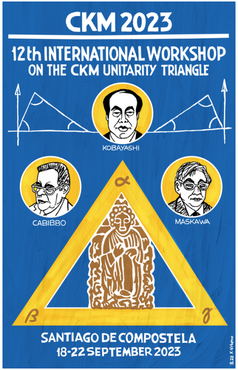
The input to V_{us} from lattice QCD including the progress on Hyperon decays

Felix Erben

CKM2023 Santiago de Compostela
20/09/2023

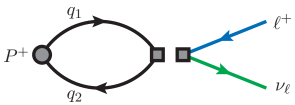


THE UNIVERSITY of EDINBURGH

European Research Council
Established by the European Commission

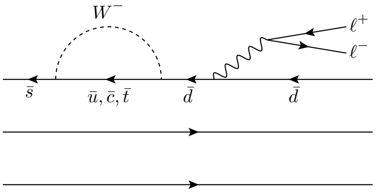
Overview

Part 1:
Precision frontier



- leptonic decays of pions and kaons
- isospin-breaking and QED corrections

Part 2:
Rare decays



- $s \to d\ell^+\ell^-$ transitions as potential window to new physics
- Here: rare hyperon decay $\Sigma^+ \to p\ell^+\ell^-$

Leptonic decays of pions and kaons

JHEP

PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: *December 23, 2022*
ACCEPTED: *February 14, 2023*
PUBLISHED: *February 27, 2023*

Isospin-breaking corrections to light-meson leptonic decays from lattice simulations at physical quark masses

Peter Boyle,[a,b] **Matteo Di Carlo,**[b] **Felix Erben,**[b] **Vera Gülpers,**[b] **Maxwell T. Hansen,**[b] **Tim Harris,**[b] **Nils Hermansson-Truedsson,**[c,d] **Raoul Hodgson,**[b] **Andreas Jüttner,**[e,f] **Fionn Ó hÓgáin,**[b] **Antonin Portelli,**[b] **James Richings**[b,e,g] **and Andrew Zhen Ning Yong**[b]

[Boyle, FE et al., JHEP 02 (2023) 242]

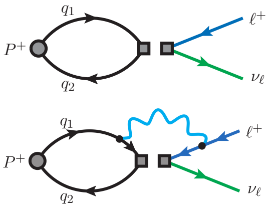
Within the Standard Model, the CKM matrix has unitarity constraints

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \qquad |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

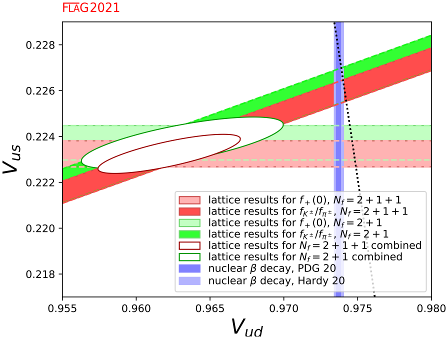
$\Rightarrow$ Precision determinations of CKM matrix elements as indirect search for new physics

- Experiment very accurately determines $|V_{us}/V_{ud}|f_{K^+}/f_{\pi^+}$

  [PDG, Prog. Theor. Exp. Phys. 2022, 083C01]
- Ratio of decay constants f_{K^+}/f_{π^+} can also be accuratley determined from lattice QCD
- sub-percent precision: **isospin-breaking** and **QED effects** need to be controlled!



- FLAG review: Comparison of lattice results for V_{us}, V_{ud} and exp. value of V_{ud} from nuclear β decay
- dotted line: ’CKM-unitarity’
- f_{K^+}/f_{π^+}:
  - 3.3σ tension with V_{ud} from PDG
  - 1.7σ agreement with V_{ud} from analysis with updated nuclear corrections [Hardy, Towner, Phys. Rev. C 102, 045501]



[FLAG Review 2021, Eur. Phys. J. C 82, 869 (2022)]

QED + ISOSPIN BREAKING

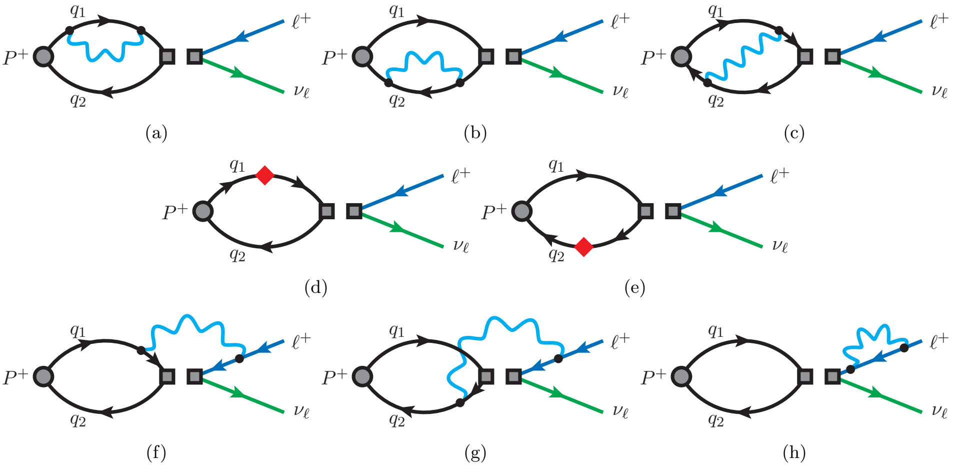
- Lattice QCD computations are defined in
  - finite volume $L^3 \times T$
  - often with periodic boundary conditions $\Psi(x) = \pm\Psi(x + L\mu)$
- Naively defining QED in such a box leads to non-zero net charge, in violation of Gauss' law

$\Rightarrow$ we use QED_L prescription: remove spatial zero-mode of the photon

[Hayakawa, Uno, Prog.Theor.Phys.120:413-441,2008]

- in QCD+QED, lepton ℓ can interact directly with the meson K/π
  - $\Rightarrow$ decay amplitude $\Gamma(P \to \ell\nu)$ cannot be factorised into QCD and non-QCD parts
  - virtual and real photon emission are individually IR divergent
- We use the RM123+Soton prescription: [Carrasco et al., Phys. Rev. D 91, 074506]
  - remove IR-divergent part of virtual-photon emission explicitly on the lattice
  - compute the same term in perturbation theory for the real-photon emission part
  - $\Rightarrow$ Both contributions are IR safe

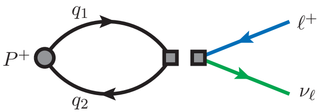
DIAGRAMS ENTERING OUR CALCULATION



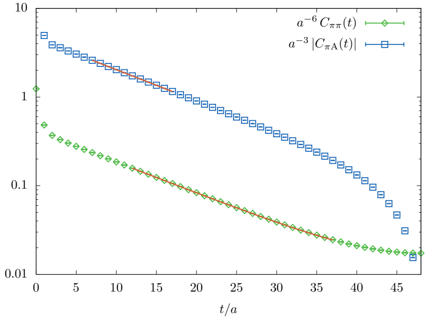
wiggly lines: photons

diamond-shaped vertices: scalar insertions

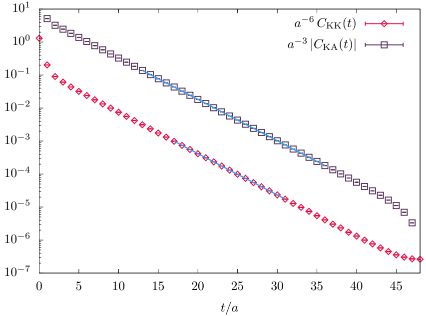
TREE LEVEL CONTRIBUTION

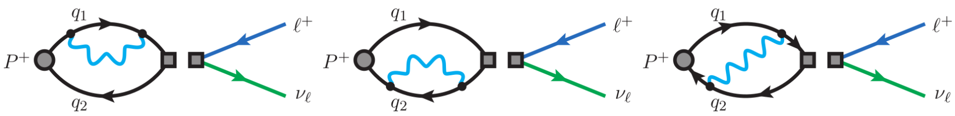


Pion

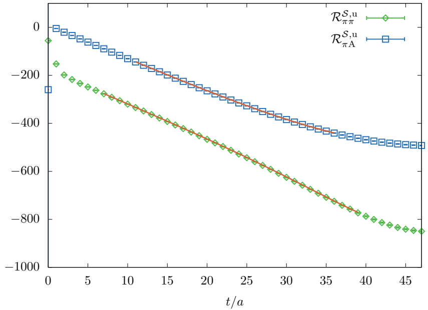


Kaon

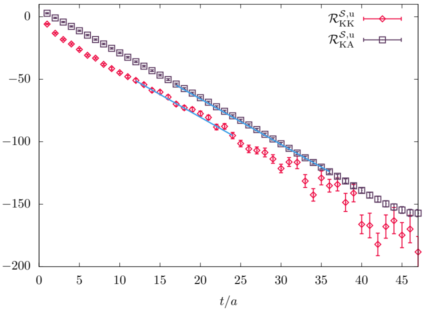



q_1
q_2
P^+
ℓ^+
ν_ℓ

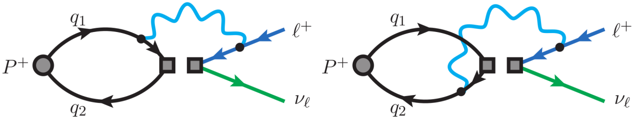
Pion


$\mathcal{R}^{\mathcal{S},\mathrm{u}}_{\pi\pi}$
$\mathcal{R}^{\mathcal{S},\mathrm{u}}_{\pi\mathrm{A}}$
0
−200
−400
−600
−800
−1000
0 5 10 15 20 25 30 35 40 45
t/a

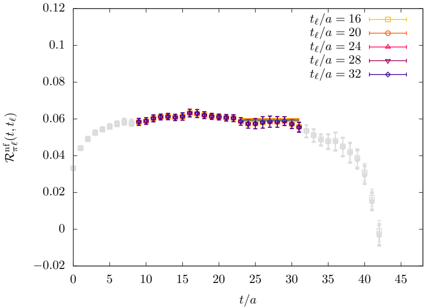
Kaon


$\mathcal{R}^{\mathcal{S},\mathrm{u}}_{\mathrm{KK}}$
$\mathcal{R}^{\mathcal{S},\mathrm{u}}_{\mathrm{KA}}$
0
−50
−100
−150
−200
0 5 10 15 20 25 30 35 40 45
t/a

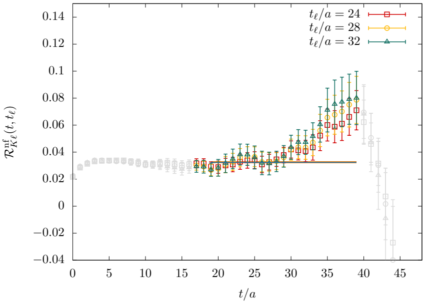
Non-factorizable contribution contribution



Pion



Kaon



Final result

$$\frac{|V_{us}|}{|V_{ud}|} = \left[\frac{\Gamma(K\to\mu\nu)}{\Gamma(\pi\to\mu\nu)}\frac{m_K}{m_\pi}\frac{(m_\pi^2-m_\mu^2)}{(m_K^2-m_\mu^2)}\right]^{1/2}\frac{f_\pi}{f_K}\left(1-\frac{1}{2}\delta R_{K\pi}\right)$$

with values from experiment, FLAG lattice average, and this calculation:

$$\delta R_{K\pi} = -0.0086(3)_{\mathrm{stat.}}(^{+11}_{-4})_{\mathrm{fit}}(5)_{\mathrm{disc.}}(5)_{\mathrm{quench.}}(39)_{\mathrm{vol.}}$$

leading to

$$\frac{|V_{us}|}{|V_{ud}|} = 0.23154(28)_{\mathrm{exp.}}(15)_{\delta R_{K\pi}}(45)_{\delta R_{K\pi},\mathrm{vol.}}(65)_{f_\pi/f_K}$$

- our uncertainty is dominated by finite-volume error due to single lattice spacing - this will improve drastically in the near future! [Matteo Di Carlo, plenary at Lattice23]
- result in agreement with only other lattice calculation [Di Carlo et al., Phys.Rev.D 100 (2019) 3, 034514]
- $|V_{us}|/|V_{ud}|$ uncertainty dominated by f_π/f_K lattice average

Rare hyperon decays

JHEP

PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: *October 14, 2022*
REVISED: *March 7, 2023*
ACCEPTED: *April 3, 2023*
PUBLISHED: *April 24, 2023*

Prospects for a lattice calculation of the rare decay $\Sigma^+ \to p\ell^+\ell^-$

Felix Erben, Vera Gülpers, Maxwell T. Hansen, Raoul Hodgson and Antonin Portelli

[FE et al., JHEP 04 (2023) 108]

EXPERIMENAL STATUS

- $s \to d$ transition is a *flavour-changing neutral current*
- within the Standard Model only allowed at loop level

$\Rightarrow$ potential window into new physics

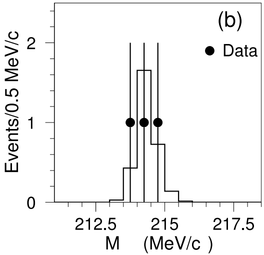
- first measured by HyperCP [HyperCP, Phys. Rev. Lett. 94, 021801]

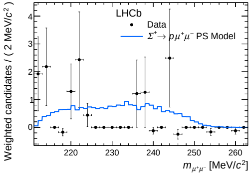
$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) = 8.6(^{+6.6}_{-5.4})_{\text{stat}}(5.5)_{\text{sys}} \times 10^{-8}$$

- 3 events, all at similar dimuon invariant mass
  $m_{\chi^0} = 214.3 \pm 0.5\mathrm{MeV}/c^2 \Rightarrow$ resonance?
- subsequent measurement by LHCb [LHCb, Phys. Rev. Lett. 120, 221803]

$$\mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) = 2.2(^{+1.8}_{-1.3}) \times 10^{-8}$$

- 10 events, no significant structure in dimuon invariant mass
- Run 2 update in progress [Paras Naik, Mon 18/09 15:10, WG 3]





PHENOMENOLOGY

- Amplitude of rare hyperon decay is long-distance dominated with form-factor decomposition

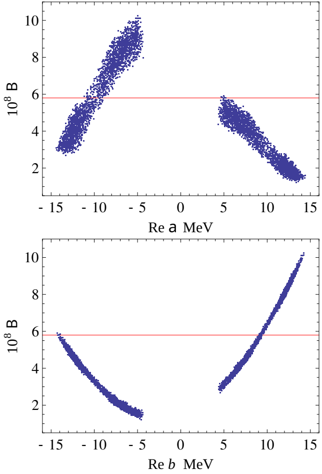
$$A_\mu^{rs} = \bar{u}_p^r \left[i\sigma_{\nu\mu} q^\nu (a + b\gamma_5) + (q^2\gamma_\mu - q_\mu \not{q})(c + d\gamma_5) \right] u_\Sigma^s$$

- Computation using ChiPT, vector-meson dominance and experimental input gives [He et al., Phys.Rev.D 72 (2005) 074003] [He et al., JHEP 10 (2018) 040]

$$1.6 \times 10^{-8} < \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) < 9.0 \times 10^{-8}$$

- $\mathrm{Re}(a)$, $\mathrm{Re}(b)$ poorly constrained from experiment $(\Sigma^+ \to p\gamma)$

$\Rightarrow$ Lattice could help considerably constraining Standard-Model estimate of $\mathcal{B}(\Sigma^+ \to p\ell^+\ell^-)$



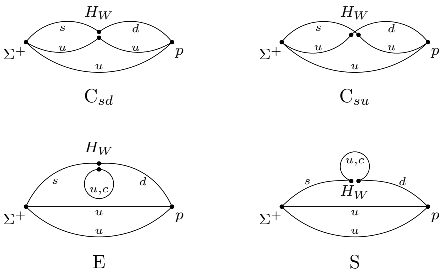
Lattice Strategy

- Full strategy laid out in theory paper [FE et al., JHEP 04 (2023) 108]
- shares challenges with the rare kaon decay $K \to \pi\ell^+\ell^-$ [Boyle, FE et al., Phys.Rev.D 107 (2023) 1, L011503]
    - calculation at physical pion mass presented by Ryan Hill [Tue 19/09 14:45 (WG 3)]
- brief outline of strategy:

$$\mathcal{A}_\mu^{rs} = \int d^4x \, \langle p(\mathbf{p}), r | T[\mathcal{H}_W(x) J_\mu(0)] | \Sigma^+(\mathbf{k}), s \rangle \,, \quad (\mathcal{A}_\mu^{rs} = \bar{u}^r \tilde{\mathcal{A}}_\mu u^s)$$

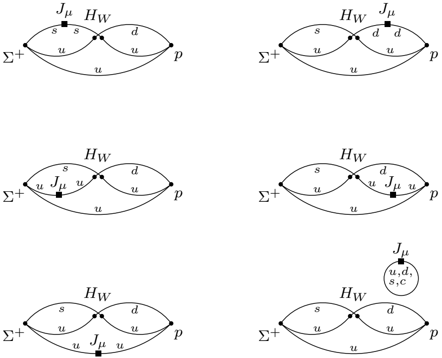
- with e.m. current J_μ and effective $s \to d$ weak Hamiltonian $\mathcal{H}_W$, involving 4-quark interactions
- similar formalism described in more detail by En-Hung Chao [Mon 18/09 16:00 (WG 3)]

4 classes of diagrams ($\mathcal{H}_W$ only)



"Non-Eye" (top), "Eye" (bottom)

total 24 diagrams with J_μ



Lattice Strategy

- Minkowski amplitude $\tilde{\mathcal{A}}_\mu$ can be extracted from finite-volume estimator

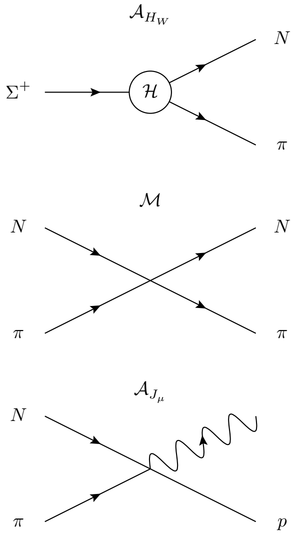
$$F_\mu(\mathbf{k},\mathbf{p})_L = i\int_0^\infty d\omega\, \frac{\rho_\mu(\omega)_L}{E_\Sigma(\mathbf{k})-\omega} - i\int_0^\infty d\omega\, \frac{\sigma_\mu(\omega)_L}{\omega - E_\Sigma(\mathbf{k})}$$

$$\text{where e.g. } \rho_\mu(\omega)_L = \sum_n \frac{C_{n,\mu}(\mathbf{k})}{2E_n(\mathbf{k})}\delta(E_n(\mathbf{k})-\omega)$$

- $F_\mu(\mathbf{k},\mathbf{p})_L$ contains poles in volume L, when $E_n(L) \to E_\Sigma$
- infinite-volume amplitude can be obtained via

$$\tilde{\mathcal{A}}_\mu(k,p) = F_\mu(\mathbf{k},\mathbf{p})_L + \Delta F_\mu(\mathbf{k},\mathbf{p})_L$$

- $\Delta F_\mu(\mathbf{k},\mathbf{p})_L$ is obtained from 3 finite-volume amplitudes $\mathcal{A}_{H_W}, \mathcal{M}, \mathcal{A}_{J_\mu}$, cancelling poles in $F_\mu(\mathbf{k},\mathbf{p})_L$ exactly



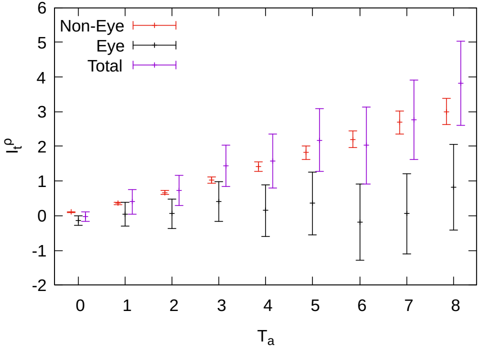
Exploratory calculation

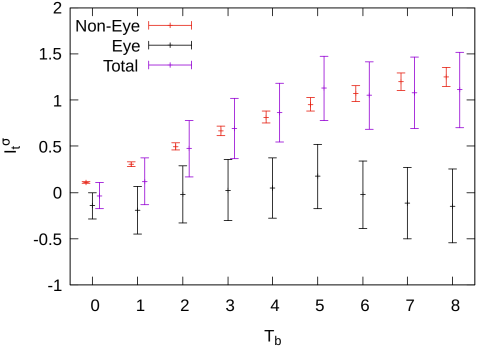
- First exploratory calculation:
  - $M_\pi = 340\text{MeV}$
  - $M_N = 1200\text{MeV}$
  - $M_\Sigma = 1370\text{MeV} < M_\pi + M_N$

$\Rightarrow$ Finite-volume corrections a lot easier to handle, no knowledge about $N\pi$ spectrum in the Δ & Roper resonance channels necessary

Exploratory calculation

- FV estimator with a temporal separation of $t_\Sigma - t_p = 16$





Plot and analysis: Raoul Hodgson

Exploratory calculation

Form Factor	Value	(Stat)	
Re a^{NE}	5	(16)	MeV
Re c^{NE}	0.009	(30)	
Re a^{Eye}	-58	(100)	MeV
Re c^{Eye}	0.034	(173)	
Re a	-53	(114)	MeV
Re c	0.018	(249)	

Analysis by Raoul Hodgson [Lattice 23]

- extracted from fit to data with $t_\Sigma - t_p \geqslant 16$
- values from phenomenology:
  - Re $a \sim 10$ MeV
  - Re $c \sim 10^{-2}$
- when including $t_\Sigma - t_p = 12$ (NE only):
  - Re $a^{NE} = 4(5)$ MeV
  - Re $c^{NE} = 0.030(9)$

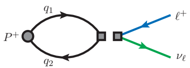
$\Rightarrow$ signal emerging

- requires fit starting from $t = 3a \sim 0.3$fm

$\Rightarrow$ large uncontrolled excited-state contributions

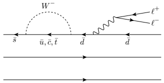
CONCLUSIONS

Leptonic decays



- indirect search for new physics via CKM unitarity
- precision calculation at M_π^{phys}
- isospin-breaking and QED corrections
- theory error still dominates
- finite-volume error drastically reduced soon!

Rare decays



- direct search for new physics via flavour-changing neutral currents
- exploratory calculation at $M_\pi = 280\text{MeV}$
- rare hyperon decay $\Sigma^+ \to p\ell^+\ell^-$
- cancellation in noisy part
- first signal emerging

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 757646.