Kaon decays and the Cabibbo Angle Anomaly

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First-row CKM unitarity

Standard-model coupling of quarks and leptons to *W*:

$$\frac{g}{\sqrt{2}}W_\alpha^+\ \left(\overline{\mathbf{U}}_L\mathbf{V}_{\rm CKM}\gamma^\alpha\mathbf{D}_L+\overline{e}_L\gamma^\alpha\nu_{e\,L}+\overline{\mu}_L\gamma^\alpha\nu_{\mu\,L}+\overline{\tau}_L\gamma^\alpha\nu_{\tau\,L}\right)\ +\ \text{h.c.}$$

Single gauge coupling

Unitary matrix

$$\Delta_{\rm CKM}\ \equiv\ |V_{ud}^2|+|V_{us}^2|+\cancel{|V_{ub}^2|}-1$$

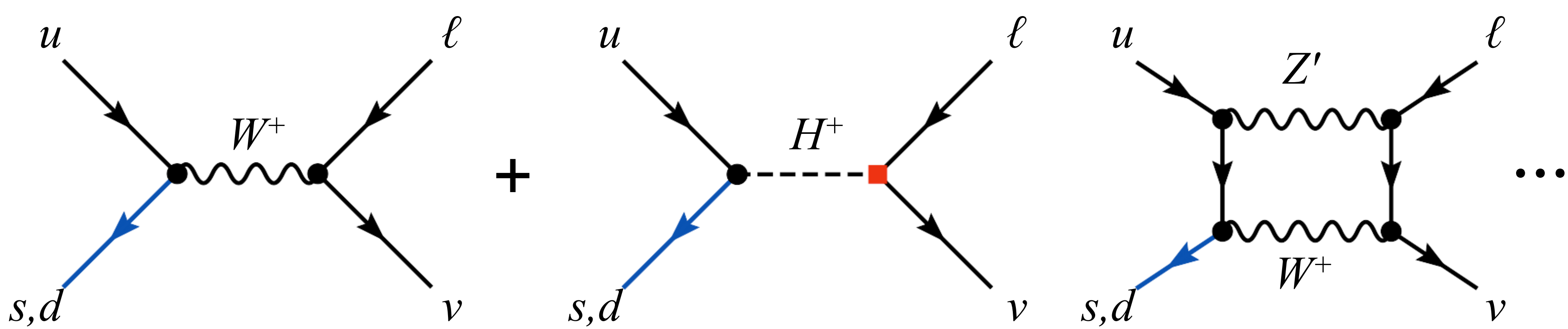
≈ 2×10^{-5}

Most precise test of CKM unitarity!

Universality: Is G_F from μ decay equal to G_F from π, K, nuclear β decay?

$$G_\mu^2=(g_\mu g_e)^2/M_W^4\quad\stackrel{?}{=}\quad G_{\rm CKM}^2=(g_q g_\ell)^2\left(|V_{ud}|^2+|V_{us}|^2\right)/M_W^4$$

Physics beyond the Standard Model can break gauge universality:



First-row CKM unitarity

Standard-model coupling of quarks and leptons to W:

$$\frac{g}{\sqrt{2}} W_\alpha^+ \left(\overline{\mathbf{U}}_L \mathbf{V}_{\mathrm{CKM}} \gamma^\alpha \mathbf{D}_L + \overline{e}_L \gamma^\alpha \nu_{e\,L} + \overline{\mu}_L \gamma^\alpha \nu_{\mu\,L} + \overline{\tau}_L \gamma^\alpha \nu_{\tau\,L}\right) \quad + \quad \text{h.c.}$$

Single gauge coupling *Unitary matrix*

$$\Delta_{\mathrm{CKM}} \equiv |V_{ud}^2| + |V_{us}^2| + \cancel{|V_{ub}^2|} - 1$$

$\approx 2\times10^{-5}$

Most precise test of CKM unitarity!

$$G_{\mathrm{CKM},ij} \sim G_\mu V_{ij} \sim (g^2/M_W^2)\, V_{ij}$$

$\delta G_{\mathrm{CKM}} \sim 1/\Lambda^2$ ← energy scale for NP

BSM effects scale as $(M_W^2/g^2)/\Lambda^2$

For measurement of Δ_{CKM} with total uncertainty σ:

- Scale probed is $\Lambda \sim (M_W/g)/\sqrt{\sigma}$
- For $\sigma \sim 10^{-4}$ → probe **Λ ~ 20 TeV**

Determination of V_{us} from $K_{\ell 3}$ data

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\text{EW}} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\text{EM}}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

S_{EW} Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios: K_S, K_L, $K^\pm$
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

- K_{e3}: Only λ_+ (or λ_+', λ_+'')
- $K_{\mu 3}$: Need λ_+ and λ_0

Inputs from theory:

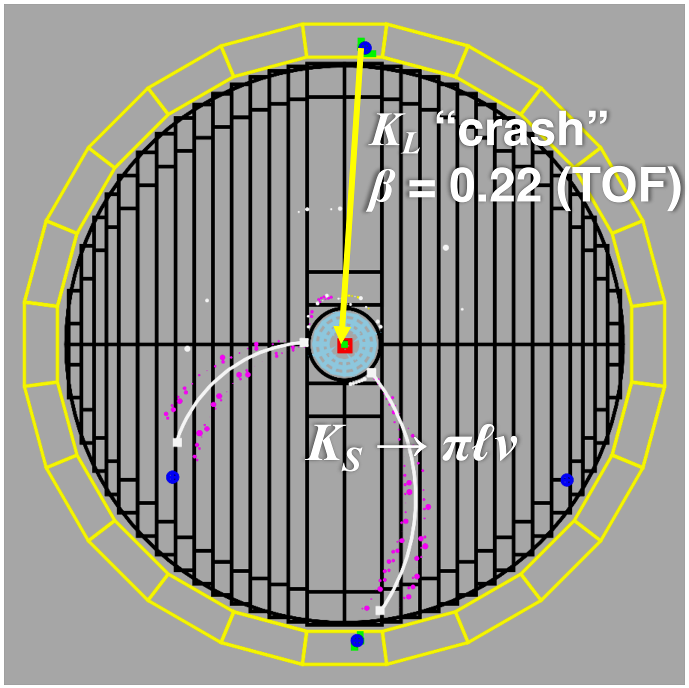
$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

$\Delta_{K\ell}^{\text{EM}}$ Form-factor correction for long-distance EM effects

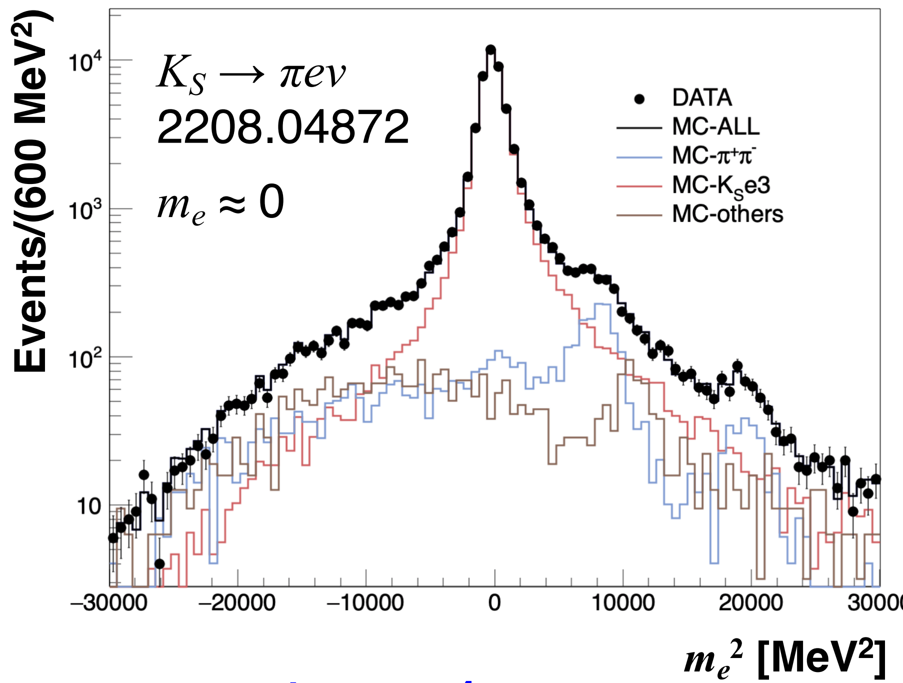
BR($K_S \to \pi\ell\nu$) from KLOE-2

K_S from $\phi \to K_L K_S$ tagged by K_L interaction in calorimeter barrel



Preselection with kinematic BDT and time-of-flight $\pi\ell$ assignment

Fit to $m_\ell^2 = (E_{KS} - E_\pi - p_{\text{miss}})^2 - \mathbf{p}_\ell^2$



KLOE-2
PLB 804 (2020)

BR($K_S \to \pi\mu\nu$) = (4.56 ± 0.20) × 10^{-4}
First measurement of this BR

KLOE-2
2208.04872

BR($K_S \to \pi e\nu$) = (7.153 ± 0.037 ± 0.043) × 10^{-4}
0.4 + 1.6 fb^{-1}: 0.8% uncertainty

Fit to K_S rate data (2022)

7 input measurements:

- **KLOE '06** BR $\pi^0\pi^0/\pi^+\pi^-$
- **NA48** $\Gamma(K_S \to \pi e \nu)/\Gamma(K_L \to \pi e \nu)$, τ_S
- **KLOE '11** τ_S
- **KTeV '11** τ_S
- **KLOE-2 '22** BR $\pi e \nu/\pi^+\pi^-$ **New!**
- **KLOE-2 '20** BR $\pi \mu \nu/\pi^+\pi^-$

2 possible constraints:

- **Σ BR = 1**
- **BR(K_{e3})/BR($K_{\mu3}$) = 0.6640(17)**
  From ratio of phase-space integrals from current fit to dispersive $K_{\ell 3}$ form factor parameters

Only sum constraint used for fit

Parameter	Value
BR($\pi^+\pi^-(\gamma)$)	69.20(5)%
BR($\pi^0\pi^0$)	30.69(5)%
BR(K_{e3})	7.15(6) × 10^{-4}
BR($K_{\mu3}$)	4.56(20) × 10^{-4}
τ_S	89.58(4) ns

χ^2/ndf = 0.36/3 (Prob = 95%)

Little correlation for K_{e3} $K_{\mu3}$ from fit

10-20% correlations with $\pi^0\pi^0/\pi^+\pi^-$

Input measurements essentially unchanged

Fit to K_L rate data (2010)

21 input measurements:

- **5 KTeV** ratios
- **NA48** BR(K_{e3}/2 track)
- **4 KLOE** BRs
  with dependence on τ_L
- **KLOE**, **NA48** BR($\pi^+\pi^-/K_{\ell 3}$)
- **KLOE**, **NA48** BR($\gamma\gamma/3\pi^0$)
- **BR($2\pi^0/\pi^+\pi^-$) from K_S fit, Re ε'/ε**
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh '72** τ_L
- **KTeV** BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)
- **E731, 2 KTeV** BR($\pi^+\pi^-\gamma_{\mathrm{DE}}/\pi^+\pi^-\gamma$)

1 constraint: Σ BR = 1

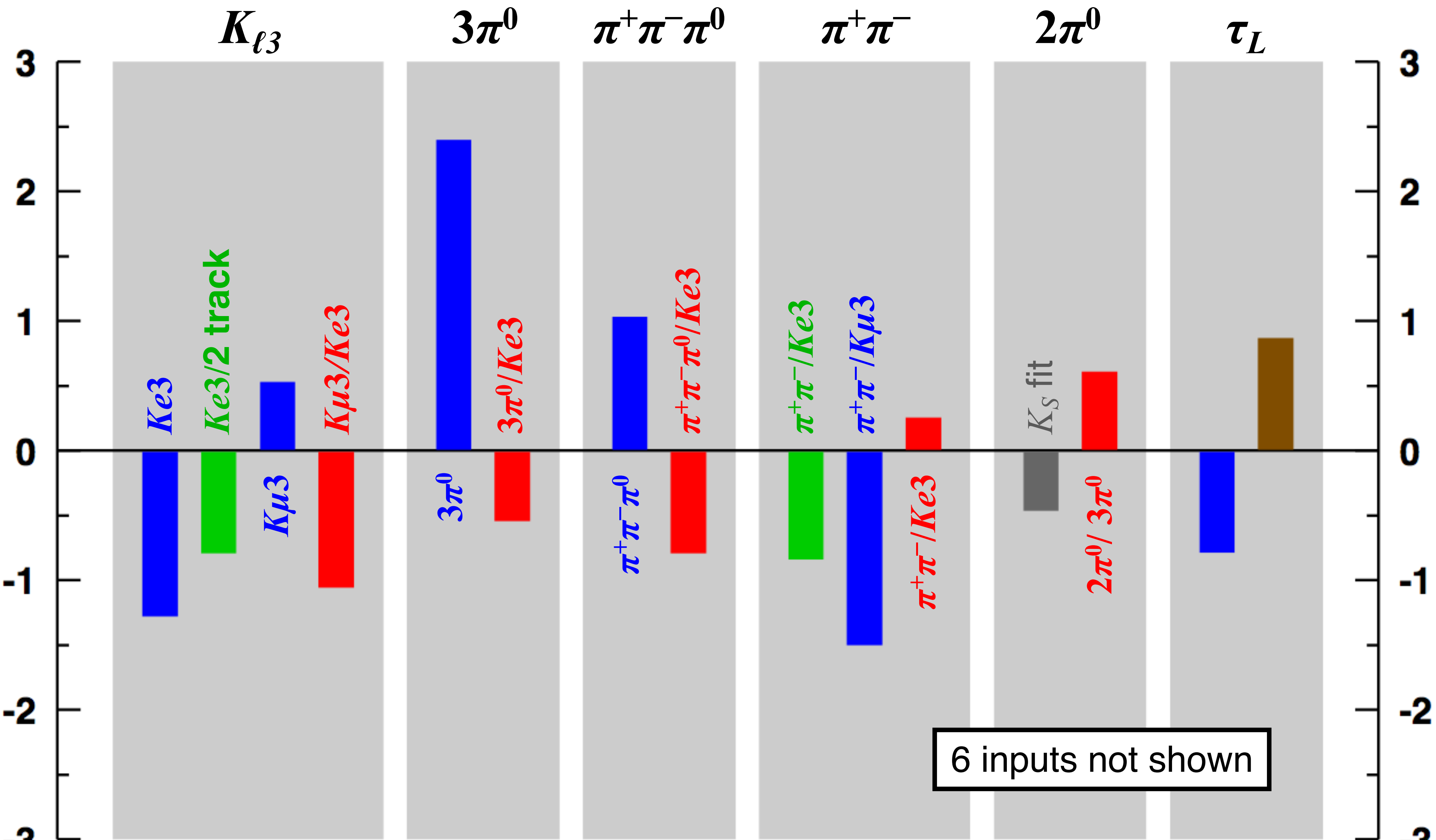
Parameter	Value	S
BR(K_{e3})	0.4056(9)	1.3
BR($K_{\mu 3}$)	0.2704(10)	1.5
BR($3\pi^0$)	0.1952(9)	1.2
BR($\pi^+\pi^-\pi^0$)	0.1254(6)	1.3
BR($\pi^+\pi^-(\gamma_{\mathrm{IB}})$)	$1.967(7) \times 10^{-3}$	1.1
BR($\pi^+\pi^-\gamma$)	$4.15(9) \times 10^{-5}$	1.6
BR($\pi^+\pi^-\gamma_{\mathrm{DE}}$)	$2.84(8) \times 10^{-5}$	1.3
BR($2\pi^0$)	$8.65(4) \times 10^{-4}$	1.4
BR($\gamma\gamma$)	$5.47(4) \times 10^{-4}$	1.1
τ_L	51.16(21) ns	1.1

χ^2/ndf = 19.8/12 (Prob = 7.0%)

Essentially same result as 2010 fit

Current PDG (since '09): **37.4/17** (0.30%)

Comparison: K_L fit result vs. input data



Measurement pulls for KTeV KLOE NA48 K_S fit Vosburgh '72

Fit to $K^{\pm}$ rate data (2014)

17 input measurements:

- **3 old τ values in PDG**
- **KLOE τ**
- **KLOE BR $\mu\nu$, $\pi\pi^0$**
- **KLOE BR K_{e3}, $K_{\mu3}$** with dependence on τ
- **NA48/2 BR $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$**
- **E865 BR K_{e3}/KDal**
- **3 old BR $\pi\pi^0/\mu\nu$**
- **KEK-246 $K_{\mu3}/K_{e3}$**
- **KLOE BR $\pi\pi\pi$, $\pi\pi^0\pi^0$**
- (Bisi '65 BR $\pi\pi^0\pi^0/\pi\pi\pi$ removed)

1 constraint: Σ BR = 1

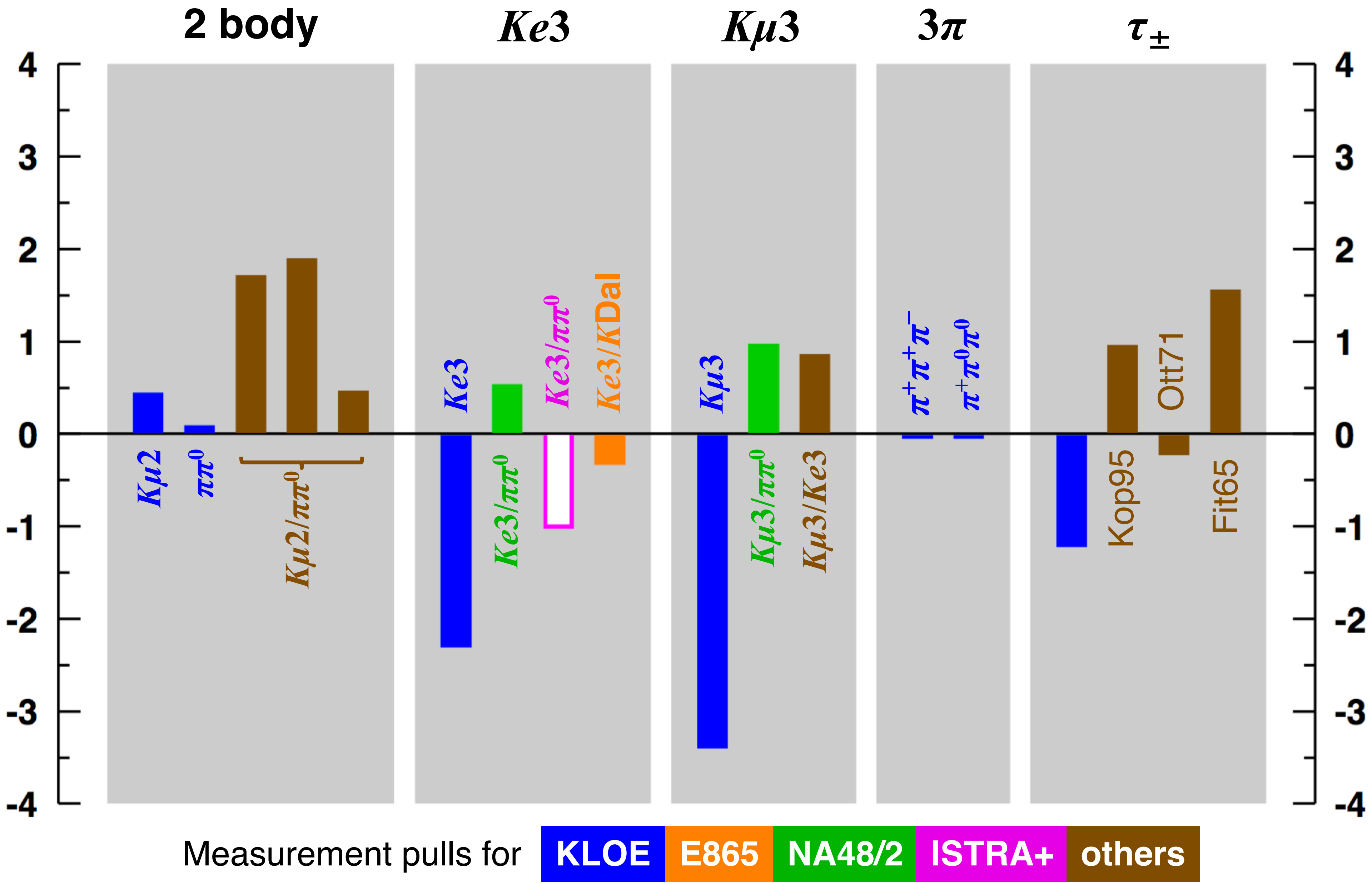
Much more selective than PDG fit
PDG '16: 35 inputs, 8 parameters

Parameter	Value	S
BR($\mu\nu$)	63.58(11)%	1.1
BR($\pi\pi^0$)	20.64(7)%	1.1
BR($\pi\pi\pi$)	5.56(4)%	1.0
BR(K_{e3})	5.088(27)%	1.2
BR($K_{\mu3}$)	3.366(30)%	1.9
BR($\pi\pi^0\pi^0$)	1.764(25)%	1.0
$\tau_{\pm}$	12.384(15) ns	1.2

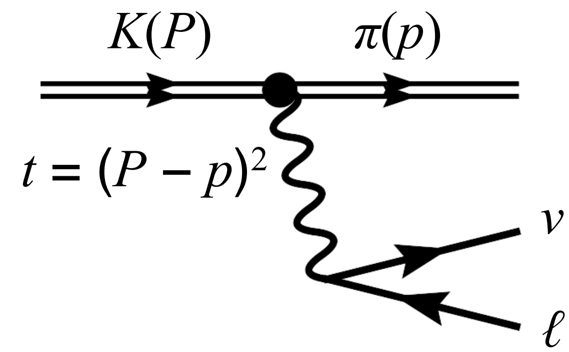
χ^2/ndf = 25.5/11 (Prob = 0.78%)
compare PDG '16: 53/28 (0.26%)

With **ISTRA+ '14 BR($K^-_{e3}/\pi^-\pi^0$)**
- **BR(K_{e3}) = 5.083(27)%**
- Negligible changes in other parameters, fit quality

Comparison: $K^{\pm}$ fit result vs. input data



$K_{\ell 3}$ form factors



Hadronic matrix element:

$$\langle \pi | J_\alpha | K \rangle = f(0) \times \left[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha \right]$$

K_{e3} decays: Only **vector form factor:** $\tilde{f}_+(t)$

$K_{\mu 3}$ decays: Also need **scalar form factor:** $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \dfrac{t}{m_K^2 - m_\pi^2}$

For V_{us}, need integral over phase space of squared matrix element:

Parameterize form factors and fit distributions in t (or related variables)

Parameterizations based on systematic expansions

Taylor expansion:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right) + \lambda''_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)^2$$

Notes:

Many parameters: λ_+', λ_+'', λ_0', λ_0''

Large correlations, unstable fits

Higher-order terms ignored

$K_{\ell 3}$ form-factor parameterizations

Parameterizations incorporating physical constraints

Pole dominance: $$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t}$$

Notes:

What does M_S correspond to?

Dispersion relations:

$$\tilde{f}_{+}(t) = \exp\left[\frac{t}{m_\pi^2}(\Lambda_+ - H(t))\right]$$

$$\tilde{f}_{0}(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2}(\ln C - G(t))\right]$$

Notes:

Allows tests of ChPT & low-energy dynamics

$H(t)$, $G(t)$ evaluated from $K\pi$ scattering data and given as polynomials

Bernard et al., PRD 80 (2009)

Uncertainties from representations $H(t)$, $G(t)$ of $K\pi$ phase-shift data contribute to fit results for Λ_+, $\ln C$

– Small compared to other uncertainties for single measurements (so far)

2010 FlaviaNet analysis used average of FF parameters from dispersive fits

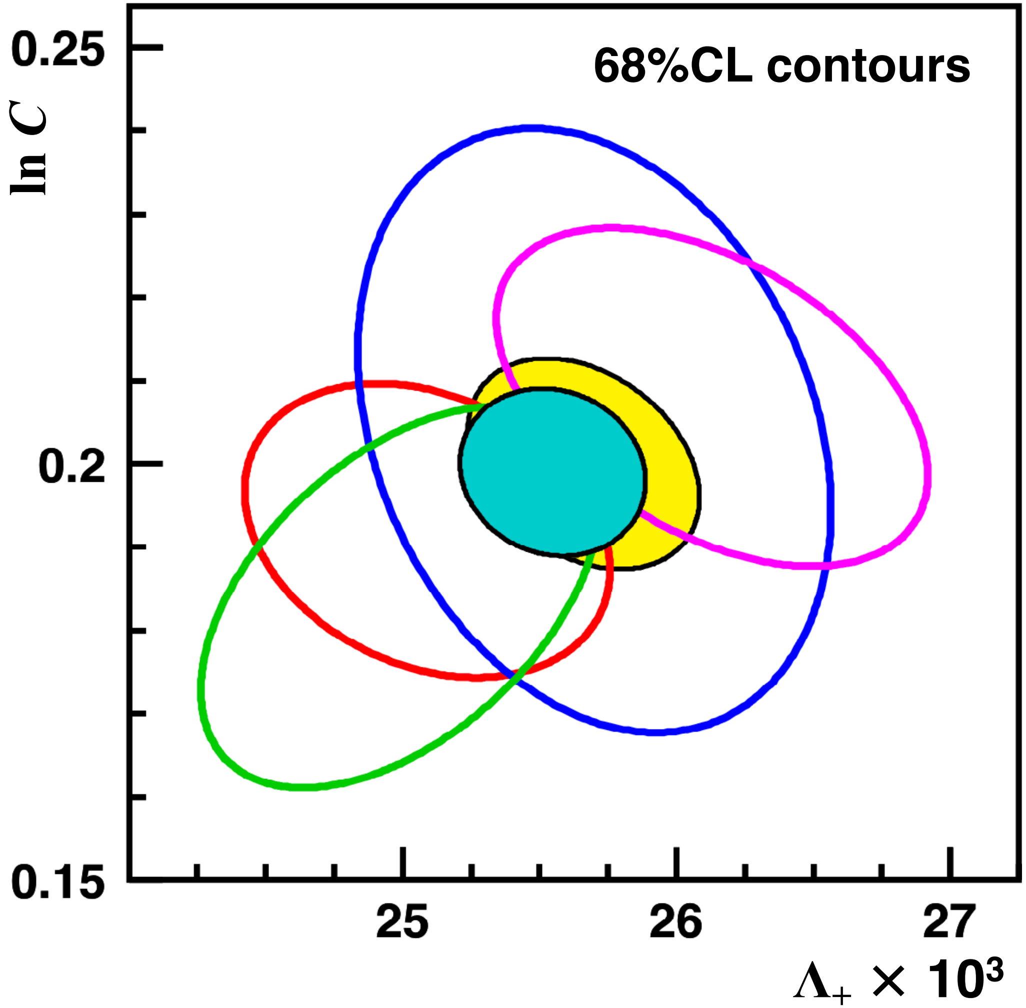
– Parameterization uncertainties beginning to dominate averages for Λ_+, $\ln C$

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Current**



$\Lambda_+ \times 10^3$	=	**25.55 ± 0.38**
$\ln C$	=	**0.1992(78)**
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Integrals

Mode	Update	2010
$K^0{}_{e3}$	**0.15470(15)**	0.15476(18)
$K^+{}_{e3}$	**0.15915(15)**	0.15922(18)
$K^0{}_{\mu 3}$	**0.10247(15)**	0.10253(16)
$K^+{}_{\mu 3}$	**0.10553(16)**	0.10559(17)

Only tiny changes in central values

Dispersive parameters for $K_{\ell 3}$ form factors

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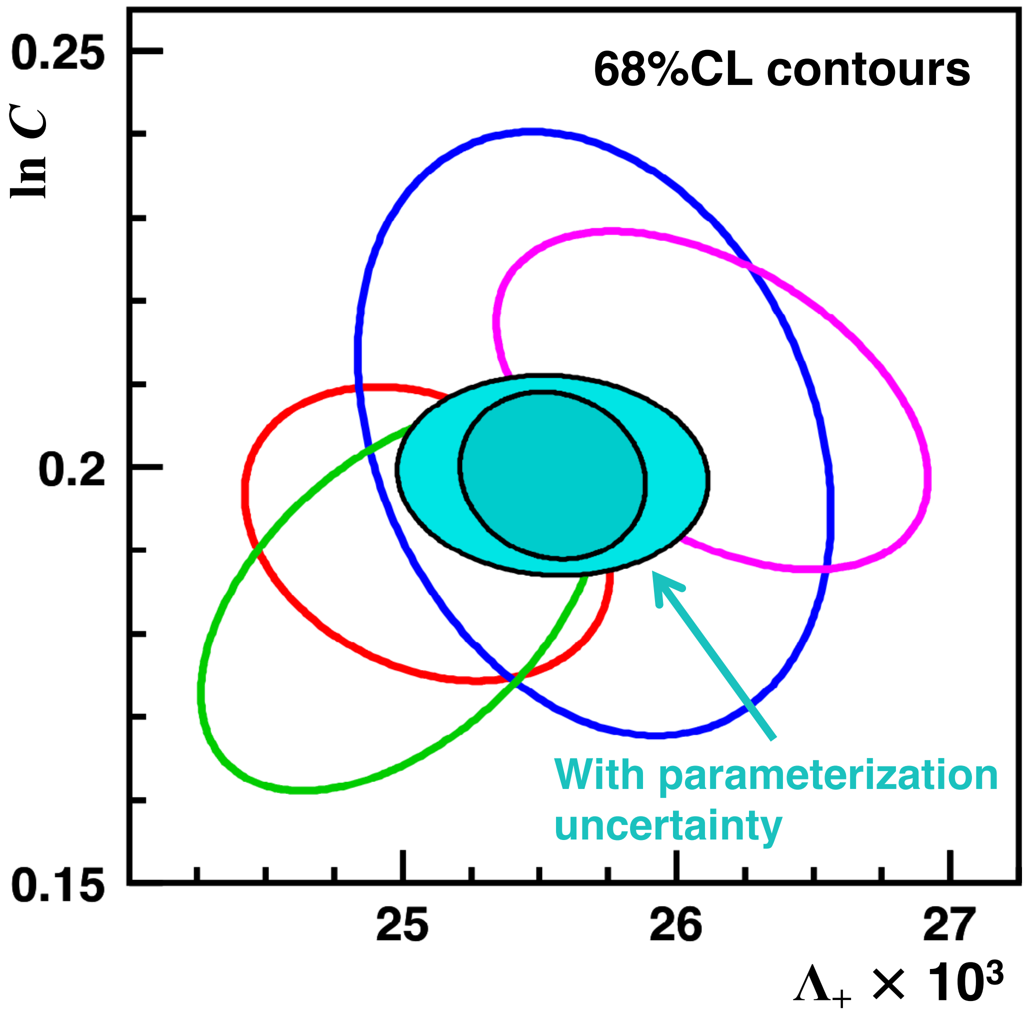

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Fit results include common uncertainty from $H(t)$, $G(t)$:

$\sigma_{\text{param}}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{\text{param}}(\ln C) = 0.0040$

KTeV, Bernard et al. '09

Confidence ellipses shown **without** common uncertainty
(except as indicated)

Long-distance EM corrections

Mode-dependent corrections $\Delta^{\mathrm{EM}}_{K\ell}$ to phase-space integrals $I_{K\ell}$ from EM-induced Dalitz plot modifications

- Values depend on acceptance for events with additional real photon(s)
- All recent measurements assumed fully inclusive

FlaviaNet analysis and updates used Cirigliano et al. '08

- Comprehensive analysis at fixed order e^2p^2

Seng et al.
JHEP 07 (2022)

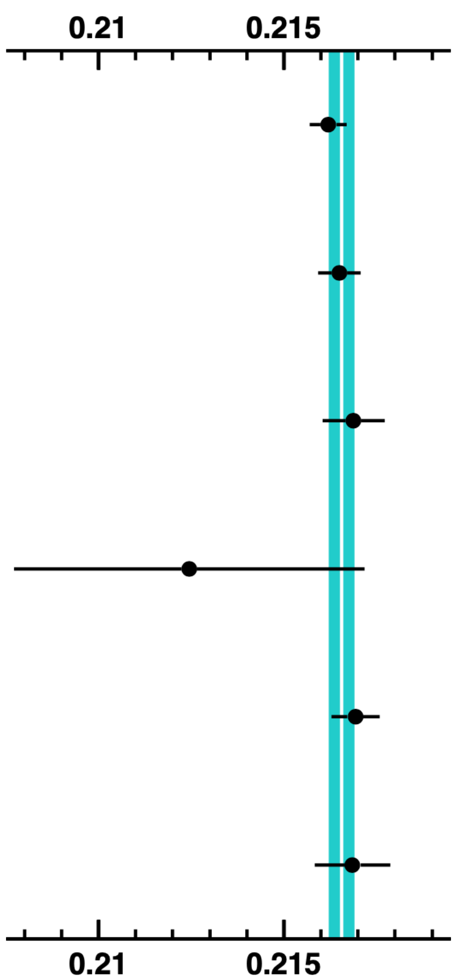
Calculation of complete EW RC using hybrid current algebra and ChPT with resummation of largest terms to all chiral orders

- Reduced uncertainties at O(e^2p^4)
- Lattice evaluation of QCD contributions to γW box diagrams
- Conventional value of S_{EW} subtracted from results for use with standard formula for V_{us}

	Cirigliano et al. '08	Seng et al. '21
$\Delta^{\mathrm{EM}}(K^0_{e3})$ [%]	0.50 ± 0.11	**0.580 ± 0.016**
$\Delta^{\mathrm{EM}}(K^+_{e3})$ [%]	0.05 ± 0.12	**0.105 ± 0.023**
$\Delta^{\mathrm{EM}}(K^+_{\mu3})$ [%]	0.70 ± 0.11	**0.770 ± 0.019**
$\Delta^{\mathrm{EM}}(K^0_{\mu3})$ [%]	0.01 ± 0.12	**0.025 ± 0.027**

$|V_{us}|\,f_+(0)$ from world data: 2022 update

$|V_{us}|\,f_+(0)$

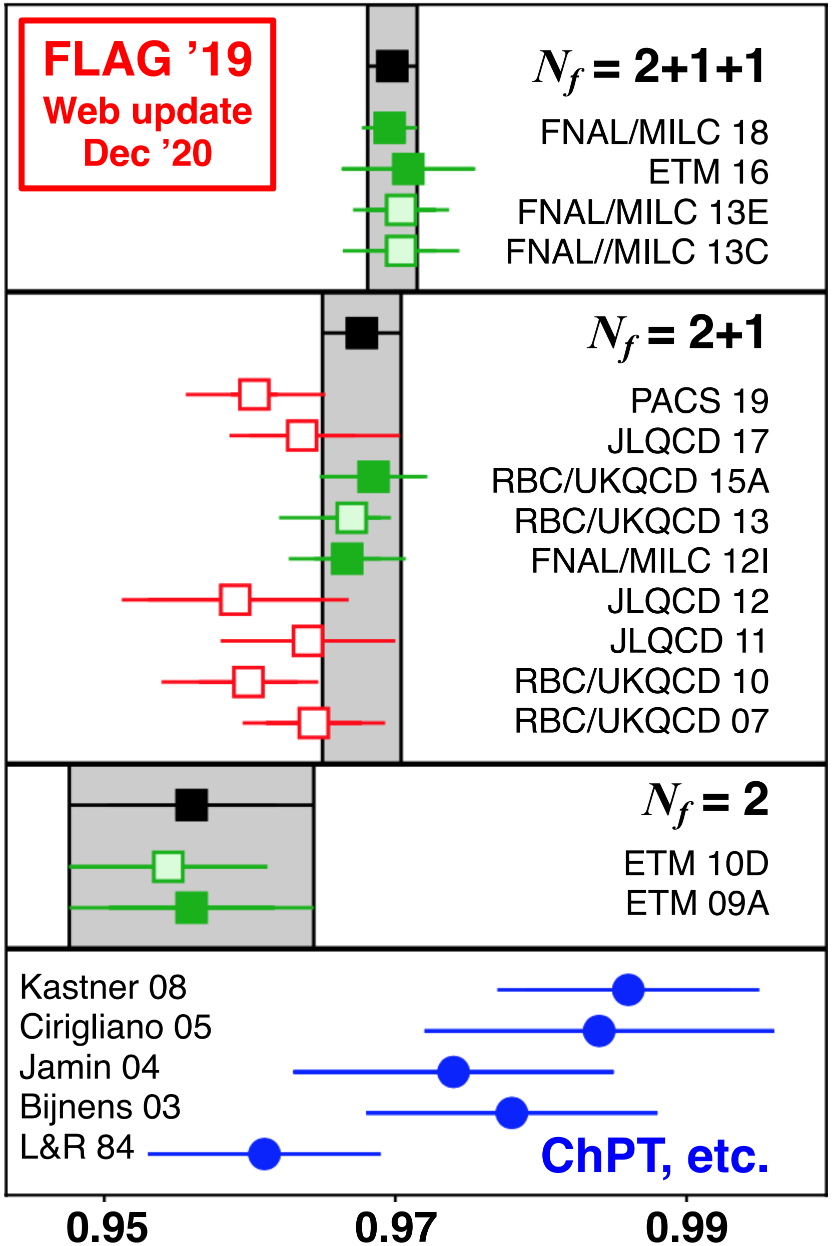


Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
K_Le3	**0.2162(5)**	**0.23**	0.09	0.20	0.02	0.05
$K_L\mu3$	**0.2165(6)**	**0.26**	0.15	0.18	0.02	0.07
K_Se3	**0.2169(8)**	**0.39**	0.38	0.02	0.02	0.05
$K_S\mu3$	**0.2125(47)**	**2.2**	2.2	0.02	0.02	0.08
$K^{\pm}e3$	**0.2169(6)**	**0.30**	0.27	0.06	0.11	0.05
$K^{\pm}\mu3$	**0.2168(10)**	**0.47**	0.45	0.06	0.11	0.08

Average: $|V_{us}|\,f_+(0)$ = 0.21656(35) **χ^2/ndf = 1.89/5 (86%)**

Evaluations of $f_+(0)$



FLAG '21 averages:

N_f = 2+1+1 **$f_+(0) = 0.9698(17)$**

Uncorrelated average of:

FNAL/MILC 18: HISQ, 5sp, $m_\pi \rightarrow 135$ MeV, new ensembles added to FNAL/MILC 13E

ETM 16: TwMW, 3sp, $m_\pi \rightarrow 210$ MeV, full q^2 dependence of f_+, f_0

N_f = 2+1 **$f_+(0) = 0.9677(27)$**

Uncorrelated average of:

FNAL/MILC 12I: HISQ, $m_\pi \sim 300$ MeV

RBC/UKQCD 15A: DWF, $m_\pi \rightarrow 139$ MeV

JLQCD 17 not included because only single lattice spacing used

ChPT **$f_+(0) = 0.970(8)$**

Ecker 15, Chiral Dynamics 15:
Calculation from Bijnens 03,
with new LECs from Bijnens, Ecker 14

Evaluations of $f_+(0)$

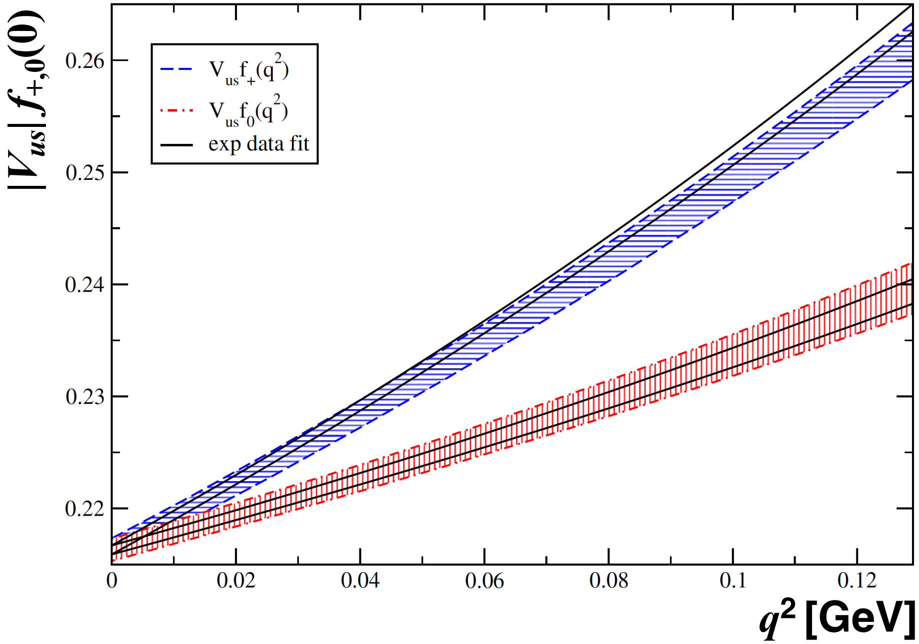
ETM
PRD 93 (2016)

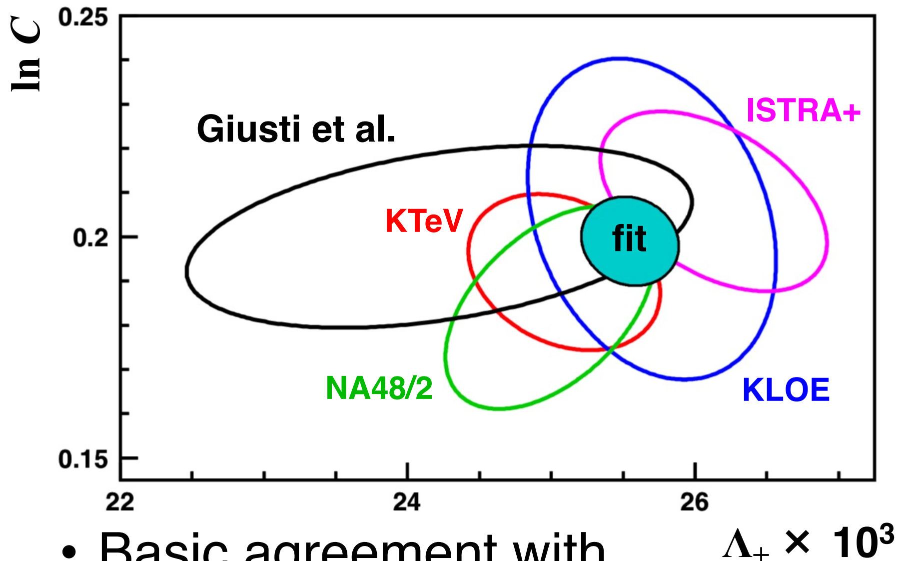
N_f **= 2+1+1**

$f_+(0) = 0.9709(44)_{st}(9)_{sy}(11)_{ext}$

Full q^2 dependence of f_+, f_0

See also:
PACS PRD 101 (2020)
ETM PRD 105 (2022)





Fit synthetic data points with dispersive parameterization

$\mathbf{\Lambda_+ = 24.22(1.16) \times 10^{-3}}$

$\mathbf{\ln C = 0.1998(138)}$

$\rho(\Lambda_+, f_+(0)) = -0.228$

$\rho(\ln C, f_+(0)) = -0.719$

$\rho(\Lambda_+, \ln C) = +0.376$

- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

V_{us}/V_{ud} and $K_{\ell 2}$ decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2} \left(1 - \frac{1}{2}\delta_{\mathrm{EM}} - \frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from experiment:

From $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

From PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

Inputs from theory:

δ_{EM} Long-distance EM corrections

$\delta_{SU(2)}$ Strong isospin breaking
$f_K/f_\pi \to f_{K\pm}/f_{\pi\pm}$

f_K/f_π Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

V_{us}/V_{ud} and $K_{\ell 2}$ decays

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- N_f = 2+1+1 Twisted-mass Wilson fermions

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)}$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld ’11:

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)}$

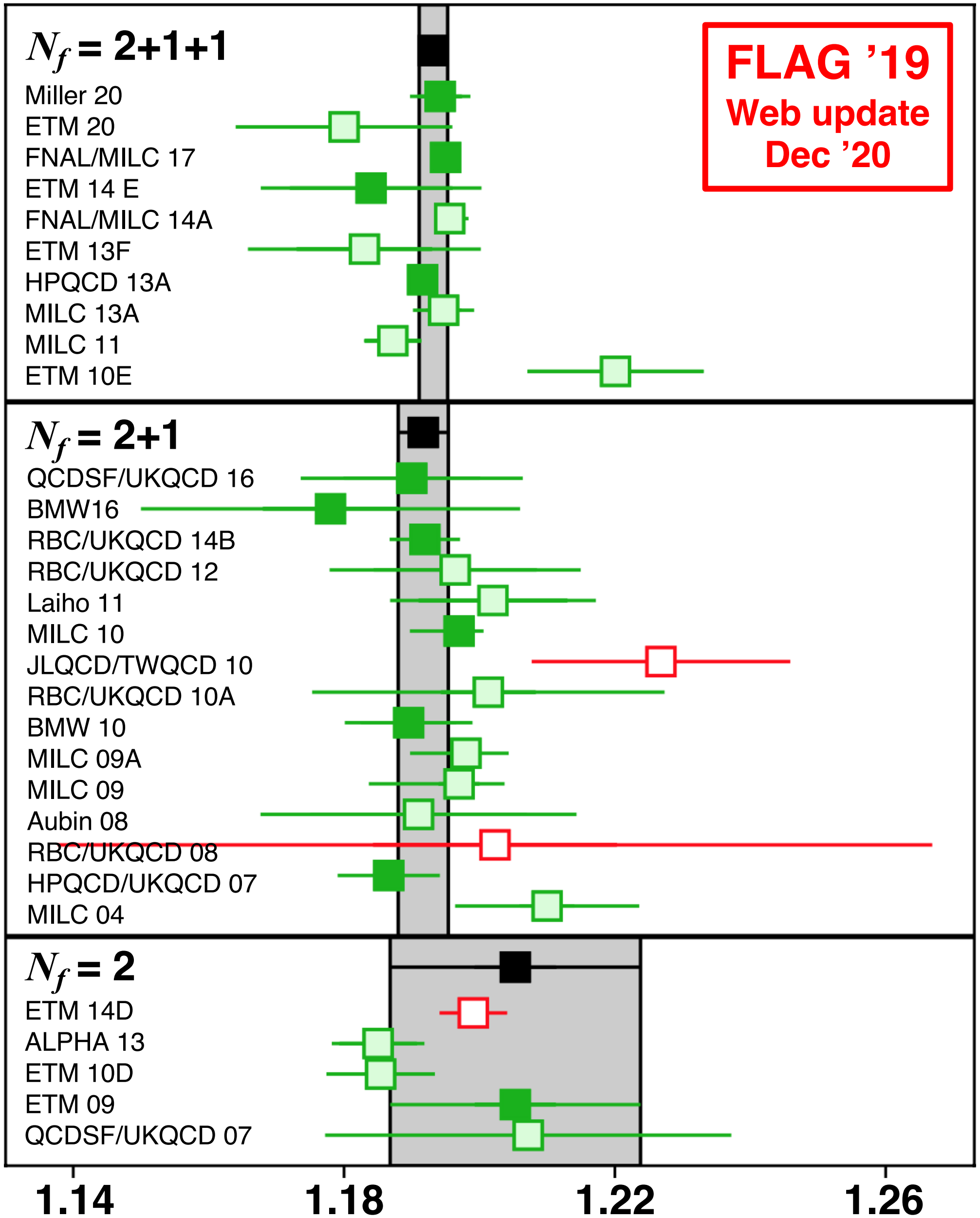
Di Carlo et al.
PRD 100 (2019)

Update, extended description, and systematics of Giusti et al.

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)}$

$\boldsymbol{|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{\mathrm{BR}}(20)_{\mathrm{corr}}}$

Lattice evaluations of f_K/f_π



N_f = 2+1+1 $\quad f_{K\pm}/f_{\pi\pm}$ = 1.1932(19)

Miller 20
- HISQ + DWF, 4sp, $m_\pi \to$ 130 MeV
- Uses FNAL/MILC ensembles

FNAL/MILC 17
- HISQ, 4sp, m_π phys
- Updates MILC 13A, FNAL/MILC 14A

ETM 14E
- TwM, 3sp, m_π = 210-450 MeV

HPQCD 13A
- HISQ, 3sp, m_π phys
- Uses FNAL/MILC ensembles

N_f = 2+1 $\quad f_{K\pm}/f_{\pi\pm}$ = 1.1917(37)

QCDSF/UKQCD 16
- Clover, 4sp, $m_\pi \to$ 220 MeV

BMW 16
- Clover, 5sp, $m_\pi \to$ 139 MeV

RBC/UKQCD 14B
- DWF, m_π = 139 MeV
- f_K and f_π separately (isospin limit)

Lattice results for f_K/f_π

Recalculate FLAG averages for results without $SU(2)$-breaking
Isospin-limit results as reported in original papers

N_f = 2+1+1

ETM 21 New!	**1.1995(44)(7)**
TM quarks, 3sp, $m_\pi \rightarrow$ physical Not yet in FLAG '21 average! Replaces ETM 14E in our average	
Miller 20	1.1964(44)
FNAL/MILC17	$1.1980(^{+13}_{-19})$
HPQCD13A	1.1948(15)(18)

f_K/f_π = 1.1978(22) S = 1.1

Average is problematic with correlations assumed by FLAG, dominated by FNAL/MILC17 (symmetrized)

Share ensembles
Partially correlated uncertainties using FLAG prescription

N_f = 2+1

QCDSF/UKQCD17	1.192(10)(13)
BMW16	1.182(10)(26)
RBC/UKQCD14B	1.1945(45)
BMW10	1.192(7)(6)
HPQCD/UKQCD07	1.198(2)(7)

f_K/f_π = 1.1946(34)*

* MILC10 omitted from average because unpublished

V_{us} from kaon decays: Summary

$K_{\ell 3}$

$f_+(0) = 0.9698(17)$

$N_f = 2+1+1$

$\mathbf{V_{us} = 0.22330(35)_{exp}(39)_{lat}(8)_{IB}}$

$\mathbf{(53)_{tot}}$ $= 0.24\%$

$K_{\mu 2}$

$f_K/f_\pi = 1.1978(22)$

$N_f = 2+1+1$

$\mathbf{V_{us}/V_{ud} = 0.23108(23)_{exp}(42)_{lat}(16)_{IB}}$

$\mathbf{(51)_{tot}}$ $= 0.22\%$

First hint of an anomaly: ***Without information from β decays***

$$\Delta^{(3)}_{\mathrm{CKM}} = |V_{us}^{K\ell 3}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K\mu 2}} \right)^2 + 1 \right] - 1$$

$\mathbf{\Delta^{(3)}_{CKM} = -0.0164(63)}$

$\mathbf{-2.6\sigma}$

Need additional information to test consistency of $K_{\ell 3}$ and $K_{\mu 2}$

$|V_{ud}|$ from $0^+ \to 0^+$: World data

$$ft = \frac{K}{G_V^2 \langle\tau\rangle^2}$$

$f(Z, Q_{EC})$	statistical rate function
$t = t_{1/2}$/BR	partial half life
$G_V = G_F V_{ud}$	vector coupling constant
$\langle\tau\rangle$	Fermi matrix element

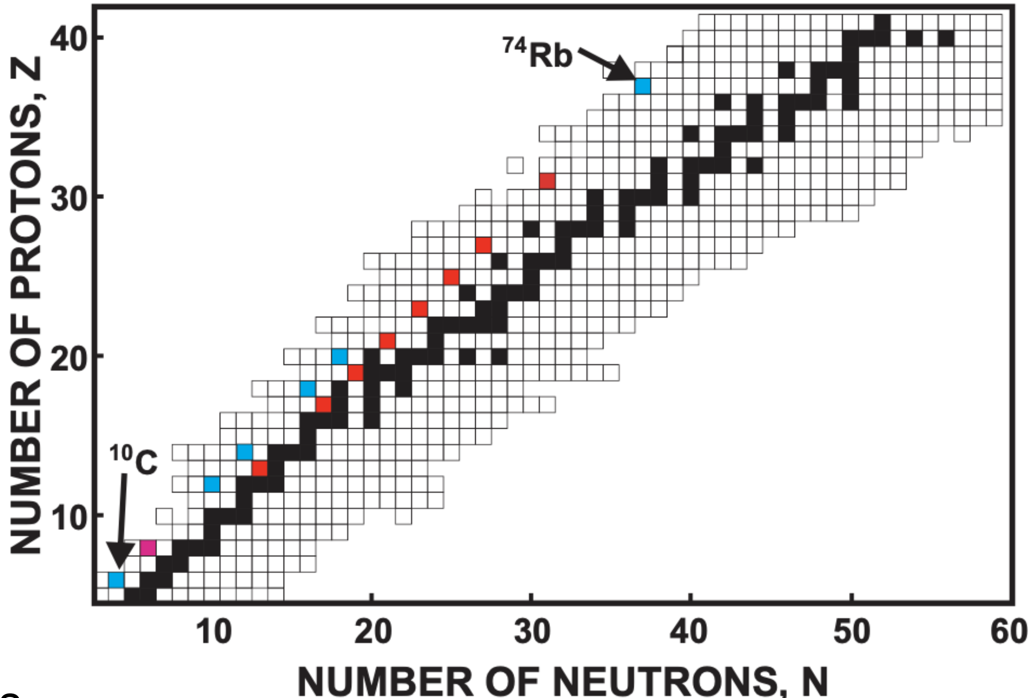
Experimentally, measure

- **BR** **branching ratios**
- $t_{1/2}$ **parent half-life**
- Q_{EC} **transition energy**

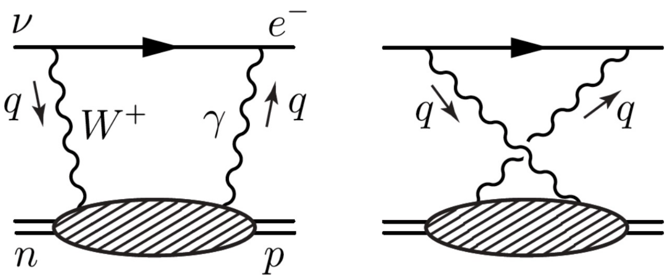
Hardy & Towner
PRC 102 (2020)

Comprehensive survey of *ft* measurements

- 9 cases with precision < 0.05%
- 6 cases with precision 0.05-0.23%
- About 220 individual measurements with compatible precison



V_{ud} and inner radiative correction Δ_R^V



Box diagrams contributing at order α/π to neutron β decay at the hadronic scale

Hardy & Towner
1807.01146

$|V_{ud}|$ = 0.97420(21) **Δ_R^V = 2.361(38)%**
Δ_R^V from Marciano & Sirlin ’06
First-row CKM unitarity respected

Seng et al.
PRD 100 (2019)

$|V_{ud}|$ = 0.97370(14) **Δ_R^V = 2.467(22)%**
New calculation of γW-box contribution to Δ_R^V using dispersion relations and DIS structure functions
3σ shift in Δ_R^V and V_{ud}: the birth of the anomaly!
Also identified need for new calculations of δ_{NS}

Czarnecki et al.
PRD 100 (2019)

$|V_{ud}|$ = 0.97389(18) **Δ_R^V = 2.426(32)%**
Improved use of Bjorken sum rule to constrain strong-interaction corrections to axial-vector component of the γW-box

Hardy & Towner
PRC 102 (2020)

$|V_{ud}|$ = 0.97373(31) **Δ_R^V = 2.454(19)%**
23 new publications, some older measurements eliminated
Use weighted average of above values for Δ_R^V
Larger uncertainty for δ_{NS}

$|V_{ud}|$ from neutron β decays

$$|V_{ud}|^2 = \frac{5024.7\text{s}}{\tau_n\,(1+3\lambda^2)\,(1+\Delta_R)}$$

τ_n	Free neutron lifetime
$\lambda = g_A/g_V$	Ratio of axial to vector couplings
Δ_R	Radiative correction (universal + outer)

- Δ_R under control to same extent as in $0^+\to0^+$
- To match precision from $0^+\to0^+$ require $\sigma_\tau \sim 0.3$ s and $\sigma_\lambda/\lambda \sim 3\times10^{-4}$
- World data set for τ and λ riddled by inconsistencies → large scale factors
  → Use recent high-precision measurements instead of averages

UCNτ
PRL 127 (2021)

$\boldsymbol{\tau_n = 877.75(28)_{\text{stat}}(^{+22}_{-16})_{\text{sys}}}$ **s** Ultra-cold neutron trap
Improves on precision of previous results by > 2x

PERKEO III
PRL 122 (2019)

$\boldsymbol{\lambda = -1.27641(45)_{\text{stat}}(33)_{\text{sys}}}$ β decay asymmetry
5x improvement on precision of world average

Combined result for $|V_{ud}|$

Cirigliano et al.
PLB 838 (2023)

Evaluation of $\Delta_R{}^V$ and Δ_R:

- Hadronic scheme for resummation of infrared logs
- Non-correlated average of contributions to γW box

V_{ud} from neutron decays uses current best measurements (not averages) for τ_n and $\lambda = g_A/g_V$

$0^+ \to 0^+$ with $\Delta_R{}^V = 2.467(27)\%$

$\boldsymbol{V_{ud}{}^{0+\to 0+} = 0.97367(11)_{\text{exp}}(13)_{\Delta RV}(27)_{\text{NS}}}$ $[32]_{\text{tot}}$, 0.033%

n decays with $\Delta_R = 3.983(27)\%$

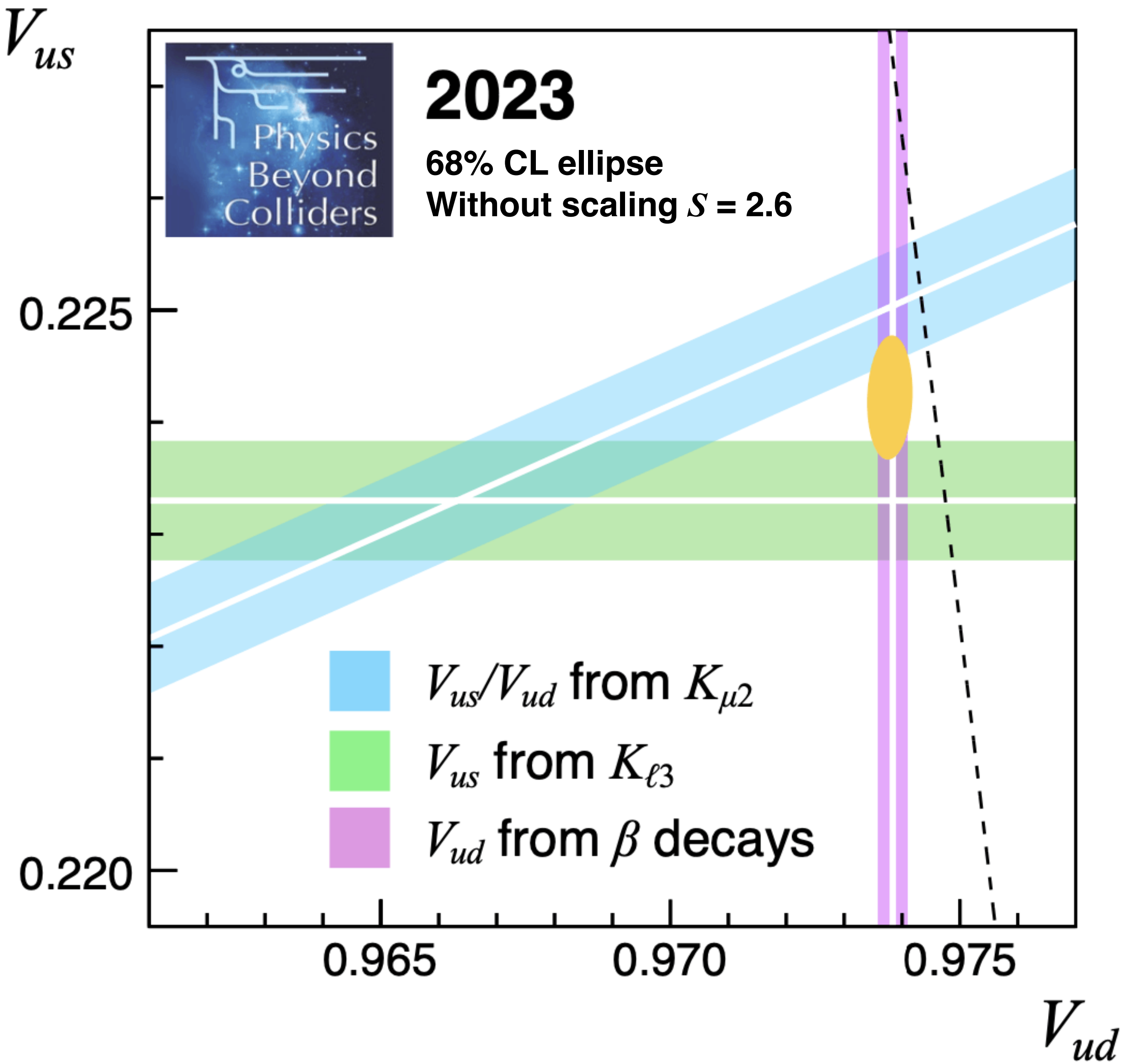
$\boldsymbol{V_{ud}{}^{n,\,\text{best}} = 0.97413(13)_{\Delta R}(35)_{\lambda}(20)_{\tau_n}}$ $[43]_{\text{tot}}$, 0.044%

0.9σ agreement ▶

Average

$\boldsymbol{|V_{ud}| = 0.97384(26)}$

Status of first-row unitarity



Fit results, no constraint

V_{ud} = **0.97378(26)**

V_{us} = **0.22422(36)**

χ^2/ndf = 6.4/2 (4.1%)

Δ_{CKM} = **–0.0018(6)**

–2.8σ

With scale factor S = **2.6**

V_{ud} = **0.9737(8)**

V_{us} = **0.2242(10)**

Status of first-row unitarity

3 observables: $|V_{us}|^{K\ell 3}$, $|V_{us}/V_{ud}|^{K\mu 2}$, V_{ud}
2 quantities to determine: V_{us}, V_{ud}

3 ways to test unitarity

$$\Delta^{(1)}_{\rm CKM} = |V_{ud}|^2 + |V_{us}^{K\ell 3}|^2 - 1$$ **= –0.00176(56) –3.1σ**

$$\Delta^{(2)}_{\rm CKM} = |V_{ud}|^2 \left[1 + \left(\left|\frac{V_{us}}{V_{ud}}\right|^{K\mu 2}\right)^2\right] - 1$$ **= –0.00098(58) –1.7σ**

$K_{\mu 2}$ result shows better agreement with unitarity than $K_{\ell 3}$ result when $|V_{ud}|$ obtained from beta decays:

$$\Delta V_{us}(K_{\ell 3} - K_{\mu 2}) = V_{us}^{K\ell 3} - V_{ud}\left(\frac{V_{us}}{V_{ud}}\right)^{K\mu 2}$$ **= –0.0174(73) –2.4σ**

$\Delta^{(3)}_{\rm CKM}$ uses no information from β decays:

$$\Delta^{(3)}_{\rm CKM} = |V_{us}^{K\ell 3}|^2 \left[\left(\frac{1}{|V_{us}/V_{ud}|^{K\mu 2}}\right)^2 + 1\right] - 1$$ **= –0.0164(63) –2.6σ**

Constraints on right-handed currents

Cirigliano et al.
PLB 838 (2023)

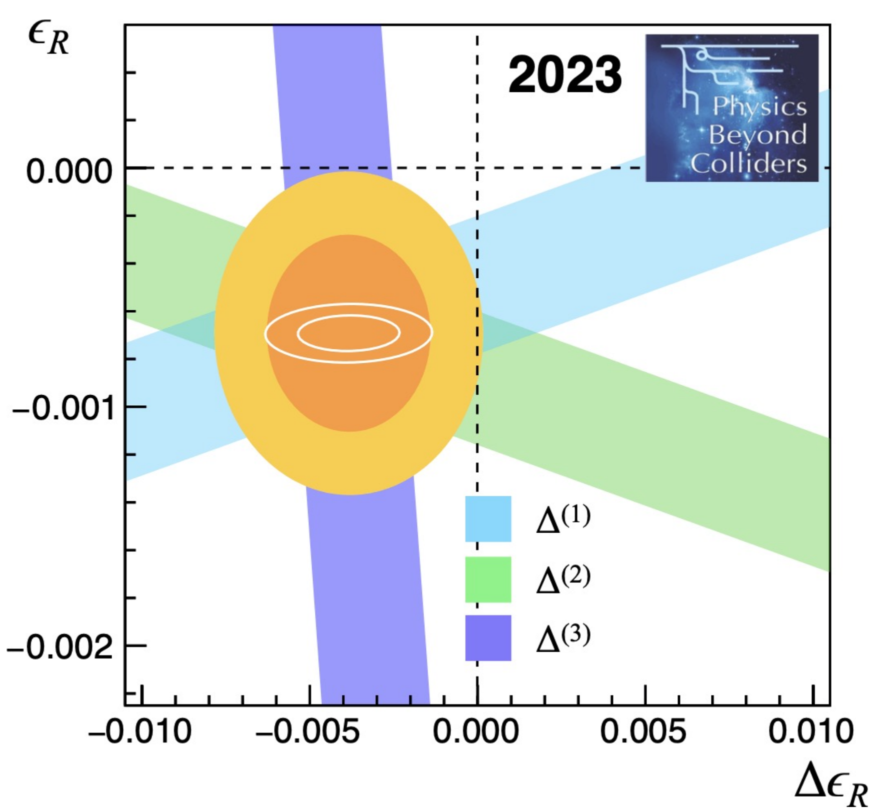
- In SM, W couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}$-$K_{\mu 2}$ difference
- Define ϵ_R = admixture of RH currents in non-strange sector
  $\epsilon_R + \Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(2)}_{\rm CKM} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(3)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)$$

$$r \equiv \left(\frac{1+\Delta^{(2)}_{\rm CKM}}{1+\Delta^{(3)}_{\rm CKM}}\right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

From current fit:

$\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)
$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)
$\epsilon_R = \Delta\epsilon_R = 0$ excluded at 3.1σ



What can NA62 contribute?

NA62 can make a precision measurement of $K_{\mu 3}/K_{\mu 2}$, with many systematics cancelling. What can this measurement alone tell us?

r is proportional to $(K_{\mu 3}/K_{\mu 2})^{-1/2}$:

$$r \equiv \left(\frac{1 + \Delta_{\mathrm{CKM}}^{(2)}}{1 + \Delta_{\mathrm{CKM}}^{(3)}} \right)^{1/2} = \frac{\left. \frac{V_{us}}{V_{ud}} \right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

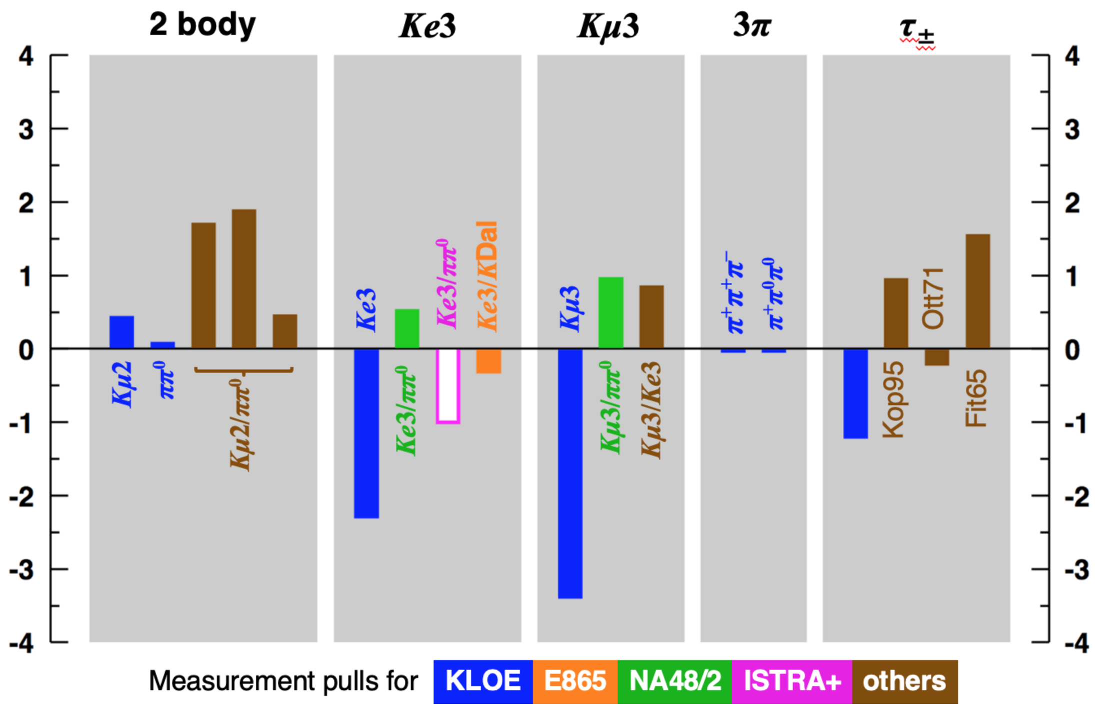
- Uses input from β decays, but provides a qualified statement about consistency of data set
- Search for right-handed currents

NA62 hypothetical $K_{\mu 3}/K_{\mu 2}$ to 0.5%:

Result	$\Delta\epsilon_R$		Remarks
Same as fit	–4.0(1.9)×10^{-3}	2.1σ	Almost same precision as result from world average
+ 1.5σ	–0.4(1.9)×10^{-3}	0.2σ	$K_{\mu 2}$, $K_{\mu 3}$, V_{ud} consistent: current tensions have experimental origin?
– 1.5σ	–7.6(1.9)×10^{-3}	4.0σ	Evidence for right-handed currents contributing to CKM non-unitarity

What can NA62 contribute?

While a high priority, $K_{\mu3}/K_{\mu2}$ is not the only measurement that NA62 can make to help clarify the inconsistencies in the first row



Other important measurements

$K_{\mu2}/\pi\pi^0$	Verify KLOE mmt with nominal 0.27% error
$K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$	K_{e3}, $K_{\mu3}$ have large scale factors
$K_{e3}/K_{\mu3}$	Test of lepton universality

- Suite of redundant measurements for good control of systematics
- Single analysis framework to maximize cancellation of systematics
- **Dedicated data-taking with minimum-bias trigger maintaining stable conditions at low intensity: statistical uncertainties < 0.1% in two weeks**

What can NA62 contribute?

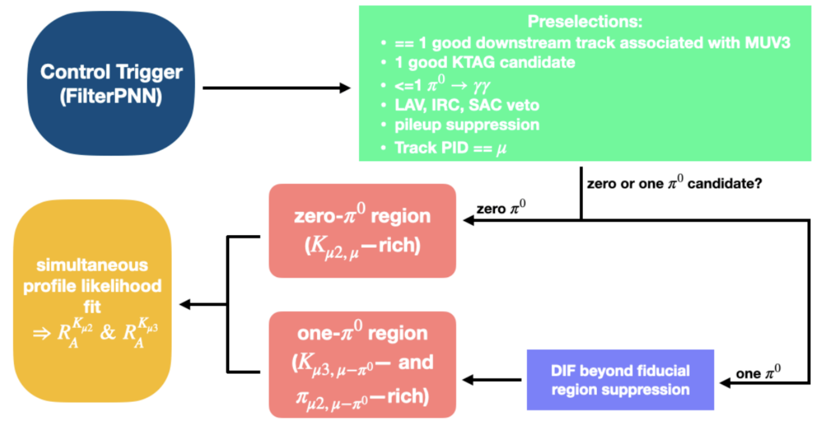
While a high priority, $K_{\mu 3}/K_{\mu 2}$ is not the only measurement that NA62 can make to help clarify the inconsistencies in the first row

Some degree of independence from the global fit can be obtained in an alternate analysis scheme to measure the following ratios:

$$R_A^{K_{\mu 2}} = \frac{K^+ \to \mu^+ \nu}{K^+ \to \pi^0 \pi^+ \to \pi^0 \mu^+ \nu}$$

$$R_A^{K_{\mu 3}} = \frac{K^+ \to \pi^0 \mu^+ \nu}{K^+ \to \pi^0 \pi^+ \to \pi^0 \mu^+ \nu}$$

$$R^{K_{\mu 3}/K_{\mu 2}} = \frac{K^+ \to \pi^0 \mu^+ \nu}{K^+ \to \mu^+ \nu}$$



- Use of coherent analysis scheme to minimize systematics from comparison of modes with/without π^0s and decays in flight

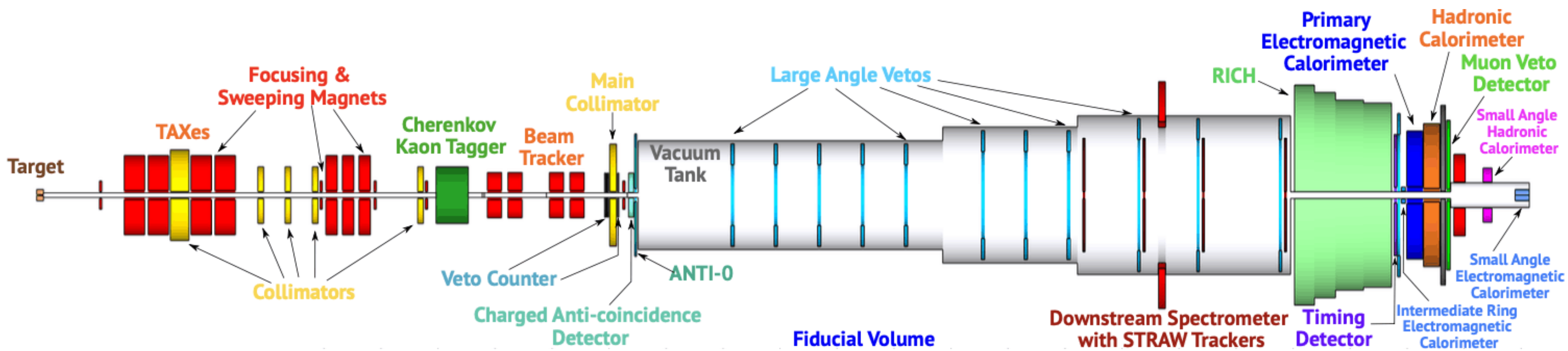
  Simultaneous fit to m^2_{miss} spectra for 0 and 1 π^0 modes
- 2 weeks of dedicated data taking would reduce statistical uncertainty to <0.1%
- Systematic uncertainties under evaluation with NA62 data from 2017-2018: expect to reach < 0.6%

From NA62 to HIKE

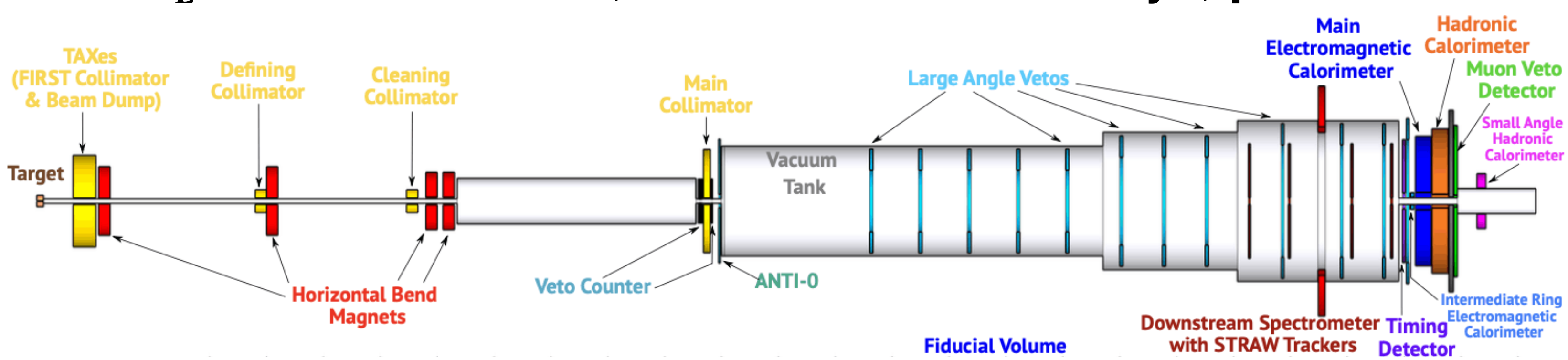
HIKE is a multi-phase, general purpose kaon experiment to extend the NA62 physics program at the CERN SPS into the HL-LHC era and beyond

High Intensity Kaon Experiments

Phase 1: $K^+ \rightarrow \pi^+ \nu\nu$ to 5%, LFV/LNV & other rare decays, precision mmts



Phase 2: $K_L \rightarrow \pi^0 \ell\ell$ to 12-18%, LFV/LNV & rare decays, precision mmts



Plus FIP searches with kaon beams and in periodic dump-mode runs

Hypothetical Phase-1 fit to $K^{\pm}$ rate data

11 input measurements:

3 old τ values in PDG

KLOE τ

KLOE BR $\pi\pi\pi$, $\pi\pi^0\pi^0$

HIKE $\pi\pi^0/\mu\nu$ to 0.4%

HIKE $K_{e3}/\pi\pi^0$ to 0.4%

HIKE $K_{\mu3}/\mu\nu$ to 0.2%

HIKE $K_{\mu3}/\pi\pi^0$ to 0.4%

HIKE $K_{\mu3}/K_{e3}$ to 0.2%

1 constraint: Σ BR = 1

Hypothetical HIKE measurements chosen to agree with V_{us} = **0.22417** (midway between current values for $K_{\ell3}$ and $K_{\mu2}$)

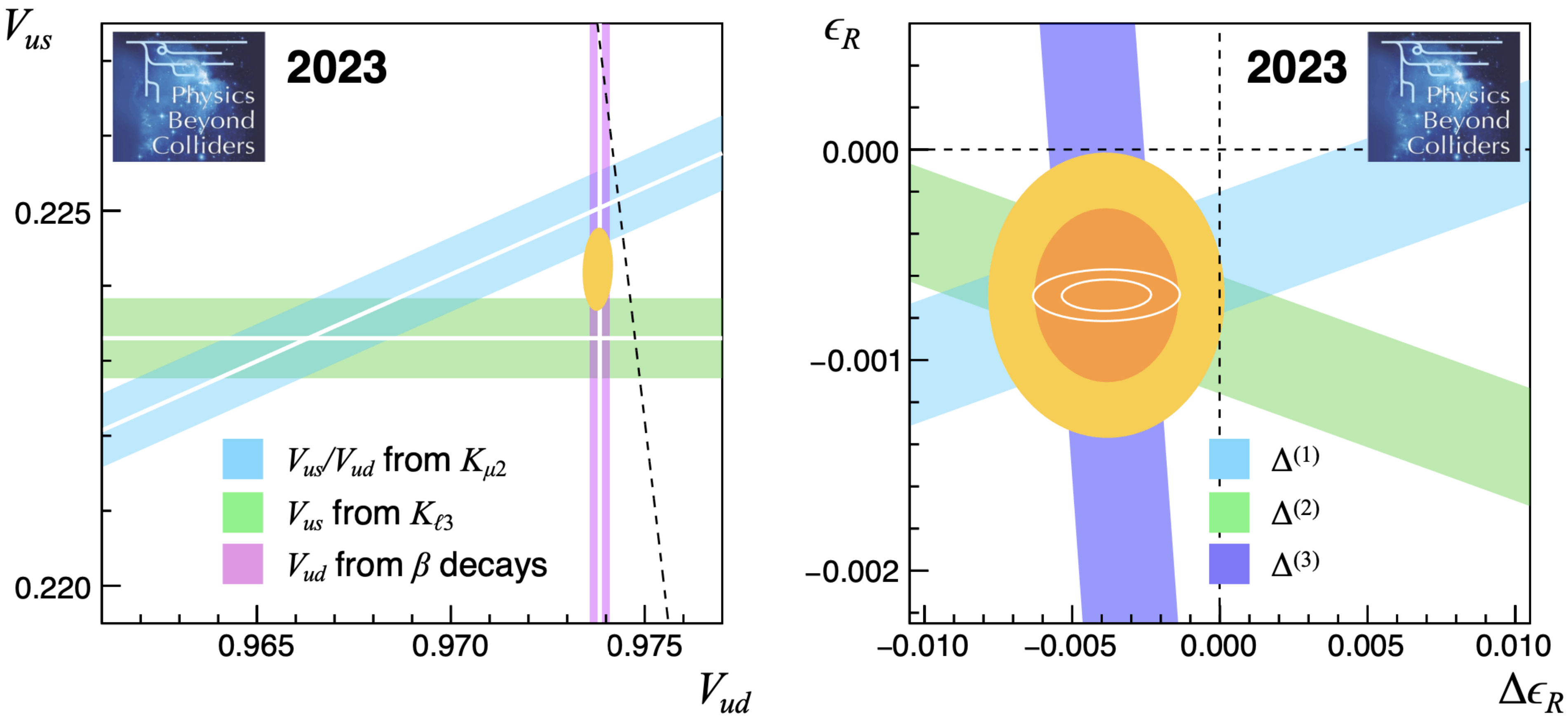
Parameter	Value	S
BR($\mu\nu$)	63.08(6)%	1.0
BR($\pi\pi^0$)	21.11(5)%	1.0
BR($\pi\pi\pi$)	5.56(4)%	1.0
BR(K_{e3})	5.109(9)%	1.0
BR($K_{\mu3}$)	3.383(5)%	1.0
BR($\pi\pi^0\pi^0$)	1.763(26)%	1.0
$\tau_{\pm}$	12.385(15) ns	1.2

χ^2/ndf = 4.90/5 (Prob = 42.8%)

compare current: 25.5/11 (0.78%)

- Can remove all most all old data except 3π and τ measurements
- Some strong correlations in fit results, esp between $\mu\nu$, $\pi\pi^0$ (–0.7) and K_{e3}, $K_{\mu3}$ (+0.5)
- **Fit constraint Σ BR = 1 significantly increases result for $\mu\nu$**

Comparison: First-row unitarity in 2023

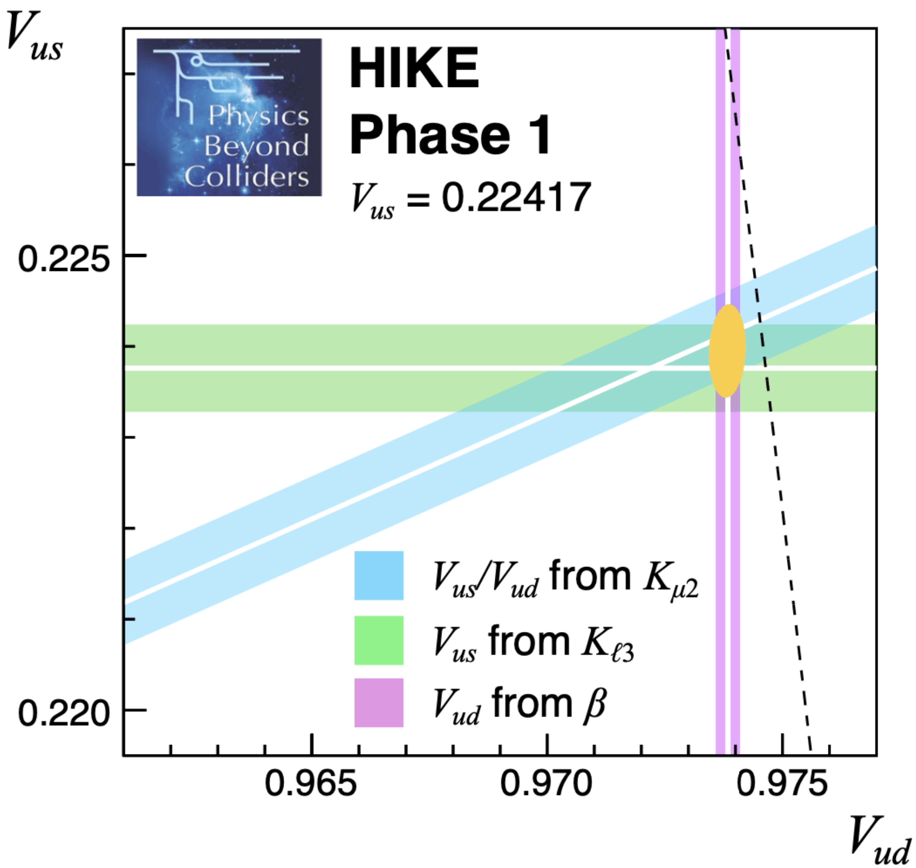


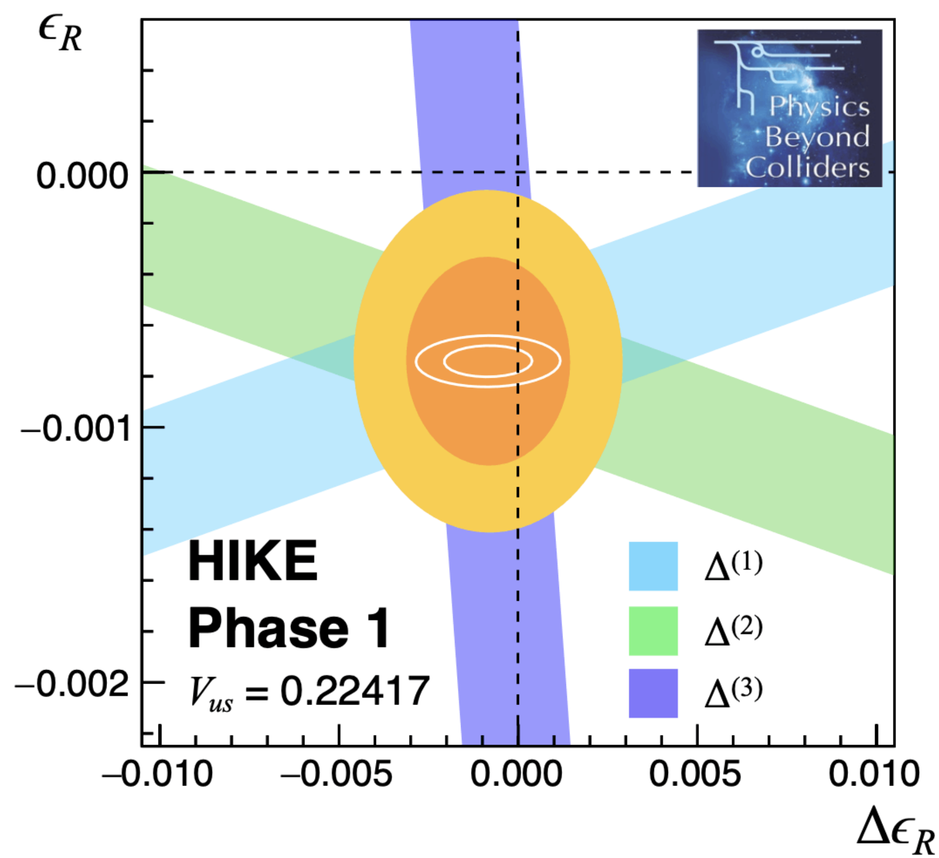
Poor agreement between $K_{\ell 3}$ & $K_{\mu 2}$

Unitarity deficit at ~2.8σ level

$\epsilon_R = \Delta\epsilon_R = 0$ excluded at 3.1σ

Scenario: HIKE Phase 1 with $V_{us} = 0.22417$





V_{us} from $K_{\ell 3}$ shifted up by 1σ

V_{us}/V_{ud} from $K_{\mu 2}$ shifted down by 1.5σ

Reasonable agreement between $K_{\ell 3}$ & $K_{\mu 2}$

$\epsilon_R = 0$ at 2.3σ level

Unitarity deficit persists at ~2.8σ level

Hypothetical Phase-2 fit to K_L rate data

Improvements to K_L fit tricky:

- HIKE measures *ratios* of BRs
- $K_{\ell 3}$ modes dominant
- $3\pi^0$, $\pi^+\pi^-\pi^0$ critical for normalization but poor cancellation of systematics with $K_{\ell 3}$ modes
- Strong constraints from CP measurements

24 input measurements:

21 inputs from current fit

Hypothetical HIKE measurements chosen to agree with V_{us} **= 0.22417:**

HIKE $K_{\mu 3}/K_{e3}$ to 0.3%

HIKE $\pi^+\pi^-/K_{e3}$ to 0.4%

HIKE $\pi^+\pi^-/\ \pi^+\pi^-\pi^0$ to 0.6%

1 constraint: Σ BR = 1

Parameter	Value	S
BR(K_{e3})	0.4064(7)	1.3
BR($K_{\mu 3}$)	0.2707(5)	1.5
BR($3\pi^0$)	0.1950(11)	1.2
BR($\pi^+\pi^-\pi^0$)	0.1244(4)	1.3
BR($\pi^+\pi^-(\gamma_{\mathrm{IB}})$)	$1.959(6) \times 10^{-3}$	1.1
BR($\pi^+\pi^-\gamma$)	$4.13(6) \times 10^{-5}$	1.6
BR($\pi^+\pi^-\gamma_{\mathrm{DE}}$)	$2.83(6) \times 10^{-5}$	1.3
BR($2\pi^0$)	$8.62(6) \times 10^{-4}$	1.4
BR($\gamma\gamma$)	$5.47(4) \times 10^{-4}$	1.1
τ_L	51.23(22) ns	1.1

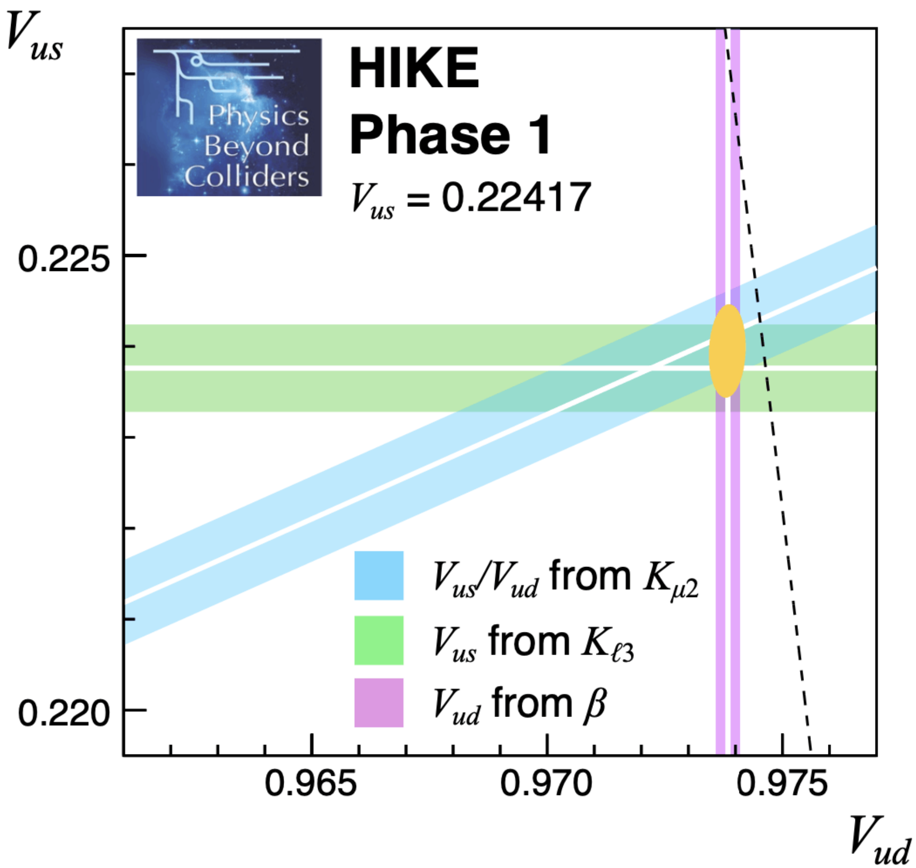
χ^2/ndf = 30.7/15 (Prob = 0.94%)

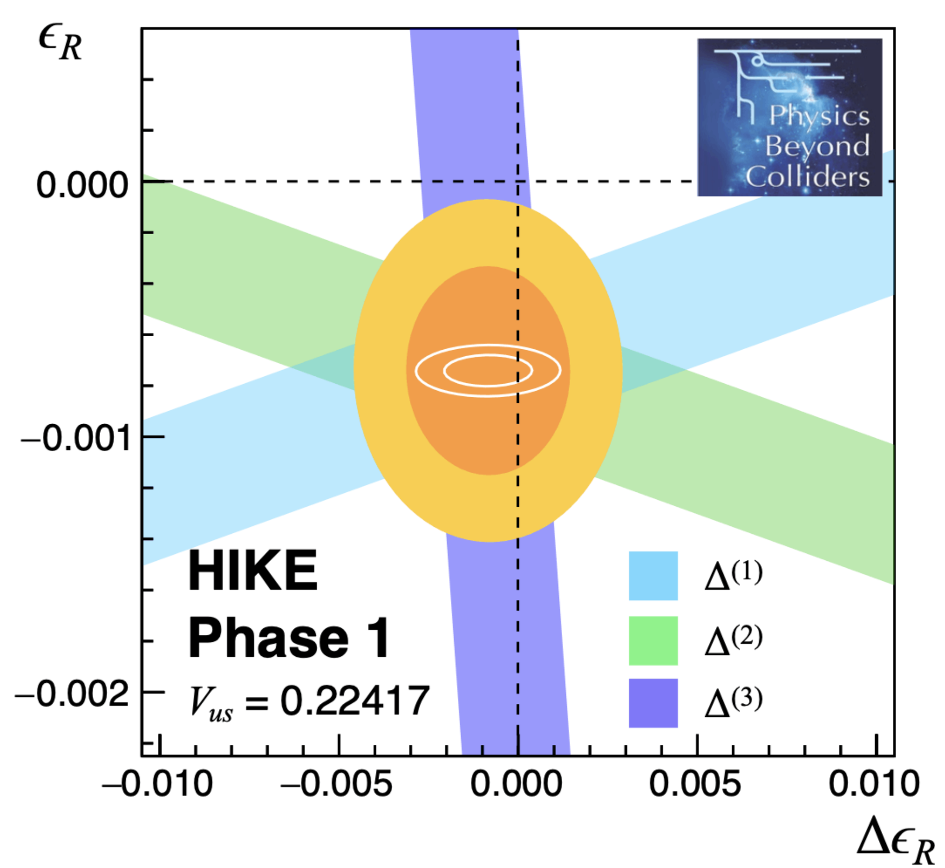
Significantly reduced errors for $K_{\ell 3}$ BRs

Most BRs change by < 1σ

Adds tension to current fit: Prob 7.0 → 0.94%

Scenario: HIKE Phase 1 with $V_{us} = 0.22417$

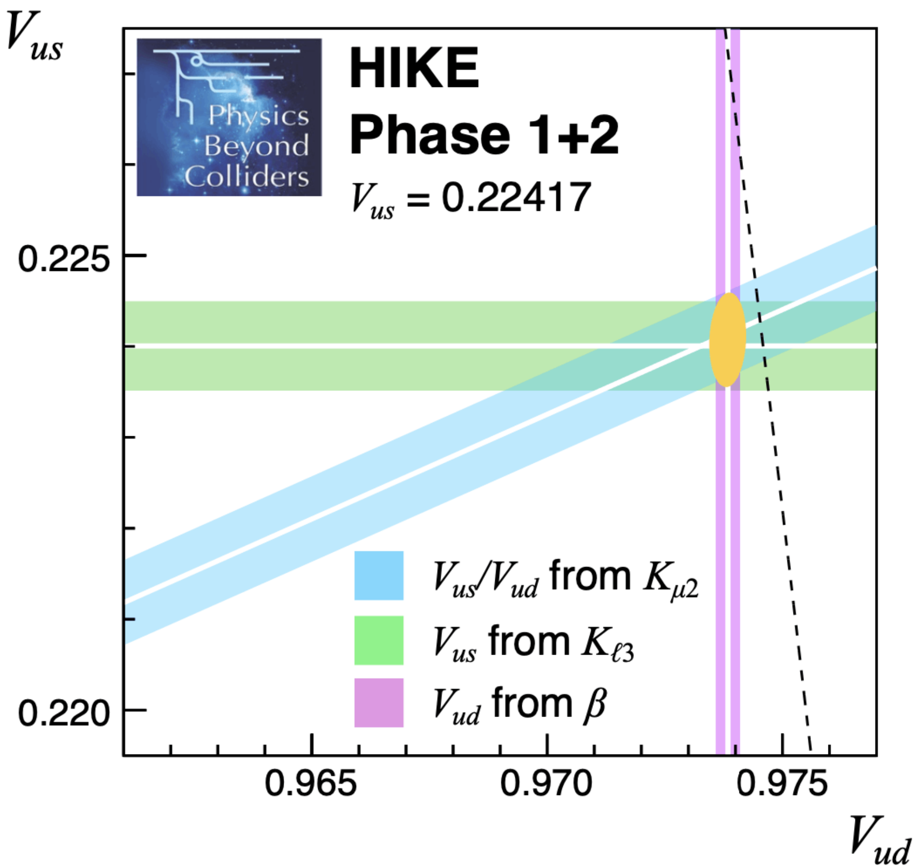


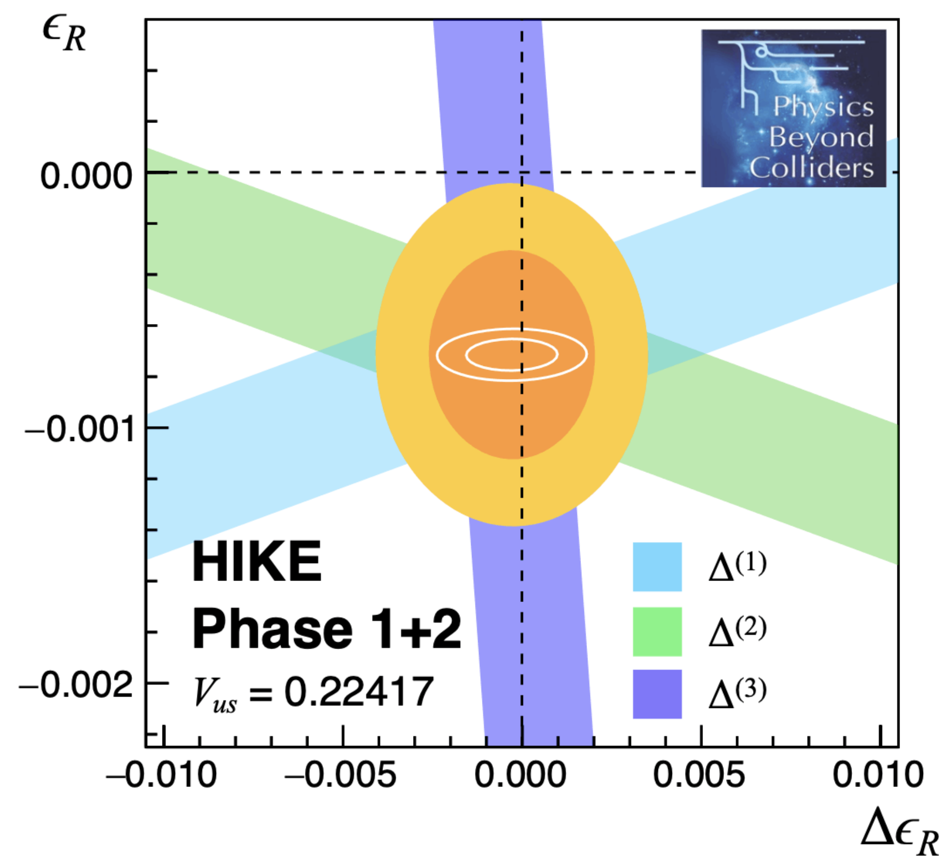


V_{us} from $K_{\ell 3}$ shifted up by 1σ
V_{us}/V_{ud} from $K_{\mu 2}$ shifted down by 1.5σ

Reasonable agreement between $K_{\ell 3}$ & $K_{\mu 2}$
$\epsilon_R = 0$ at 2.3σ level
Unitarity deficit persists at $\sim 2.8\sigma$ level

Scenario: Phase 1+2 with V_{us} = 0.22417





Good agreement between $K_{\ell 3}$ & $K_{\mu 2}$
ϵ_R = 0 at 2.2σ level
Unitarity deficit persists at ~2.7σ level

Precision in kaon sector strongly motivates further progress on V_{ud}, especially in theoretical calculation of radiative corrections!

Status of first-row unitarity

Experimental results from kaons	With $|V_{ud}|(\beta)$ and $N_f = 2+1+1$ lattice
$|V_{us}|\, f_+(0) = 0.21656(35)$	$\Delta^{(1)}_{\mathrm{CKM}} = -0.00176(56) = -3.1\sigma$
$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$	$\Delta^{(2)}_{\mathrm{CKM}} = -0.00098(58) = -1.7\sigma$

Fit to both gives $\Delta_{\mathrm{CKM}} = -2.8\sigma$ and 3.1σ evidence for right-handed currents

$K_{\mu 2}$ result shows better agreement with unitarity than $K_{\ell 3}$ result when $|V_{ud}|$ obtained from beta decays

New measurement of $K_{\mu 3}/K_{\mu 2}$ (e.g. from NA62) could be very helpful in distinguishing if origin of discrepancy is experimental

- Other measurements of main K BRs also very important!

Precision in kaon sector strongly motivates further progress on V_{ud}, especially in theoretical calculation of radiative corrections!

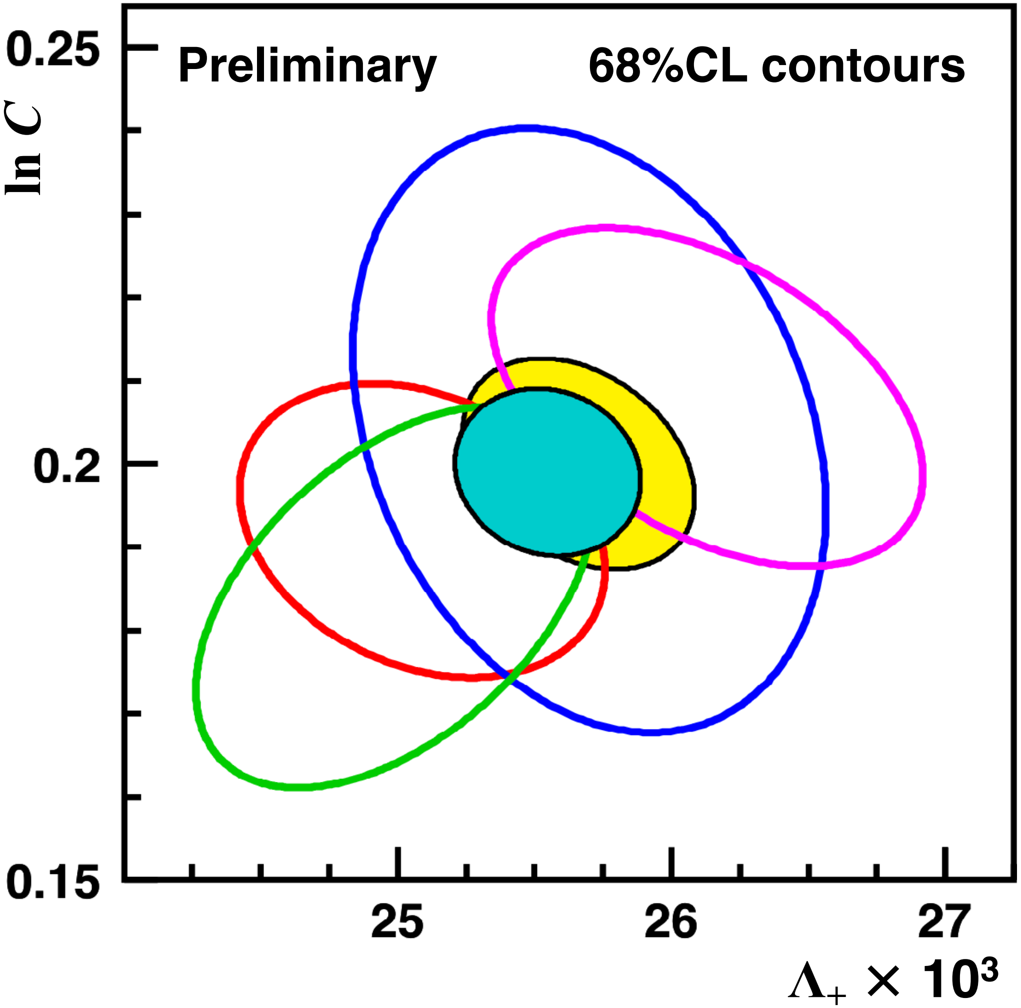
Kaon decays and the Cabibbo Angle Anomaly

Additional information

12th International Workshop on the CKM Unitarity Triangle (CKM 2023)
Santiago de Compostela, 20 September 2023

Phase-space integrals 2021

Averages of form-factor parameters for dispersive parameterization Λ_+ and ln C
Integrals calculated from average values



$$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$$
$$\ln C = 0.1992(78)$$
$$\rho(\Lambda_+, \ln C) = -0.110$$
$$\chi^2/\text{ndf} = 7.5/7\ (38\%)$$

Integrals	
$K^0{}_{e3}$	**0.15470(15)**
$K^+{}_{e3}$	**0.15915(15)**
$K^0{}_{\mu3}$	**0.10247(15)**
$K^+{}_{\mu3}$	**0.10553(16)**

Correlation matrix for integrals

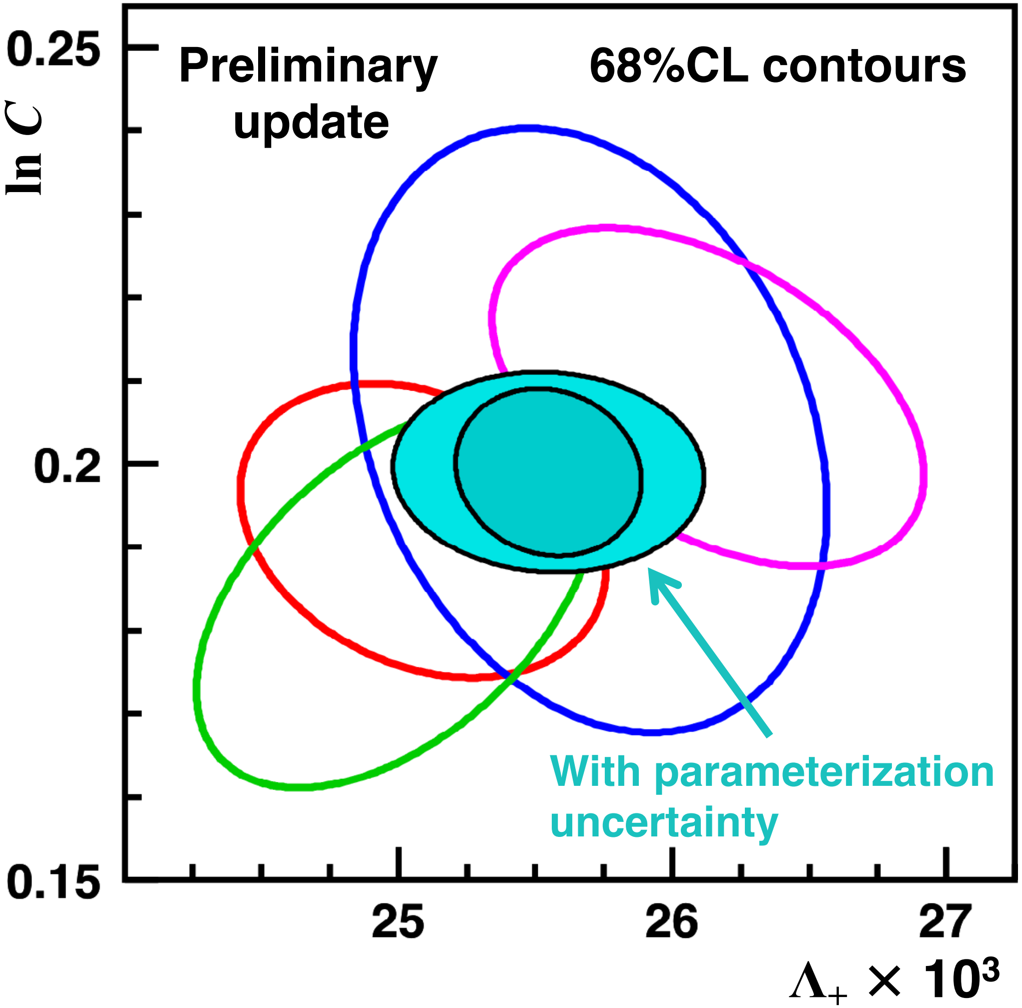
$K^0{}_{e3}$	1	1	0.530	0.521
$K^+{}_{e3}$		1	0.530	0.521
$K^0{}_{\mu3}$			1	1
$K^+{}_{\mu3}$				1

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from KTeV KLOE ISTRA+ NA48/2

NA48 K_{e3} data included in fits but not shown

2010 fit Current



$$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$$
$$\ln C = 0.1992(78)$$
$$\rho(\Lambda_+, \ln C) = -0.110$$
$$\chi^2/\text{ndf} = 7.5/7\ (38\%)$$

Fit results include common uncertainty from $H(t)$, $G(t)$

Without common uncertainty:

$\sigma(\Lambda_+)$	$(0.38 \rightarrow 0.22) \times 10^{-3}$
$\sigma(\ln C)$	$0.0078 \rightarrow 0.0067$
$\sigma(K_{e3}$ int)	$0.10\% \rightarrow 0.09\%$
$\sigma(K_{\mu 3}$ int)	$0.15\% \rightarrow 0.11\%$

$K_{\ell 3}$ data and lepton universality

For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\rm obs}}{(R_{\mu e})_{\rm SM}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\,\rm obs}}{[|V_{us}|\,f_+(0)]^2_{e3,\,\rm obs}} = \frac{g_\mu^2}{g_e^2}$$

Modes	2004 BRs*,†	Current
K_L	1.054(14)	**1.002(5)**
$K^\pm$	1.014(12)	**0.999(9)**
Avg	1.030(9)	**1.001(4)**

*Assuming current values for form-factor parameters and $\Delta^{\rm EM}$ †K_S not included

As statement on lepton universality

Compare to other precise tests:

$\pi \to \ell\nu$ $(r_{\mu e})$ = **1.0020(19)**

PDG with PIENU '15 result

$\tau \to \ell\nu\nu$ $(r_{\mu e})$ = **1.0036(28)**

HFLAV May '19 unofficial prelim.

As statement on calculation of $\Delta^{\rm EM}$

Confirmed at per-mil level

$SU(2)$-breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1 + \frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_{p^4} = 0.252$
NLO in strong interaction
$O(e^2p^2)$ term $\varepsilon_{EM}^{(4)} \sim 10^{-6}$

Cirigliano et al., ’02; Gasser & Leutwyler, ’85

$=$ **+2.52(11)%** **Calculated using:**

$Q = 22.5(5)$
$m_s/\hat{m} = 27.23(10)$

FLAG ’21, $N_f = 2+1+1$ avg.
Good agreement with ChPT
Cf. Colangelo et al., $Q = 22.1(7)$ from $\eta \to 3\pi$

$M_K = 494.2(3)$
$M_\pi = 134.8(3)$

Isospin-limit meson masses from FLAG ’17
No difference if $M_K = 494.58$, $M_\pi = m_\pi$

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require **$\Delta^{SU(2)}$ = 2.76(33)%**

Impact of hypothetical $K_{\mu 3}/K_{\mu 2}$ result

	current fit	$K_{\mu 3}/K_{\mu 2}$ BR at 0.5%			$K_{\mu 3}/K_{\mu 2}$ BR at 0.2%		
		central	$+2\sigma$	-2σ	central	$+2\sigma$	-2σ
χ^2/dof	25.5/11	25.5/12	31.8/12	32.1/12	25.5/12	35.6/12	35.9/12
p-value [%]	0.78	1.28	0.15	0.13	1.28	0.04	0.03
BR($\mu\nu$) [%]	63.58(11)	63.58(09)	63.44(10)	63.72(11)	63.58(08)	63.36(10)	63.80(11)
$S(\mu\nu)$	1.1	1.1	1.3	1.4	1.2	1.6	1.7
BR($\pi\pi^0$) [%]	20.64(7)	20.64(6)	20.73(7)	20.55(8)	20.64(6)	20.78(7)	20.50(10)
$S(\pi\pi^0)$	1.1	1.2	1.3	1.5	1.2	1.5	2.0
BR($\pi\pi\pi$) [%]				5.56(4)			
$S(\pi\pi\pi)$				1.0			
BR(K_{e3}) [%]	5.088(27)	5.088(24)	5.113(25)	5.061(31)	5.088(23)	5.128(24)	5.046(32)
$S(K_{e3})$	1.2	1.2	1.2	1.6	1.3	1.3	1.8
BR($K_{\mu 3}$) [%]	3.366(30)	3.366(13)	3.394(16)	3.336(27)	3.366(7)	3.411(13)	3.320(18)
$S(K_{\mu 3})$	1.9	1.2	1.5	2.6	1.1	2.2	3.1
BR($\pi\pi^0\pi^0$) [%]				1.764(25)			
$S(\pi\pi^0\pi^0)$				1.0			
$\tau_{\pm}$ [ns]	12.384(15)	12.384(15)	12.382(15)	12.385(15)	12.384(15)	12.381(15)	12.386(15)
$S(\tau_{\pm})$				1.2			

Hypothetical $K_{\mu 3}/K_{\mu 2}$ measurement to 0.2% giving result $\pm 2\sigma$ from current fit:

- Changes BR(K_{e3}) and BR($K_{\mu 3}$) by $\pm 1.5\sigma$
- Changes BR($K_{\mu 2}$) by $\mp 2\sigma$ (i.e. in opposite direction)

Impact of $K_{\mu 3}/K_{\mu 2}$ on unitarity tests

$\Delta^{(1)}_{\rm CKM} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1$ — V_{us} from $K_{\ell 3}$ + V_{ud} from β decays

$\Delta^{(2)}_{\rm CKM} = \left|V_{ud}^{\beta}\right|^2 + \left|V_{us}^{K_{\ell 2}/\pi_{\ell 2},\beta}\right|^2 - 1$ — V_{us}/V_{ud} from $K_{\mu 2}$ + V_{ud} from β decays

$\Delta^{(3)}_{\rm CKM} = \left|V_{ud}^{K_{\ell 2}/\pi_{\ell 2},K_{\ell 3}}\right|^2 + \left|V_{us}^{K_{\ell 3}}\right|^2 - 1$ — V_{us} from $K_{\ell 3}$ + V_{us}/V_{ud} from $K_{\mu 2}$

	current fit	$K_{\mu 3}/K_{\mu 2}$ BR at 0.5%			$K_{\mu 3}/K_{\mu 2}$ BR at 0.2%		
		central	$+2\sigma$	-2σ	central	$+2\sigma$	-2σ
$\left.\frac{V_{us}}{V_{ud}}\right\rvert_{K_{\ell 2}/\pi_{\ell 2}}$	0.23108(51)	0.23108(50)	0.23085(51)	0.23133(51)	0.23108(49)	0.23071(51)	0.23147(52)
$V_{us}^{K_{\ell 3}}$	0.22330(53)	0.22337(51)	0.22360(52)	0.22309(54)	0.22342(49)	0.22386(52)	0.22287(52)
$\Delta^{(1)}_{\rm CKM}$	−0.00176(56)	−0.00173(55)	−0.00162(56)	−0.00185(56)	−0.00171(55)	−0.00151(56)	−0.00195(56)
	-3.1σ	-3.1σ	-2.9σ	-3.3σ	-3.1σ	-2.7σ	-3.5σ
$\Delta^{(2)}_{\rm CKM}$	−0.00098(58)	−0.00098(58)	−0.00108(58)	−0.00087(58)	−0.00098(58)	−0.00114(58)	−0.00081(58)
	-1.7σ	-1.7σ	-1.9σ	-1.5σ	-1.7σ	-2.0σ	-1.4σ
$\Delta^{(3)}_{\rm CKM}$	−0.0164(63)	−0.0157(60)	−0.0118(62)	−0.0202(63)	−0.0153(59)	−0.0083(62)	−0.0233(62)
	-2.6σ	-2.6σ	-1.9σ	-3.2σ	-2.6σ	-1.4σ	-3.8σ

- $\Delta^{(3)}{}_{\rm CKM}$ has no inputs from β decays
- Less sensitive as an absolute unitarity test but clearly shows impact of new measurements of V_{us}

V_{us} from τ decays

Based mainly on work by HFLAV and talks by Alberto Lusiani

V_{us} from exclusive τ decays

$$\Gamma(\tau \to K\nu_\tau) = \frac{G_F^2}{16\pi\hbar}|V_{us}|^2 f_{K^\pm}^2 m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{\mathrm{EW}}(1 + \delta R_K)$$

$$\frac{\Gamma(\tau \to K\nu_\tau)}{\Gamma(\tau \to \pi\nu_\tau)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_{K^\pm}^2}{f_{\pi^\pm}^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2}(1 + \delta R_{K/\pi})$$

Inputs from experiment:

HFLAV ’22 Fit:

BR($K^-\nu_\tau$) = 0.006957(96)
BR($\pi^-\nu_\tau$) = 0.10808(53)
BR($K^-\nu_\tau/\pi^-\nu_\tau$) = 0.06437(92)

Radiative corrections:

Arroyo-Ureña et al., PRD104 (2021)
Large-N_C expansion

δR_K = (–0.15 ± 0.57)%
$\delta R_{K/\pi}$ = (0.10 ± 0.80)%

Results:

HFLAV ’22 web update
A. Lusiani, ELECTRO ’22

$\tau \to K\nu_\tau$

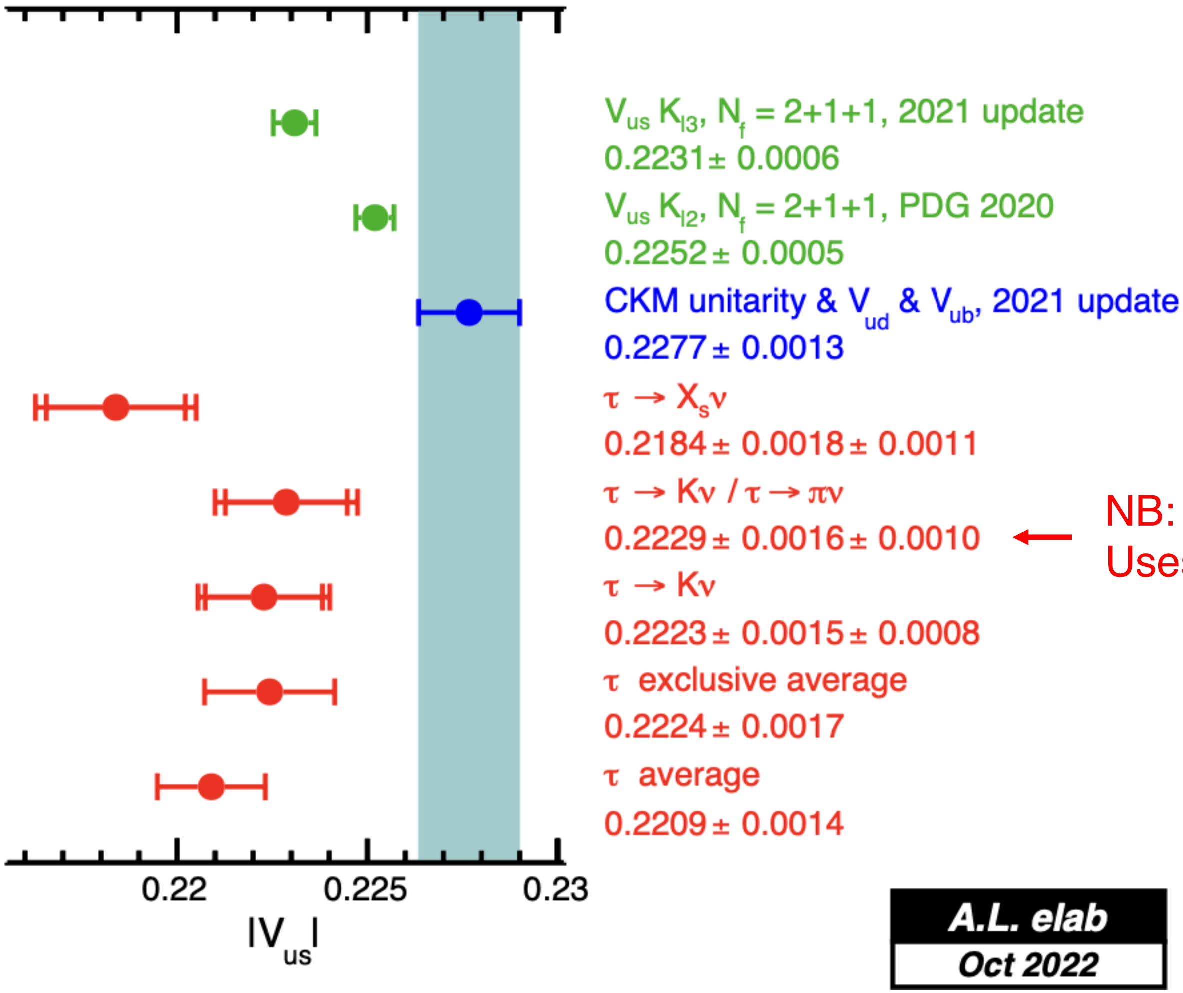
$V_{us} =$ **0.2219(17)**

$\tau \to K\nu_\tau/\tau \to \pi\nu_\tau$

V_{ud}/V_{us} = **0.2290(17)**

N_f = 2+1+1, FLAG ’19:
$f_{K\pm}$ = 155.7 ± 0.3 MeV
$f_{K\pm}/f_{\pi\pm}$ = 1.1932(21)

V_{us} from exclusive τ decays



NB: Not independent!
Uses V_{ud} from $0^+ \to 0^+$

A. Lusiani, update of HFLAV τ avergages for Electro 2022

V_{us} from inclusive hadronic τ decays

$$R_\tau = \frac{\Gamma(\tau^- \to [\text{hadrons}]^- \nu_\tau(\gamma))}{\Gamma(\tau^- \to e^- \bar{\nu}_e \nu_\tau(\gamma))}$$

$$= R_{\tau\,\text{non-strange}} + R_{\tau\,\text{strange}}$$

vector + axial current

$SU(3)$ breaking:

$$\delta R_\tau^{\text{th}} = \frac{R_{\tau\,\text{non-strange}}}{|V_{ud}|^2} - \frac{R_{\tau\,\text{strange}}}{|V_{us}|^2}$$

Experimental inputs:

$R_{\tau\ \text{non-strange}}/|V_{ud}|^2 \approx 3.7$

$R_{\tau\ \text{strange}} \approx 0.17$

Theoretical inputs:

$\delta R_\tau^{\text{th}} = 0.238(33)$ for $m_s(m_\tau) = 93.0 \pm 8.5$ MeV

- OPE with fixed-order or contour-improved perturbation theory for contributions up to $D = 2$

  E. Gamiz et al., hep-ph/0612154v1

$\delta R_\tau^{\text{th}}$ from finite-energy sum rules (FESR):

$$R_\tau^w(s_0) \equiv \int_0^{s_0} ds\, \frac{w(s)}{w_\tau(s)} \frac{dR_\tau(s)}{ds}$$

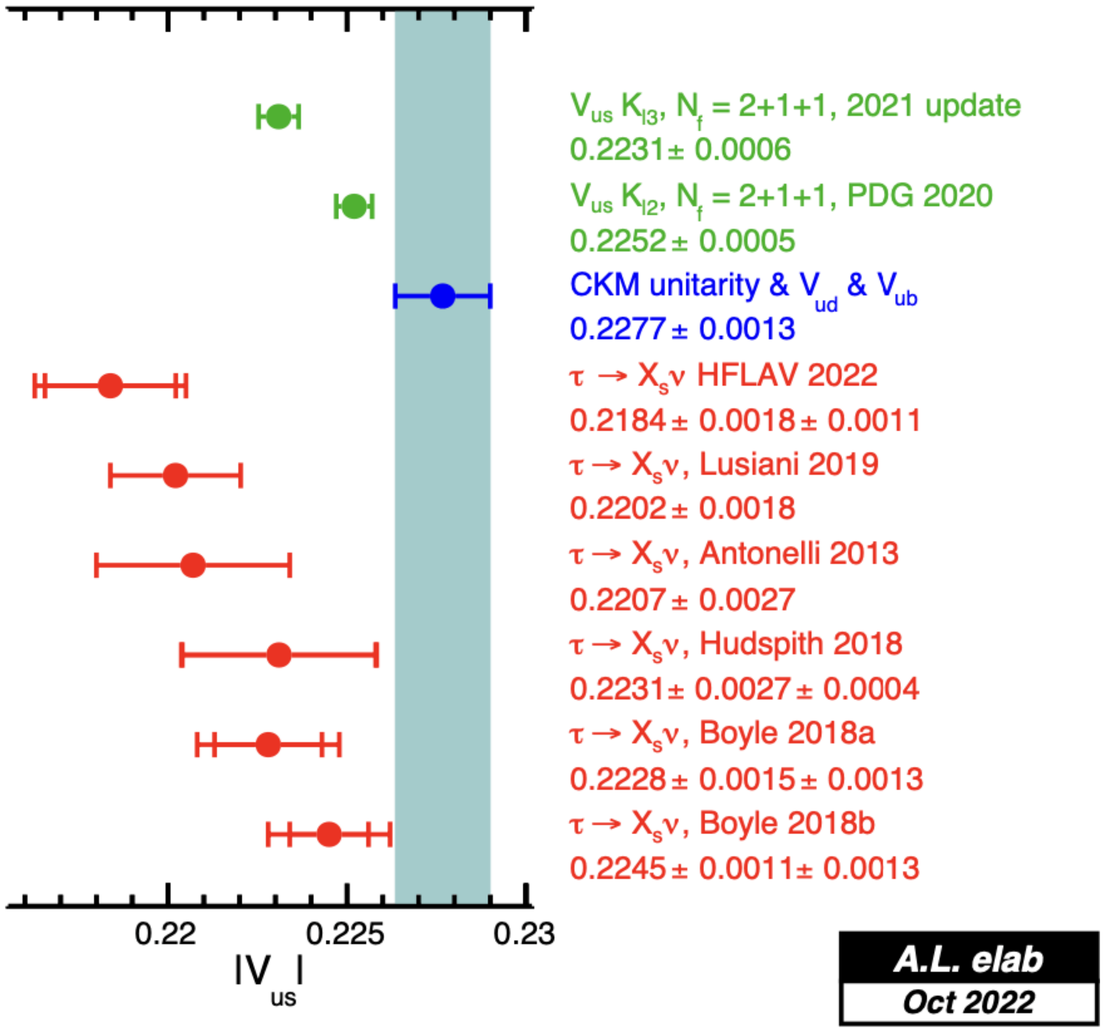
$$\delta R_\tau^w(s_0) = \frac{R^w_{\tau\,\text{non-strange}}(s_0)}{|V_{ud}|^2} - \frac{R^w_{\tau\,\text{strange}}(s_0)}{|V_{us}|^2}$$

- $\delta R_\tau^w(s_0)$ has contributions up to $D = 8$
- $\delta R_\tau^w(s_0)$ has substantial dependence on s_0, w if contributions with $D > 4$ not negligible

  Hudspith et al., 2017
- Can use lattice QCD inputs for $D = 6, 8$ contributions

  Boyle et al. (RBC/UKQCD), 2018

V_{us} from inclusive hadronic τ decays



A. Lusiani, update of HFLAV τ averages for Electro 2022

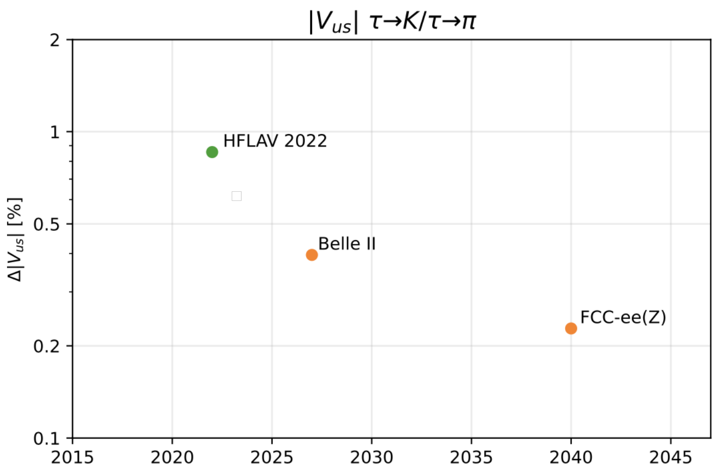
V_{us} from τ decays: Status and prospects

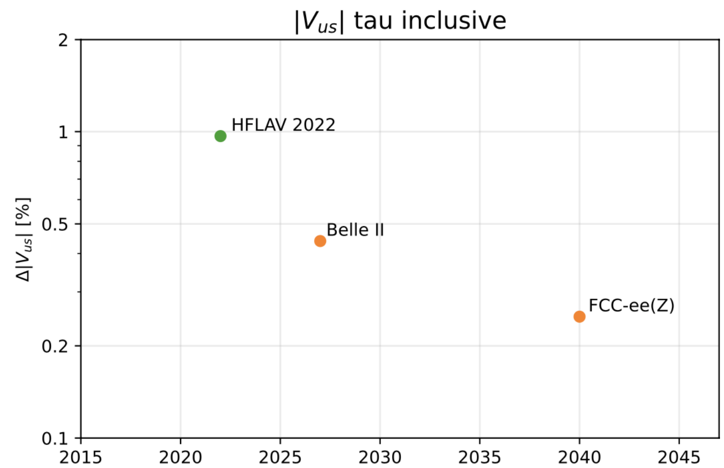
τ average	**V_{us} = 0.2209(14)**	0.63%

Currently uncertainty about 3x larger than for K decays
Significance of CAA is about the same (τ value for V_{us} a bit lower)

Prospects for improvement:

A. Lusiani, Electro 2022





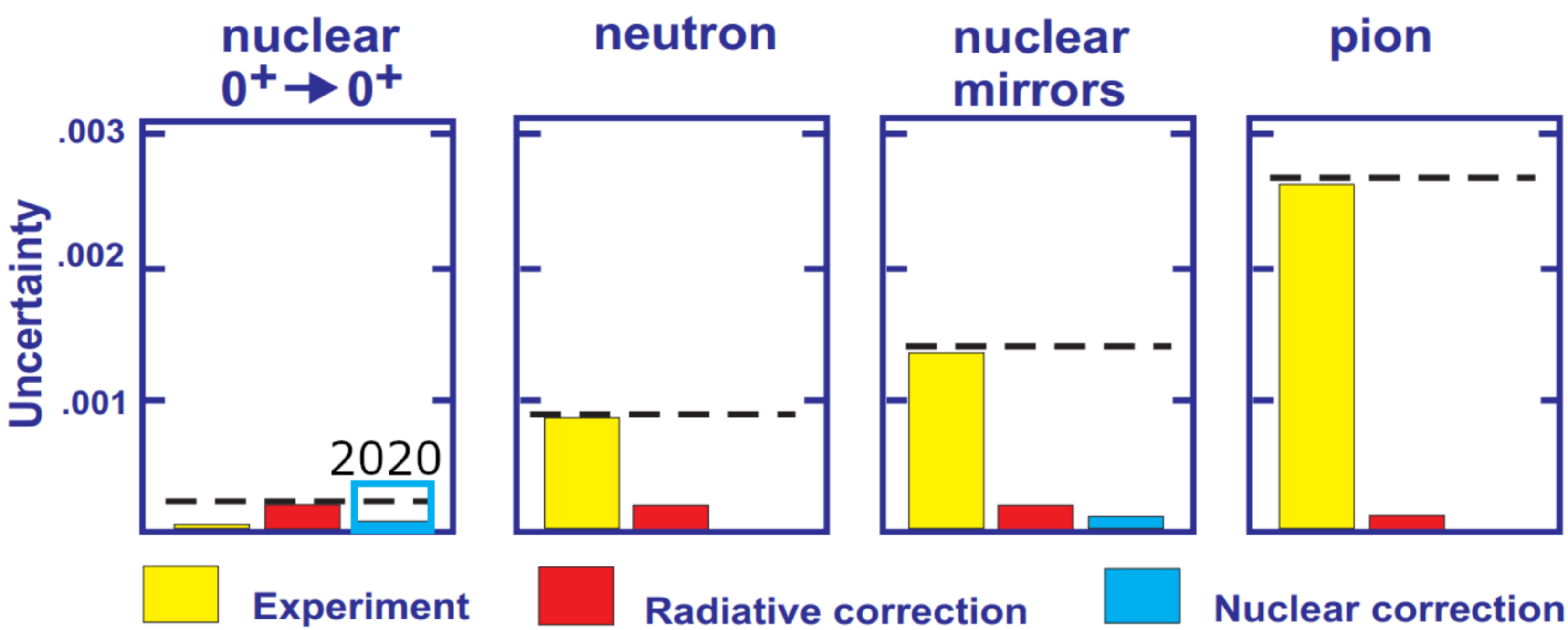
Enormous increases in statistics expected from Belle II (50-100x *BABAR*, Belle)
To be competitive with K decays, need statistics from FCC-ee (1000x ALEPH)

$$V_{ud}$$

Experimental determination of V_{ud}

1. **$0^+ \to 0^+$ nuclear β decays** (superallowed Fermi transitions)
2. **Neutron β decay**
3. **$T = 1/2$ nuclear mirror decays**
4. **Pion β decay**

J. Hardy, Amherst 2019



$|V_{ud}|$ from $0^+ \to 0^+$: Corrections

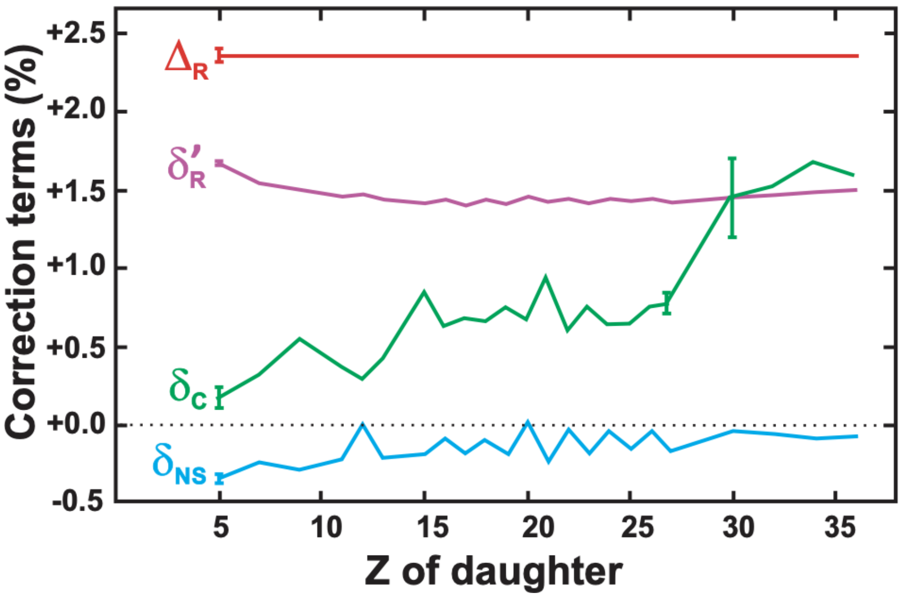
$$\mathcal{F}t \equiv ft(1+\delta_R')(1+\delta_{NS}-\delta_C) = \frac{K}{2G_V^2\left(1+\Delta_R^V\right)}$$

Δ_R Universal radiative correction
High-energy γW box + ZW box amplitudes

$\delta_R{}'$ Long-distance radiative correction
One-photon bremsstrahlung + low-energy γW box

δ_C Coulomb correction
Charge-dependent mismatch between parent and daughter analog states (members of same isospin triplet)

δ_{NS} Nuclear structure
O(α) axial photonic contributions



Consistency check: CVC demands equivalence of $\mathcal{F}t$ values after corrections

$|V_{ud}|$ from pion β decays

$$\Gamma_{\pi\beta} = \frac{G_\mu^2 |V_{ud}|^2}{30\pi^3}\left(1 - \frac{\Delta}{2M_+}\right)^3 \Delta^5 f(\epsilon, \Delta)(1 + \delta)$$

$\Delta = m_{\pi+} - m_{\pi-}$
$\epsilon = (m_e/\Delta)^2$
$f(\epsilon, \Delta)$ = Fermi function

- Experimentally, need to measure BR($\pi^+ \to \pi^0 e\nu$) and lifetime $\tau_{\pi+}$
- Radiative correction $\delta \sim 3.3\%$, very well controlled
  New lattice calculation (Feng et al., 2020)

PIBETA
PRL 93 (2004)

BR($\pi^+ \to \pi^0 e\nu$) = 1.036(4)$_{\text{stat}}$(4)$_{\text{sys}}$(3)$_{\pi e2}$ × 10^{-8}
Decays at rest

$\boldsymbol{V_{ud}^{\pi\beta} = 0.97386(281)_{BR}(9)_\tau(14)_\delta(28)_f}$

Cirigliano et al.
2208.11707

$\tau_{\pi+}$ from
PDG 2022

- Phase-2 goal of recently proposed PIONEER experiment:
  Reduce uncertainty on BR: 0.6% → **0.02%** (competitive with $0^+ \to 0^+$)
- Completely independent of $0^+ \to 0^+$: No nuclear-structure corrections and different radiative corrections

What can we learn today from $\pi^+ \to \pi^0 e\nu$?

Czarnecki, Marciano, Sirlin, PRD 101 (2020)

$$\frac{\Gamma(K_L \to \pi e\nu(\gamma))}{\Gamma(\pi^+ \to \pi^0 e\nu(\gamma))} = \frac{1}{3}\left(\frac{m_{K^0}}{m_{\pi^+}}\right)^5 \left(\frac{V_{us} f_+^K(0)}{V_{ud} f_+^\pi(0)}\right)^2 \left(\frac{I_K}{I_\pi}\right)\left(\frac{1+\mathrm{RC}_K}{1+\mathrm{RC}_\pi}\right)$$

Ratio not sensitive to short-distance EW radiative corrections

- $(1 + \mathrm{RC}_K - \mathrm{RC}_\pi) = 1.000(2)_K(1)_\pi$
- Cancellation of S_{EW} and short-distance radiative corrections
- $\Delta_{\mathrm{EM}}{}^K$ (long-distance correction) fortuitously cancels (?) when using K_{Le3}

Consider K_{Le3} mode as an example:

Most precise value of V_{us}

$$\frac{V_{us} f_+^K(0)}{V_{ud} f_+^\pi(0)} = 0.22221(53)_{\Gamma(K)}(64)_{\Gamma(\pi)}(22)_{\mathrm{RC}}(12)_{\mathrm{int}} \quad \mathbf{= 0.22221(87)}$$

	K_{Le3}/π_{e3}*	$K_{\mu2}/K_{\pi2}$†	$K_{\ell3}$* & $V_{ud}(\beta)$
V_{us}/V_{ud}	**$0.2291(9)_{\mathrm{exp}}(4)_{\mathrm{lat}}$**	0.2311(5)	$0.22930(54)_{us}(6)_{ud}$
diff with K_{Le3}/π_{e3}	–	**$+1.7\sigma$**	**$+0.2\sigma$**

*with $f_+^K(0) = 0.9698(17)$ and $f_+^\pi(0) = 1$ in $SU(2)$ limit †with $f_K/f_\pi = 1.1978(22)$

New physics implications of Δ_{CKM}

Model independent SMEFT approach

Cirigliano et al., NPB 830 (2010)
González-Alonso et al., PPNP 104 (2019)

Effective Lagrangian for $\mu \sim 1$ GeV with general set of dim-6 operators giving rise to (semi)leptonic transitions

$$\mathscr{L}_{\mathrm{eff}} = \mathscr{L}_{\mathrm{eff,SM}} + \mathscr{L}_{\mathrm{eff,NP}}^{\mathrm{dim\text{-}6}}$$

$$\mathscr{L}_{\mathrm{eff,NP}}^{\mathrm{dim\text{-}6}} = \sum_i \frac{C_i}{\Lambda^2}\mathscr{O}_i$$

Consider the **flavor-blind** limit (or similar: minimal flavor violation, etc.)
New physics appears as a small difference between G_F and G_μ

From comparison of operators for $d \to ul\nu$ and $\mu \to e\nu\nu$

$$\Delta_{\mathrm{CKM}} = 2\frac{v^2}{\Lambda^2}\left[C_{Hq}^{(3)} - C_{H\ell}^{(3)} + C_{\ell\ell} - C_{\ell q}^{(3)}\right]$$

Δ_{CKM} provides important constraints for EW fits

Types of SM extensions that can generate non-zero C contributing to Δ_{CKM}

NP in μ decay	NP in β decay
Scalar singlet	Vector boson triplet
Vector boson singlet	Vector-like quarks
Vector boson triplet	Vector-like leptons
Vector-like leptons	Leptoquarks

See, e.g.:
Manzari 2111.04519
Bagnaschi et al., JHEP 2022 308