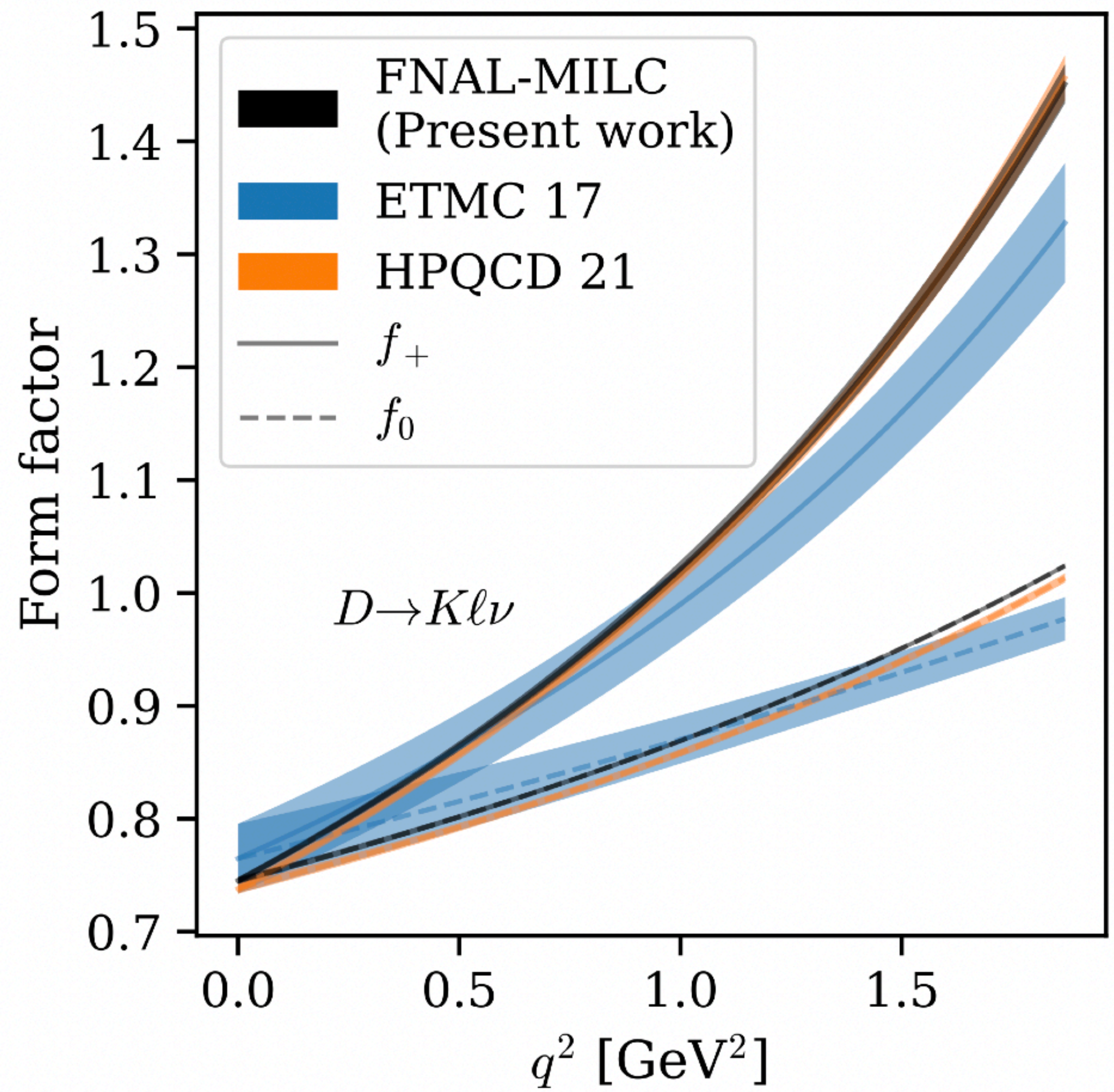
Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

$D_{(s)} \to K/\pi\,\ell\nu$ and $|V_{cd}|$, $|V_{cs}|$

ETMC
PRD 96 (2017) 5, 054514
arXiv:1706.03017

HPQCD
PRD 104 (2021) 3, 034505
arXiv:2104.09883



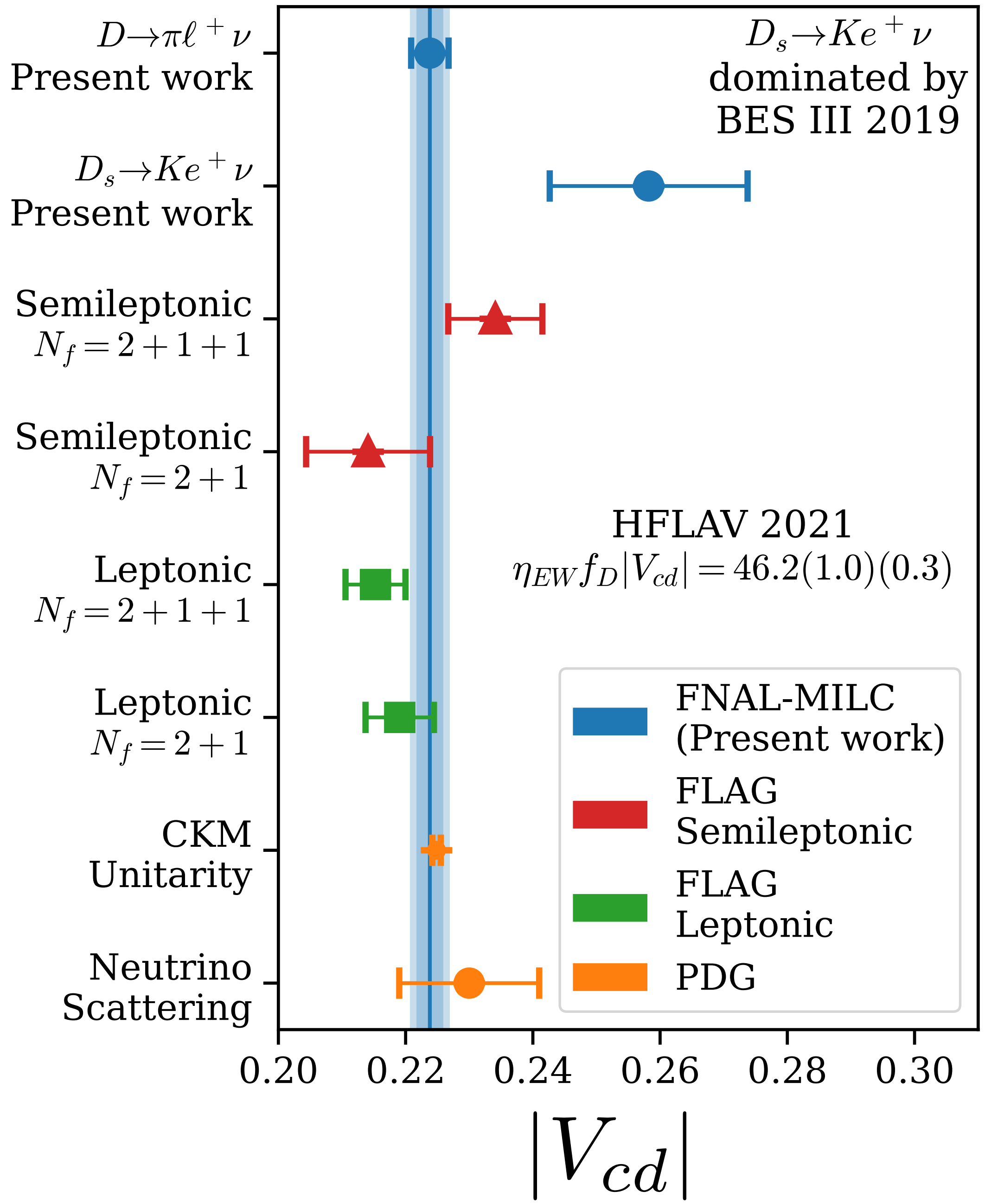


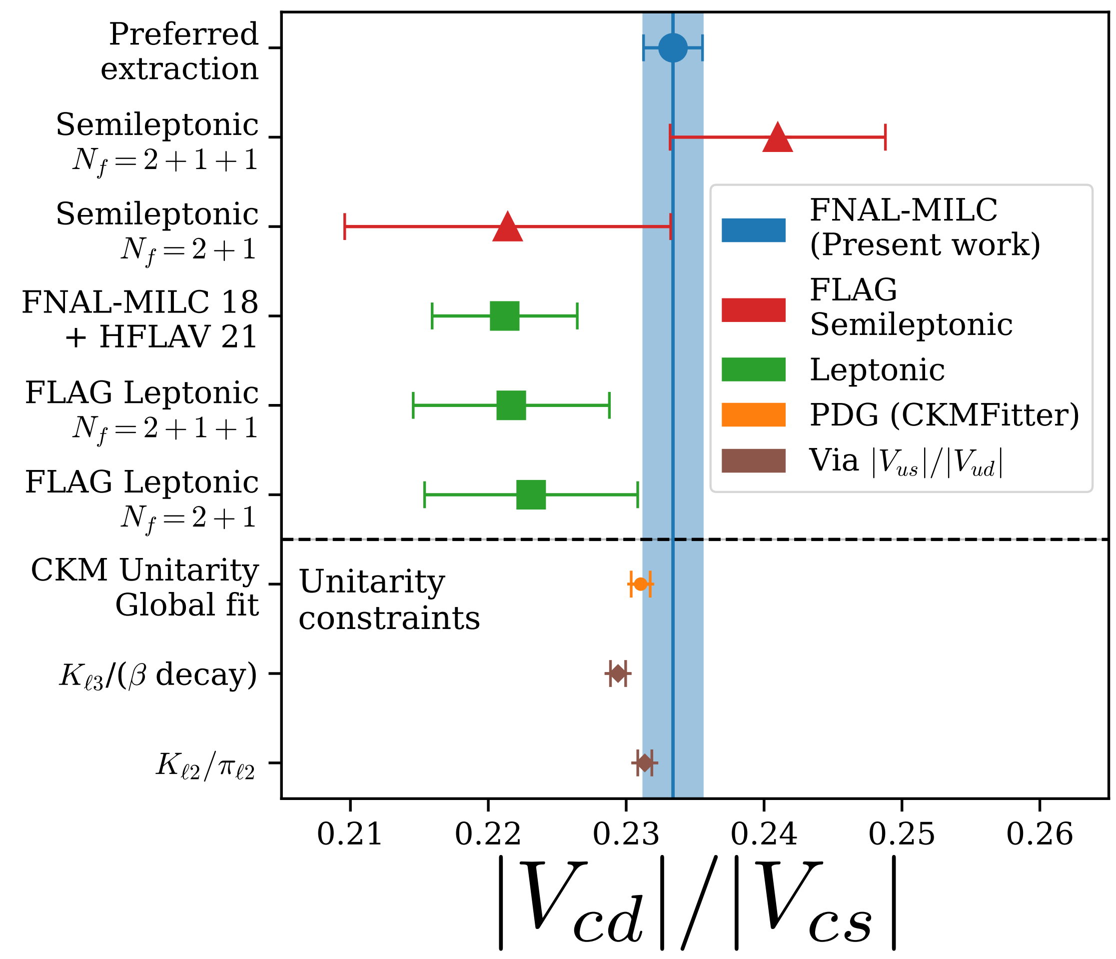
D-meson Semileptonic Decays

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

$D_{(s)} \to K/\pi \ell \nu$ and $|V_{cd}|$, $|V_{cs}|$

PDG particle data group — HFLAV — FLAG Flavour Lattice Averaging Group





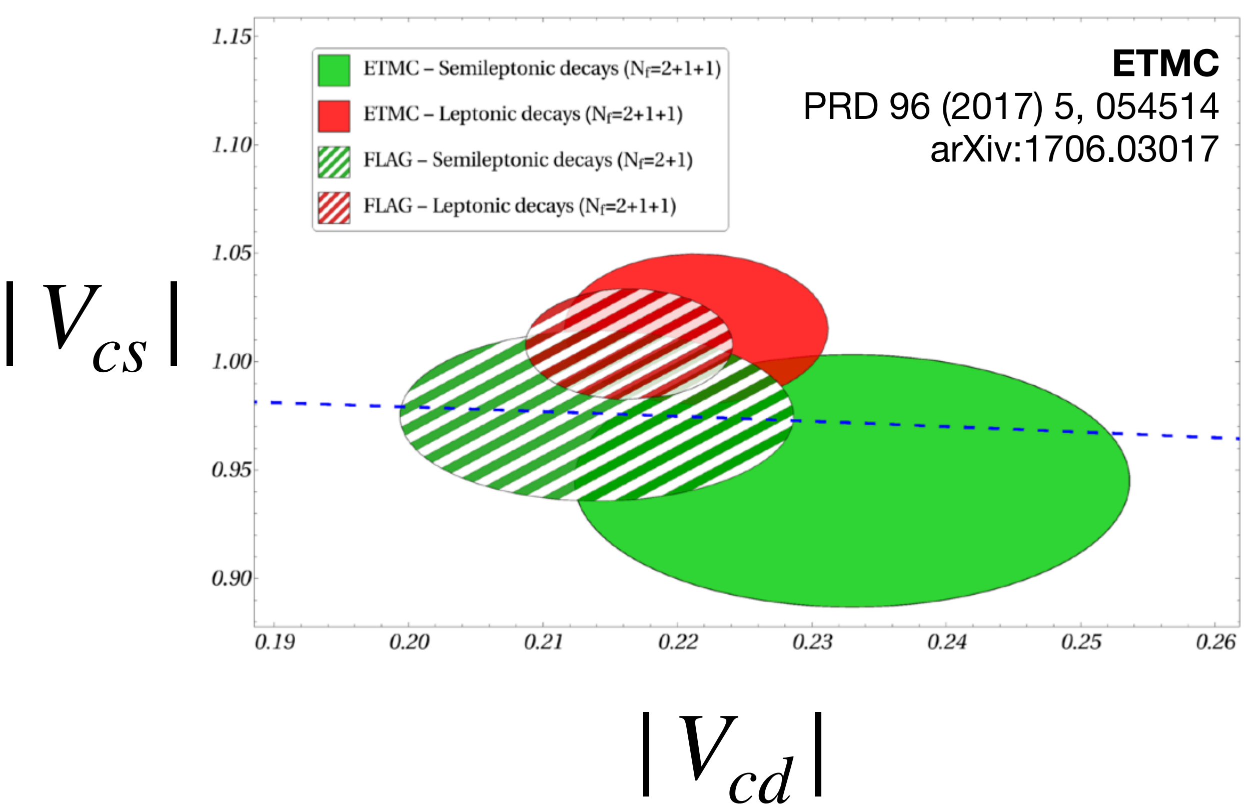
D-meson Semileptonic Decays

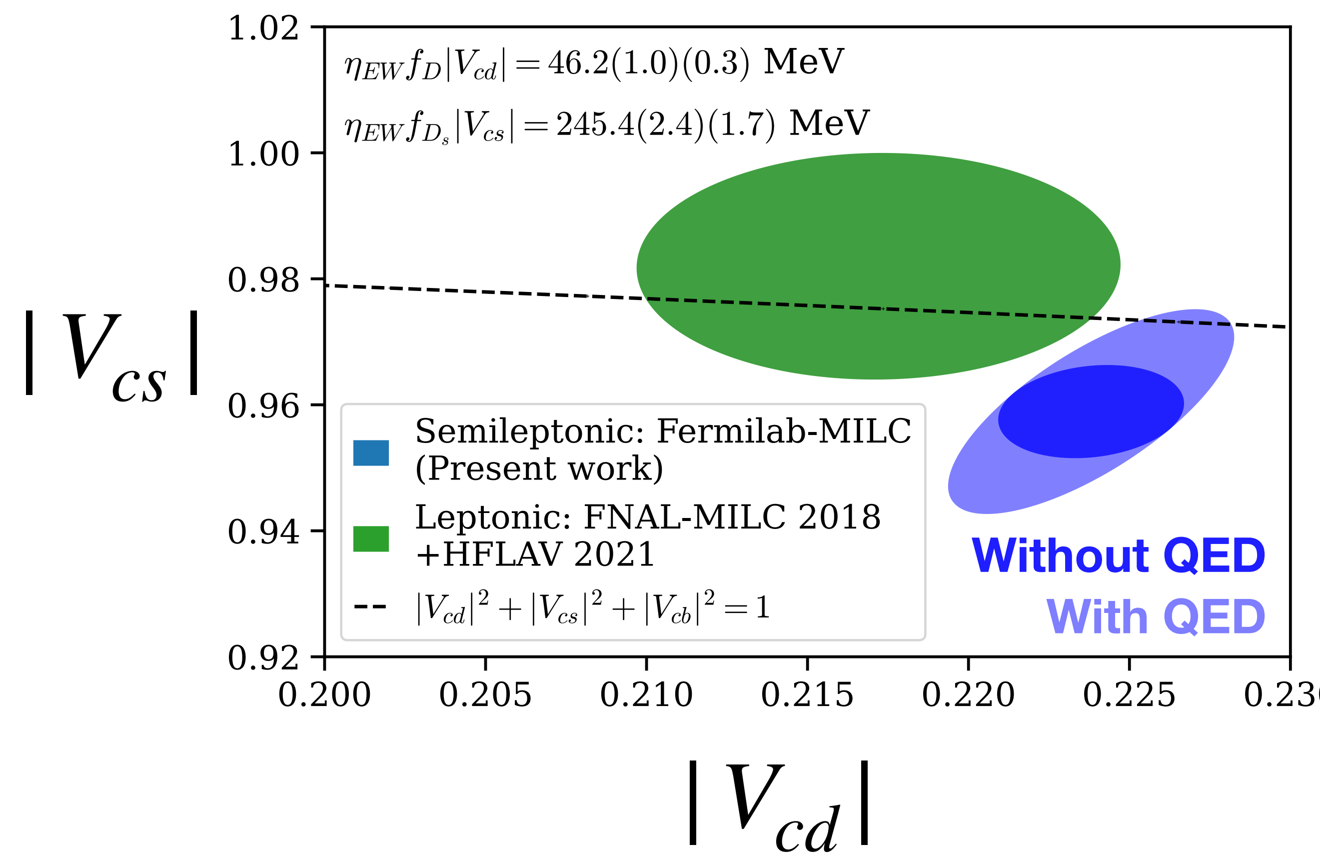
Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

Second-row unitarity tests

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = -0.0286(44)^{\mathrm{EXP}}(78)^{\mathrm{QCD}}[194]^{\mathrm{QED}}(28)^{\mathrm{EW}}$$

- Consistent with unitarity at ≈1σ
- Uncertainty still dominated by theory
- QCD uncertainty subdominant to QED
- $|V_{cd}|/|V_{cs}|$: qualitatively similar arrangement to what was seen by ETMC 2017





D-meson Semileptonic Decays

Lepton Flavor Universality Ratios

Fermilab-MILC [WJ]
PRD 107 (2023) 9, 094516
arXiv:2212.12648

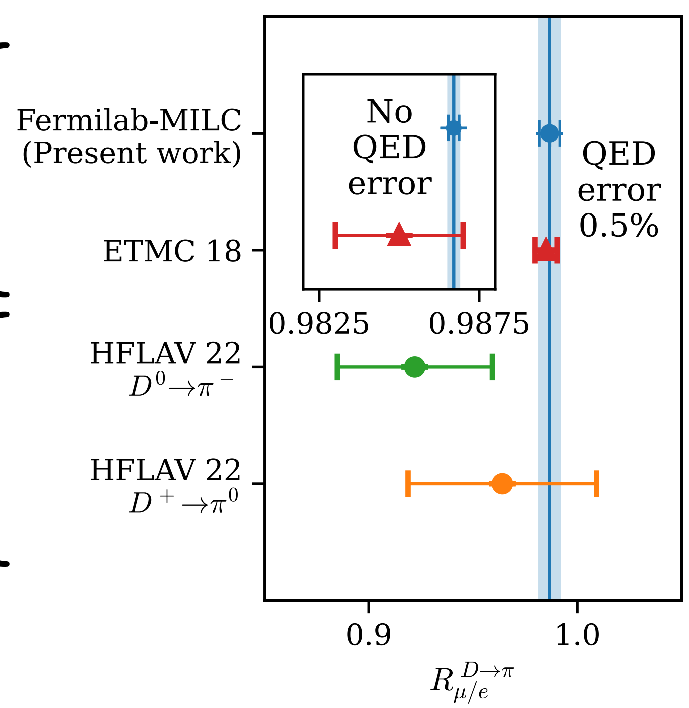
$$R^{H\to L}_{\mu/e} \equiv \frac{\mathscr{B}(H \to L\mu\nu)}{\mathscr{B}(H \to Le\nu)}$$

- CKM factors cancel in the ratio
  → pure theoretical SM predictions are available
- Theoretical uncertainties cancel in the ratio
  → lattice QCD gives very precise results

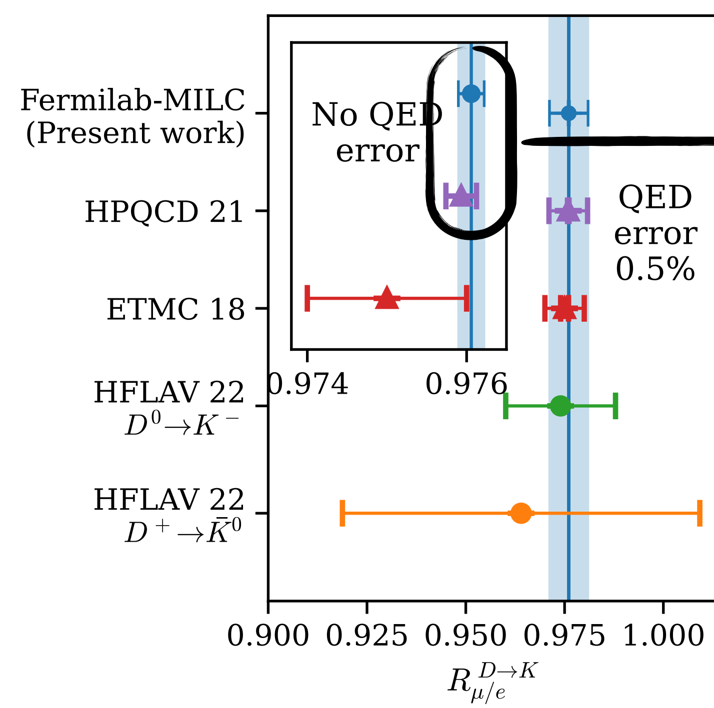
LQCD

Experiment

$D \to \pi$



$D \to K$



LQCD agreement at ≈ 0.01%

D-meson Semileptonic Decays

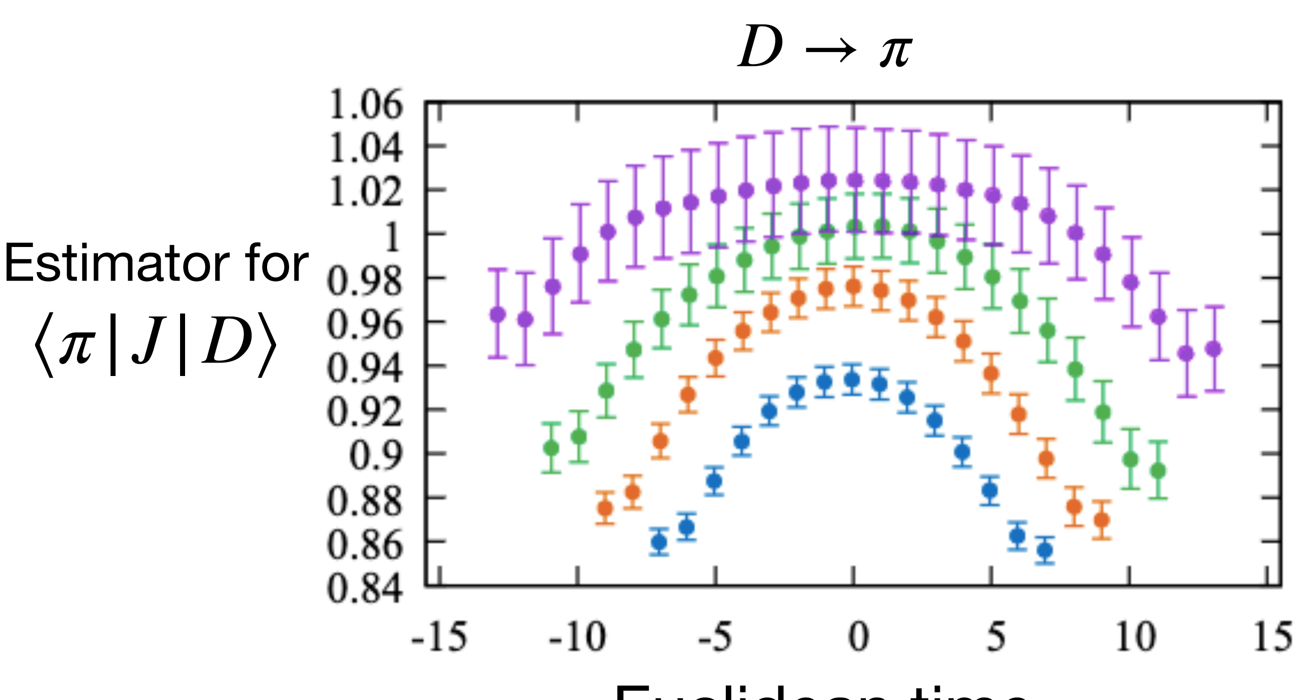
Unpublished & In-progress Calculations

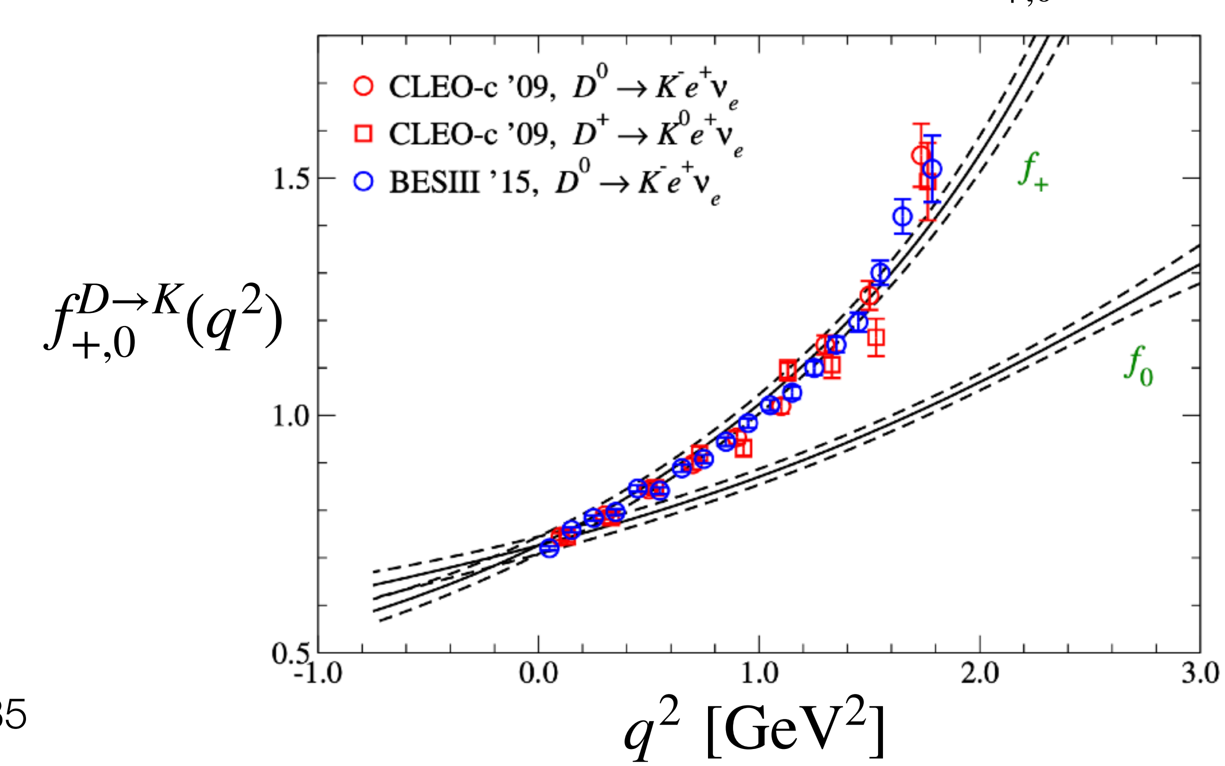
RBC/UKQCD @ Lattice 2021 [arXiv:2201.02680]

- (N_f=2+1) RBC/UKQCD domain-wall quarks
- Valence: domain wall
- Preliminary results on a single ensemble: 1/a ≈ 1.78 GeV
- Results indicate that percent-scale errors are achievable
- Plans in place to extend calculation to additional ensembles
- Precise DWF results will give a valuable check on the recent HISQ results for $D_{(s)} \to K/\pi \, \ell \nu$

JLQCD @ Lattice 2017 [arXiv:1711.11235]

- Unpublished but quite mature/complete results
- (N_f=2+1) JLQCD ensembles with domain-wall quarks
  - 14 total ensembles
  - $1/a \in \{2.5,\ 3.6,\ 4.5\}$ GeV
  - $M_\pi \in [230,\ 500]$ MeV
  - Valence: domain wall
- Form factors in the continuum limit are reported
- Excellent control over systematic effects
- $f_{+,0}^{D\to K/\pi}(0)$ at $\approx 6\,\%$ precision
- 1σ agreement with recent HISQ results for $f_{+,0}^{D\to K/\pi}(0)$





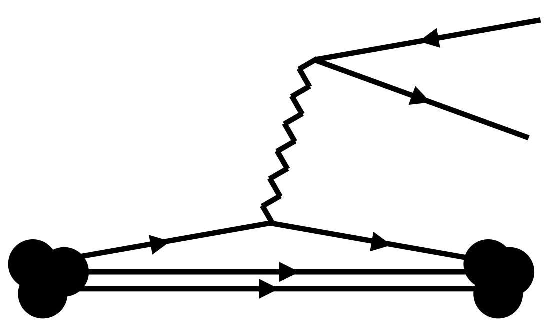
Summary & Outlook

- **Lattice QCD calculations have achieved:**
  - Sub-percent precision for leptonic decays
  - Percent level precision for D-meson semileptonic decays
  - 5-20% precision for D-baryon semileptonic decays
- **Enabling "technologies" for high precision include:**
  - Ensembles with physical mass pions: $M_\pi \approx 140 \text{ MeV}$
  - Relativistic light-quark action(s) for charms: absolutely normalized currents
  - Highly improved actions: small discretization effects for charm
- **Precise LQCD + latest experimental results give:**
  - CKM matrix elements $|V_{cd}|$ and $|V_{cs}|$ at O(1%)
  - Improved tests of second-row unitarity
  - Precise SM predictions of LFU ratio

Backup

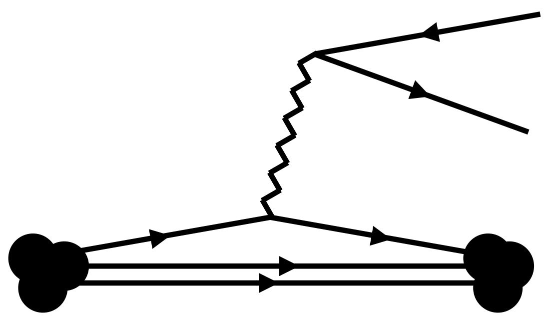
Semileptonic Decays

of D-baryons



$\mathbf{V_{ud}}$ $\pi \to \ell\nu$	$\mathbf{V_{us}}$ $K \to \ell\nu$ $K \to \pi\ell\nu$	$\mathbf{V_{ub}}$ $B \to \ell\nu$ $B \to \pi\ell\nu$ $\Lambda_b \to p\ell\nu$
$\mathbf{V_{cd}}$ $D \to \ell\nu$ $D \to \pi\ell\nu$ $D_s \to K\ell\nu$ $\Lambda_c \to N\ell\nu$	$\mathbf{V_{cs}}$ $D_s \to \ell\nu$ $D \to K\ell\nu$ $\Lambda_c \to \Lambda\ell\nu$ $\Xi_c \to \Xi\ell\nu$	$\mathbf{V_{cb}}$ $B \to D\ell\nu$ $B \to D^\star\ell\nu$ $\Lambda_b \to \Lambda_c\ell\nu$
$\mathbf{V_{td}}$ $\langle B_d \vert \bar{B}_d \rangle)$	$\mathbf{V_{ts}}$ $\langle B_s \vert \bar{B}_s \rangle)$	$\mathbf{V_{tb}}$

Semileptonic Decays
of D-baryons



$$\langle \Lambda \,|\, \bar{s}\gamma^{\mu}(1-\gamma_5)c \,|\, \Lambda_c \rangle$$

Vector form factors: $f_{+,0,\perp}$

Axial form factors: $g_{+,0,\perp}$

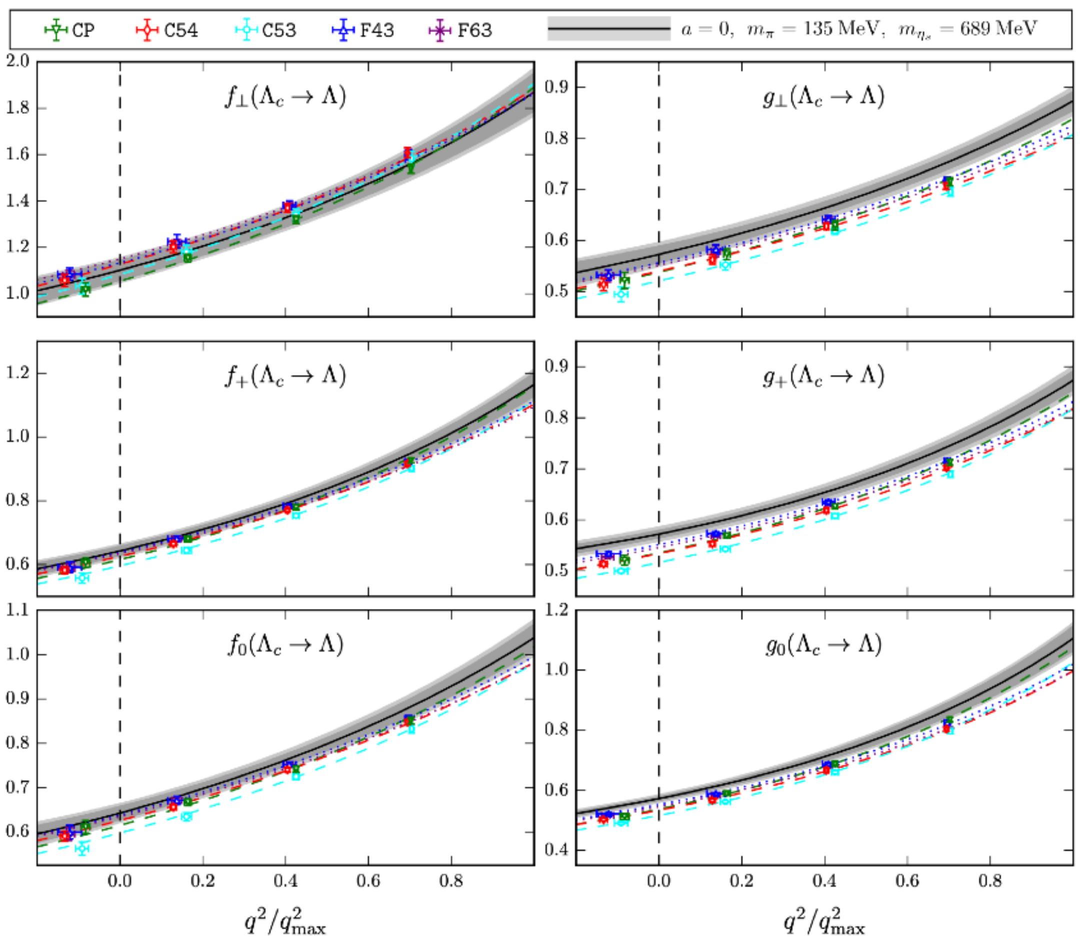
D-baryon semileptonic decays

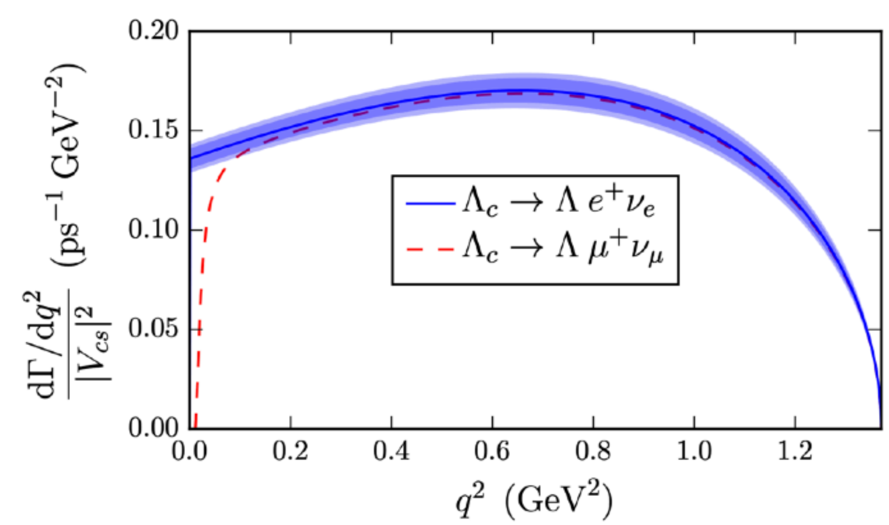
S. Meinel
PRL 118 (2017) 8, 082001
arXiv:1611.09696

$\Lambda_c \to \Lambda \ell \nu$

- 5x ensembles, $N_f = 2+1$ domain wall fermions
  - $a \in \{0.09, 0.11\}$ fm
  - $M_\pi \in \{139 - 350\}$ MeV
- Valence charm: Columbia RHQ (clover action, tuned to give J/ψ dispersion relation)
- "Mostly non-perturbative" renormalization
- First-ever determination of $|V_{cs}|$ [≈6%] from baryon decays when combined with measurements from BESIII

$$|V_{cs}| = \begin{cases} 0.951(24)_{\rm LQCD}(14)_{\tau_{\Lambda_c}}(56)_{\mathcal{B}}, & \ell = e, \\ 0.947(24)_{\rm LQCD}(14)_{\tau_{\Lambda_c}}(72)_{\mathcal{B}}, & \ell = \mu, \\ 0.949(24)_{\rm LQCD}(14)_{\tau_{\Lambda_c}}(49)_{\mathcal{B}}, & \ell = e, \mu, \end{cases}$$



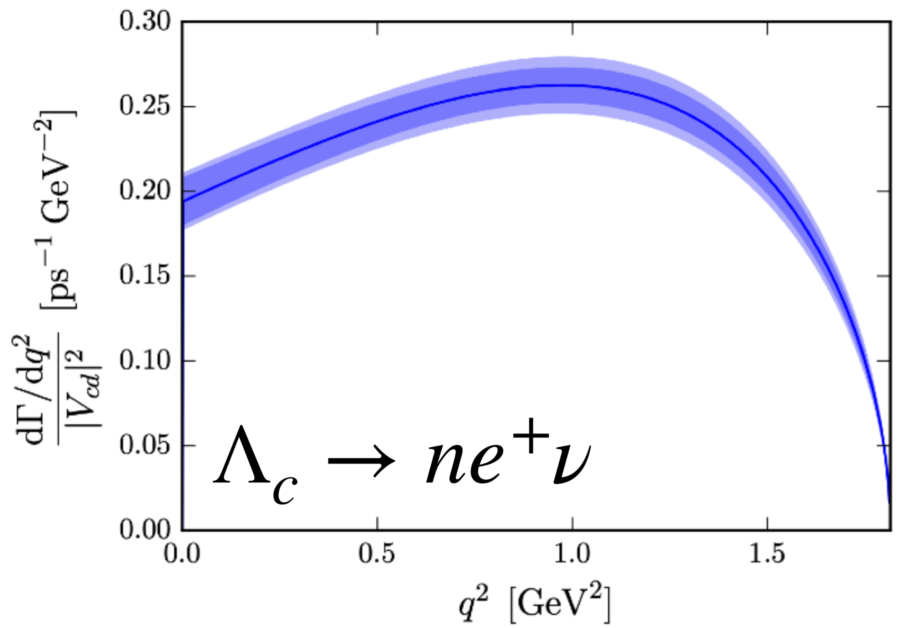


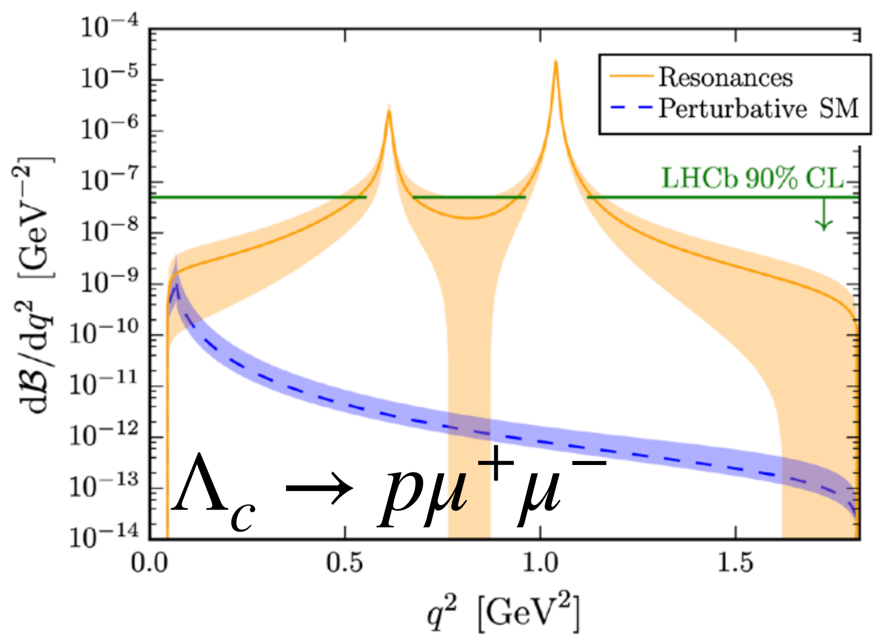
D-baryon semileptonic decays

S. Meinel
PRD 97 (2018) 3, 034511
arXiv:1712.05783

$\Lambda_c \to N$ form factors

- Isospin limit: same form factors for $\Lambda_c \to p^+$, $\Lambda_c \to n$
- 6x ensembles, $N_f = 2+1$ domain wall fermions
  - $a \in \{0.09, 0.11\}$ fm
  - $M_\pi \in \{240 - 350\}$ MeV
- Valence charm: Columbia RHQ
- "Mostly non-perturbative" renormalization
- SM predictions for charged-current $\Lambda_c \to n\ell^+\nu$ rates [≈6.4%]
  - $\Gamma(\Lambda_c \to ne^+\nu)/|V_{cd}|^2 = (0.405 \pm 0.016_{\text{stat}} \pm 0.020_{\text{syst}})\ \text{ps}^{-1}$
  - Tough to measure experimentally (n and ν in final state)
  - Results larger by factor of ≈1.5–2 compared to other calculations [quark models, sum rules, SU(3)]
- Rare neutral-current decay:
  - LHCb 2018: $\mathscr{B}(\Lambda_c \to p^+\mu^+\mu^-) < 7.7 \times 10^{-8}$ [90%]
  - Comparison to LQCD with additional assumptions
    - SM Wilson coefficients at NLO
    - Breit-Wigner model for intermediate $\phi/\omega/\rho$





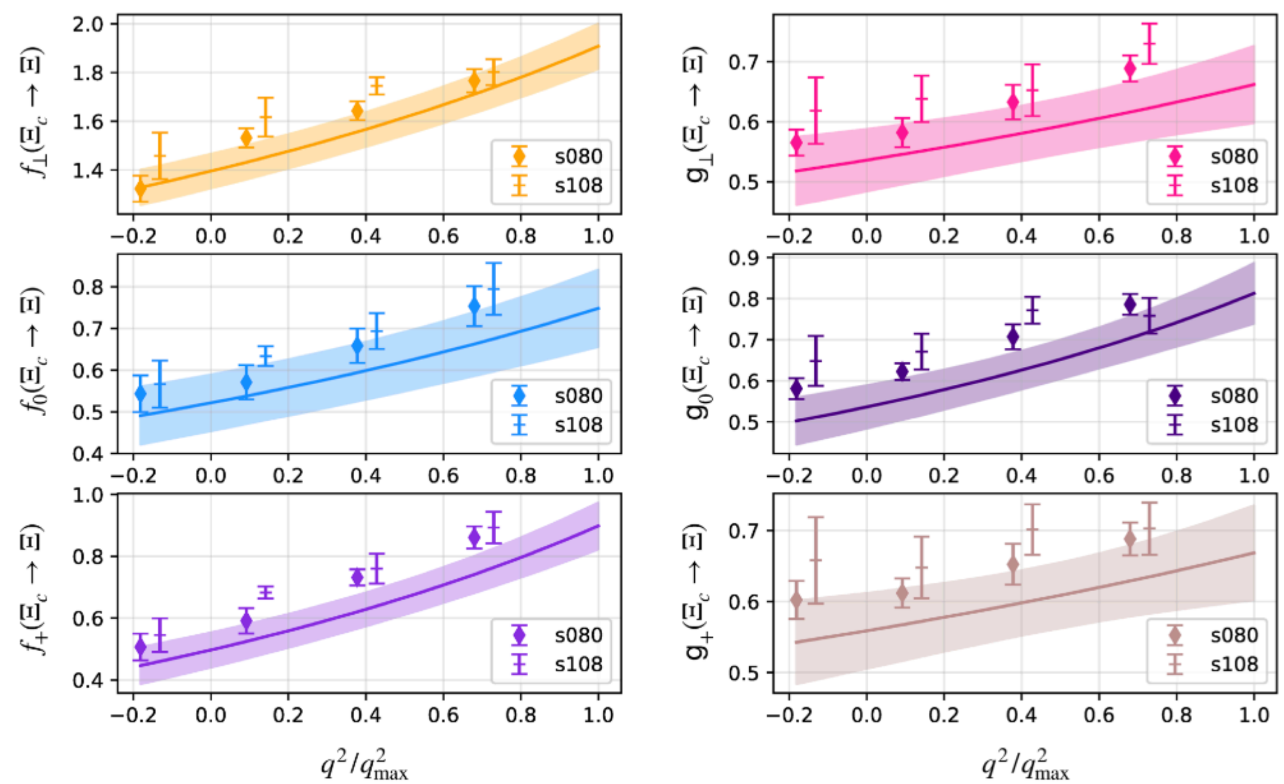
LHCb
PRD 97 (2018) 9, 091101
arXiv:1712.07938

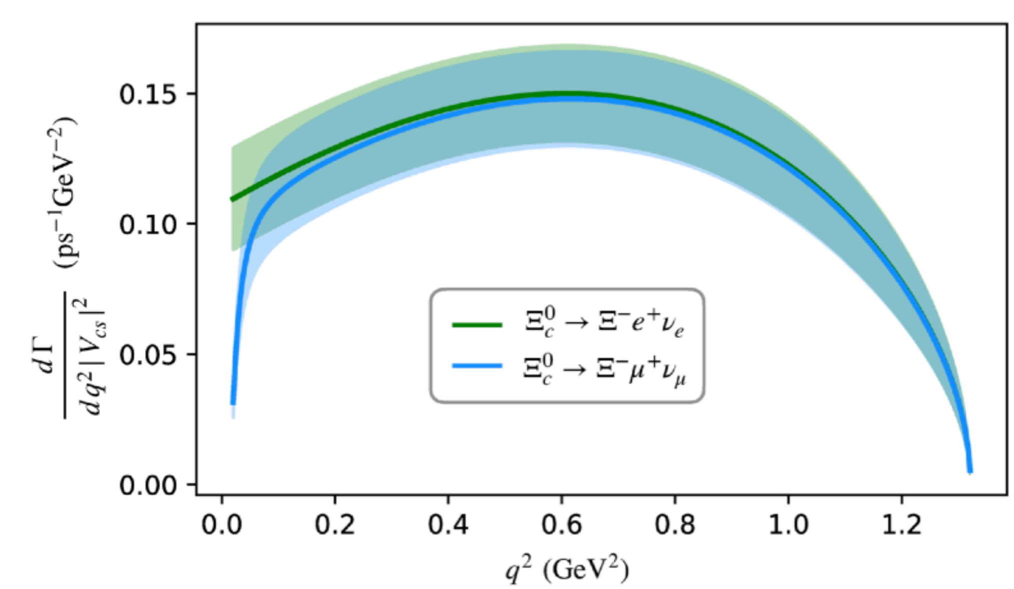
D-baryon semileptonic decays

Q.-A. Zhang et al.
Chin.Phys.C 46 (2022) 1, 011002
arXiv:2103.07064

$\Xi_c \to \Xi \ell \nu$ form factors

- 2x ensembles with N_f=2+1 Wilson clover quarks
  - $a \in \{0.11, 0.08\}$ fm
  - $M_\pi \approx 300$ MeV
- Continuum extrapolation is given
- No chiral extrapolation to physical pion mass
- Extractions of $|V_{cs}|$:
  - Using ALICE branching-fraction measurements: $|V_{cs}| = 0.983(0.060)^{\mathrm{stat}}(0.065)^{\mathrm{syst}}(0.167)^{\mathrm{exp}}$ [≈19%]
  - Using Belle branching-fraction measurements $|V_{cs}| = 0.834(0.051)^{\mathrm{stat}}(0.056)^{\mathrm{syst}}(0.127)^{\mathrm{exp}}$ [≈18%]

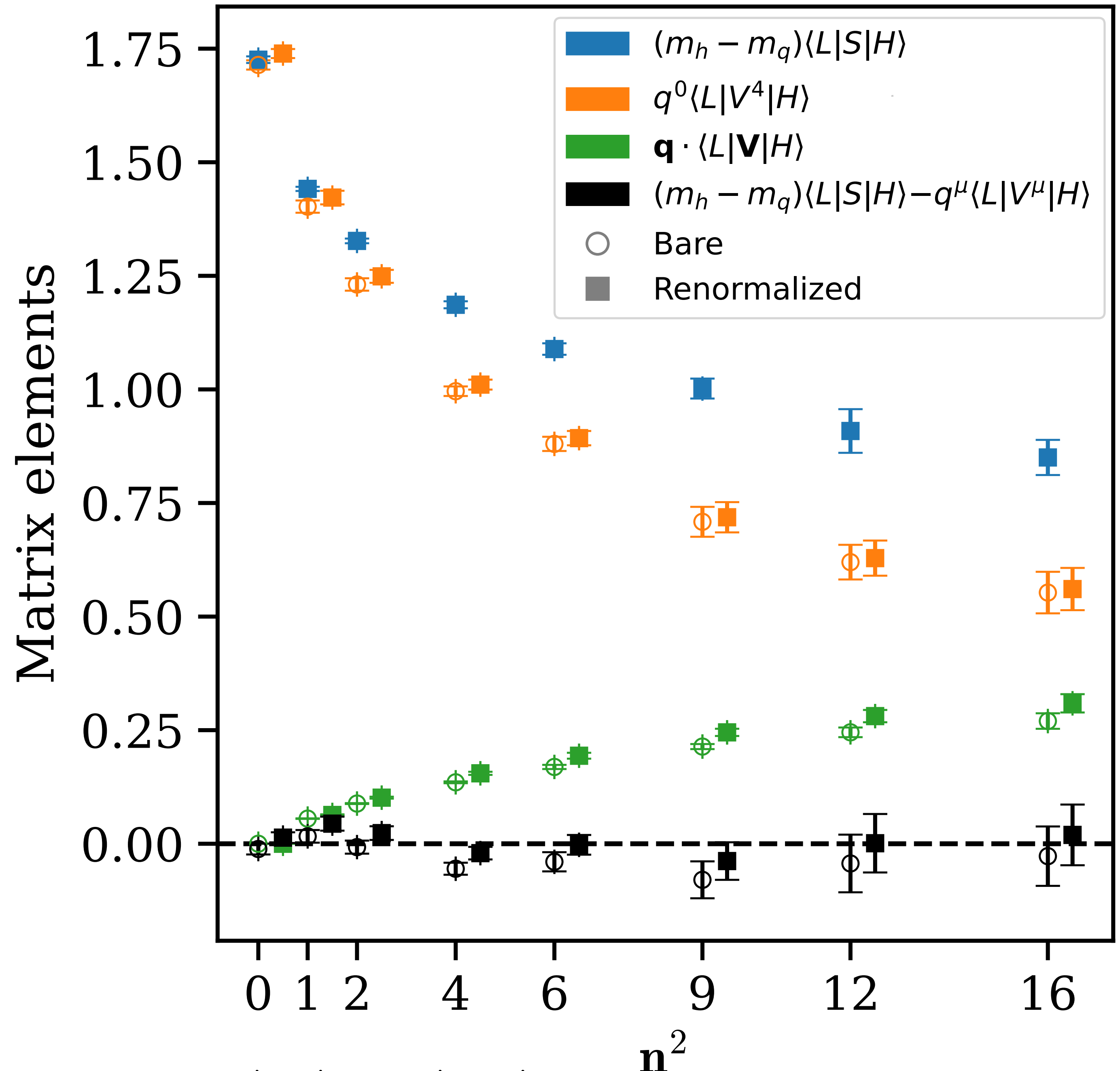




Renormalization semileptonic decays

Example $D \to \pi \ell \nu$

- Recall $\mathcal{J} = Z_J J$
- PVCV: $\partial_\mu \mathcal{V}^\mu = (m_1 - m_2)\mathcal{S}$
- For the HISQ action, the local scalar density is absolutely normalized.
- Imposing PCVC in a global fit gives values for Z_{V_0} and Z_{V_i}
- In terms of D→π matrix elements, PCVC reads:

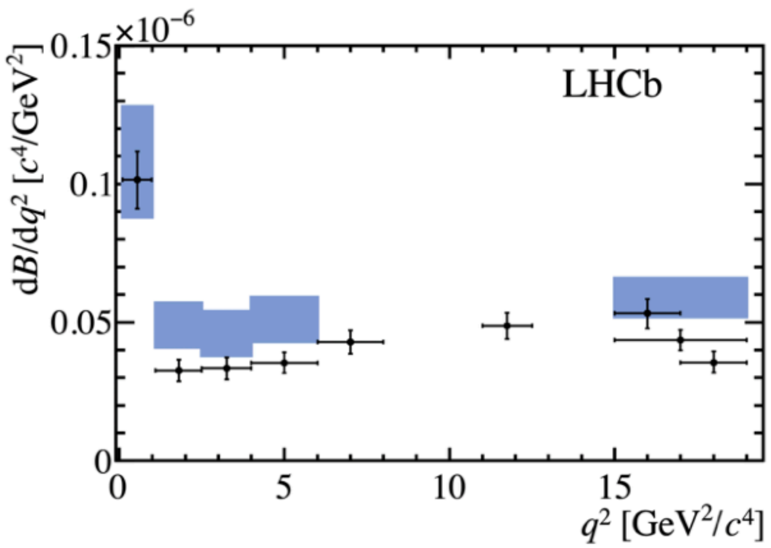


$$Z_{V^0}(M_D - E_\pi)\left\langle \pi \right| V^0 \left| D \right\rangle + Z_{V^i}\boldsymbol{q}\cdot\left\langle \pi \right| \boldsymbol{V} \left| D \right\rangle = (m_c - m_d)\left\langle \pi \right| S \left| D \right\rangle$$

Branching fraction tensions

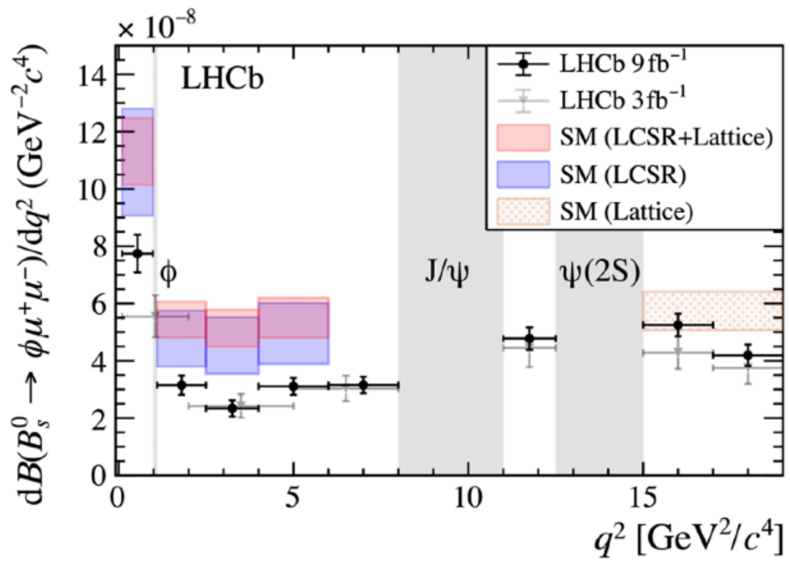
$B^0 \to K^{\star 0}\mu\mu$

LHCb *JHEP* 11 (2016) 047
LHCb *JHEP* 04 (2017) 142



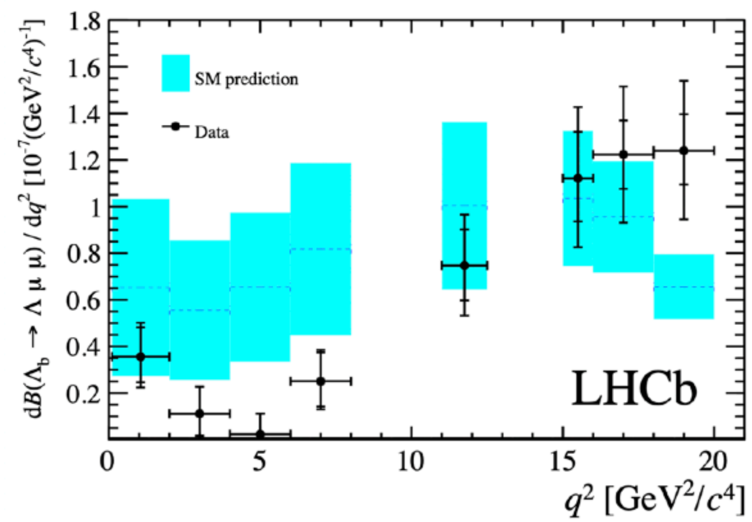
$B_s^0 \to \varphi\mu\mu$

LHCb *JHEP 09 (2015) 179*
LHCb *PRL* 127 (2021) 15, 151801

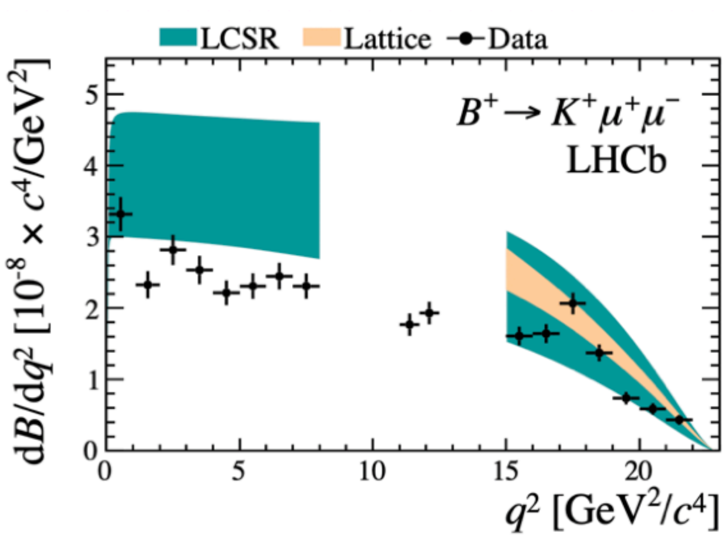


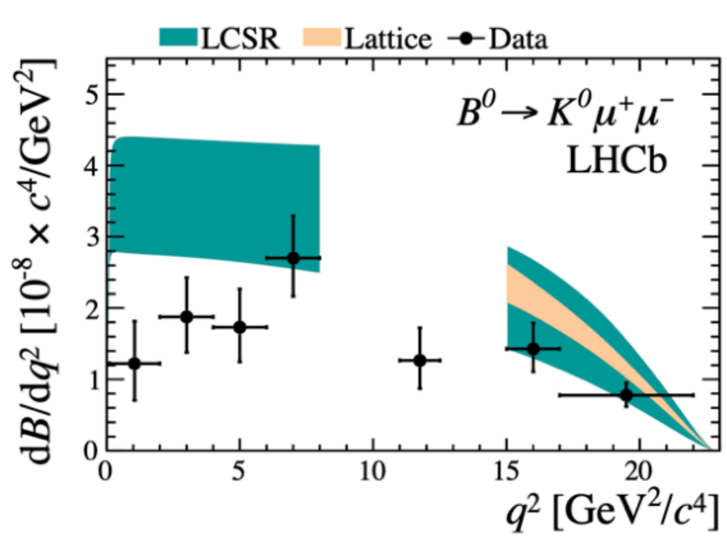
$\Lambda_b^0 \to \Lambda^0\mu\mu$

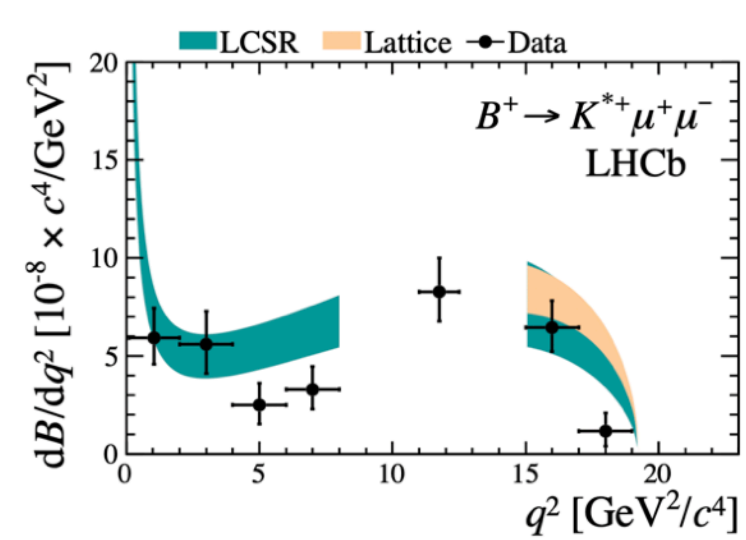
LHCb *JHEP* 06 (2015) 115



LHCb *JHEP* 06 (2014) 133



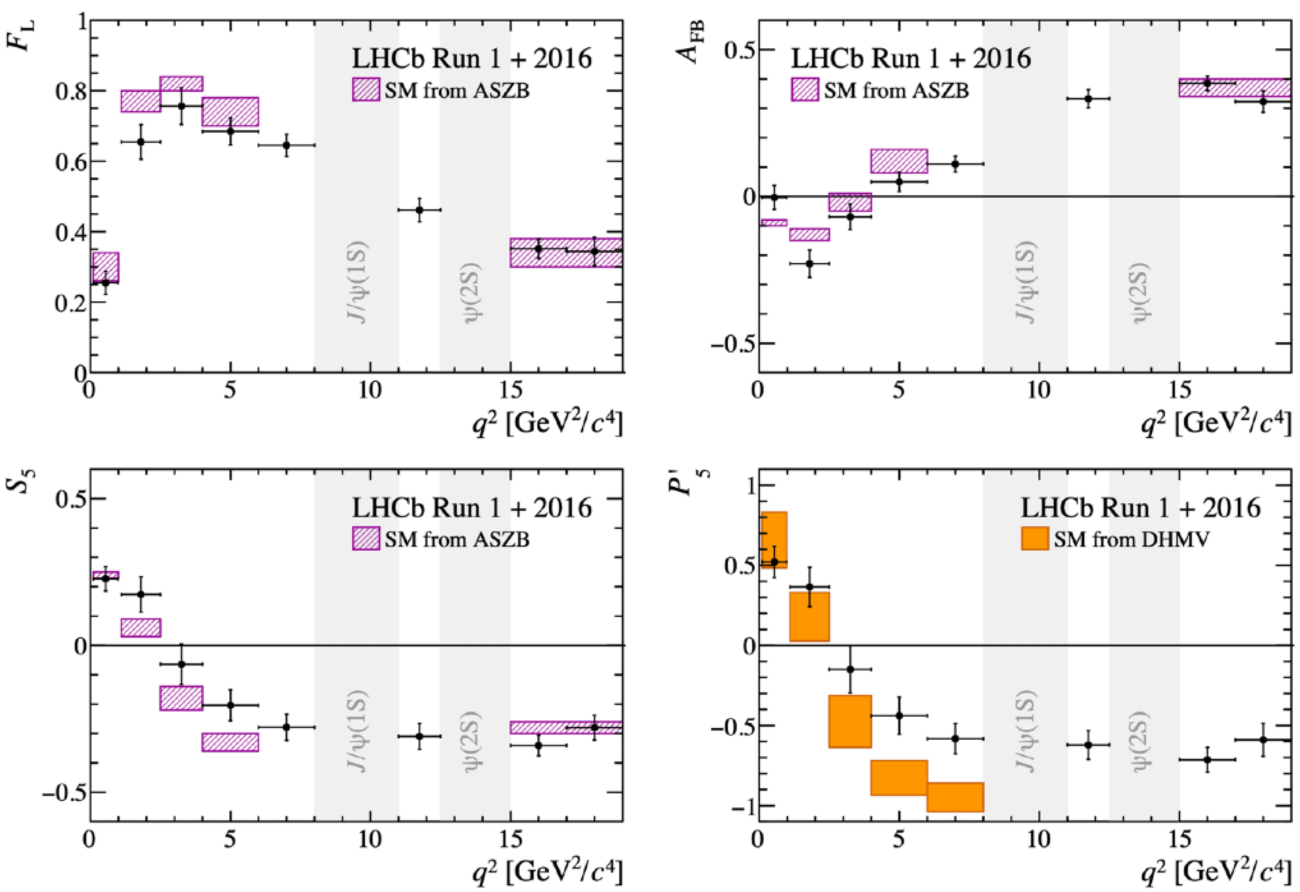




Angular Tensions

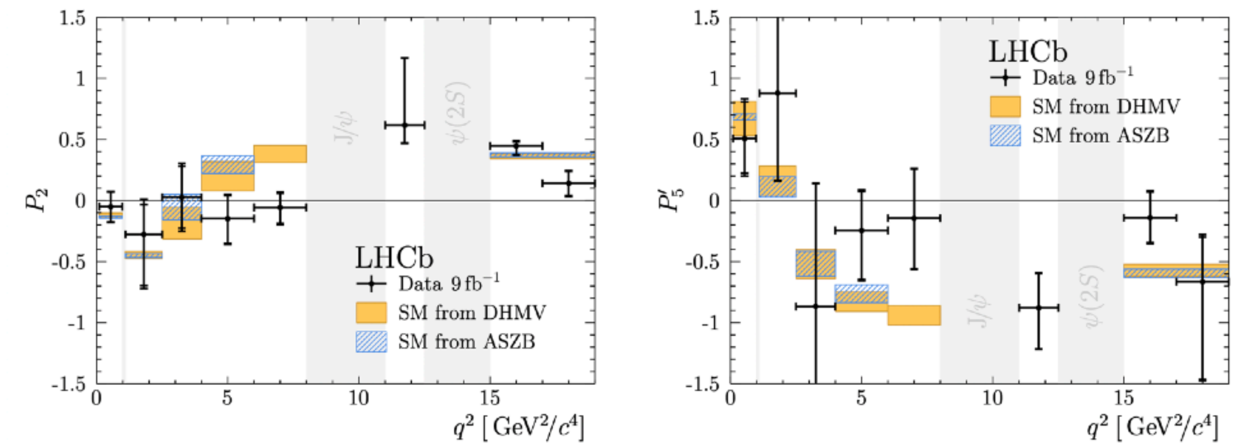
$B^0 \to K^{\star 0}\mu\mu$

LHCb *PRL* 125 (2020) 011802



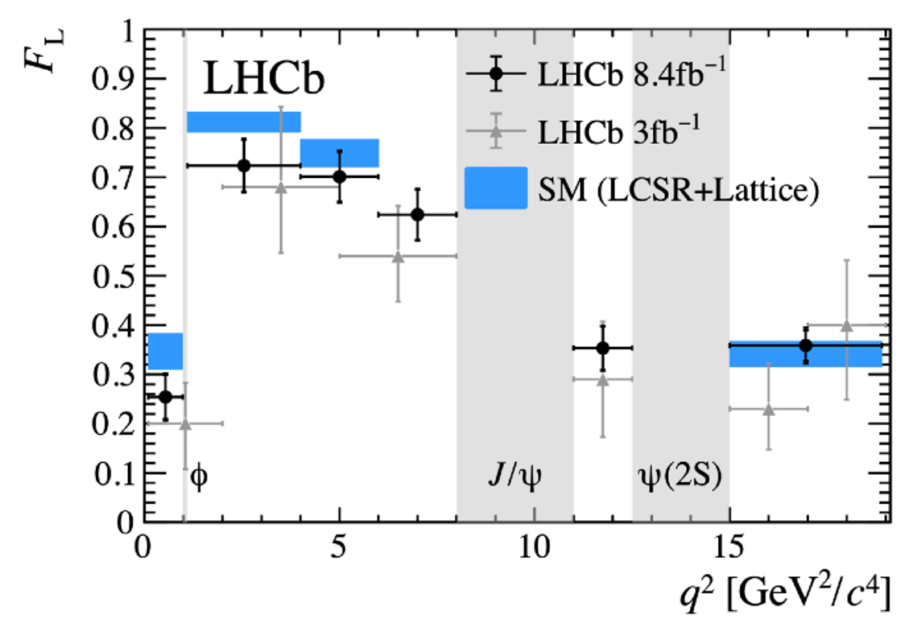
$B^+ \to K^{\star +}\mu\mu$

LHCb PRL 126 (2021) 161802



$B_s^0 \to \varphi\mu\mu$

LHCb *JHEP* 11 (2021) 043

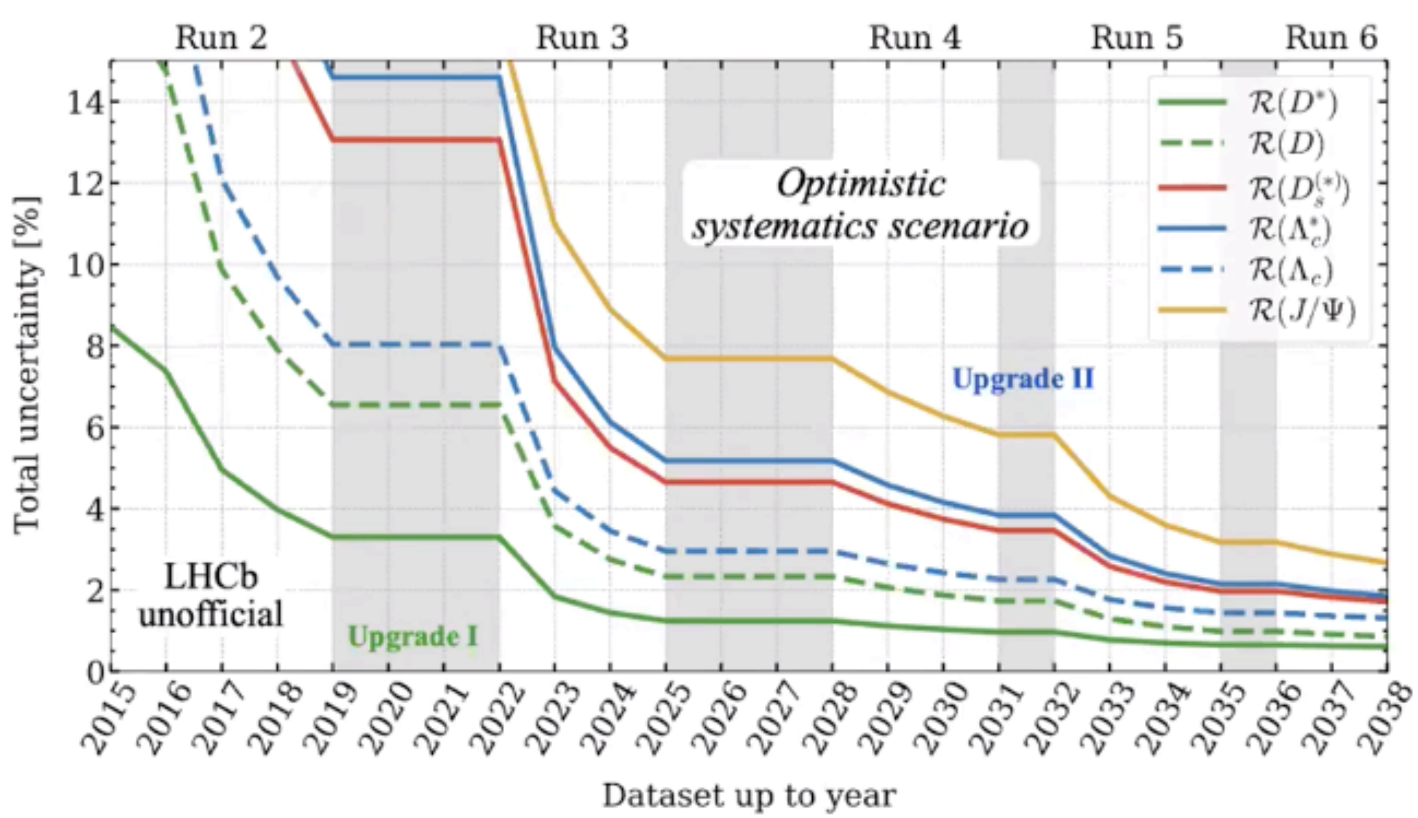


Note: Evidence for LFUV in $b \to s\ell\ell$ is gone after LHCb arXiv:2212.09153

Culprit: residual from mis-ID of hadronic backgrounds

Additional Experimental Prospects

- LHCb: pp at LHC
  - ~10^{12} b-hadrons to date (cf. ~10^{7} at LEP)
- Belle II: $e^{+}e^{-}$ around $\Upsilon(4s)$ ~ 10.5 GeV
  - Goal: 50 ab^{-1} (50x Belle), roughly 215 fb^{-1} to date



Many exciting first measurements.
For example:

- BESIII: Form factors for $D_s \to K^{(*)} e \nu$
  - PRL 122, 061801 arXiv:1811.02911
- LHCb: Rare CKM suppressed $B \to \pi \mu^+ \mu^-$
  - JHEP 10 (2015) 034 arXiv:1509.00414

LQCD precision achievements over time

CSS2013: Snowmass on the Mississippi
S. Butler et al [arXiv:1311.1076]

		2013		2013 Expected		Achieved
Quantity	CKM element	~~Present~~ expt. error	2007 forecast lattice error	~~Present~~ lattice error	2018 lattice error	2021 FLAG avg
f_K/f_π	$\|V_{us}\|$	0.2%	0.5%	0.5%	0.15%	0.18%
$f_+^{K\pi}(0)$	$\|V_{us}\|$	0.2%	–	0.5%	0.2%	0.18%
f_D	$\|V_{cd}\|$	4.3%	5%	2%	$< 1\%$	0.3%
f_{D_s}	$\|V_{cs}\|$	2.1%	5%	2%	$< 1\%$	0.2%
$D \to \pi \ell \nu$	$\|V_{cd}\|$	2.6%	–	4.4%	2%	4.4%
$D \to K \ell \nu$	$\|V_{cs}\|$	1.1%	–	2.5%	1%	0.6%
$B \to D^* \ell \nu$	$\|V_{cb}\|$	1.3%	–	1.8%	$< 1\%$	1.7%
$B \to \pi \ell \nu$	$\|V_{ub}\|$	4.1%	–	8.7%	2%	3%
f_B	$\|V_{ub}\|$	9%	–	2.5%	$< 1\%$	0.7%
ξ	$\|V_{ts}/V_{td}\|$	0.4%	2-4%	4%	$< 1\%$	1.3%
ΔM_s	$\|V_{ts}V_{tb}\|^2$	0.24%	7–12%	11%	5%	4.5%
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	$< 1\%$	1.3%

Systematic inclusion of QED now becomes necessary

Recently improved!

Broad community effort to:
- keep pace with experimental needs
- achieve ~1% precision

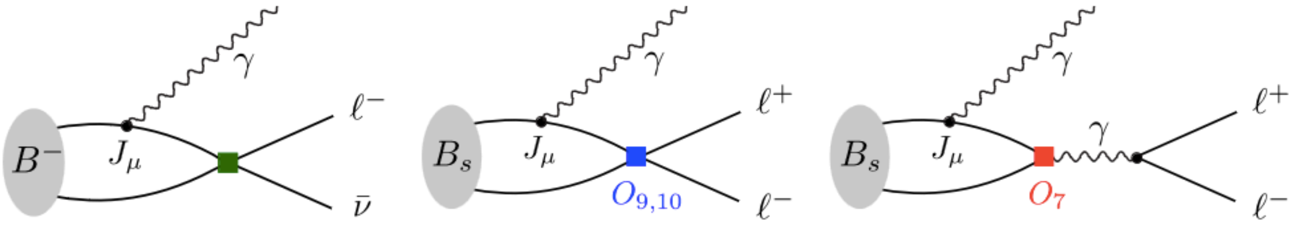
- LQCD precision: expected improvements from ~10 years ago have largely been achieved.
- In-progress calculations expect to reach ≲ 1% level for semileptonic B-decays

Radiative Leptonic Decays

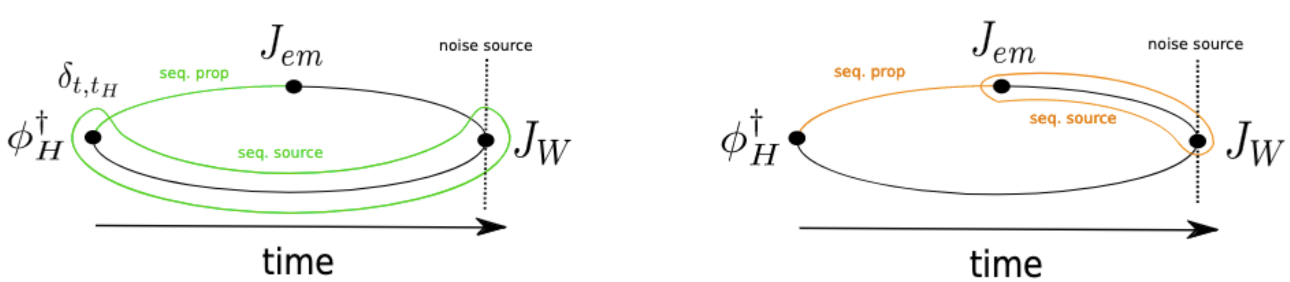
$D_s \to \ell\nu\gamma, K \to \ell\nu\gamma$

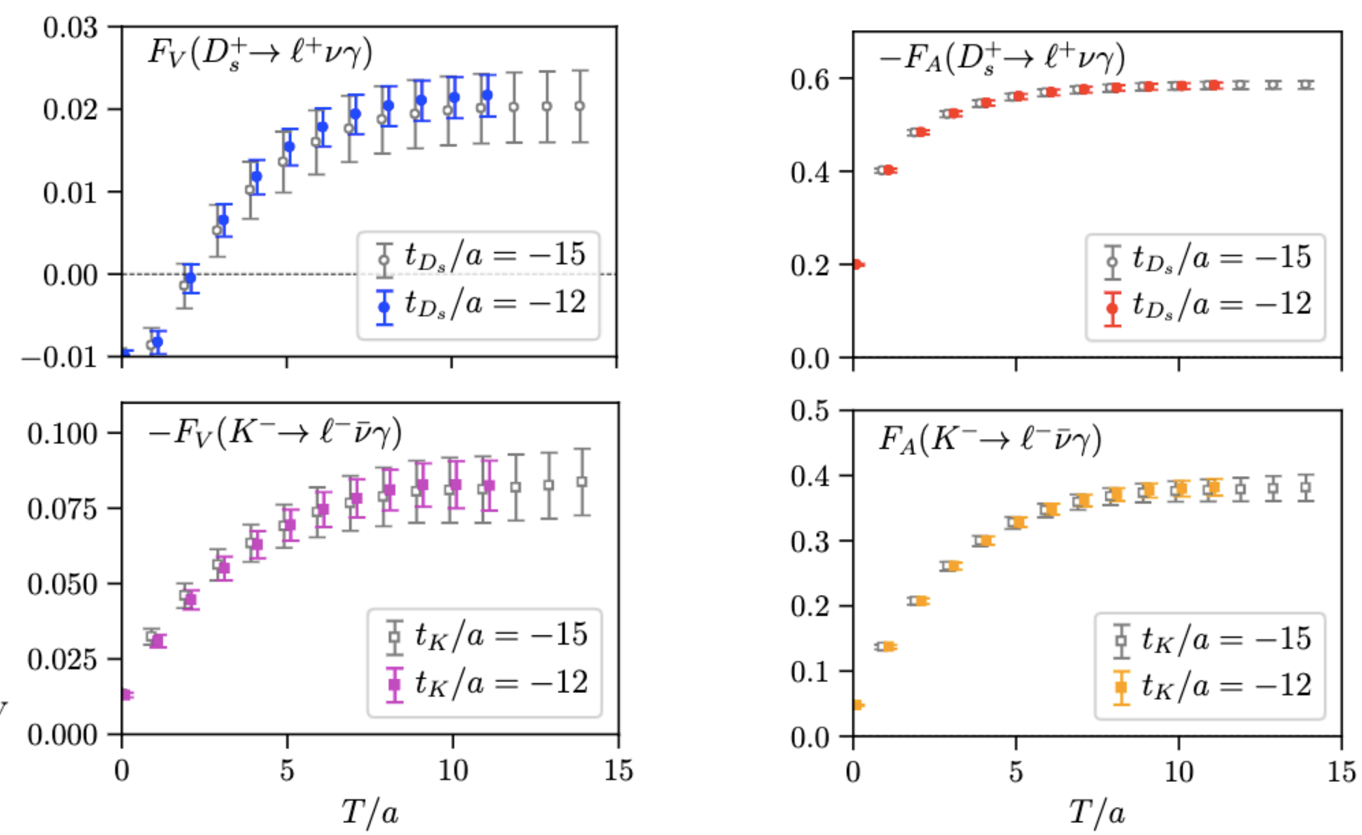
Kane, Lehner, Meinel, Soni
Lattice 2019
arXiv:1907.00279
Kane, Giusti, Lehner, Meinel, Soni
Lattice 2021
arXiv:2110.13196

- Radiative decays probe weak interaction and hadronic structure
- Example: $B \to \ell\nu\gamma$ is sensitive to the LCDA parameter λ_B
- Radiative leptonic decays probe all Wilson coefficients in the Weak effective Hamiltonian
- Exploratory calculations developing methods



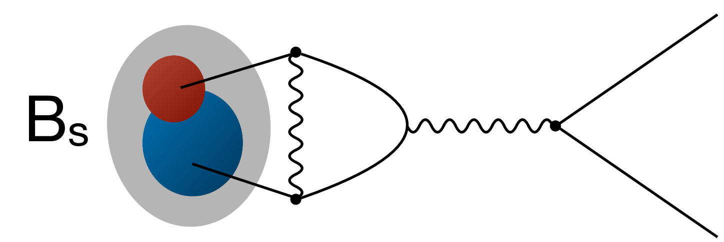
$$T_{\mu\nu} = -i \int d^4x e^{ip_\gamma \cdot x} \langle 0 | T\{J_\mu(x) J_\nu^{\text{weak}}(0)\} | H(\mathbf{p}) \rangle$$





Rare Decay $B_s \to \mu^+\mu^-$

Uncertainty Breakdown



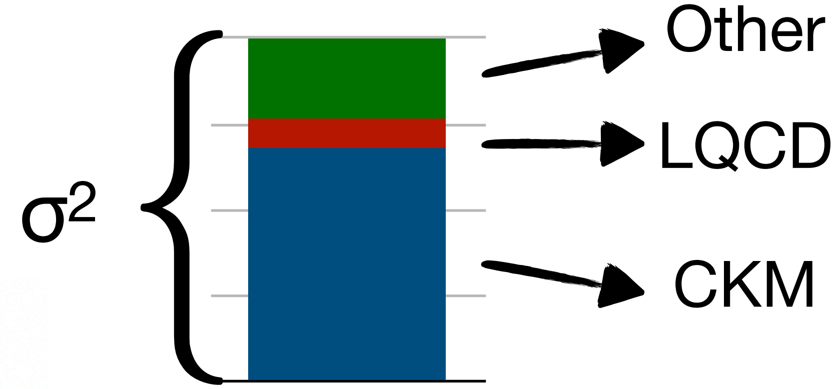
SM prediction for rare leptonic decay rate

Beneke et al, arXiv:1908.07011, JHEP 2019

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = 3.660(38) \times 10^{-9}$$

$$\overline{\mathrm{Br}}^{(0)}_{s\mu} = \begin{pmatrix} 3.599 \\ 3.660 \end{pmatrix} \Big[1 + \begin{pmatrix} 0.032 \\ 0.011 \end{pmatrix}_{f_{B_s}} + 0.031|_{\mathrm{CKM}} + 0.011|_{m_t} + 0.006|_{\mathrm{pmr}} + 0.012|_{\mathrm{non\text{-}pmr}} \, {}^{+0.003}_{-0.005}|_{\mathrm{LCDA}} \Big] \cdot 10^{-9}$$

- Parametric uncertainties
  - Long distance (f_{B_s}) and short distance (CKM, m_t)
- Non-QED parametric (Γ_q, α_s)
- Non-QED non-parametric (μ_W, μ_b, and higher order)
- QED parametric: B-meson LCDA parameters (λ_B, $\sigma_{1,2}$)



Lattice QCD value for f_{B_s} is now a sub-dominant source of uncertainty

Chiral-continuum analysis

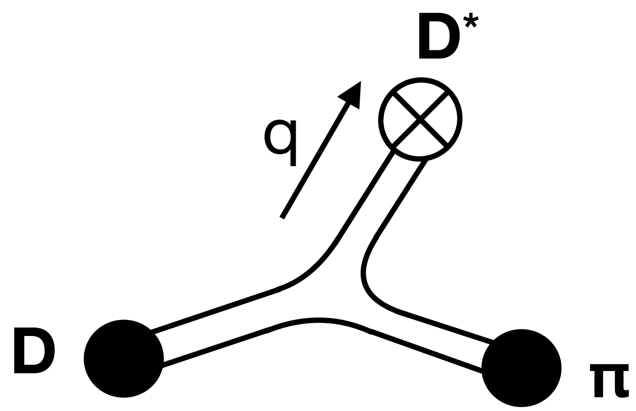
Heavy-meson rooted staggered chiral perturbation theory

- With simulations at and above the physical pion mass, the chiral fits are ***interpolations,*** not extrapolations

- The shape of the form factors can be modeled with EFT combining:
    - Chiral symmetry $$\Sigma = \exp(2i\phi/f)$$
    - HQET spin symmetry $$H^a = \frac{1+\not{v}}{2}\left[P_\mu^{*a}(v)\gamma^\mu - P^a(v)\gamma_5\right]$$
    - Light-quark discretization effects $$\frac{1}{16}\sum_{\text{tastes }\xi} M_\xi^2 \log\left(\frac{M_\xi^2}{\Lambda^2}\right)$$

Chiral-continuum analysis

Heavy-meson rooted staggered chiral perturbation theory



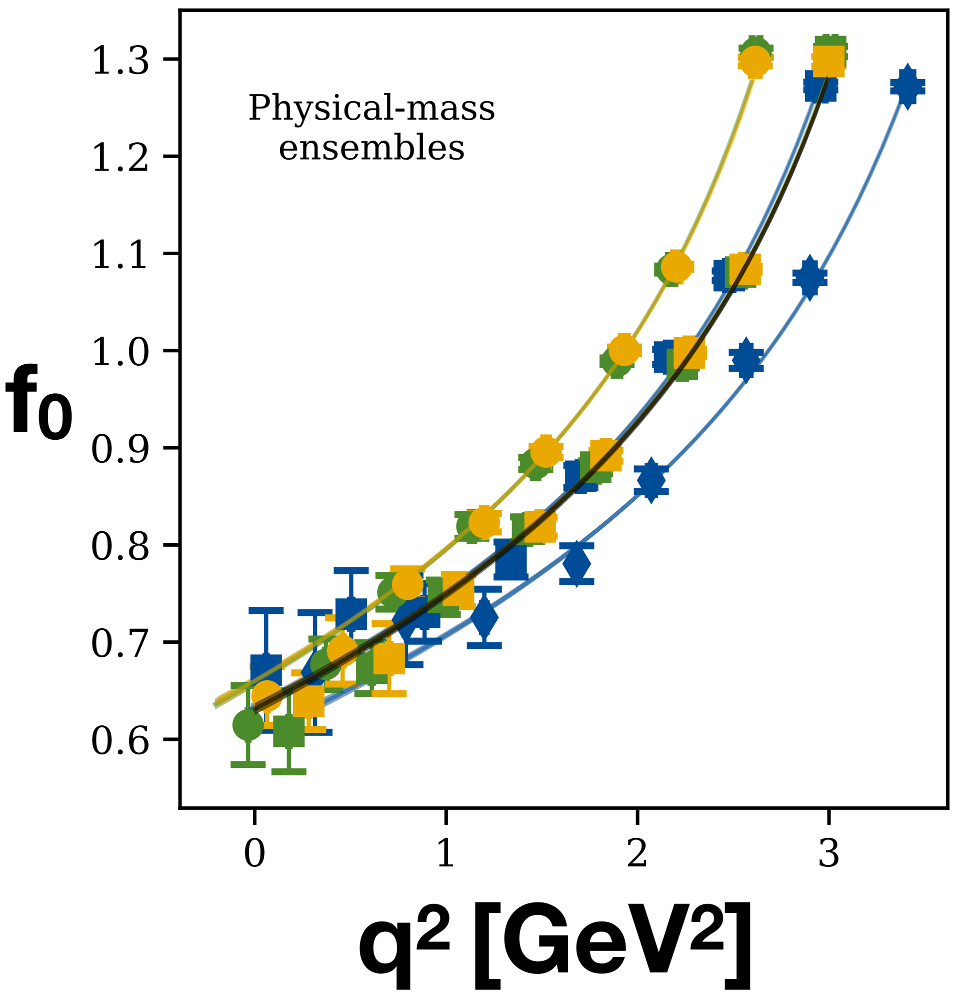
$$\sim \frac{1}{M_{D^*}^2 - q^2} \propto \frac{1}{E_\pi + \Delta}$$

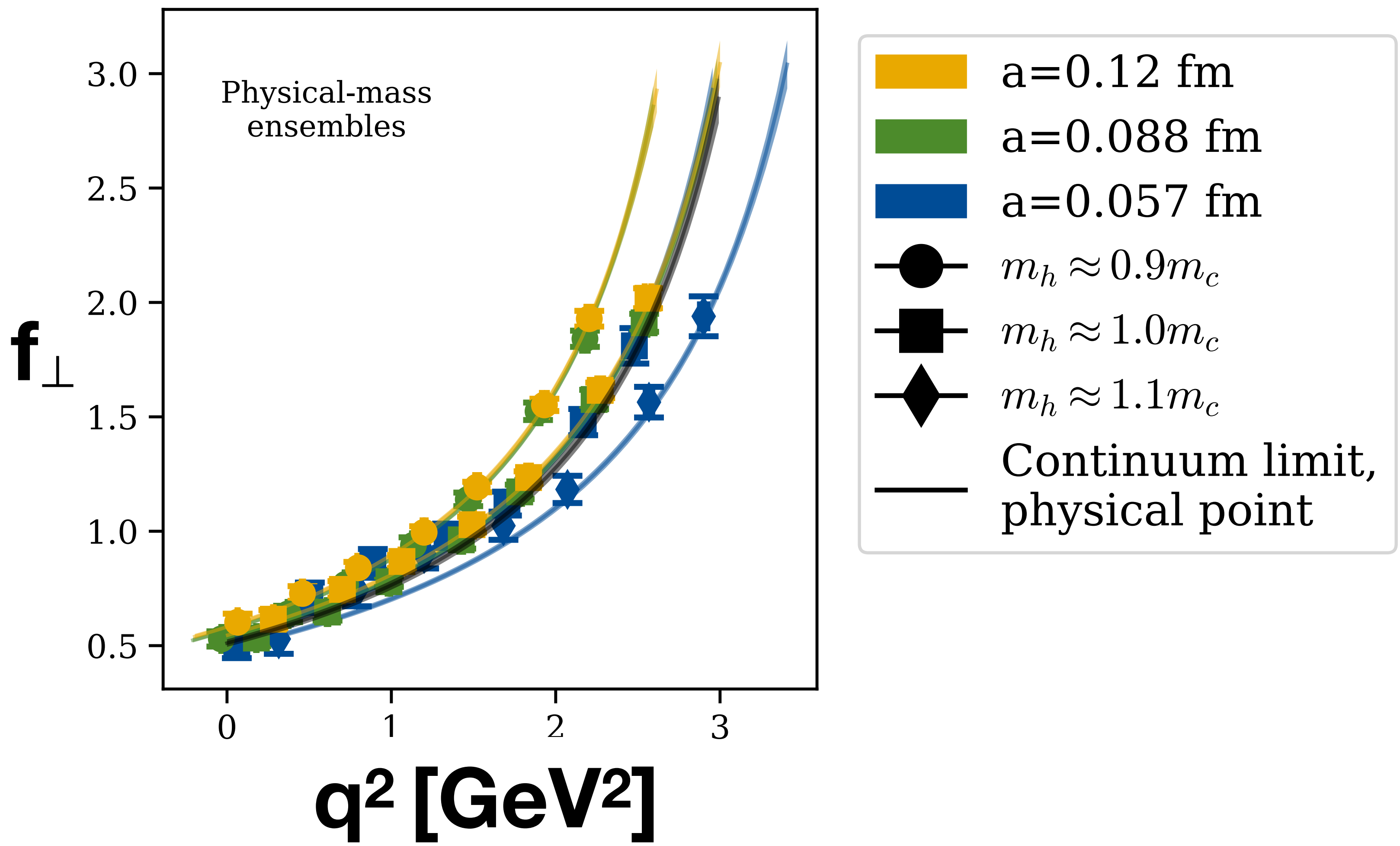
$$q^2 = M_D^2 + M_\pi^2 - 2M_D E_\pi$$

- Basically: $f = \frac{\text{const}}{E + \Delta} \times \left(1 + \delta f_{\text{logs}} + \sum_i c_i \chi_i + \delta f_{\text{artifacts}} \right)$
- Logs computed through NLO in HMRSχPT
- Analytic terms included through N²LO (consistent w/power counting)
- Lattice artifacts included from O(a²)

Chiral-continuum analysis: D→π

Example: $f_0(q^2)$ and $f_\perp(q^2)$



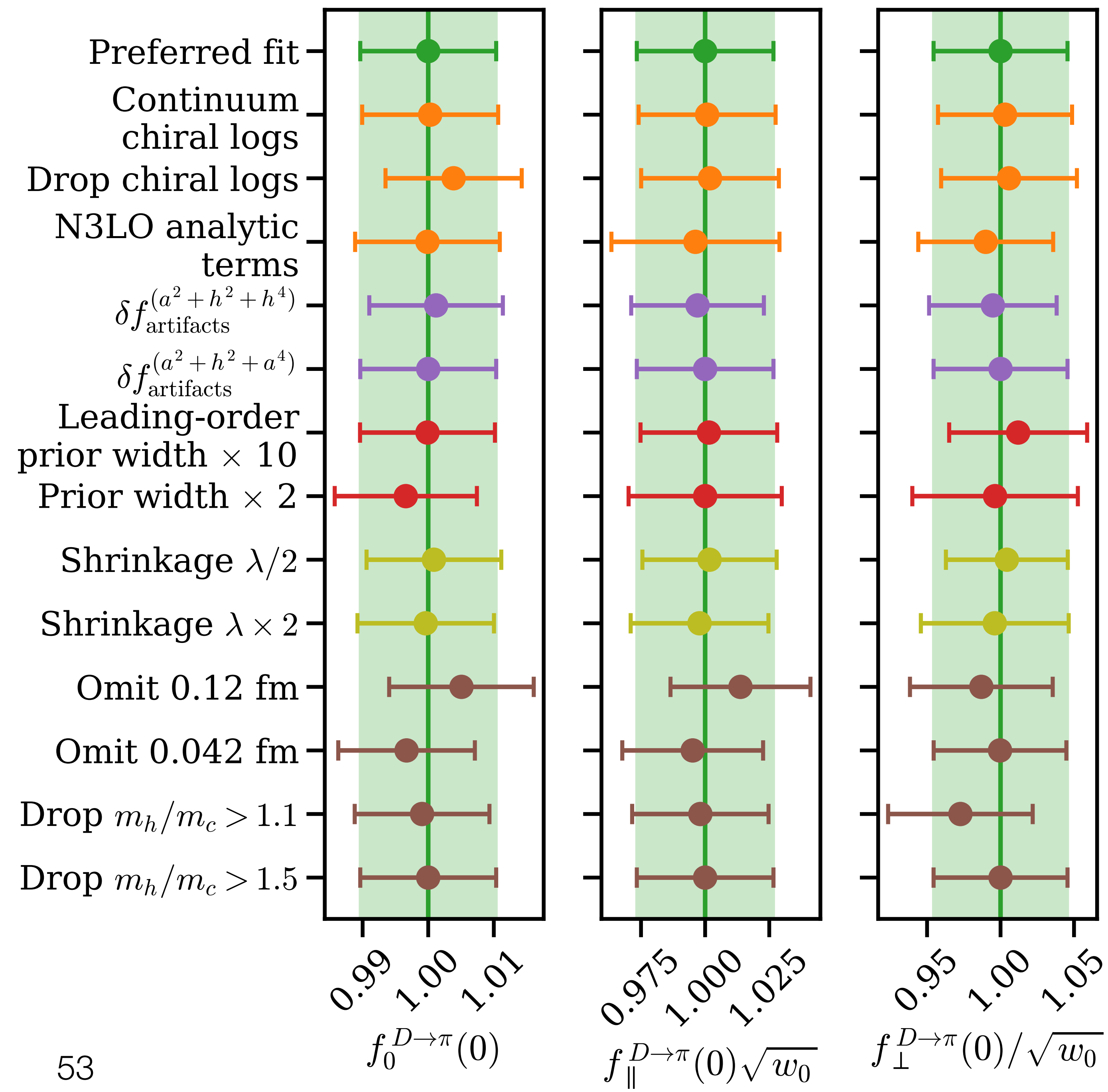


- Displayed: physical-mass ensembles only (but all ensembles included in fit)
- All fits have good quality of fit (e.g., χ^2/DOF ~ 1)
- Curve collapse at $m_h/m_c \approx 1.0$ suggests a mild approach to continuum limit

Chiral-continuum analysis: D→π

Stability of results

- Preferred analysis
- EFT variations
- Analytic discretization-term variations
- Statistical analysis variations
- Data variations



The z-expansion

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

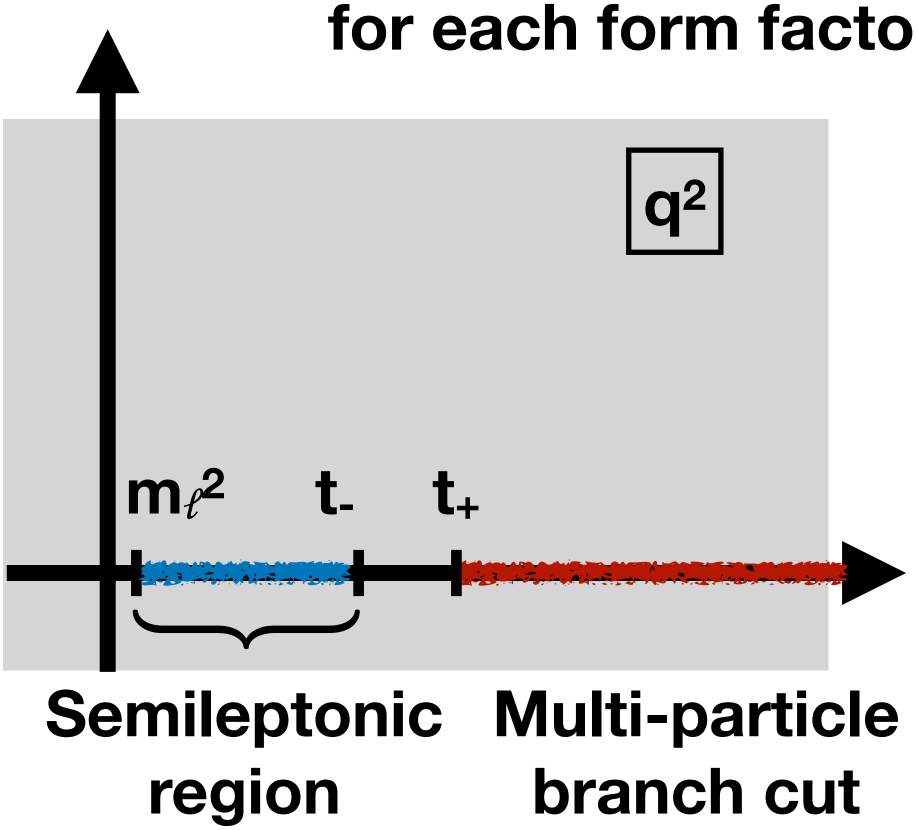
$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

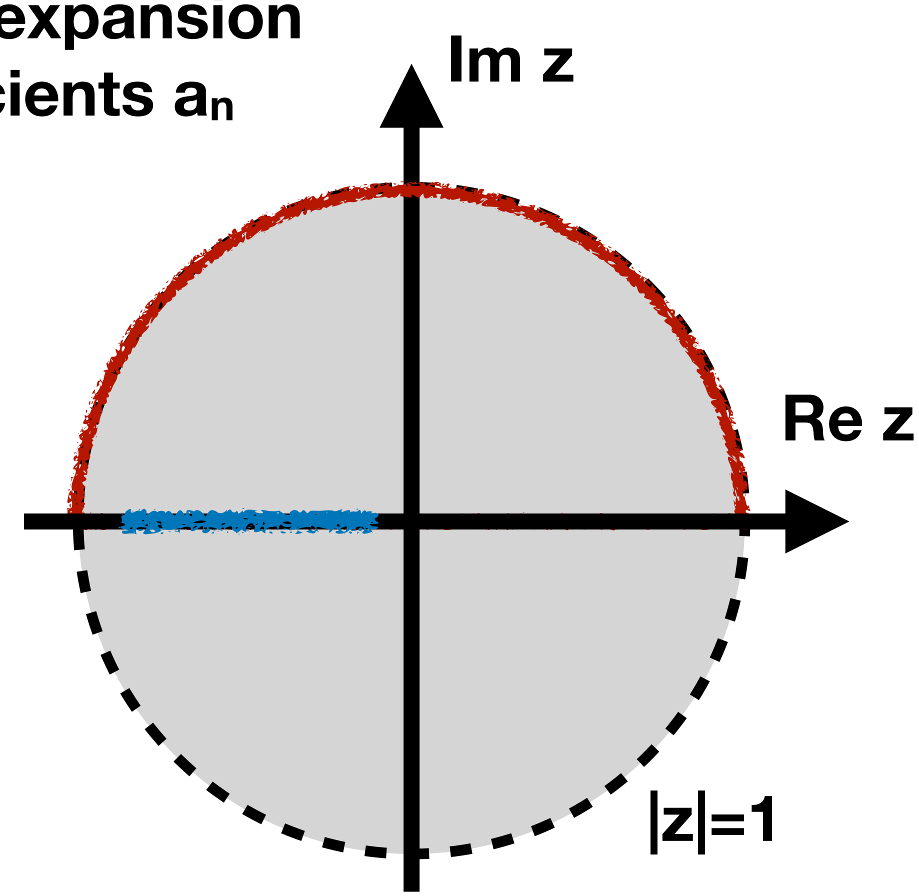
$$t_{\pm} = (M_D \pm M_\pi)^2$$

"Blashke factors" (contain poles)

"Outer functions" (computed analytically for each form factor)

LQCD calculations give the expansion coefficients a_n





Results at the physical point

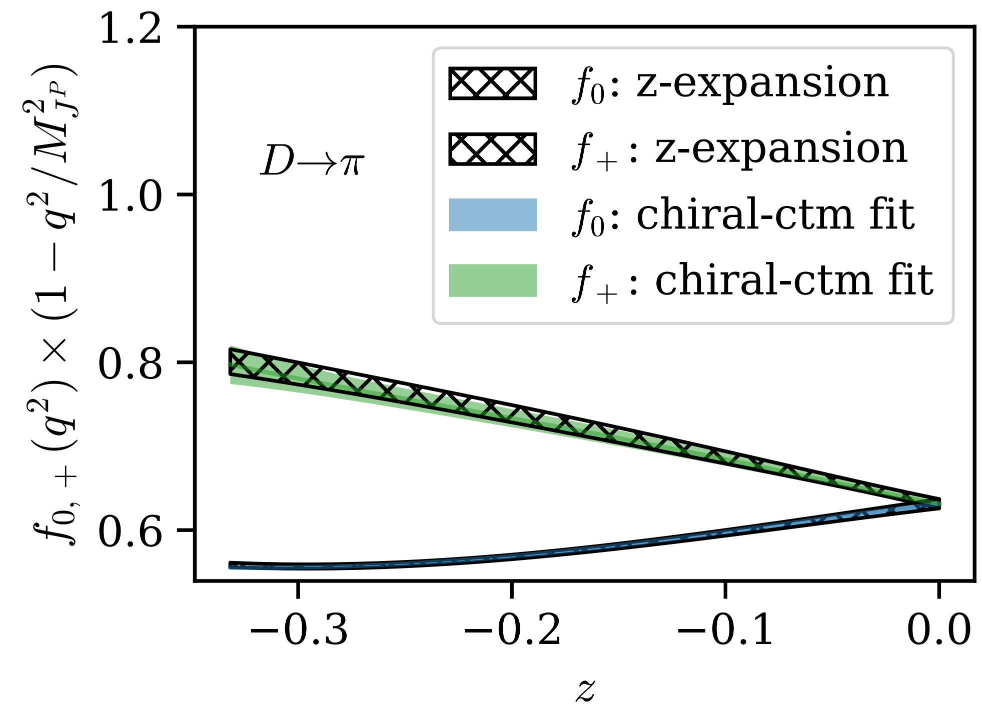
- Re-express final results using the model-independent z-expansion

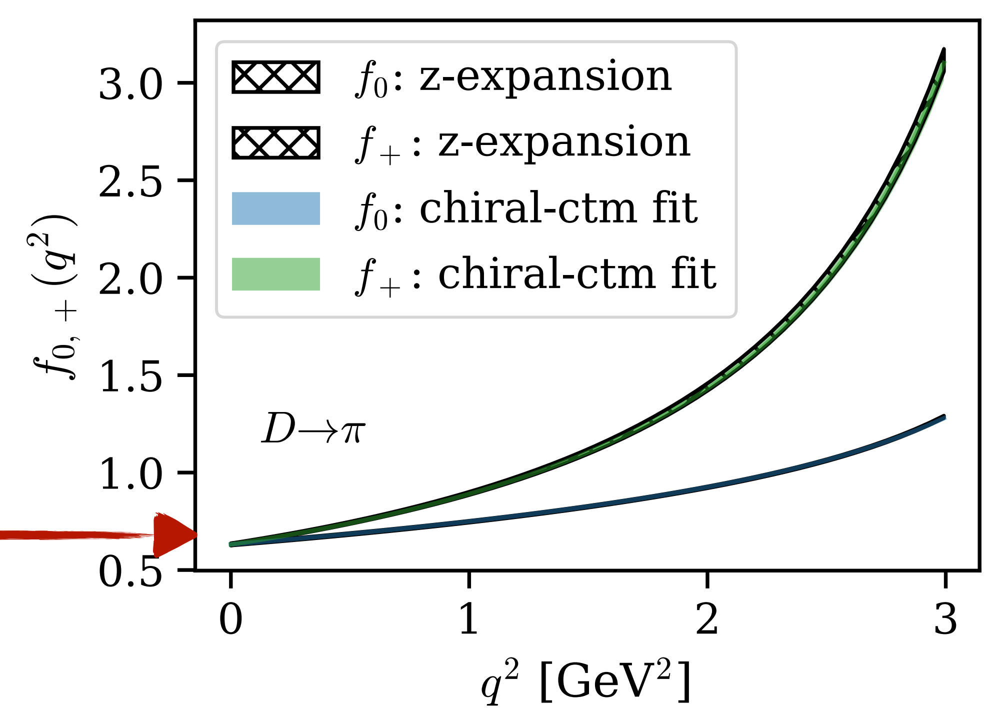
- For D-decays, the z-expansion is not an extrapolation — just a convenient change of variables

$$f_0(z) = \frac{1}{\left(1 - \frac{q^2(z)}{M_{0+}^2}\right)} \sum_{n=0}^{M-1} b_m z^m,$$

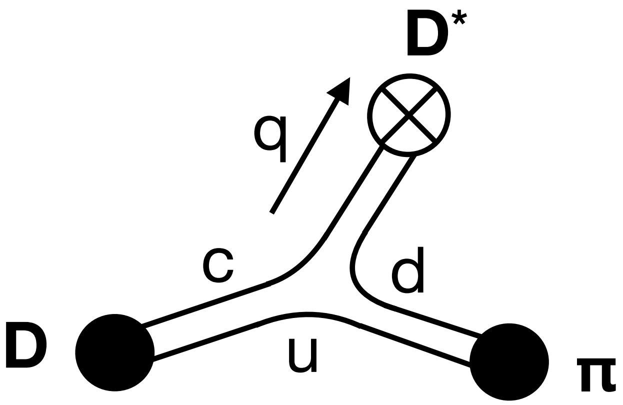
$$f_+(z) = \frac{1}{\left(1 - \frac{q^2(z)}{M_{1-}^2}\right)} \sum_{n=0}^{N-1} a_n \left(z^n - \frac{n}{N}(-1)^{n-N} z^N\right)$$

- Kinematic identity: $f_+(0) = f_0(0)$

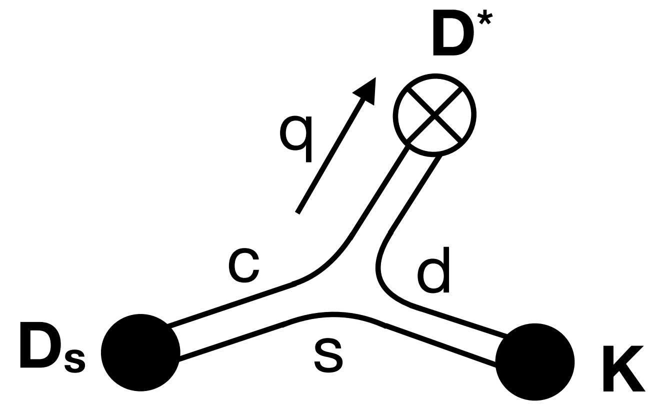




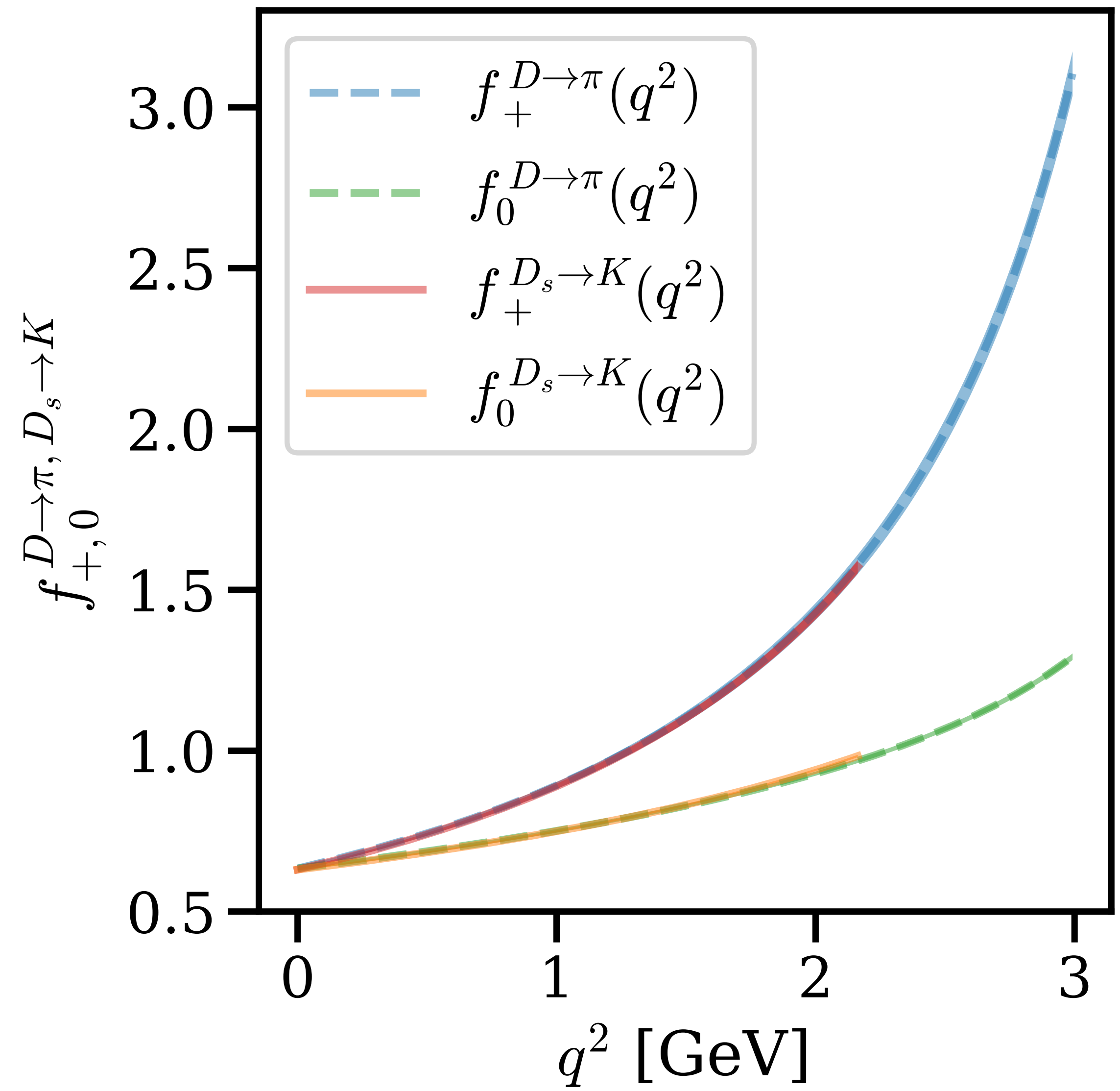
Spectator dependence: D→π vs D_s→K



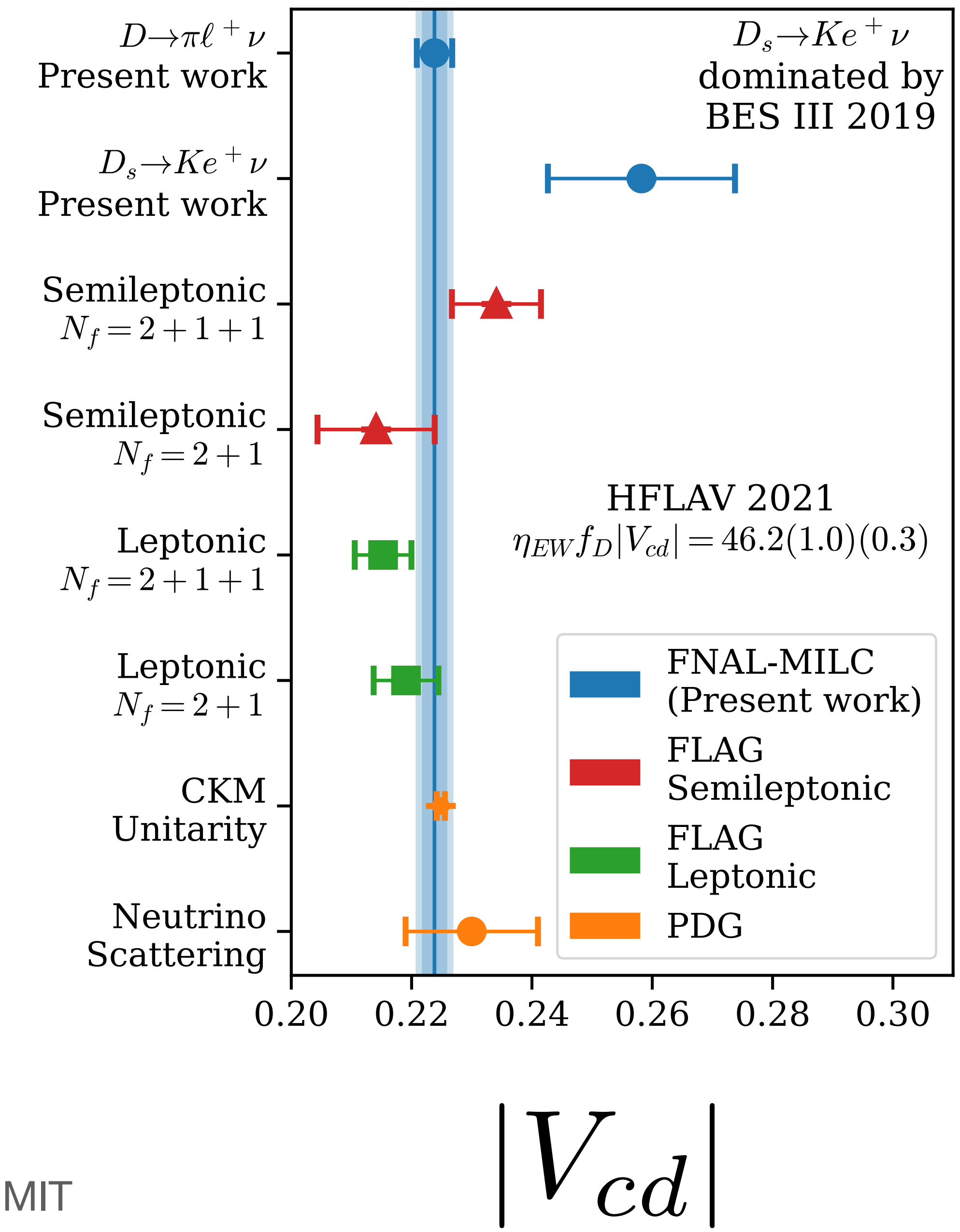
$$\sim \frac{1}{M_{D^*}^2 - q^2} \sim$$

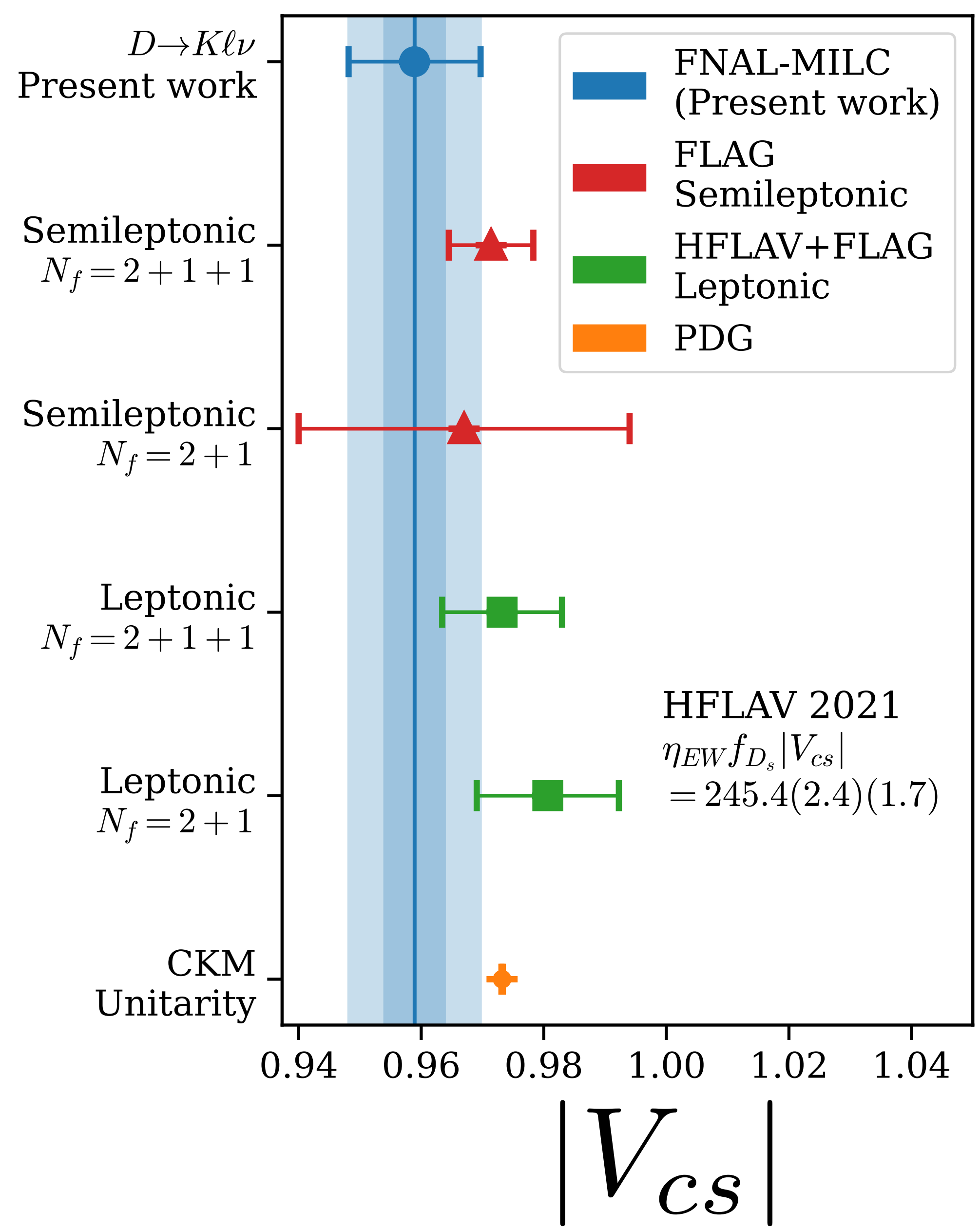


- D→π and D_s→K only differ by the mass of the spectator quark
- Vector and scalar form factors agree at ≲ 2% level throughout the kinematic range
- Older unpublished results by HPQCD are consistent with our findings



Comparison to Literature: $|V_{cd}|$ and $|V_{cs}|$





Chiral-continuum fit formulae

$$f_P(E) = \frac{c_0}{E + \Delta^*_{yx,\mathrm{P}}} \times \Big[1 + \delta f_{\mathrm{P,logs}} + c_l \chi_l + c_H \chi_H + c_E \chi_E$$
$$+ c_{l^2} (\chi_l)^2 + c_{H^2} (\chi_H)^2 + c_{E^2} (\chi_E)^2$$
$$+ c_{lH} \chi_l \chi_H + c_{lE} \chi_l \chi_E + c_{HE} \chi_H \chi_E$$
$$+ \delta f^{(a^2)}_{\mathrm{artifacts}}\Big],$$

$$\chi_l = \frac{(M_\pi^{\mathrm{meas.}})^2}{8\pi^2 f^2}$$

$$\chi_E = \frac{\sqrt{2}E}{4\pi f}$$

$$\chi_H = \frac{(M_{D_{(s)}}^{\mathrm{meas.}})^2 - (M_{D_{(s)}}^{\mathrm{PDG}})^2}{8\pi^2 f^2}$$

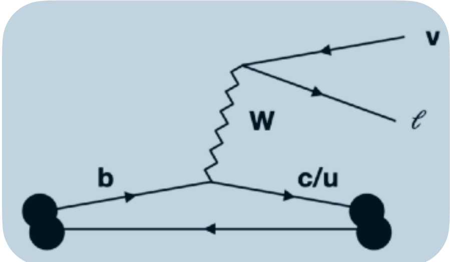
$$\delta f^{SU(2)}_{\mathrm{P,logs}} = \left(-\frac{1}{16}\sum_\xi \mathcal{I}_1(M_{\pi,\xi}) + \frac{1}{4}\mathcal{I}_1(M_{\pi,I}) + \mathcal{I}_1(M_{\pi,V}) - \mathcal{I}_1(M_{\eta,V}) + [V \to A]\right)$$
$$\times \begin{cases} \frac{1+3g^2}{(4\pi f)^2}, D \to \pi \\ \frac{3g^2}{(4\pi f)^2}, D \to K \\ \frac{1}{(4\pi f)^2}, D_s \to K \end{cases}$$

$$x_{a^2} = \frac{a^2\bar{\Delta}}{8\pi^2 f^2}$$

$$x_h = \frac{2}{\pi} a m_h.$$

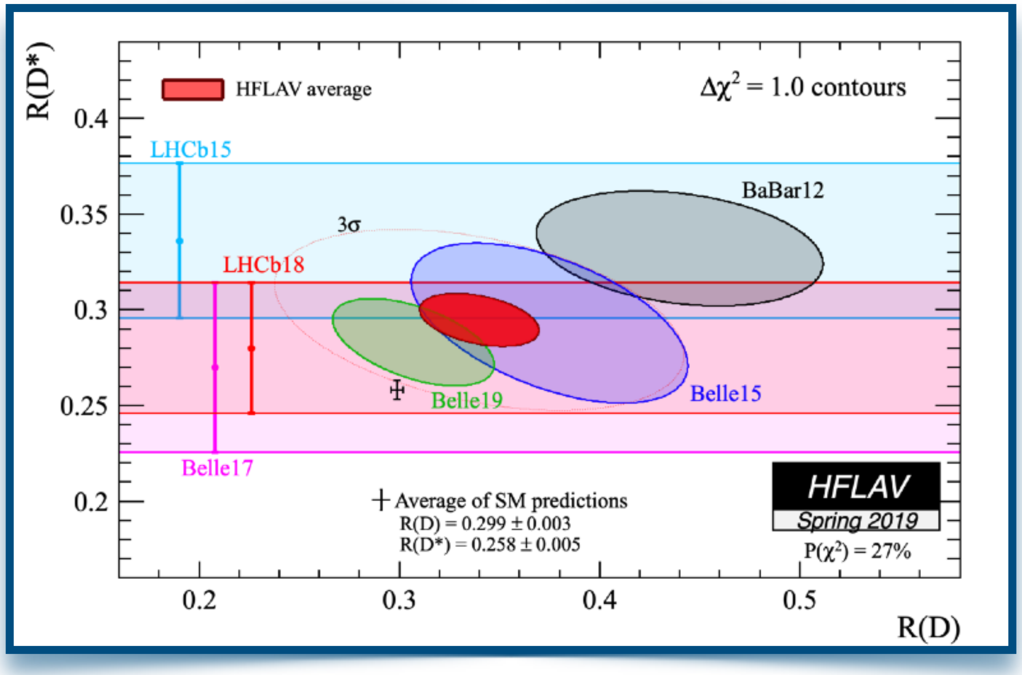
$$M^2_{\pi,\xi} = M^2_{uu,\xi} = M^2_{dd,\xi}$$
$$M^2_{ij,\xi} = \mu(m_i + m_j) + \Delta_\xi$$
$$M^2_{\eta,V(A)} = M^2_{uu,V(A)} + \frac{1}{2}\delta'_{V(A)}$$
$$\bar{\Delta} = \frac{1}{16}\sum_\xi \Delta_\xi.$$

Experimental Motivation: B-anomalies

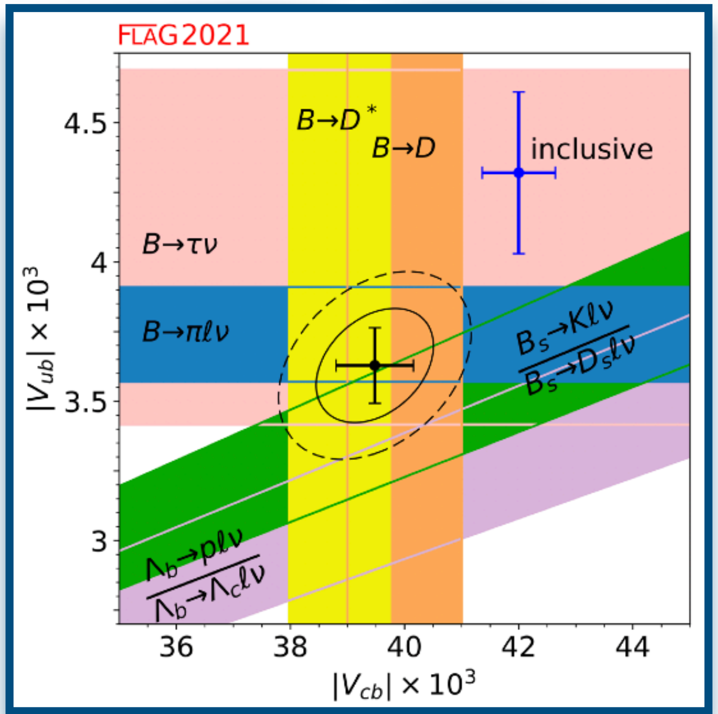


- **Tree level**: Lepton Flavor Universality: $R(D)$, $R(D^\star)$

$$R(D) = \frac{\mathscr{B}(B \to D\tau\bar{\nu})}{\mathscr{B}(B \to D\mu\bar{\nu})}$$

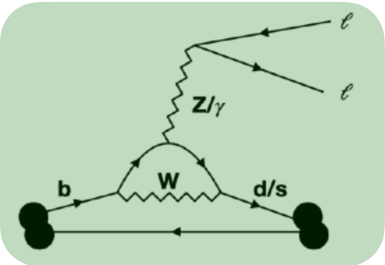


- **Tree level**: Exclusive (LQCD) vs Inclusive (OPE+HQE) determinations of CKM matrix elements
  - $|V_{cb}|$ from $B \to D^\star \ell \nu$, $B \to D\ell\nu$
  - $|V_{ub}|$ from $B \to \pi\ell\nu$



- **Loop level**: $b \to s\ell\ell$ FCNC branching fractions:
  - $B^0 \to K^{\star 0}\mu\mu$, $B_s^0 \to \varphi\mu\mu$, $\Lambda_b^0 \to \Lambda^0\mu\mu$, $B^+ \to K^+\mu\mu$, $B^0 \to K^0\mu\mu$, $B^+ \to K^{\star +}\mu\mu$
- **Loop level**: $b \to s\ell\ell$ FCNC angular observables
  - $B^0 \to K^{\star 0}\mu\mu$, $B^+ \to K^{\star +}\mu\mu$, $B_s^0 \to \varphi\mu\mu$

Experimental Motivation: B-anomalies



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- **Loop level**: $b \to s\ell\ell$ FCNC angular observables
  - $B^0 \to K^{\star 0}\mu\mu$, $B^+ \to K^{\star +}\mu\mu$, $B_s^0 \to \varphi\mu\mu$

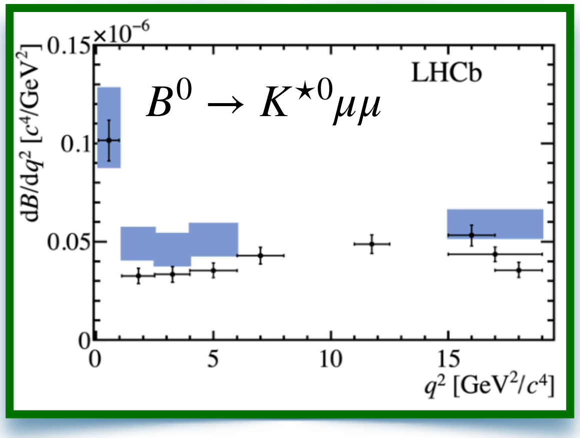
$B^0 \to K^{\star 0}\mu\mu$

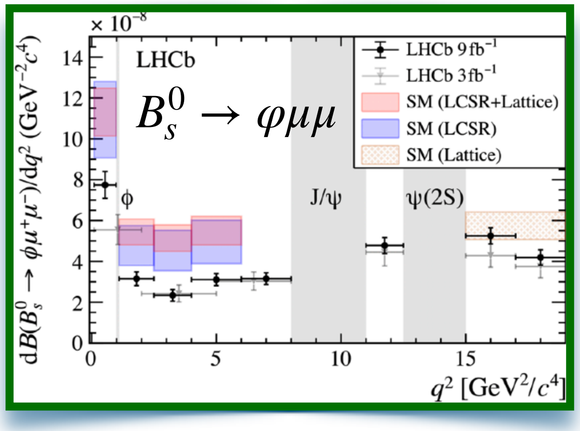
LHCb *JHEP* 11 (2016) 047
LHCb *JHEP* 04 (2017) 142
LHCb PRL 125 (2020) 011802

$B_s^0 \to \varphi\mu\mu$

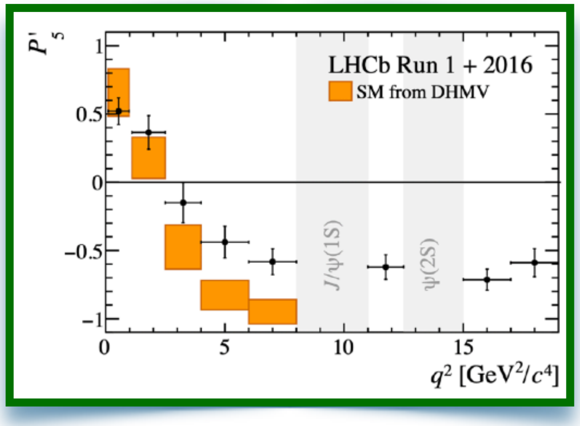
LHCb *JHEP 09 (2015) 179*
LHCb *PRL* 127 (2021) 15, 151801
LHCb JHEP 11 (2021) 043

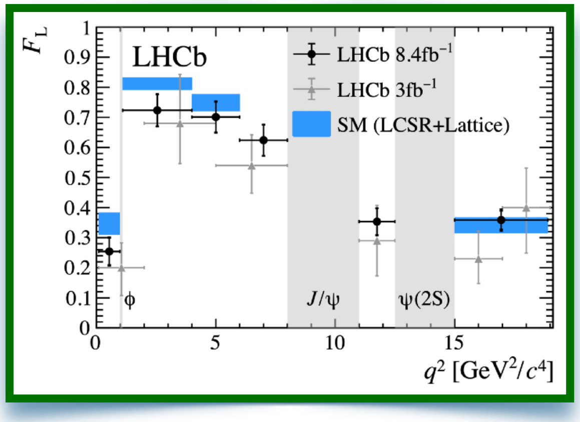
Branching fractions





Angular distributions





Experimental Motivation: CKM Unitarity

First-row unitarity?

- PDG 2022: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)^{|V_{ud}|^2}(4)^{|V_{us}|^2}$
- Quoted value has 2σ tension with unity, using as inputs
  - $|V_{ud}|$ from super-allowed $0^+ \to 0^+\ \beta$ decays
  - $|V_{us}|$ from semileptonic decay: $K_{\ell 3} \equiv K \to \pi \ell \nu$
  - Tension increases to ≈3σ if nuclear-structure uncertainties from $|V_{ud}|$ are ignored
- Similar ≈2-3σ tension if $|V_{us}|/|V_{ud}|$ taken from ratio of leptonic decays K_{l2}/π_{l2}

- Historically, similarly precise tests of second-row unitarity have been limited by experimental and theoretical precisions.
- Today's talk: recent progress in the second row via semileptonic decays

E. Blucher and W.J. Marciano
PDG 2022: 67. V_{ud}, V_{us}, the Cabibbo Angle, and CKM, Unitarity

Lattice QCD: particle masses

- 2-point correlation functions encode particle masses
- Analogy with condensed matter
  - Correlation length $\lambda \longleftrightarrow$ Particle mass 1/m

$$\langle (\bar{q}\,q)_t (\bar{q}\,q)_0 \rangle \sim \exp(-mt)$$

Lattice QCD: particle masses

- Hadronic spectrum $\longleftrightarrow$ QCD 2pt correlation functions

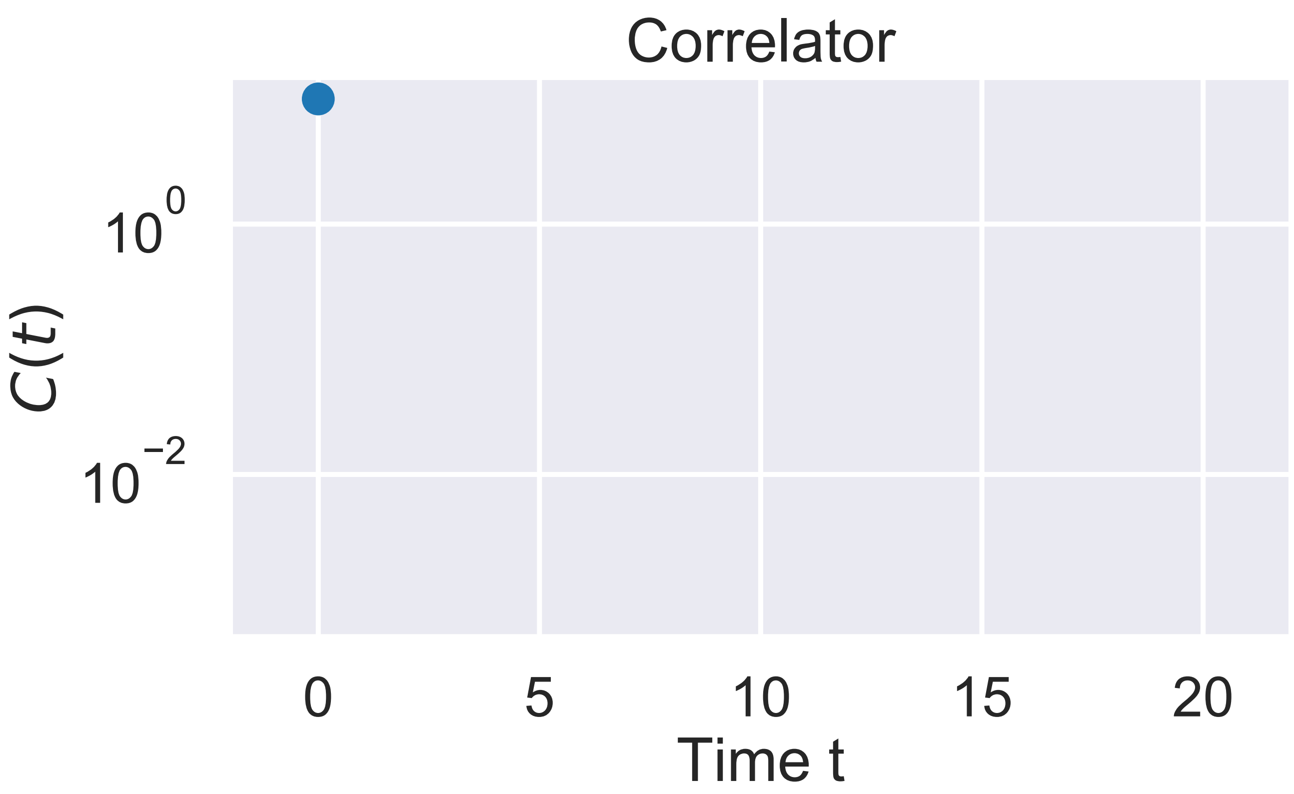
$$\begin{aligned}\langle O(t)O(0)\rangle &= \langle 0|\, e^{Ht}O(0)e^{-Ht}O(0)\,|0\rangle \\ &= \sum_n e^{-E_n t}\,\langle 0|\,O(0)\,|n\rangle\,\langle n|\,O(0)\,|0\rangle \\ &= \sum_n e^{-E_n t}\,|\langle 0|\,O(0)\,|n\rangle|^2 \\ &= \sum_n |Z_n|^2 e^{-E_n t}\end{aligned}$$

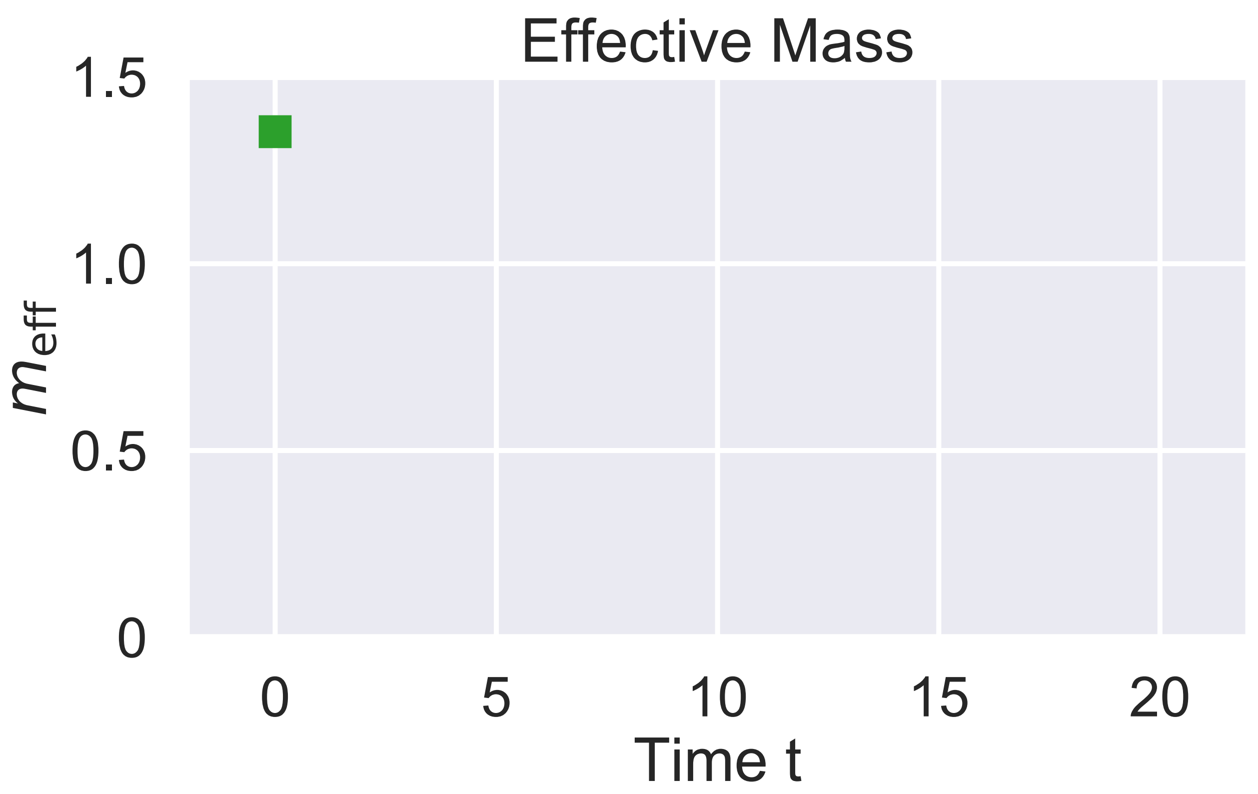
"Operators couple to an infinite tower of states."

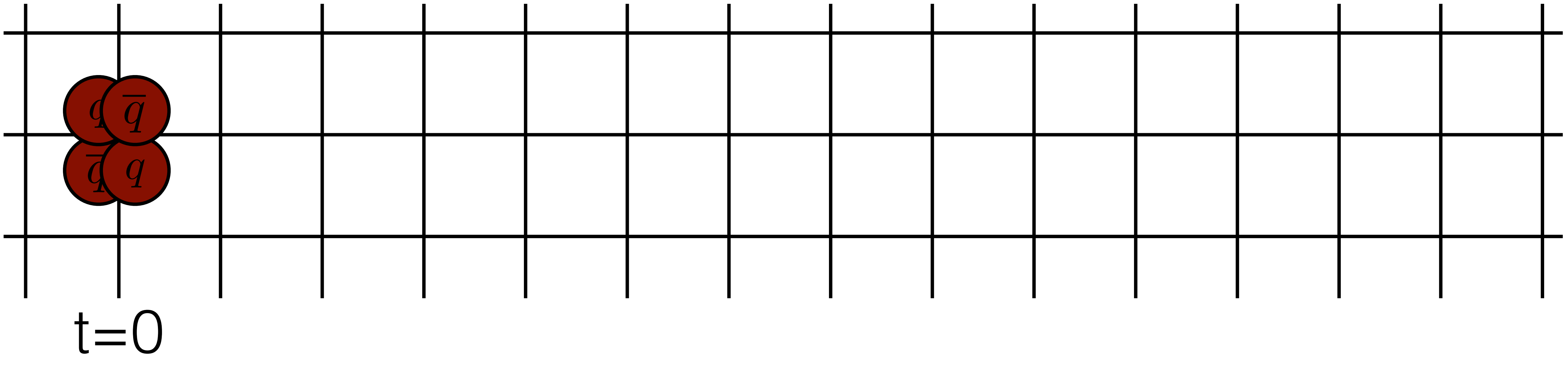
$$m_{\text{eff}}(t) = \log C(t)/C(t+1) \overset{t\to\infty}{=} m_0$$

"The ground state asymptotically dominates the Euclidean 2pt function."

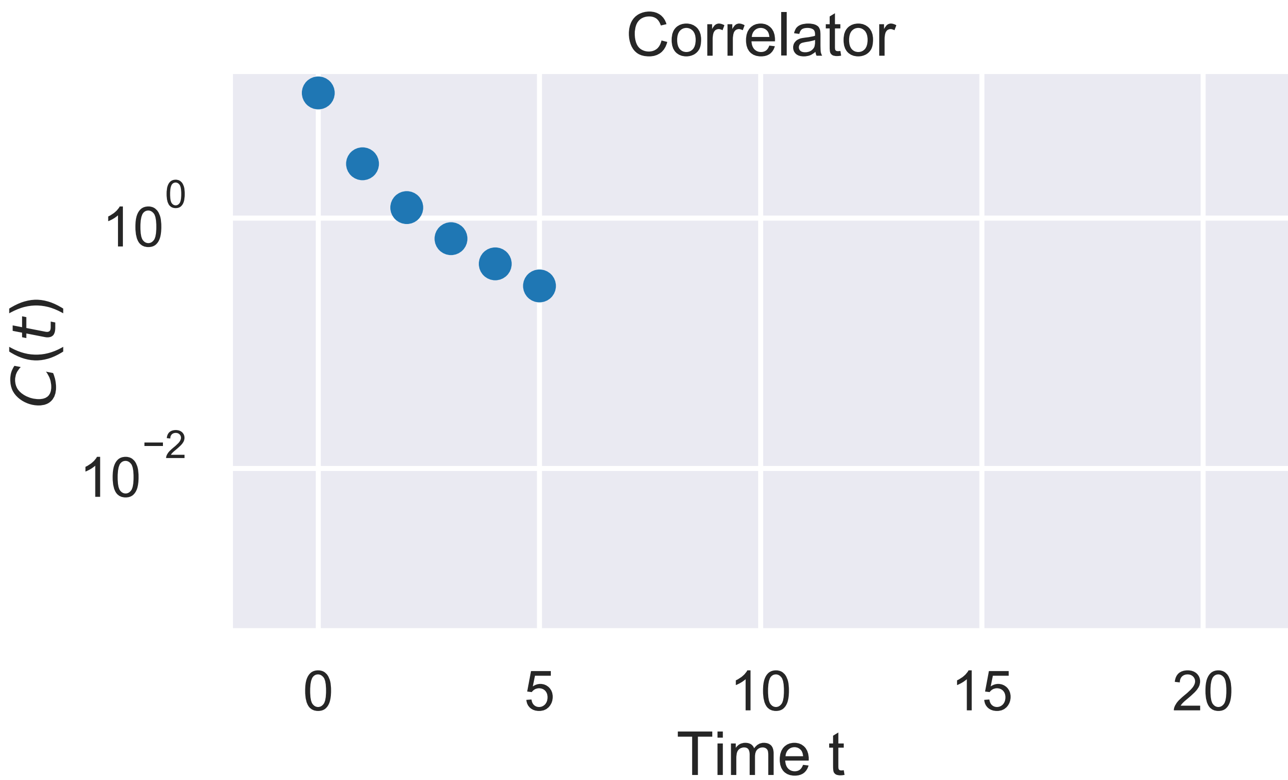
Lattice QCD: particle masses

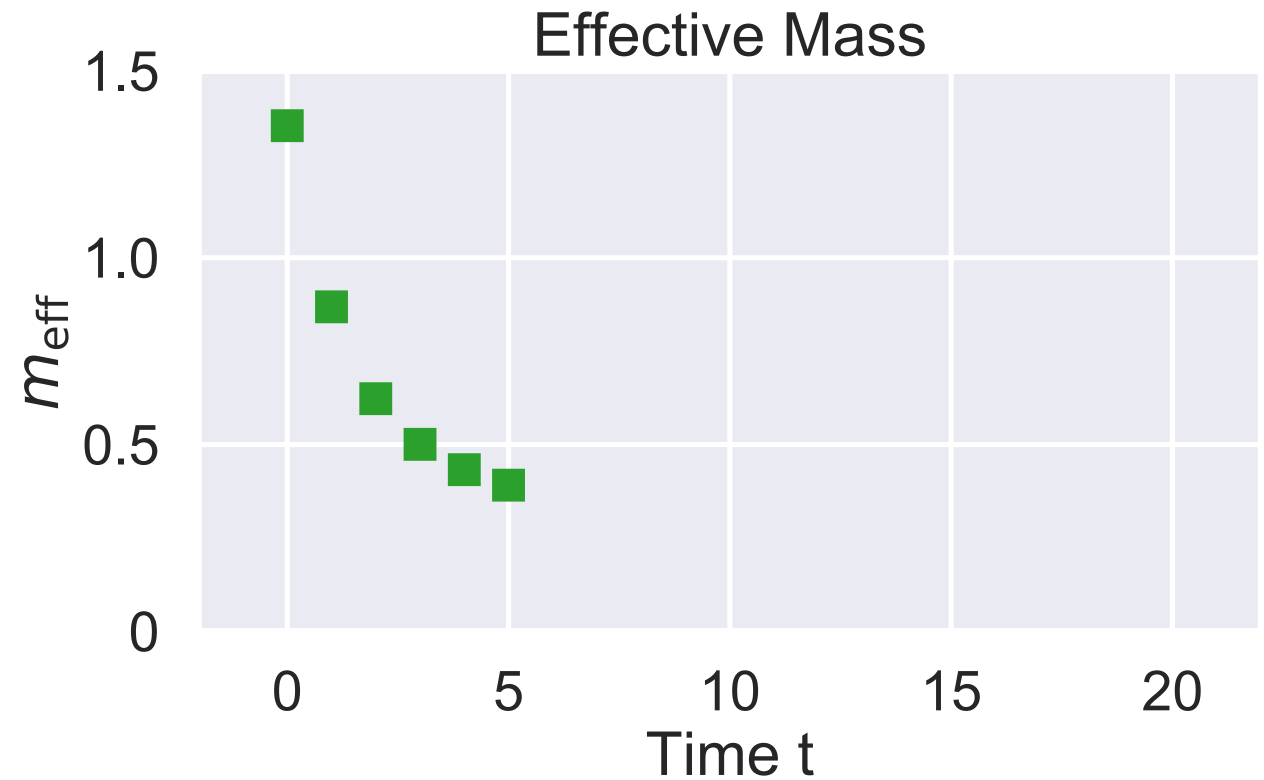


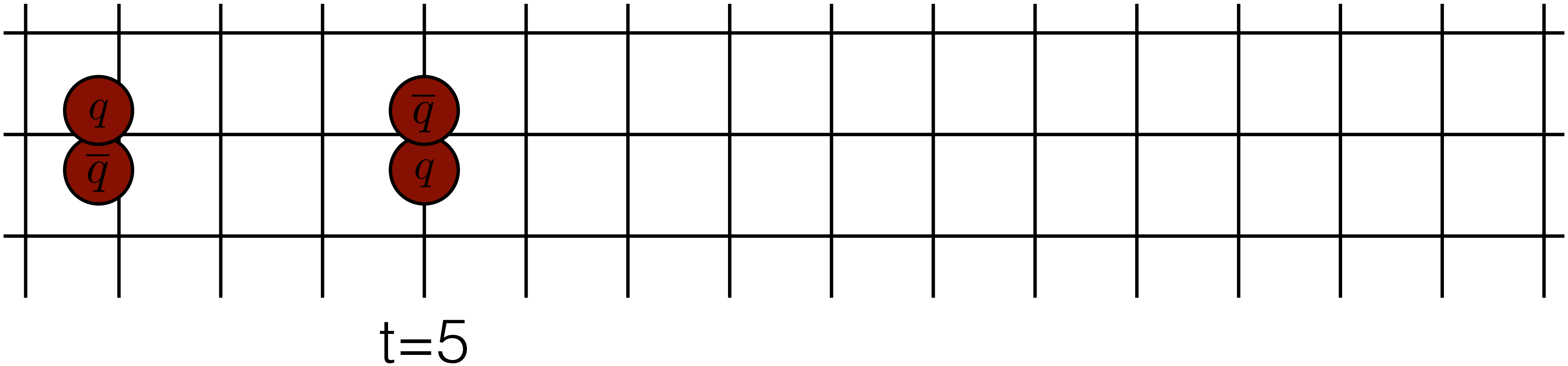




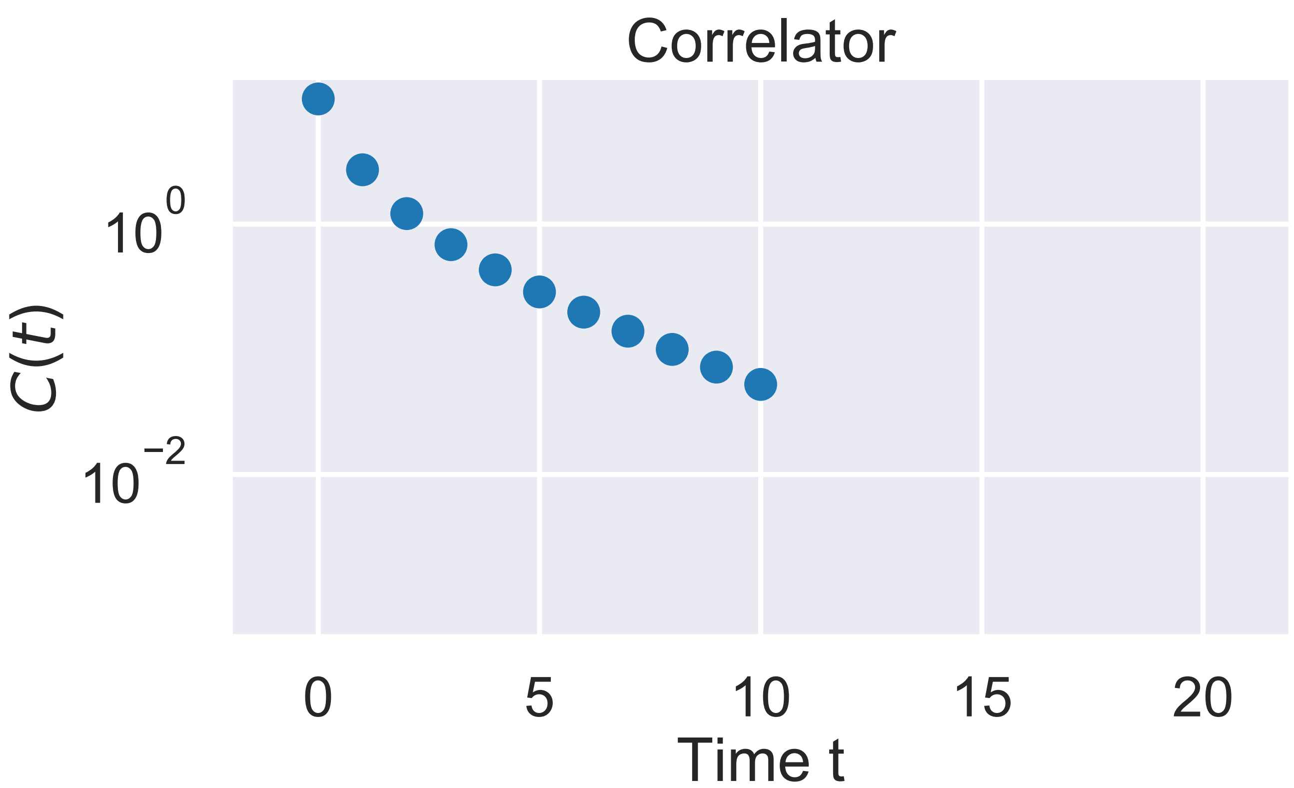
Lattice QCD: particle masses

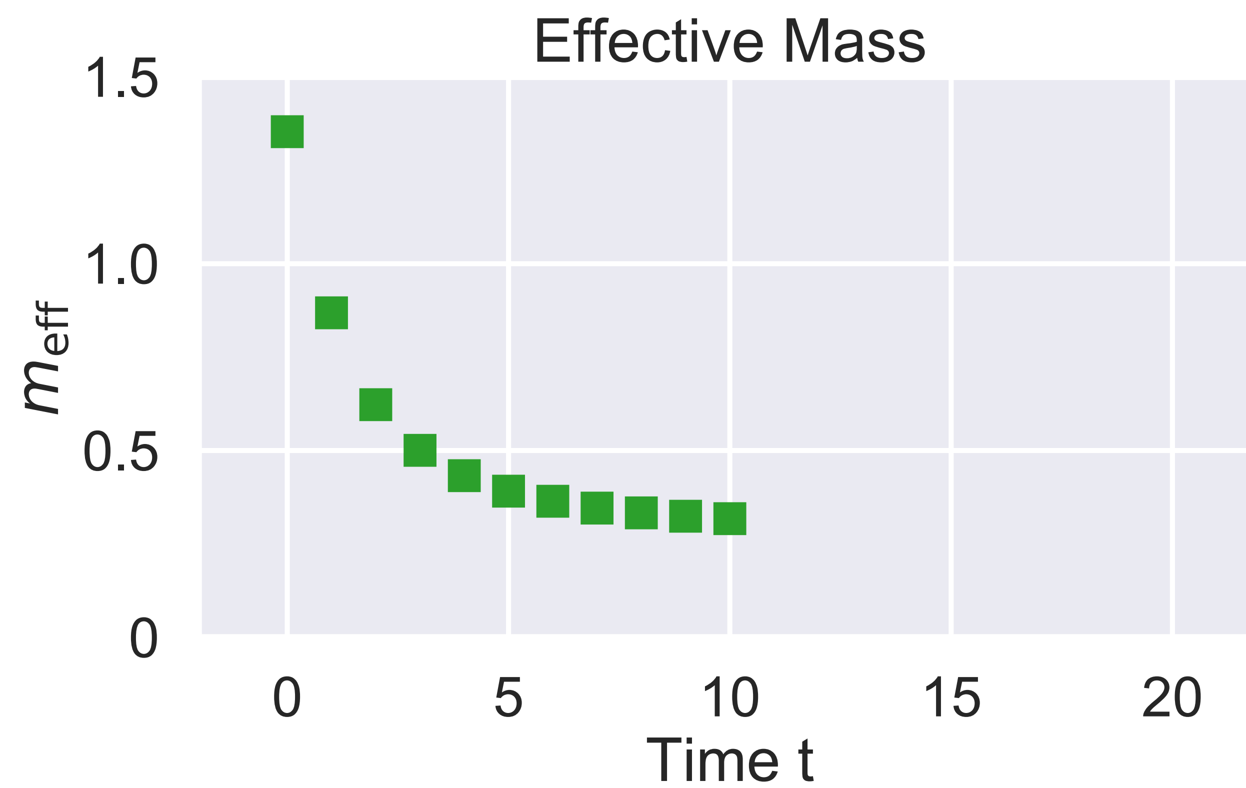


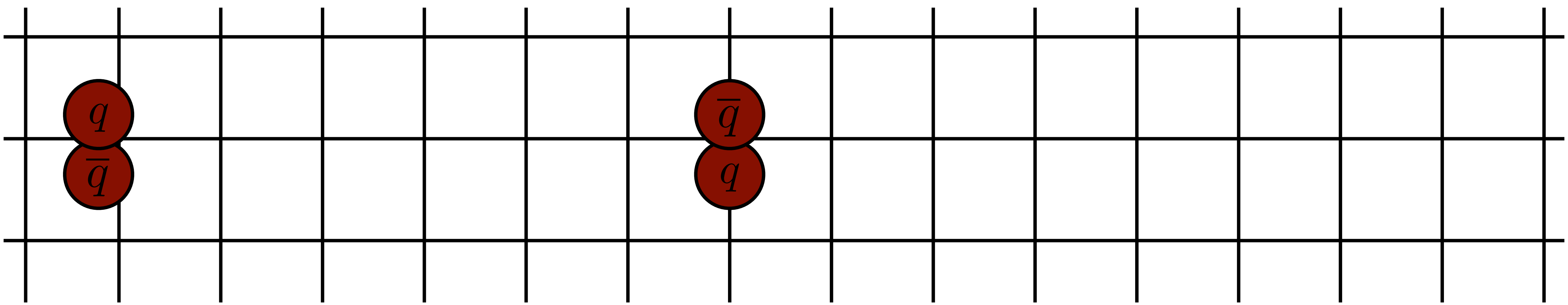




Lattice QCD: particle masses

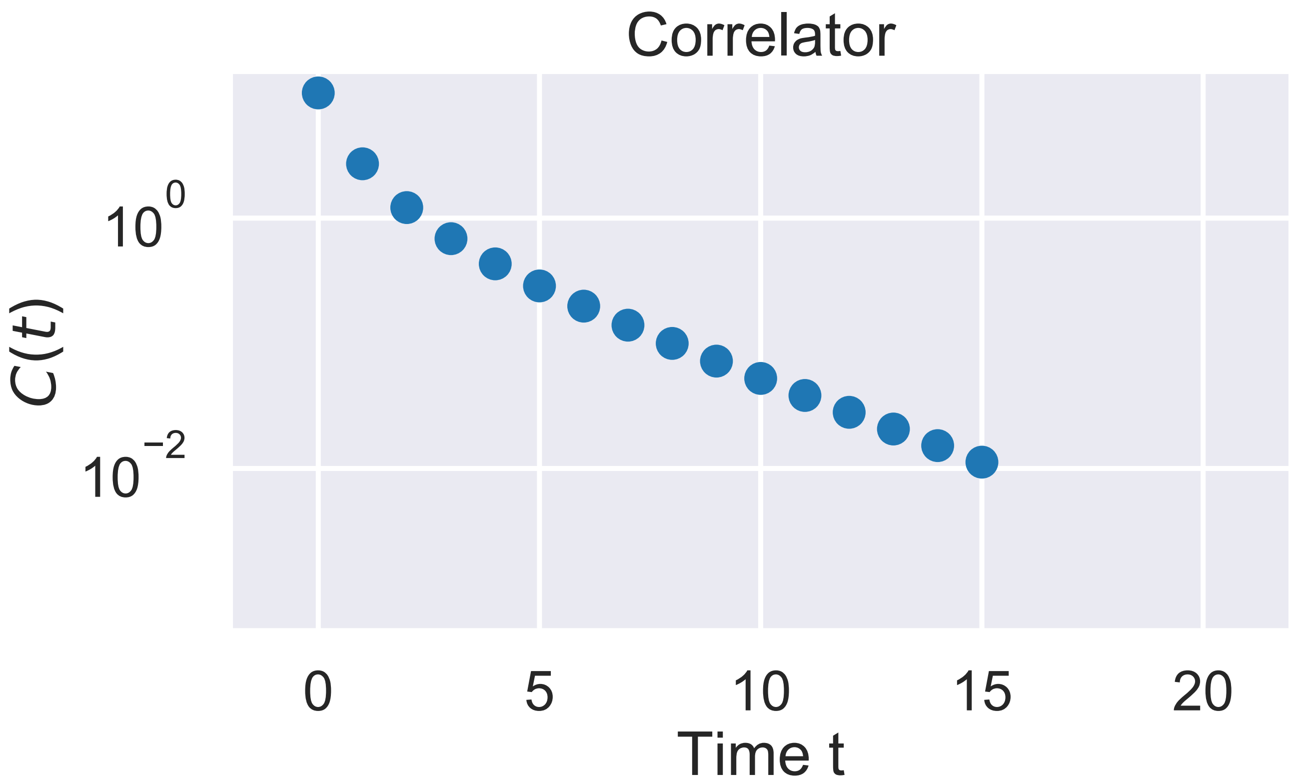


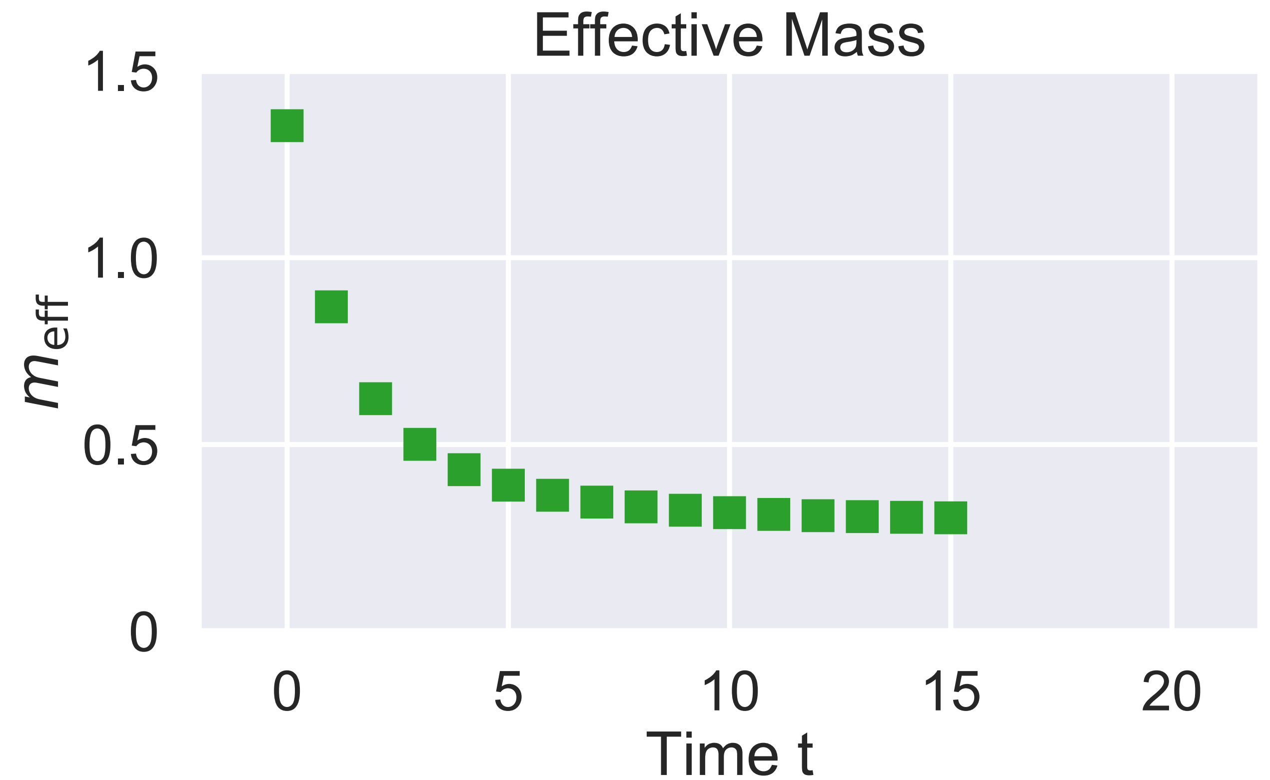


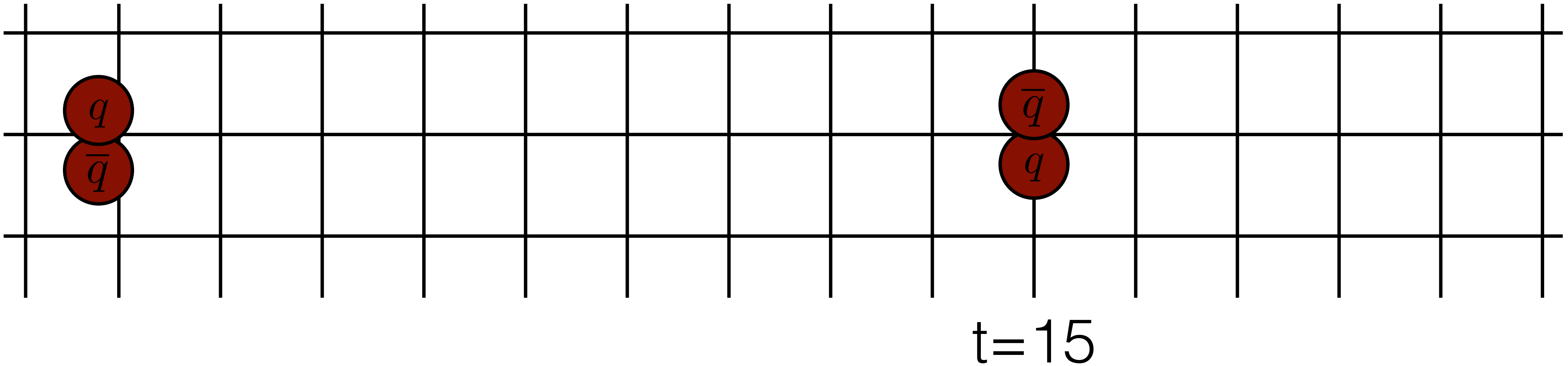


t=10

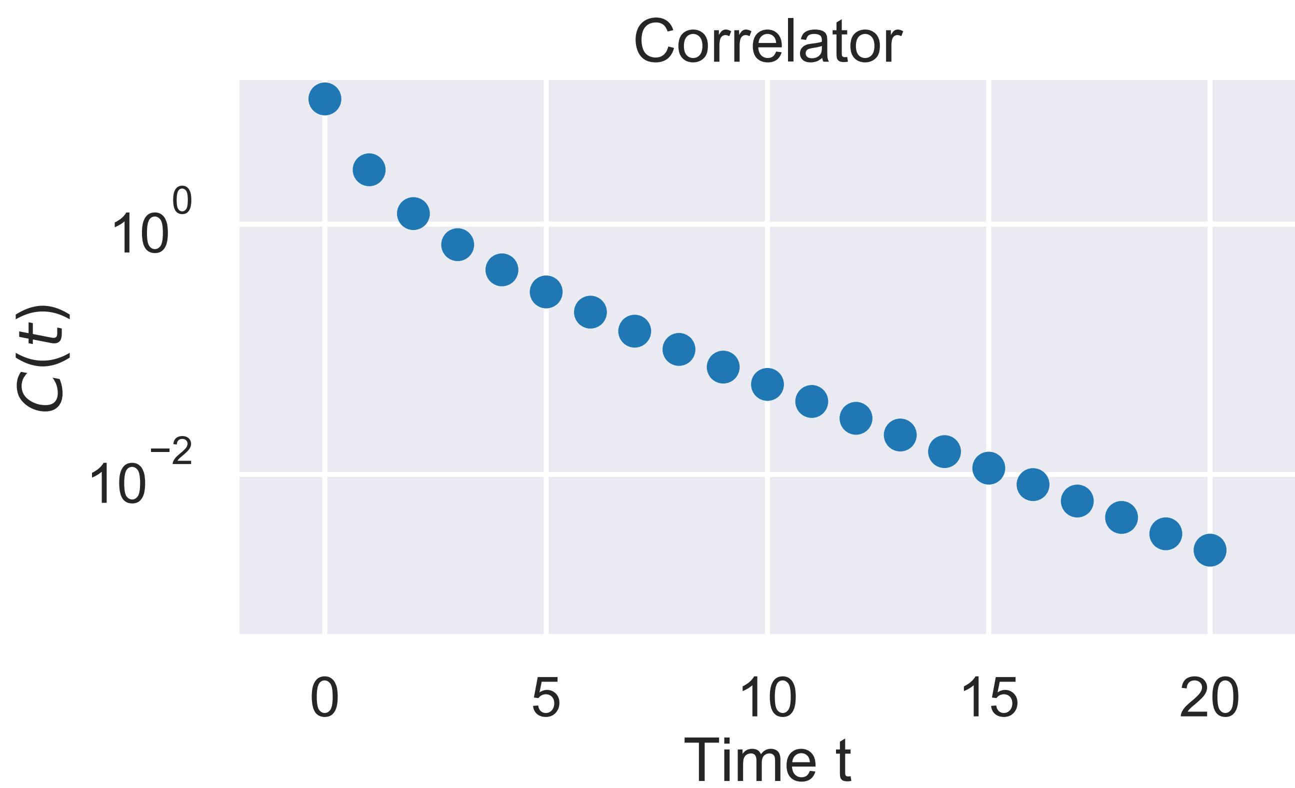
Lattice QCD: particle masses

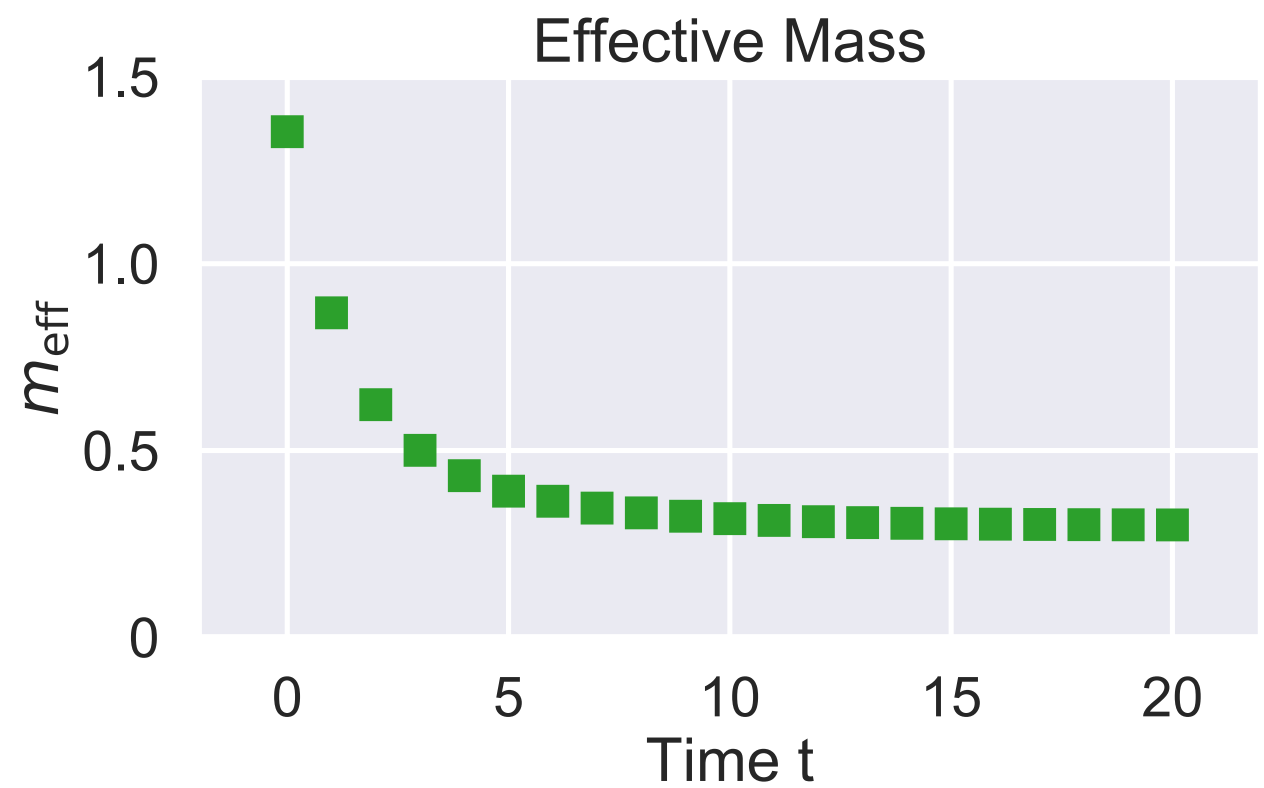


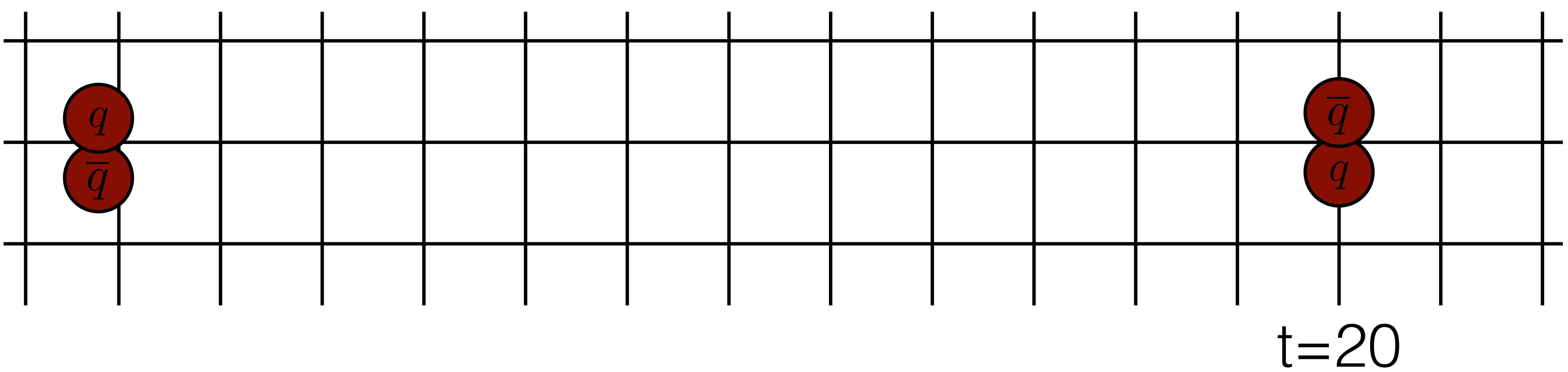




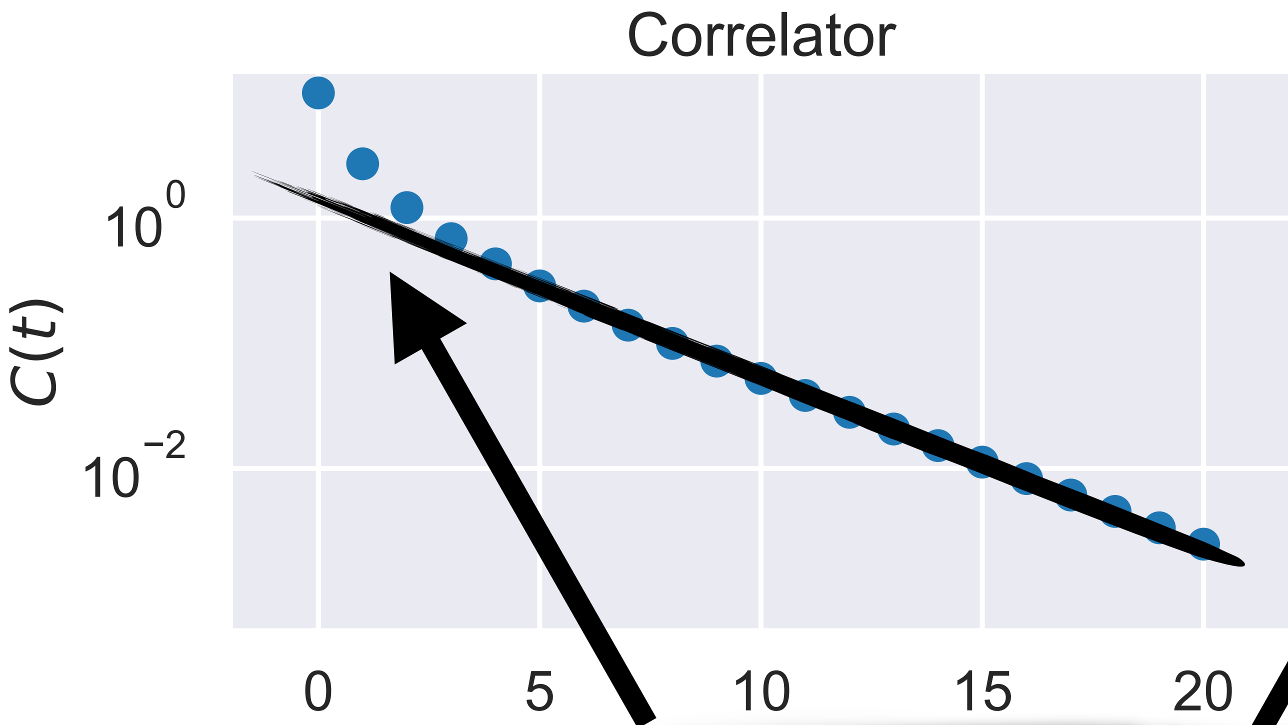
Lattice QCD: particle masses

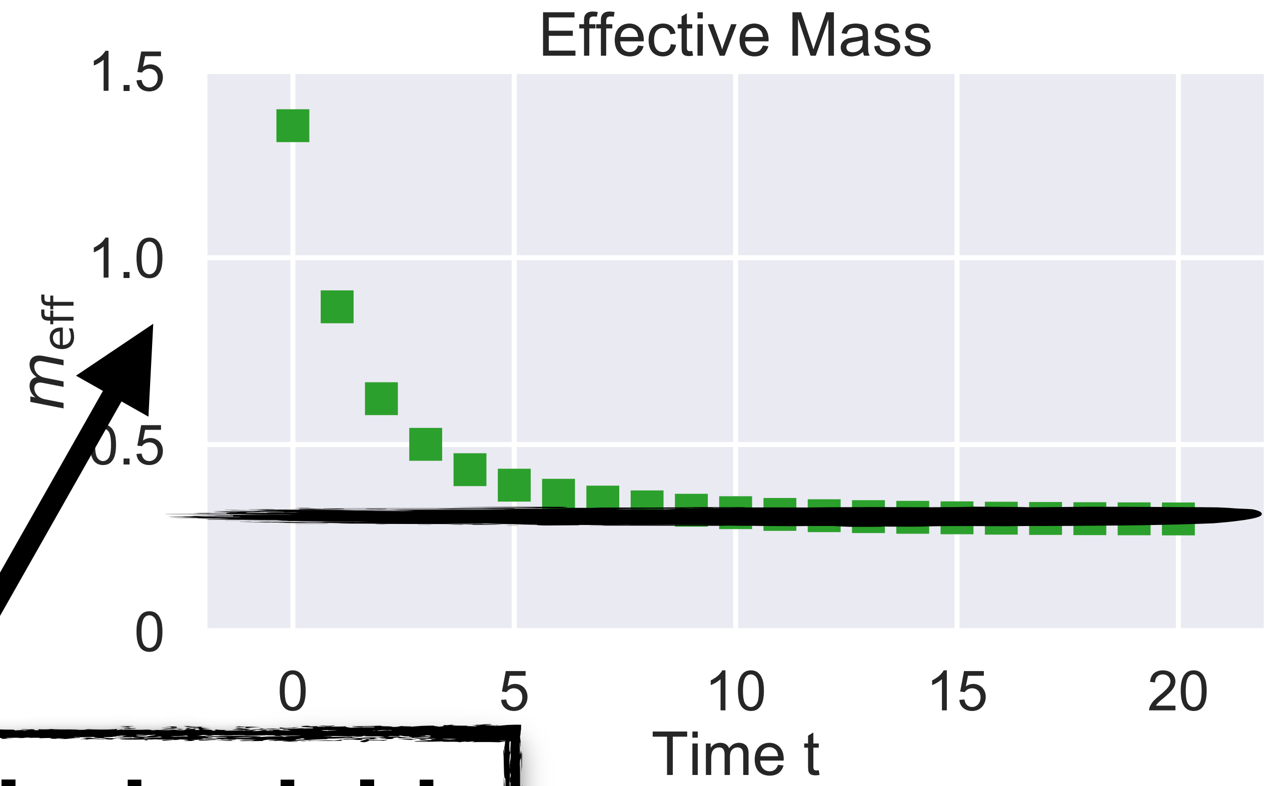




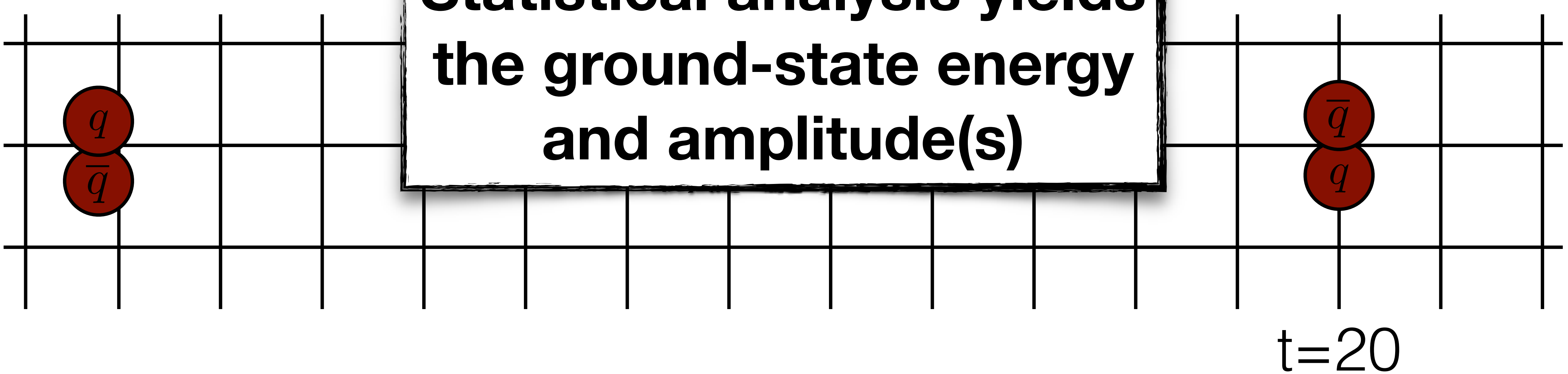


Lattice QCD: particle masses





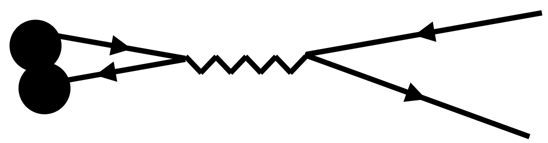
Statistical analysis yields the ground-state energy and amplitude(s)



Quark Flavor and Lattice QCD

Loop level: Flavor-Changing Neutral Currents

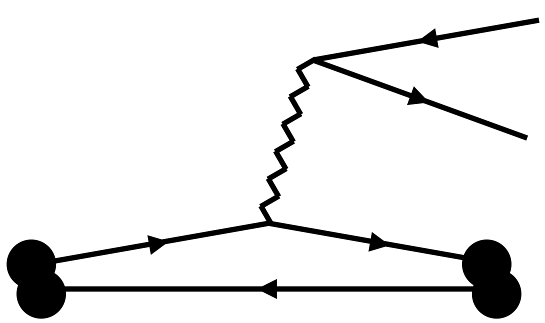
Leptonic decays



(Decay constants)

$$\langle 0|\, A^\mu \,|H(P)\rangle = i f_H\, p^\mu$$

Semi-leptonic decays



(Form factors)

$$f_J(p) \propto \langle \text{final}|\, J(p)\, |\text{initial}\rangle$$

$B_s \to \ell^+\ell^-$

$B \to K\ell\nu$

$B \to K^\star\ell\nu$ $\quad b \to s\ell\ell$

$\Lambda_b \to \Lambda\ell\nu$

$\Lambda_c \to p\mu^+\mu^-$ $\quad c \to u\ell\ell$

Hard-to-compute (=presently incalculable) long-distance charm loops render rare charm decays very difficult theoretically