Effective Field Theory for neutron β decay

Wouter Dekens

with
V. Cirigliano, E. Mereghetti, and O. Tomalak

arXiv:2306.03138

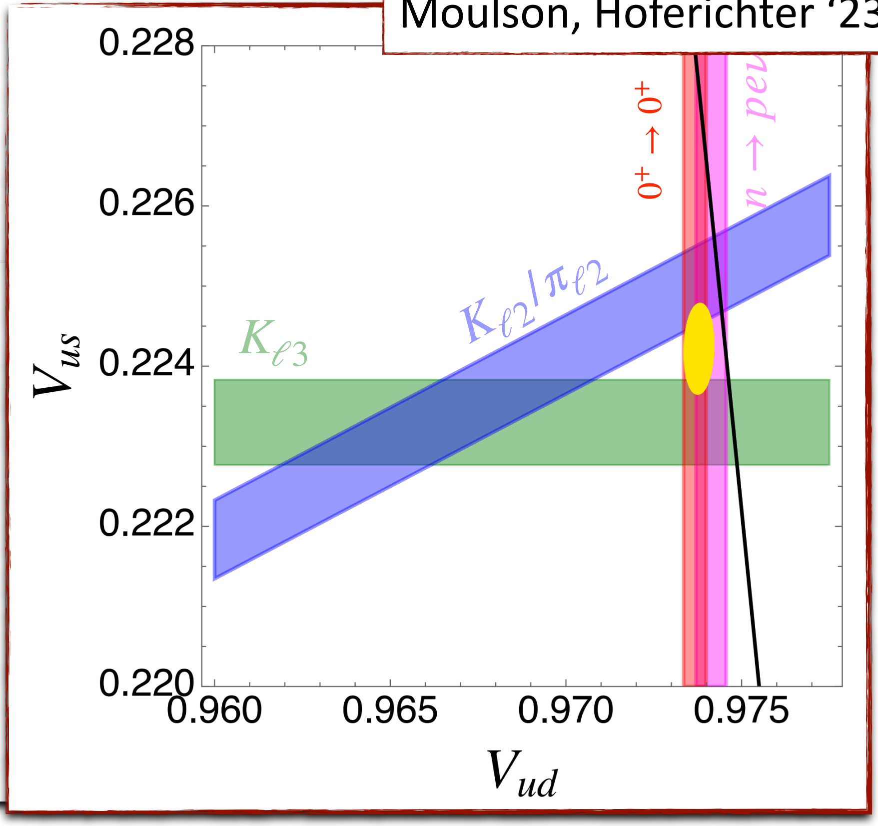
UNIVERSITY of WASHINGTON

INSTITUTE for NUCLEAR THEORY

β decays

- Helped identify V-A structure of SM
- β decays determine V_{ud}
- CKM unitarity tests

Cirigliano, Crivellin, Moulson, Hoferichter ‘23



β decays

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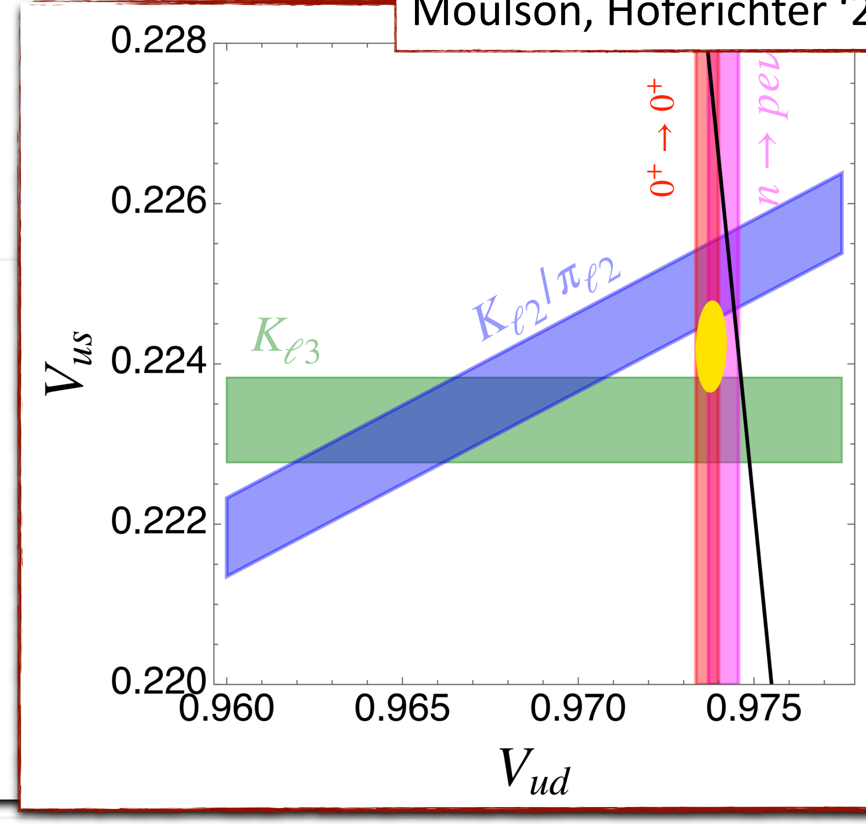

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- Individual **neutron** measurements close to $0^+ \to 0^+$ precision

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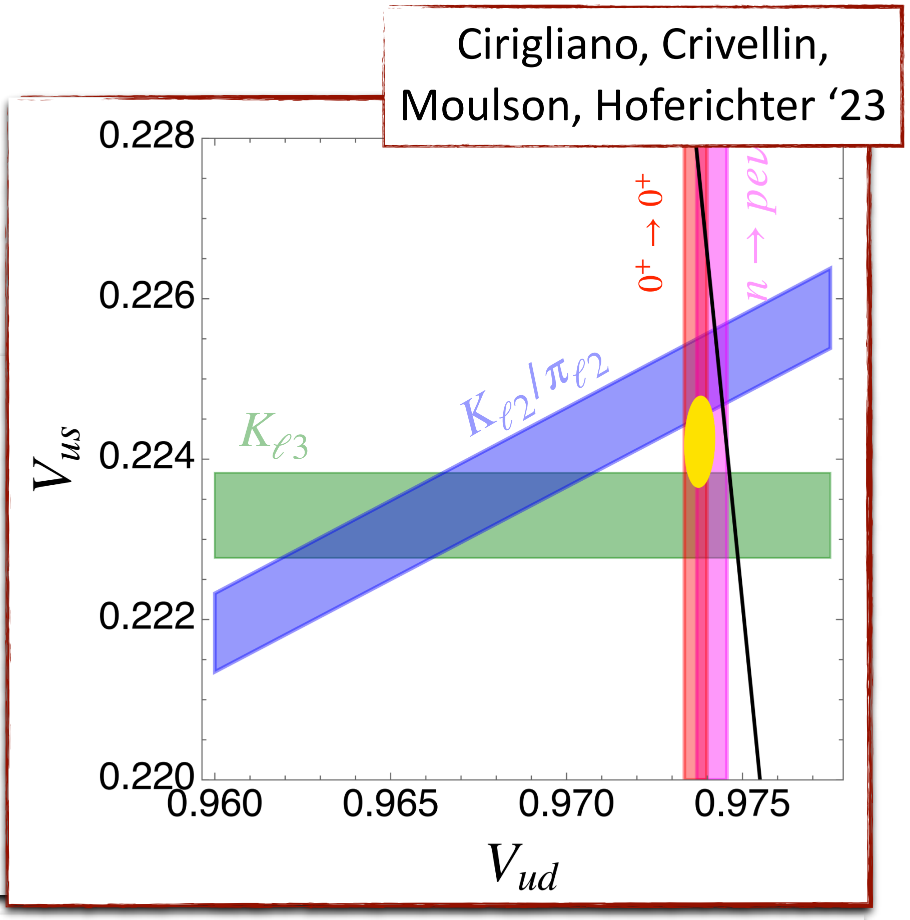

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This talk:

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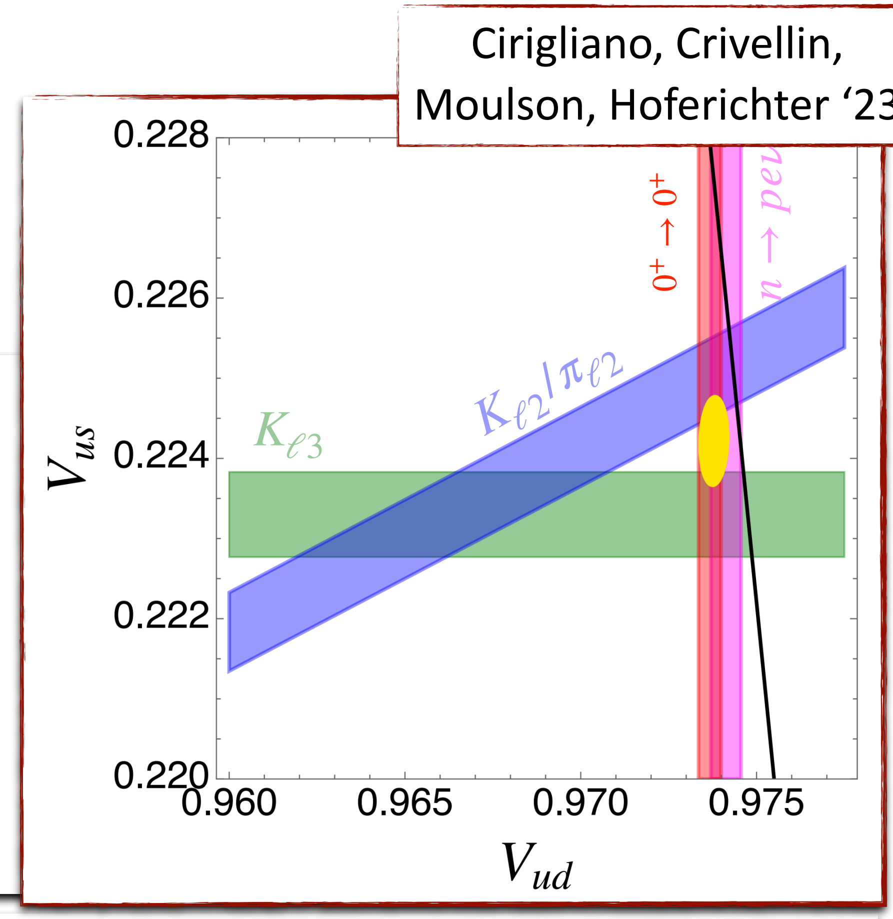

This talk:

- Individual **neutron** measurements close to $0^+ \to 0^+$ precision

See M. Gorchtein's talk tomorrow

β decays

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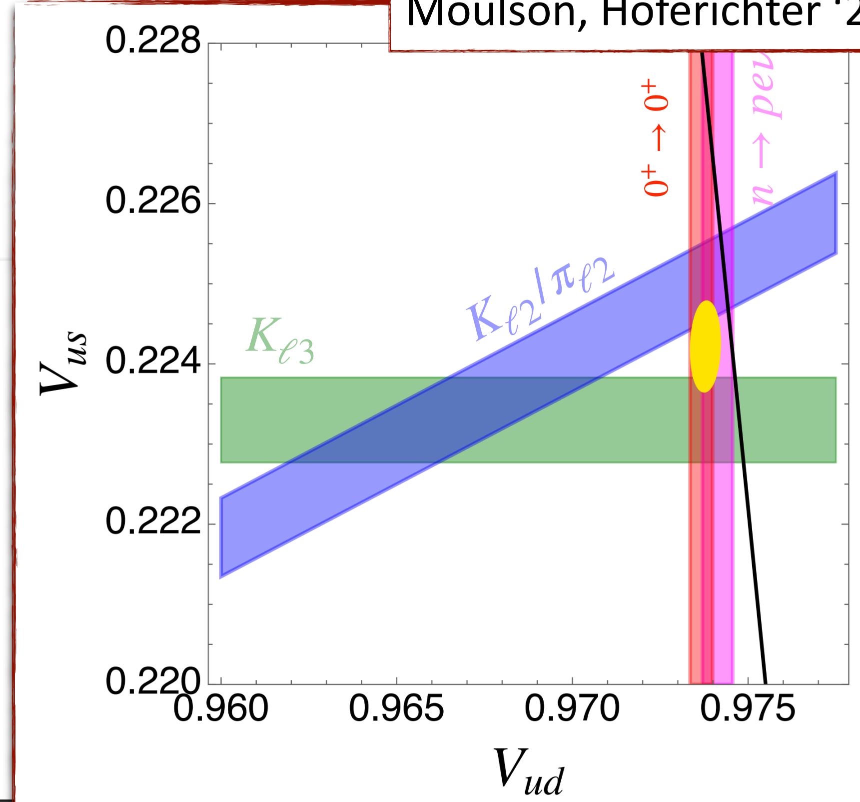

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This talk:

- Individual **neutron** measurements close to $0^+ \to 0^+$ precision
- Experimentally, few $\times 10^{-4}$ uncertainty
  - Could probe BSM scales $\Lambda \sim \mathcal{O}(10)$ TeV
  - Requires control of SM prediction to $\mathcal{O}(\alpha, \alpha_s \alpha \ln, \alpha^2 \ln)$

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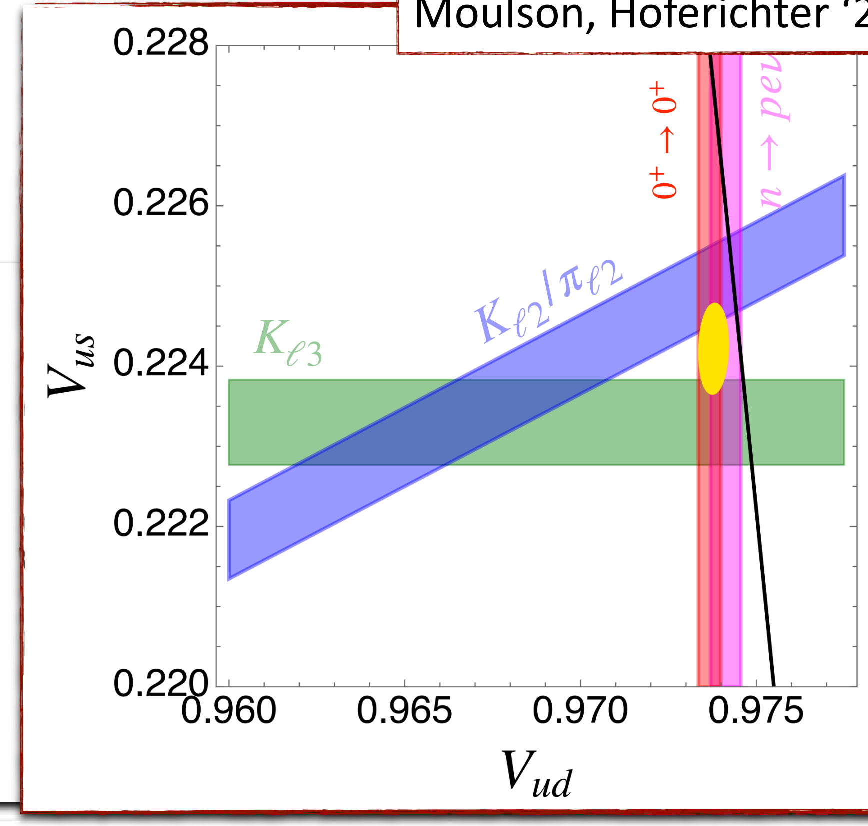

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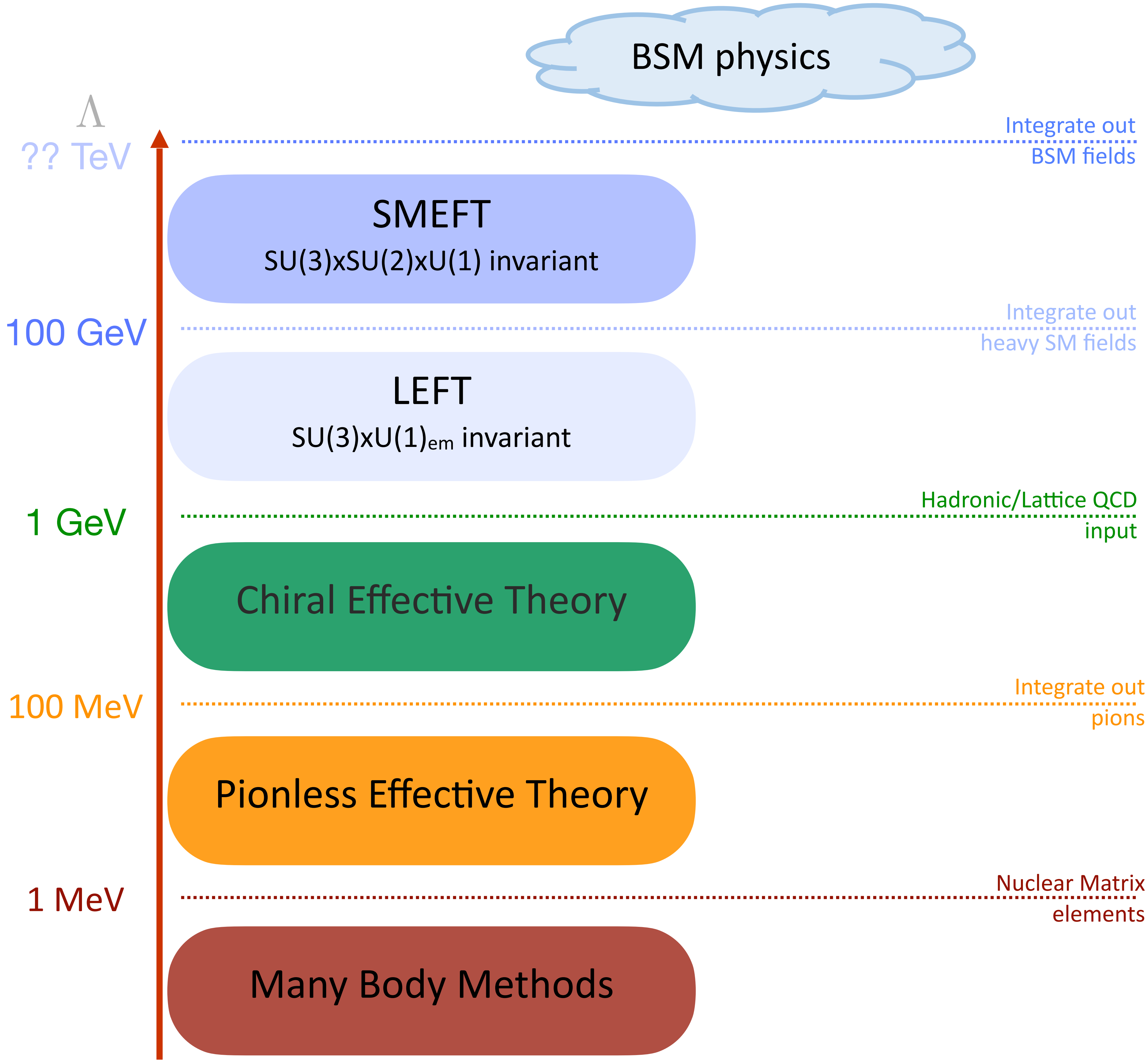
- Important to quantify SM uncertainty:
  - A lot of renewed interest
  - Formalism based on current-algebra pioneered in '60-'70s
    - Does not take full advantage of modern Effective Field Theory methods

Sirlin, Czarnecki, Marciano...

Seng, Gorchtein, Patel, Ramsey-Musolf '18, '19; Czarnecki, Marciano, Sirlin '19; Hayen, '20; Shiells, Blunden, Melnitchouk, '21; Ma et al, '23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Integrate out
BSM fields
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SU(3)xSU(2)xU(1) invariant
100 GeV
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heavy SM fields
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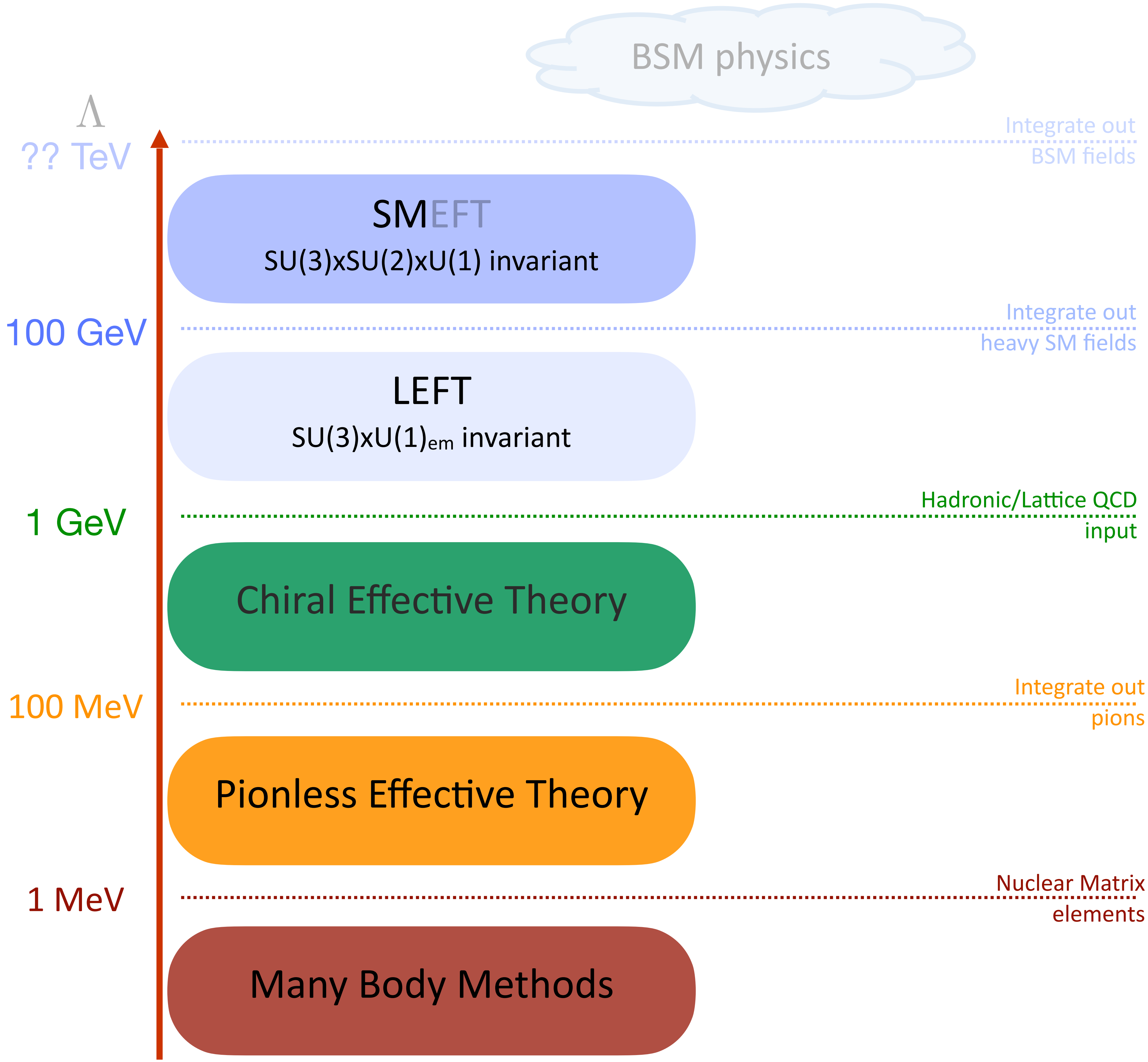

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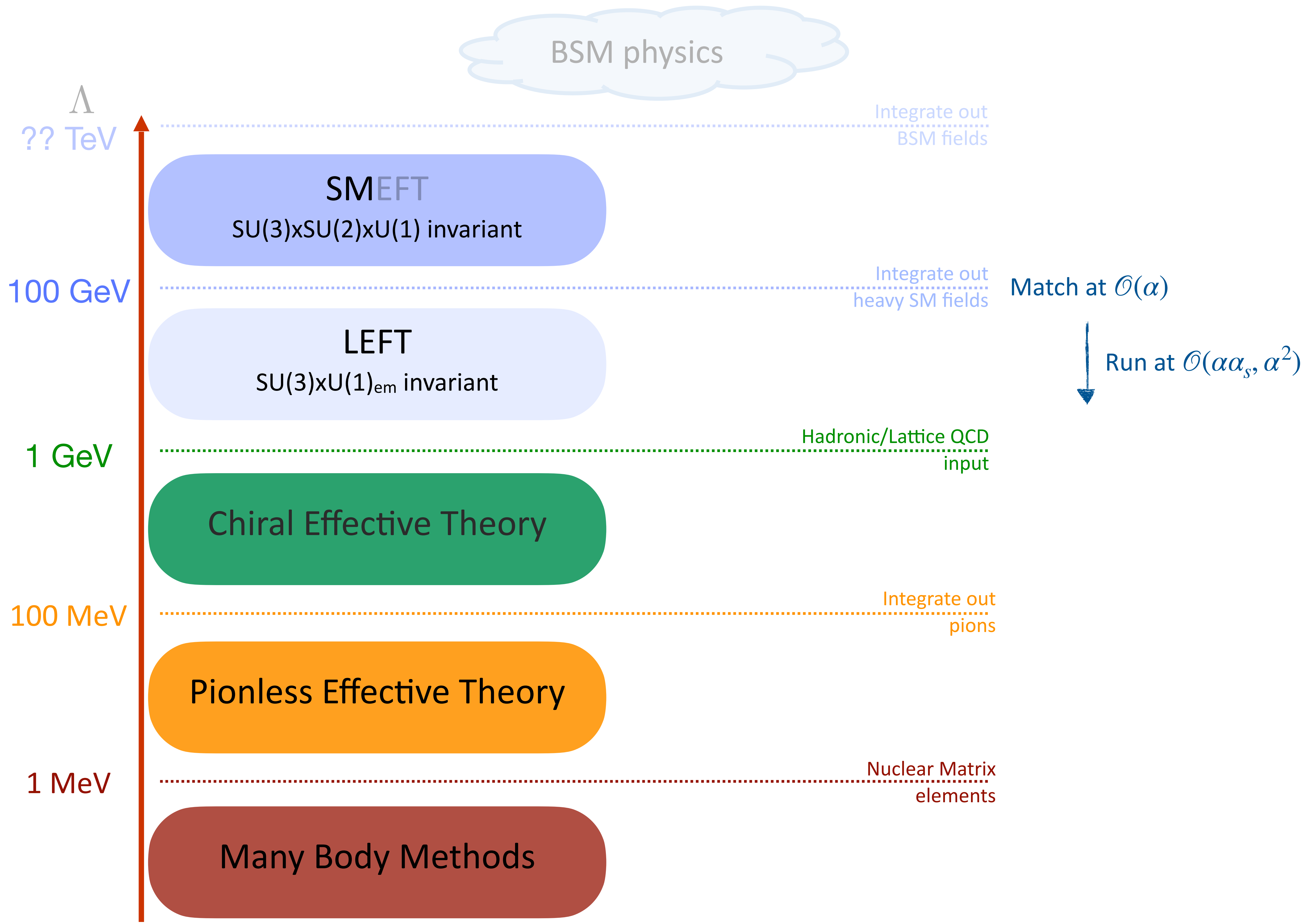

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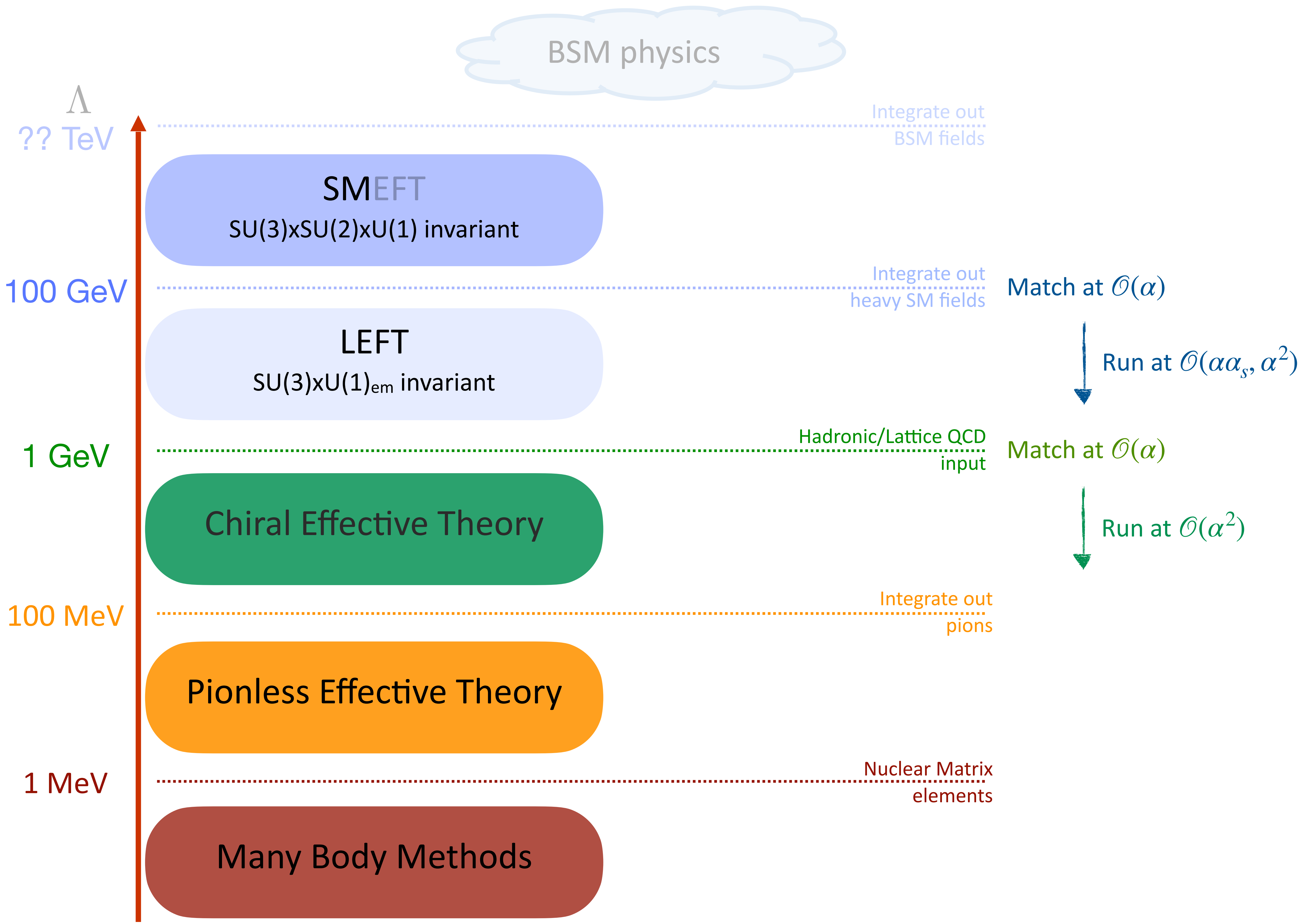

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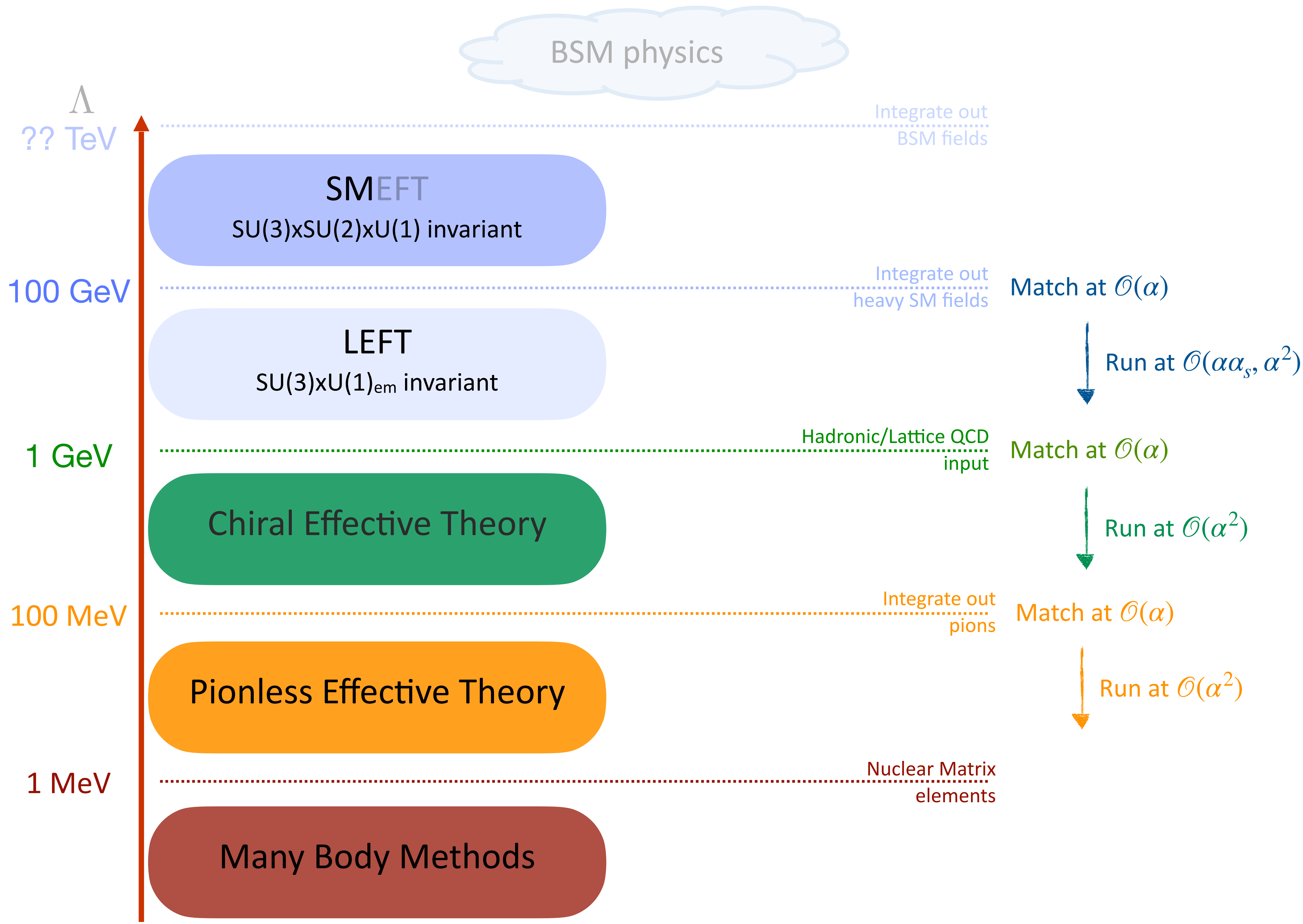

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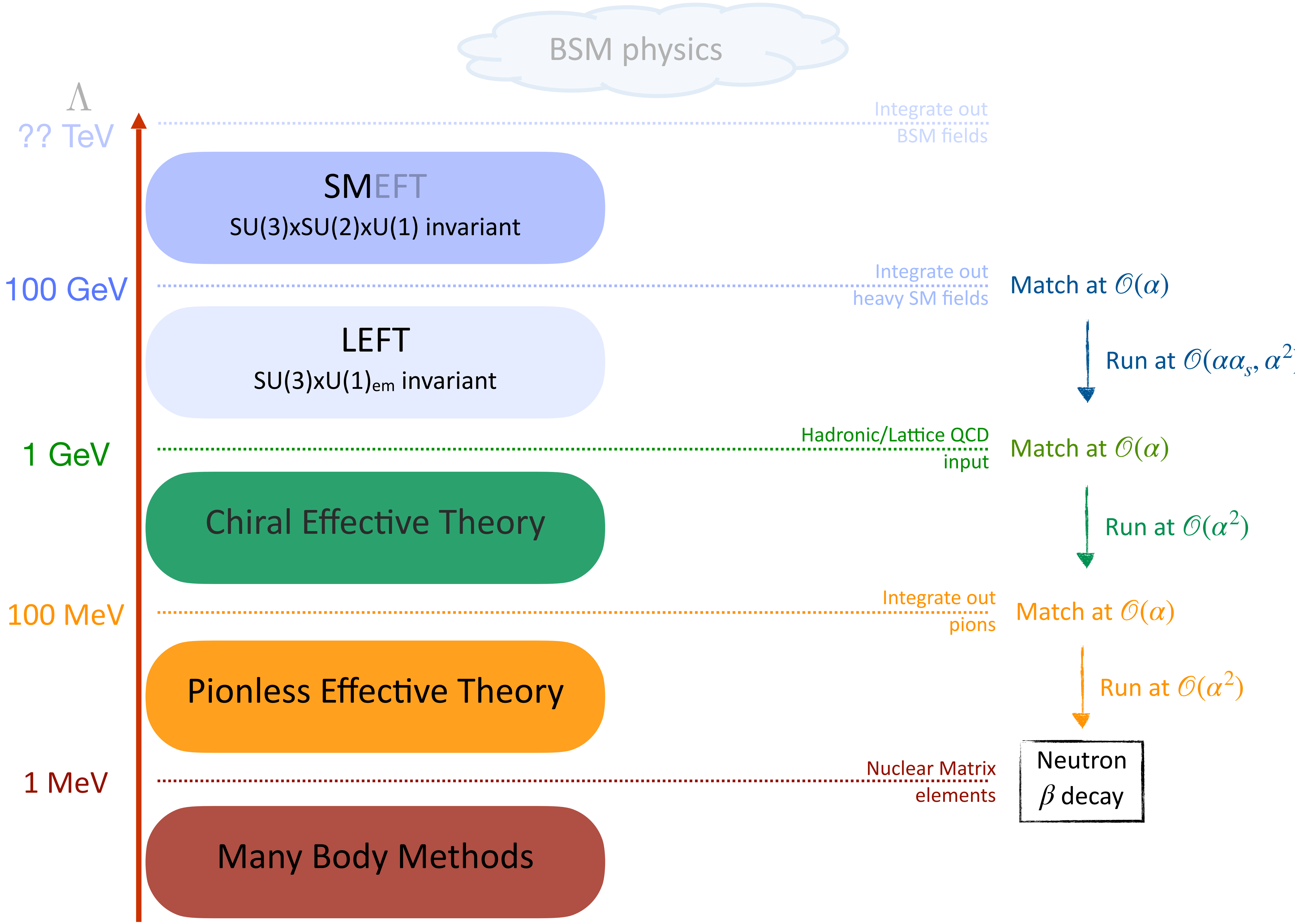
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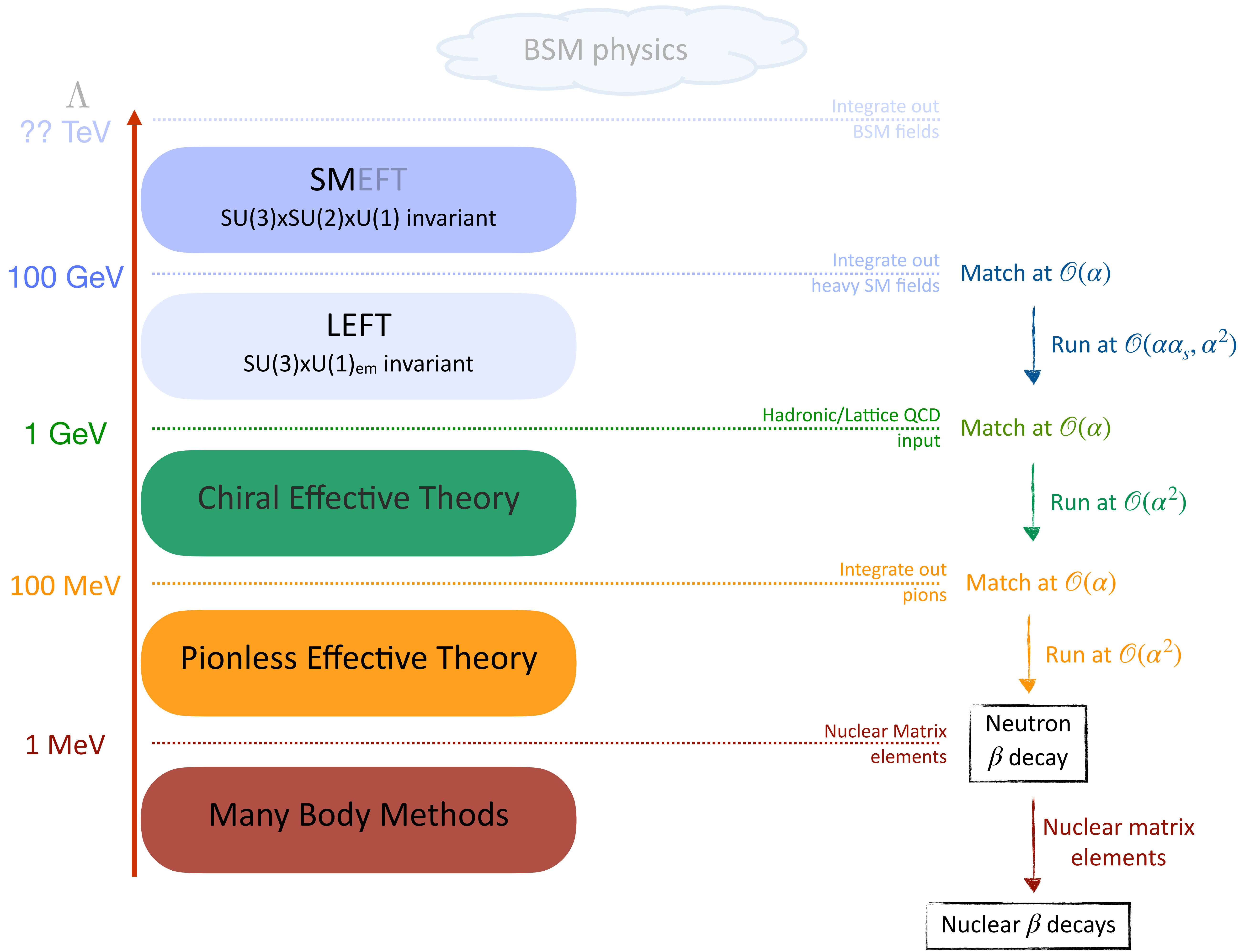
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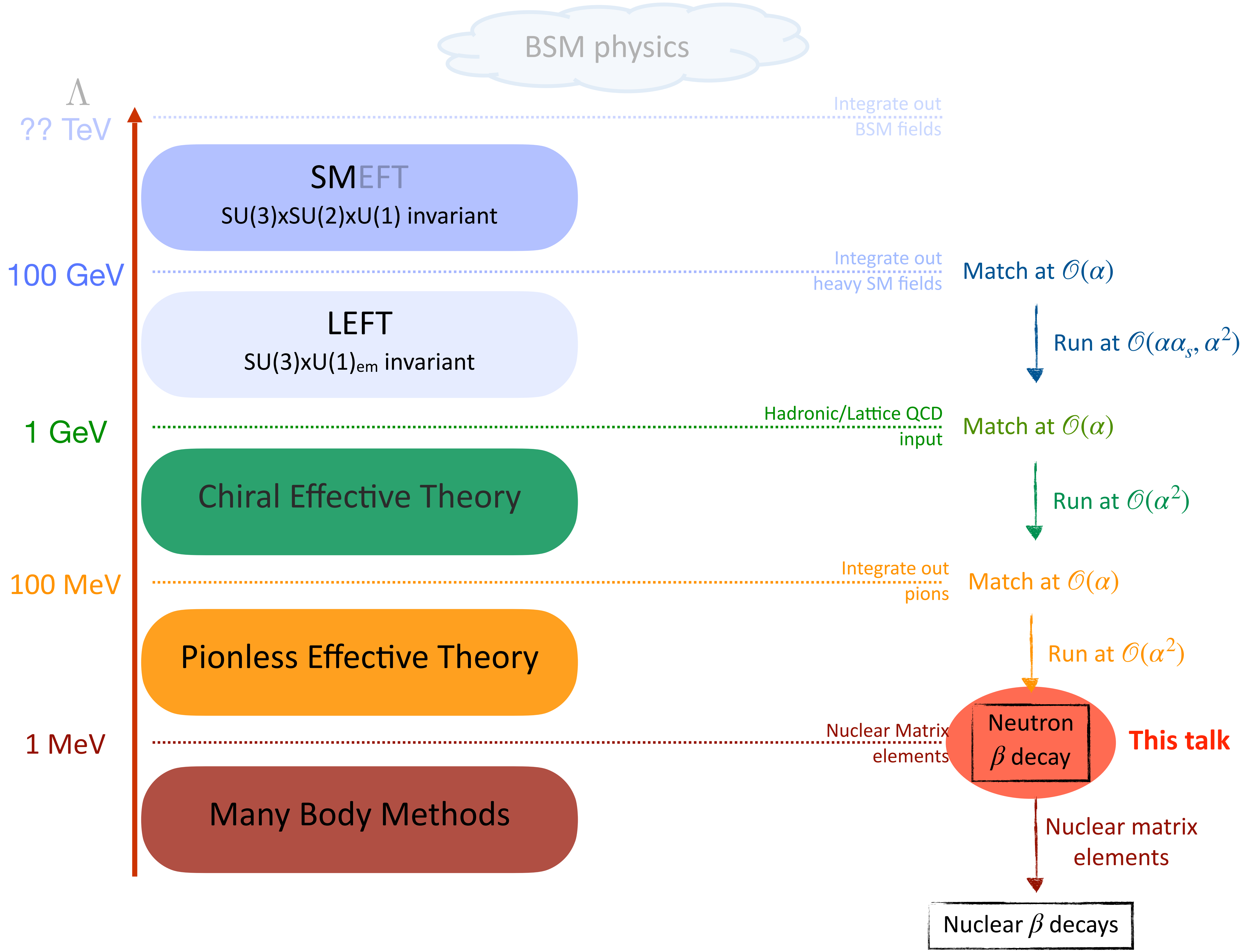

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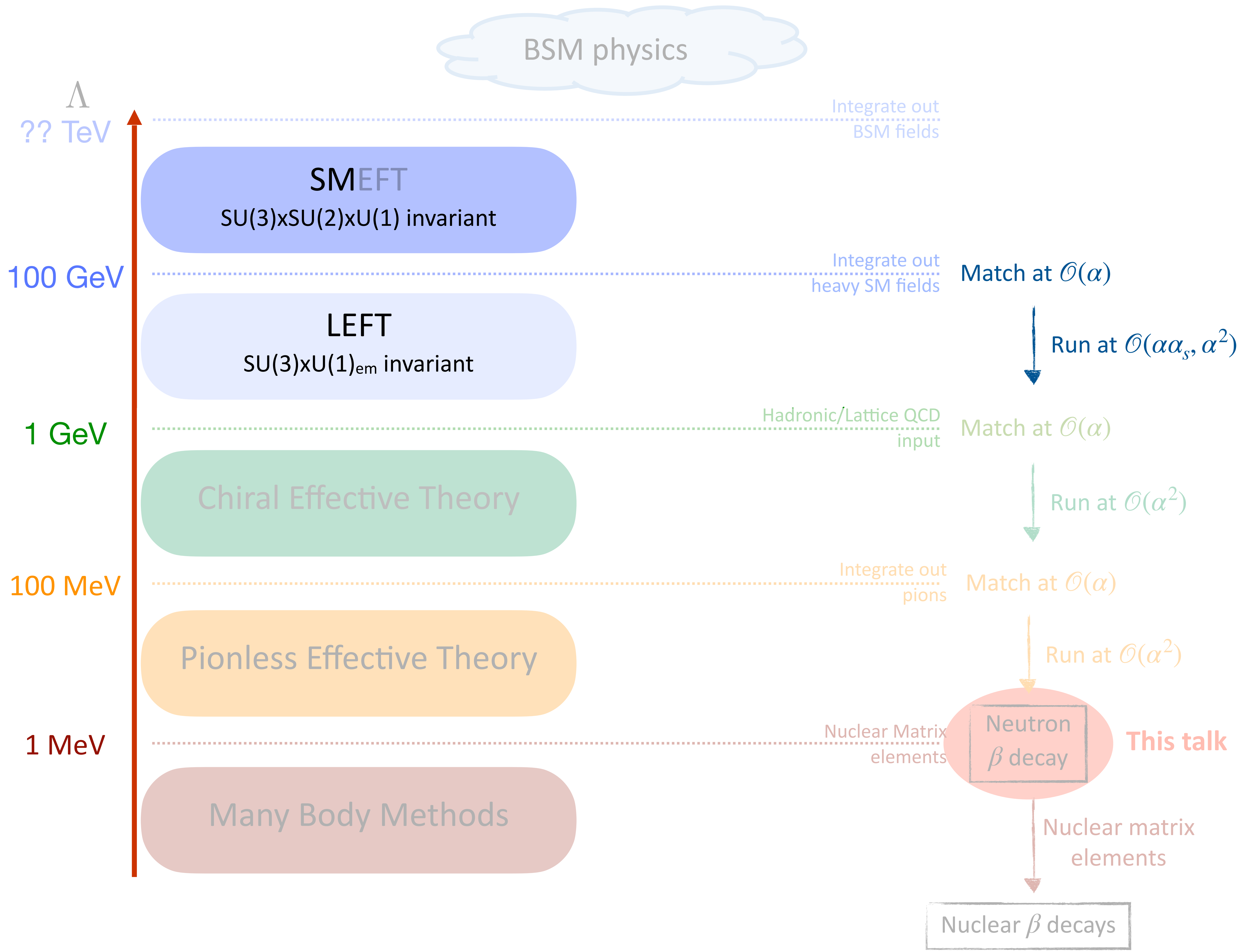
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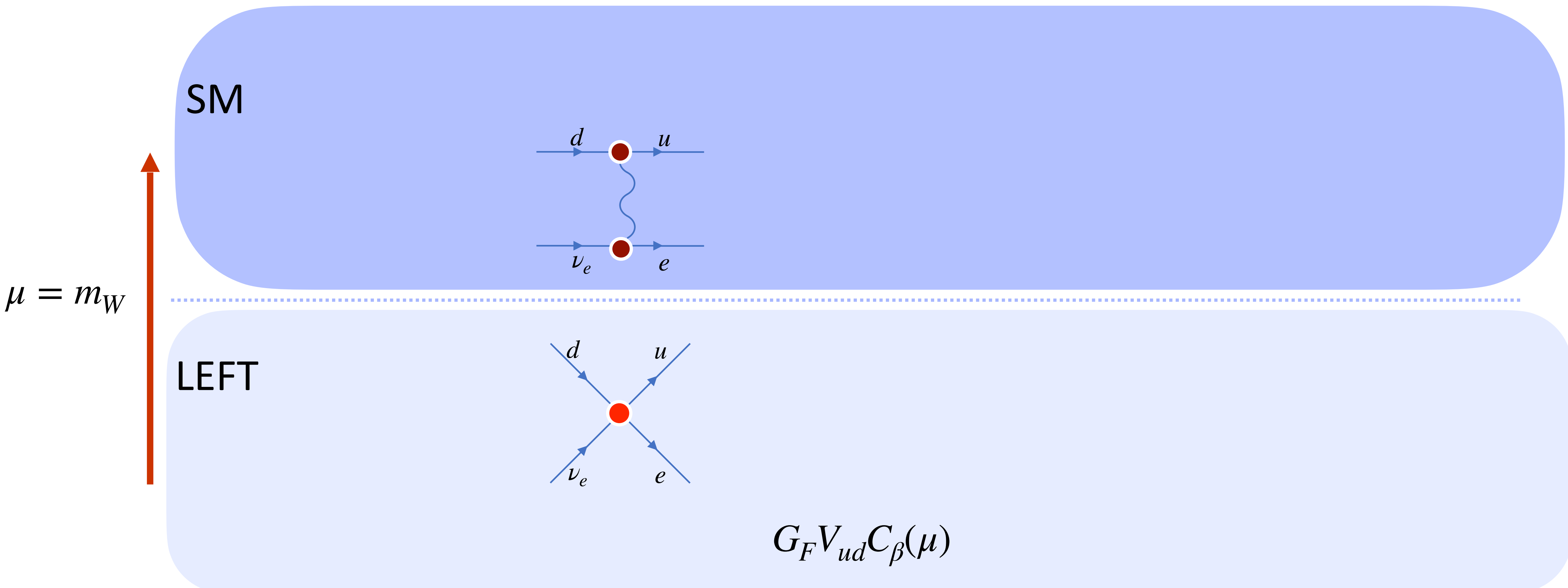
Matching

SM-LEFT: tree level

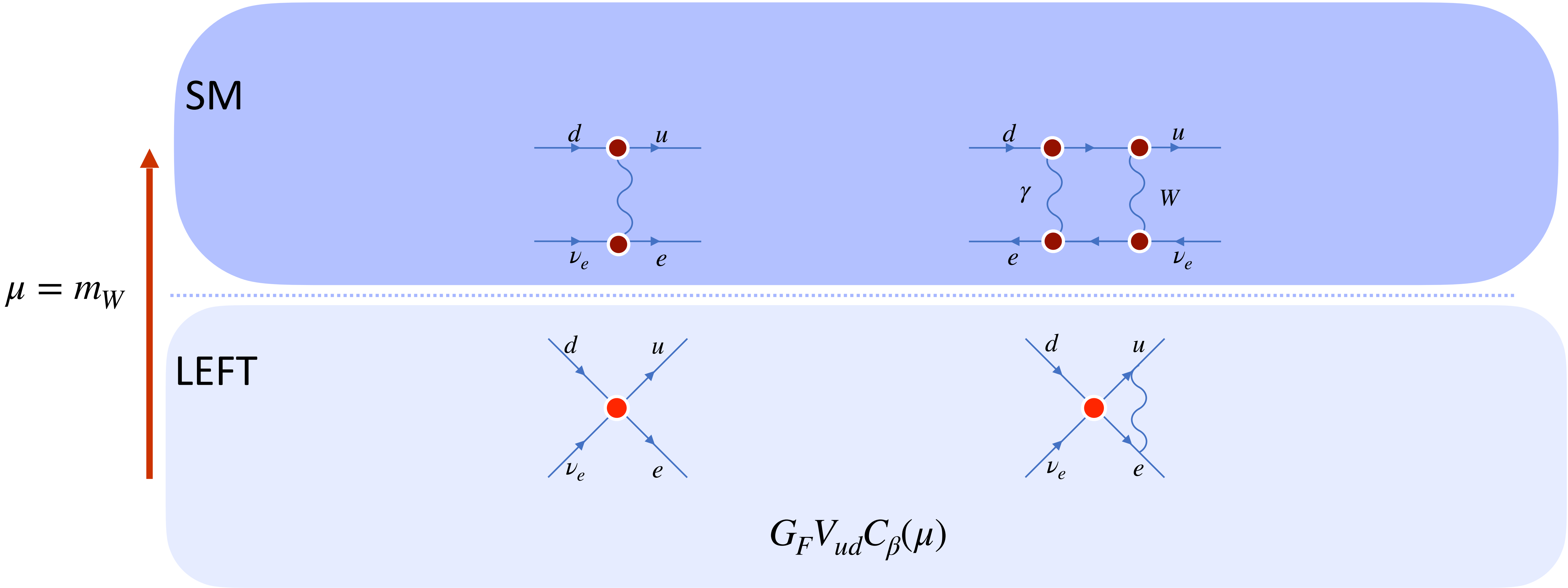


- G_F obtained from μ decay

$$C_\beta(m_W) = 1$$

Matching

SM-LEFT: loop level



Hill & Tomalak, '20;
Stoffer, WD, '19;

- G_F obtained from μ decay

$$C_\beta(m_W) = 1 + \frac{\alpha}{\pi}\left[\ln\frac{m_Z}{m_W} - \frac{3}{4} + \frac{a}{6}\right] + \mathcal{O}(\alpha^2, \alpha_s\alpha)$$

Matching

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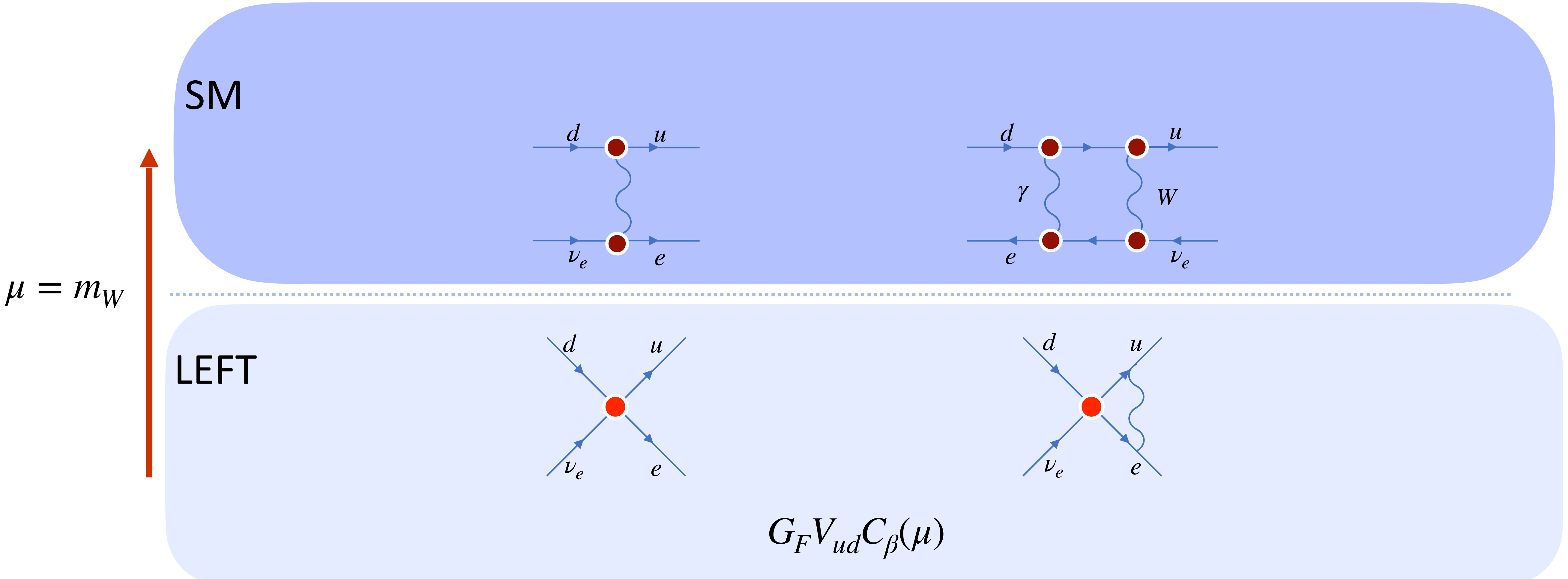

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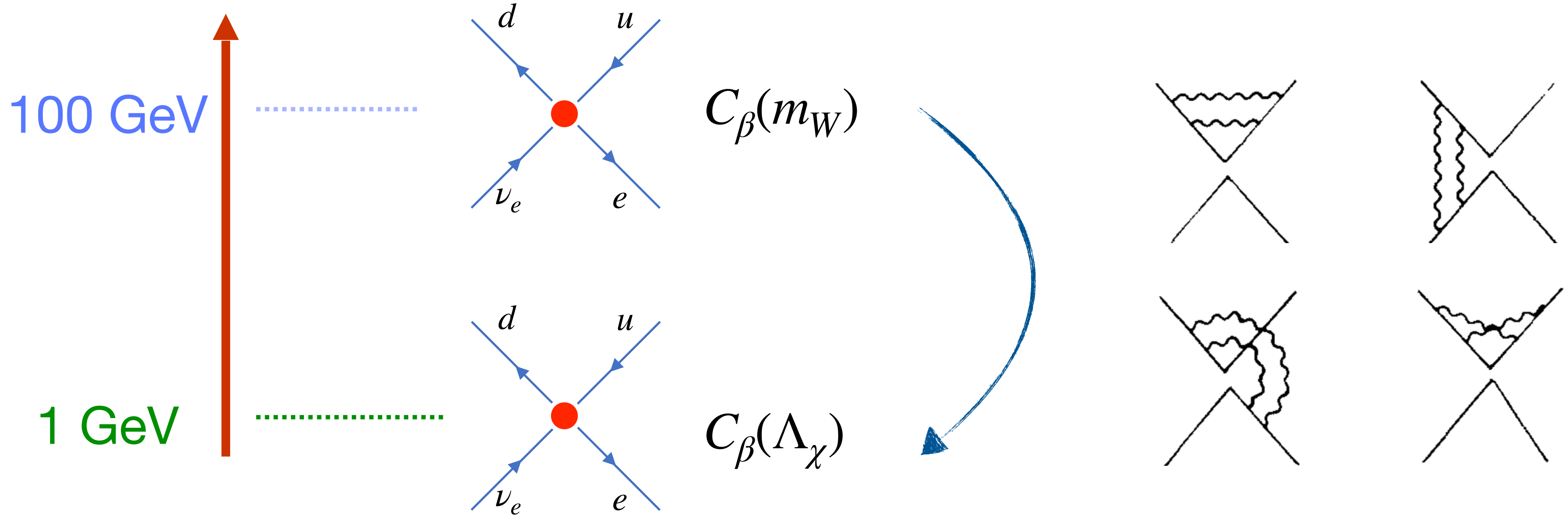
- Depends on the (evanescent) scheme, allows for checks later on

Running

LEFT



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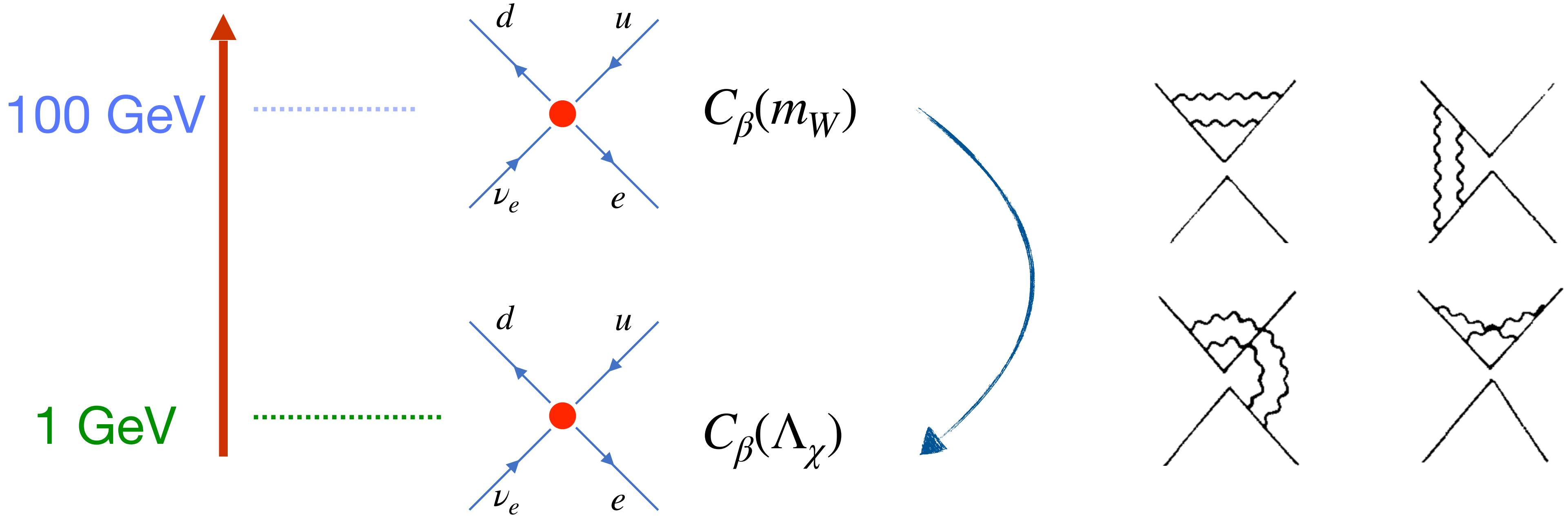

Anomalous Dimensions

$$C_\beta(\mu_\chi) = \left[1 + \mathcal{O}\left(\alpha^n \ln^n \frac{m_W}{\mu_\chi}\right) + \mathcal{O}\left(\alpha\alpha_s^n \ln^n \frac{m_W}{\mu_\chi}\right) + \mathcal{O}\left(\alpha^{n+1} \ln^n \frac{m_W}{\mu_\chi}\right)\right] C_\beta(m_W)$$

Running

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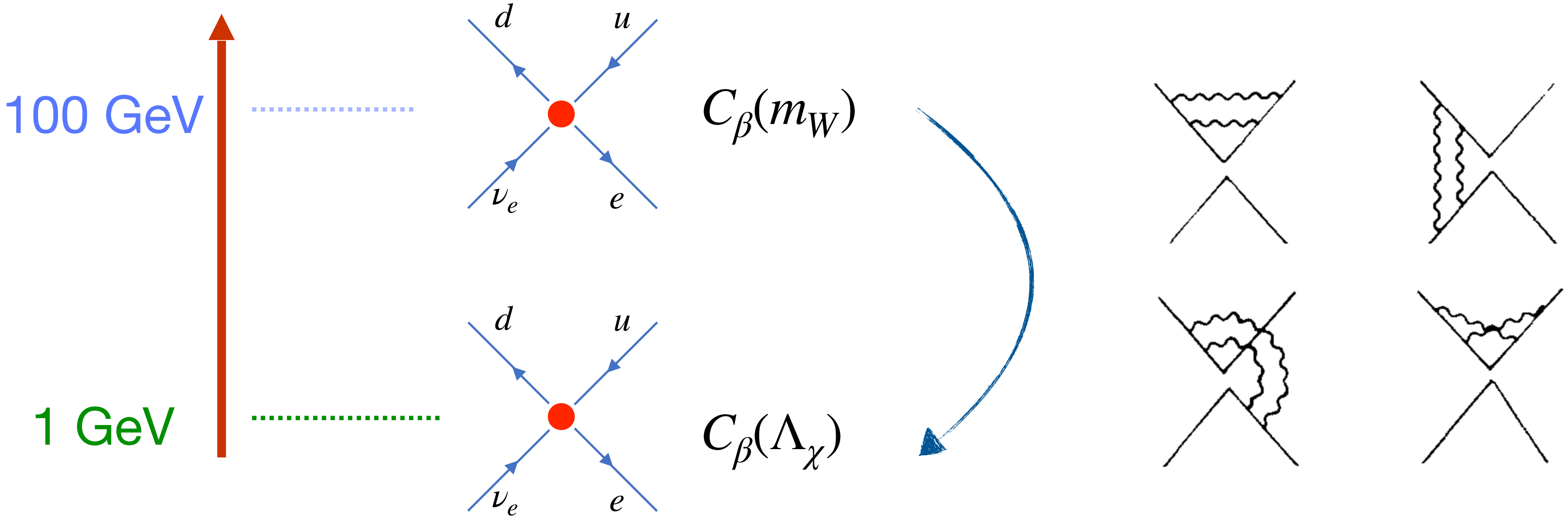

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γ_0

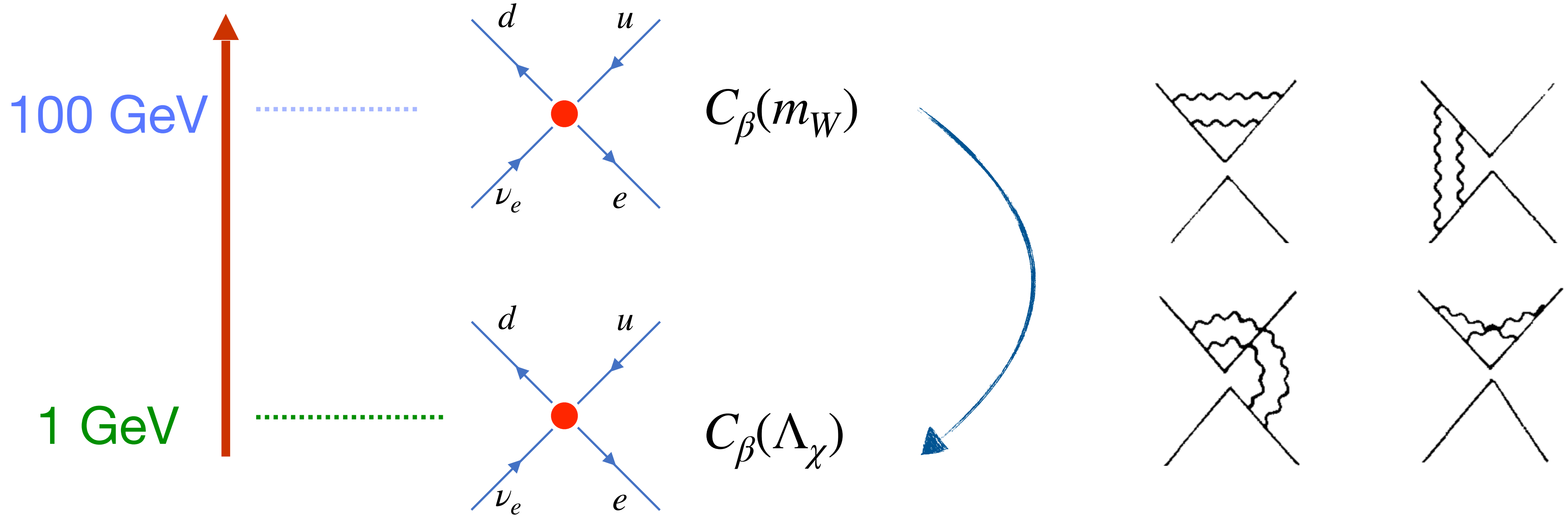
- γ_0, γ_{se} known

Sirlin, '82; Erler '04; Hill & Tomalak '20

Buras & Weisz, '90

Running

LEFT



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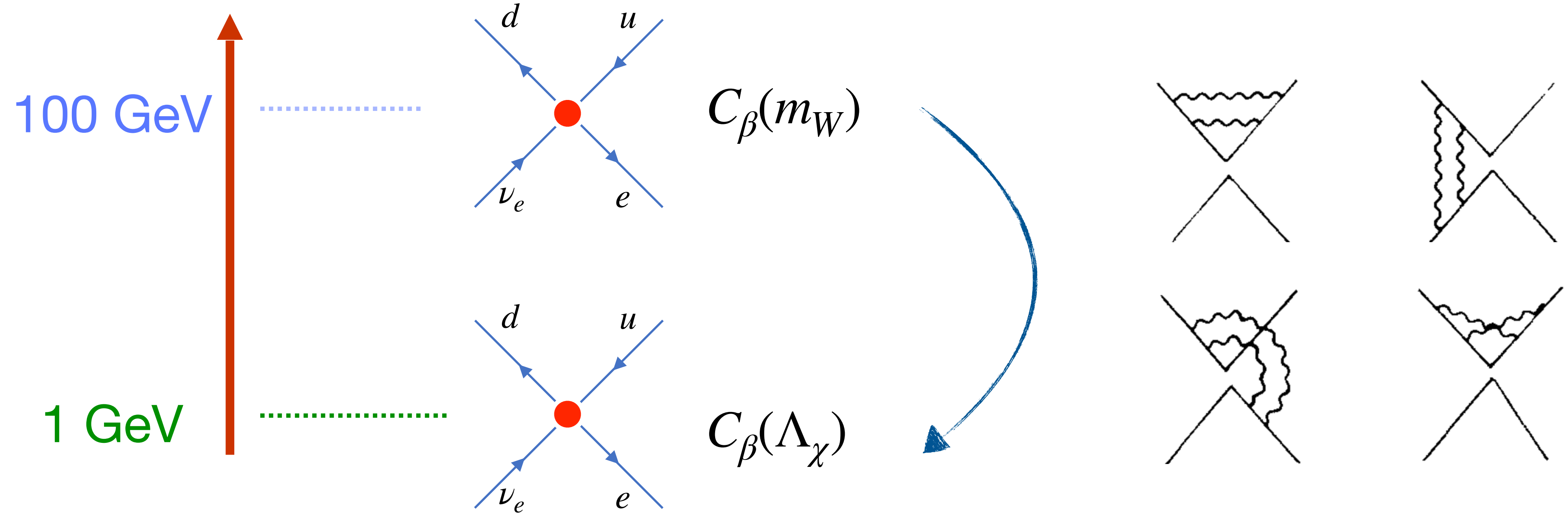

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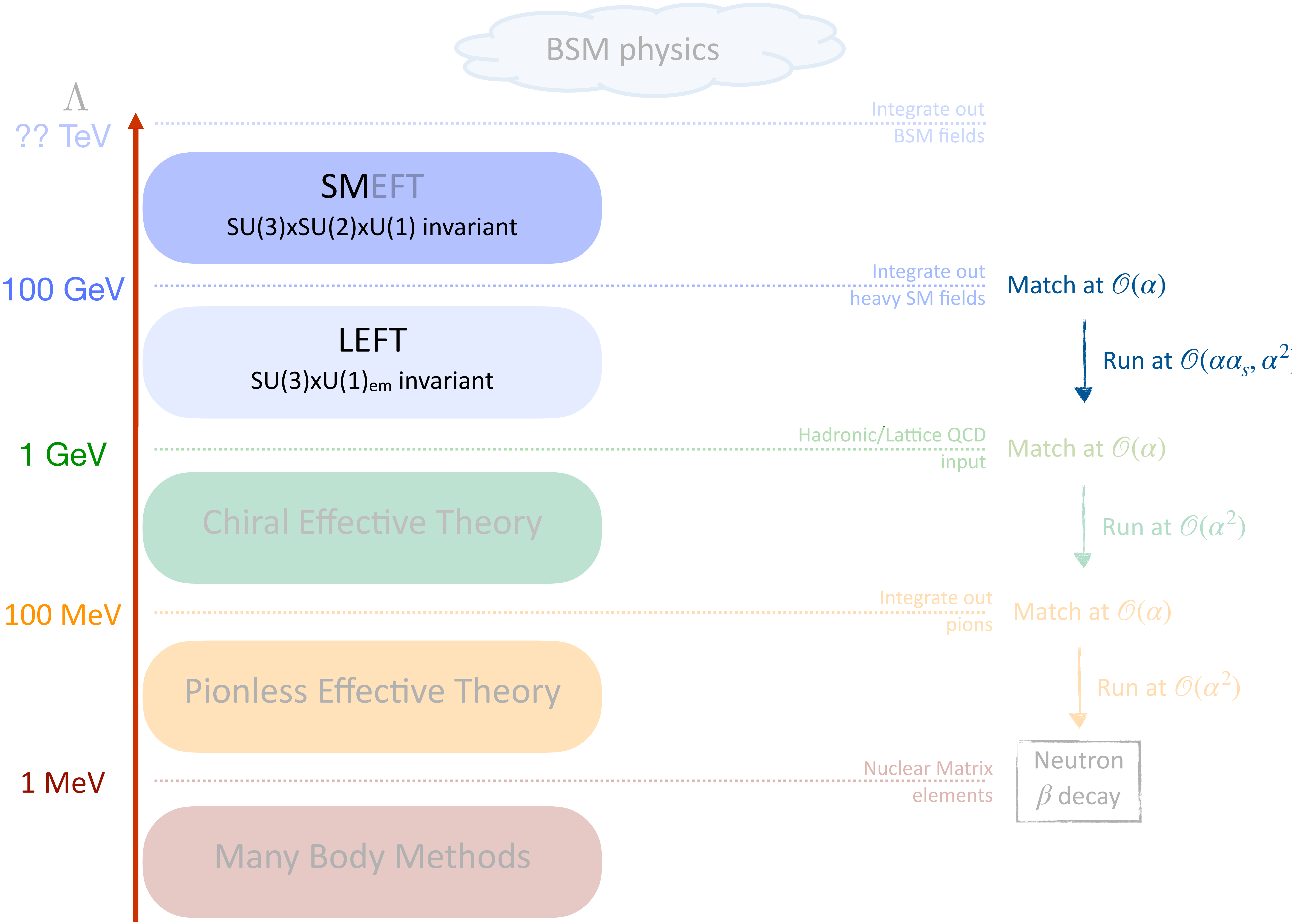
γ_0 γ_{se} γ_1

- γ_0, γ_{se} known — Sirlin, '82; Erler '04; Hill & Tomalak '20
- γ_1 can be adopted from 2-loop QCD results — Buras & Weisz, '90
    - Discrepancy with literature
    - Scheme dependence — Czarnecki, Marciano, Sirlin, '04



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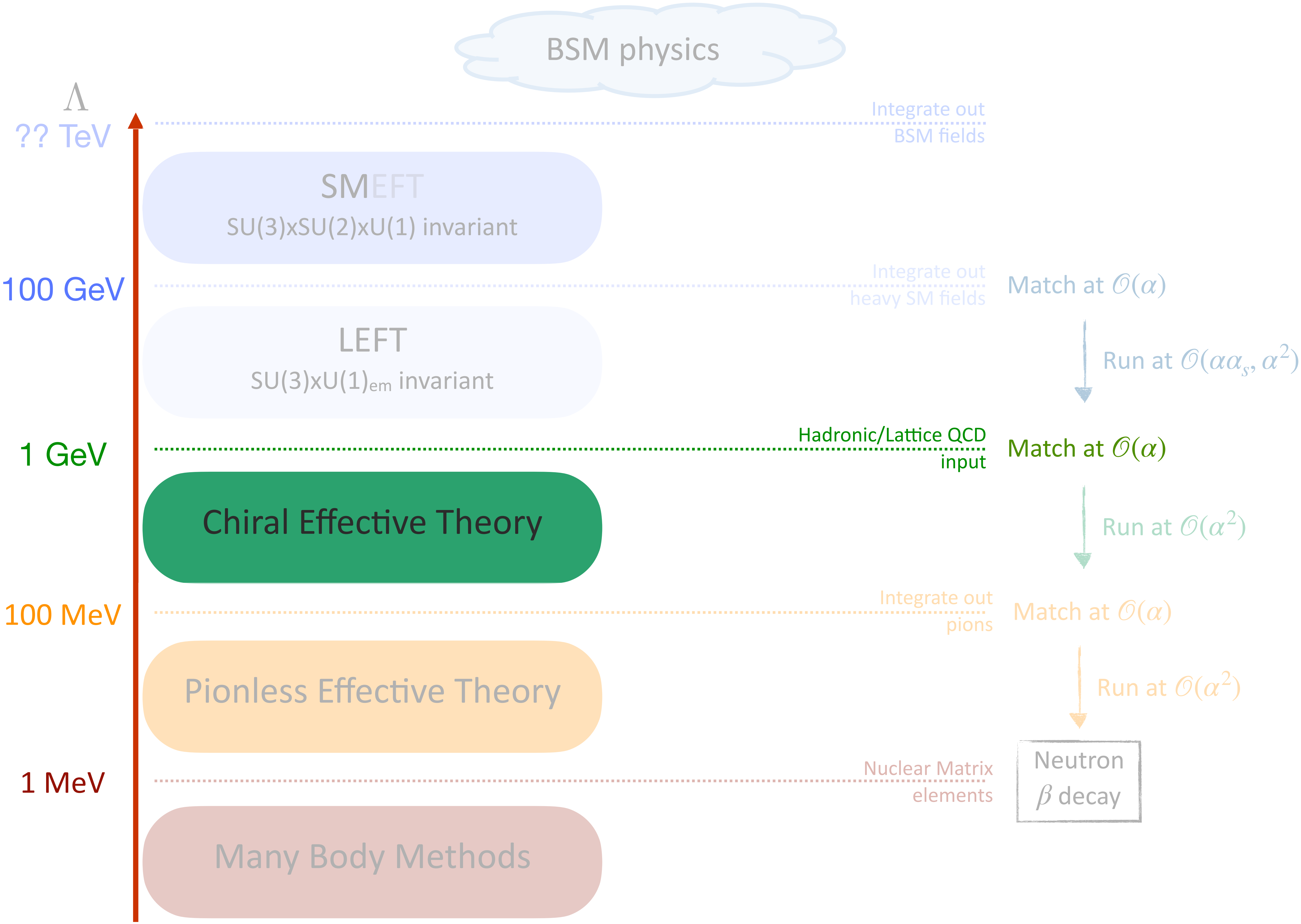
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Matching

LEFT-ChEFT

LEFT

- In terms of quarks, gluons, leptons, photons $\mathscr{L}_{\text{LEFT}}(q, g, e, \nu, \gamma)$

1 GeV

ChPT

- In terms of nucleons, pions, leptons, photons $\mathscr{L}_{\chi}(N, \pi, e, \nu, \gamma)$

Form of operators determined by chiral symmetry

The operators come with unknown low energy constants (LECs)

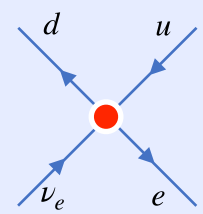
Manohar, Georgi, `84; Weinberg, `90, `91

Matching

LEFT-ChEFT: Tree level

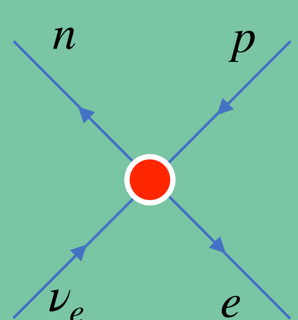
1 GeV

LEFT



$$\mathscr{L}_{\rm LEFT} = \bar{e}_L \gamma_\mu \nu_L \bar{q}_L \boldsymbol{q}_W \gamma^\mu q_L$$

ChPT



$$\mathscr{L}_\chi = -\sqrt{2} G_F V_{ud} \bar{N} v^\mu \tau^+ N \, \bar{e}_L \gamma_\mu \nu_L$$

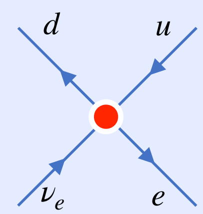
Matching

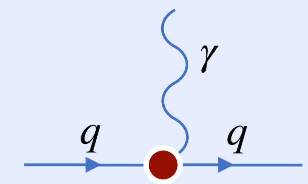
LEFT-ChEFT: $\mathcal{O}(e^2)$

Meissner, Steininger, '97
Gasser et. al. '02

1 GeV

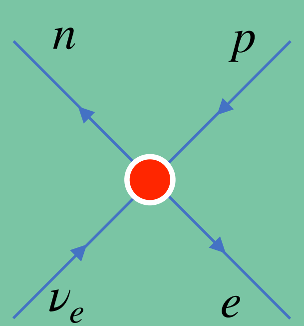
LEFT





$$\mathcal{L}_{\rm LEFT} = \bar{e}_L\gamma_\mu\nu_L\bar{q}_L\boldsymbol{q}_W\gamma^\mu q_L - e\bar{q}A_\mu\gamma^\mu\left(\boldsymbol{q}_RP_R + \boldsymbol{q}_LP_L\right)q$$

ChPT



$$\mathcal{L}_\chi = -\sqrt{2}G_FV_{ud}\bar{N}v^\mu\tau^+N\,\bar{e}_L\gamma_\mu\nu_L + \mathcal{O}(e^2)\left[\boldsymbol{q}_L\otimes\boldsymbol{q}_W + \boldsymbol{q}_{L,R}\otimes\boldsymbol{q}_{L,R}\right]$$

- Include QED: promote $\boldsymbol{q}_{L,R,W} \to \boldsymbol{q}_{L,R,W}(x)$ to spurions

Physical values
$\boldsymbol{q}_W = \tau^+$
$\boldsymbol{q}_{L,R} = Q_{\rm em}$

Matching

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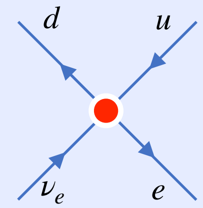
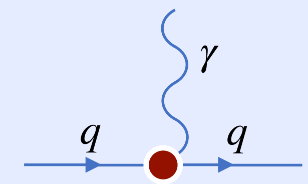
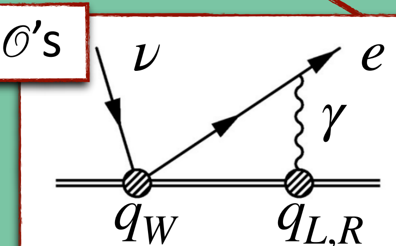


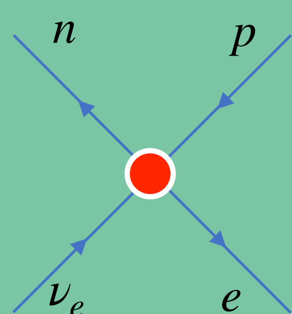


$$\mathscr{L}_{\mathrm{LEFT}} = \bar{e}_L\gamma_\mu\nu_L\bar{q}_L\boldsymbol{q}_W\gamma^\mu q_L - e\bar{q}A_\mu\gamma^\mu\left(\boldsymbol{q}_R P_R + \boldsymbol{q}_L P_L\right)q$$

ChPT

Weak $\mathcal{O}$'s





$$\mathscr{L}_\chi = -\sqrt{2}G_F V_{ud}\bar{N}v^\mu\tau^+N\,\bar{e}_L\gamma_\mu\nu_L + \mathcal{O}(e^2)\left[\boldsymbol{q}_L\otimes\boldsymbol{q}_W + \boldsymbol{q}_{L,R}\otimes\boldsymbol{q}_{L,R}\right]$$

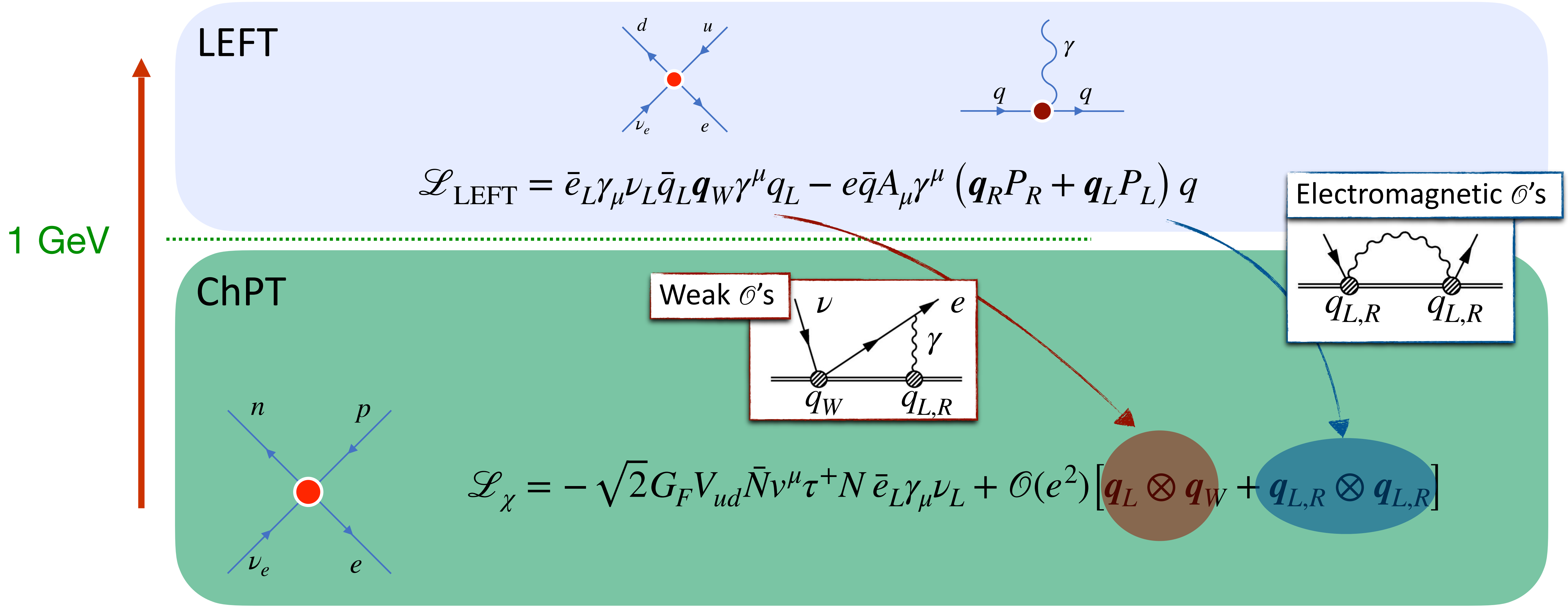
- Include QED: promote $\boldsymbol{q}_{L,R,W} \to \boldsymbol{q}_{L,R,W}(x)$ to spurions
- ChPT contains interactions with $\mathcal{O}(e^2)$ LECs

Physical values
$\boldsymbol{q}_W = \tau^+$
$\boldsymbol{q}_{L,R} = Q_{\mathrm{em}}$

Matching

LEFT-ChEFT: $\mathcal{O}(e^2)$

Meissner, Steininger, '97
Gasser et. al. '02



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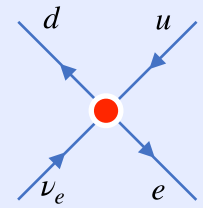
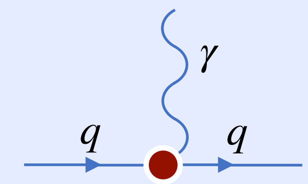
1 GeV

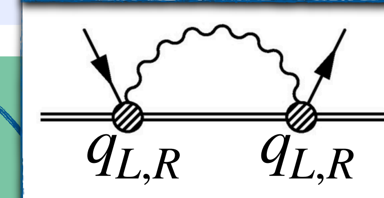
LEFT





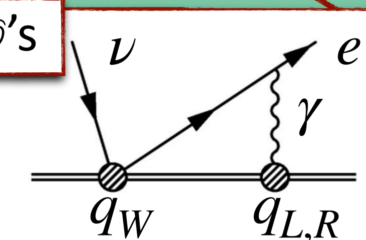
$$\mathscr{L}_{\rm LEFT} = \bar{e}_L \gamma_\mu \nu_L \bar{q}_L \boldsymbol{q}_W \gamma^\mu q_L - e\bar{q} A_\mu \gamma^\mu \left(\boldsymbol{q}_R P_R + \boldsymbol{q}_L P_L\right) q$$

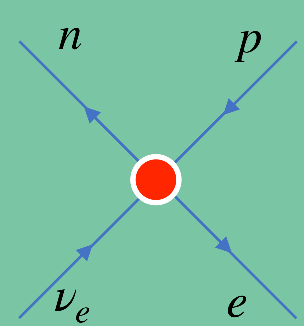
Electromagnetic $\mathcal{O}$'s



ChPT

Weak $\mathcal{O}$'s





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- Include QED: promote $\boldsymbol{q}_{L,R,W} \to \boldsymbol{q}_{L,R,W}(x)$ to spurions

- ChPT contains interactions with $\mathcal{O}(e^2)$ LECs
  - Determine the effective vector coupling

Physical values
$\boldsymbol{q}_W = \tau^+$
$\boldsymbol{q}_{L,R} = Q_{\rm em}$

$$g_V = \left[1 + e^2\left[2(V_1 + V_2 + V_3 + V_4) - X_6/2 - g_9\right]\right] C_\beta$$

Matching

LEFT-ChEFT: LECs

- Determine the LECs by equating

Descotes-Genon, Moussallam, ’05
Moussalam, ‘97

$$Z_{\text{LEFT}}(\boldsymbol{q}_L, \boldsymbol{q}_R, \boldsymbol{q}_W) = \int [D\psi][D\bar{\psi}]e^{iS_{\text{LEFT}}} \longleftrightarrow Z_\chi(\boldsymbol{q}_L, \boldsymbol{q}_R, \boldsymbol{q}_W) = \int [DN][D\bar{N}]e^{iS_\chi}$$

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Perturbative `loops'

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LEFT
RG/matching

Matching
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RG/matching

- Separates hadronic from $\mu > 1\text{GeV}$ scales
- Allows one to check scheme independence

Matching

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Matching

LEFT-ChEFT: hadronic input

Needed non-perturbative input:

$$\overline{\Box}^{V}_{\rm Had}(\mu_0) = -e^2 \int \frac{i\mathrm{d}^4 q}{(2\pi)^4} \frac{\nu^2 + Q^2}{Q^4} \; \frac{T_3(\nu, Q^2)}{2m_N \nu} \quad + \ldots$$

Matching

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Structure function T_3 arises through the correlator:

$$T_3(q, v) \sim \varepsilon^{\mu\nu\alpha\beta} q_\beta v_\alpha \int_x e^{iq\cdot x} \langle N | T[\bar{q}\gamma^\mu q(x)\, \bar{q}\gamma^\nu \gamma_5 \tau^a q] | N \rangle \sim \frac{\delta^2 Z_{\mathrm{LEFT}}}{\delta \boldsymbol{q}_W \delta \boldsymbol{q}_V}$$

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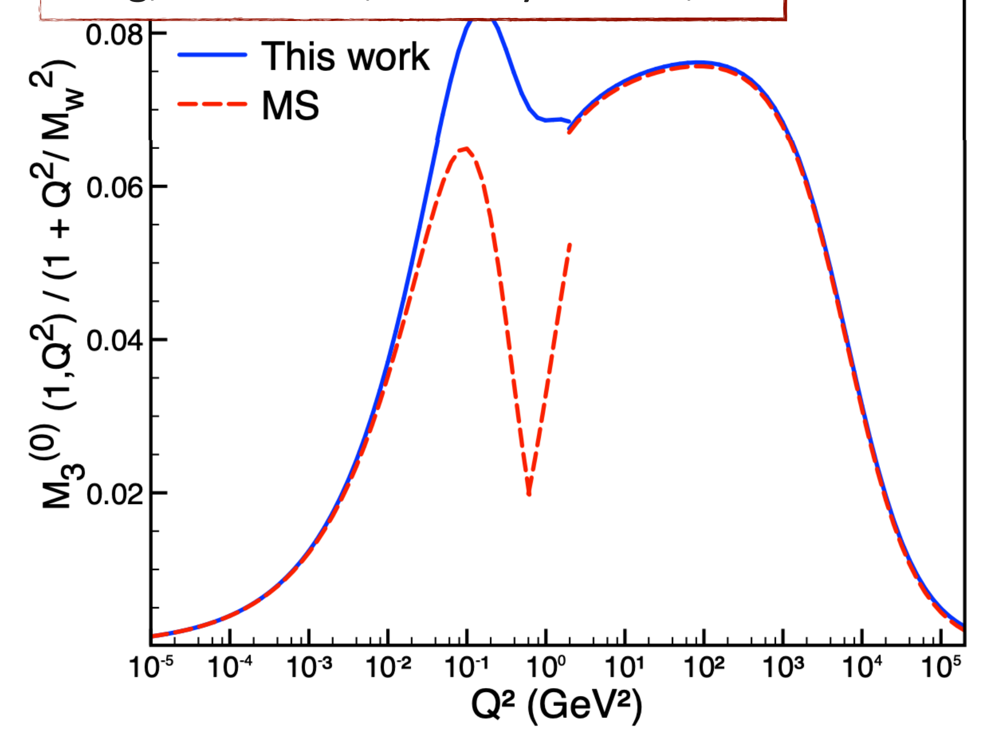
$\overline{\Box}^V_{\rm Had}$ requires knowledge of T_3

- $Q^2 \leq 2$ GeV: elastic & inelastic regions
- $Q^2 \geq 2$ GeV: operator product expansion

Seng, Gorchtein, Ramsey-Musolf, '19



Matching
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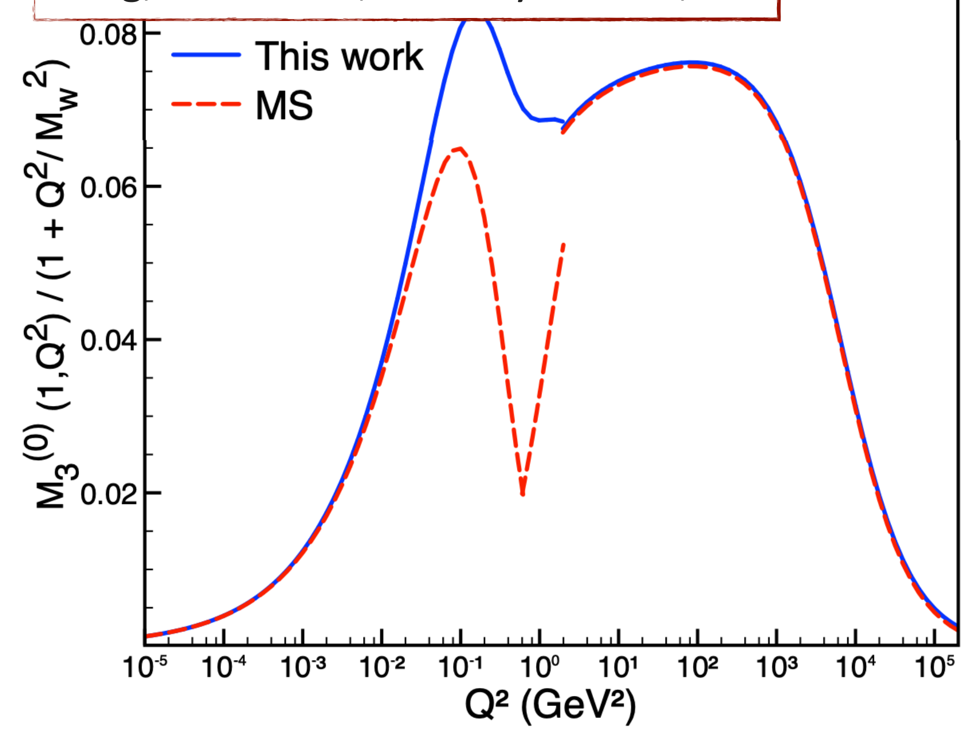
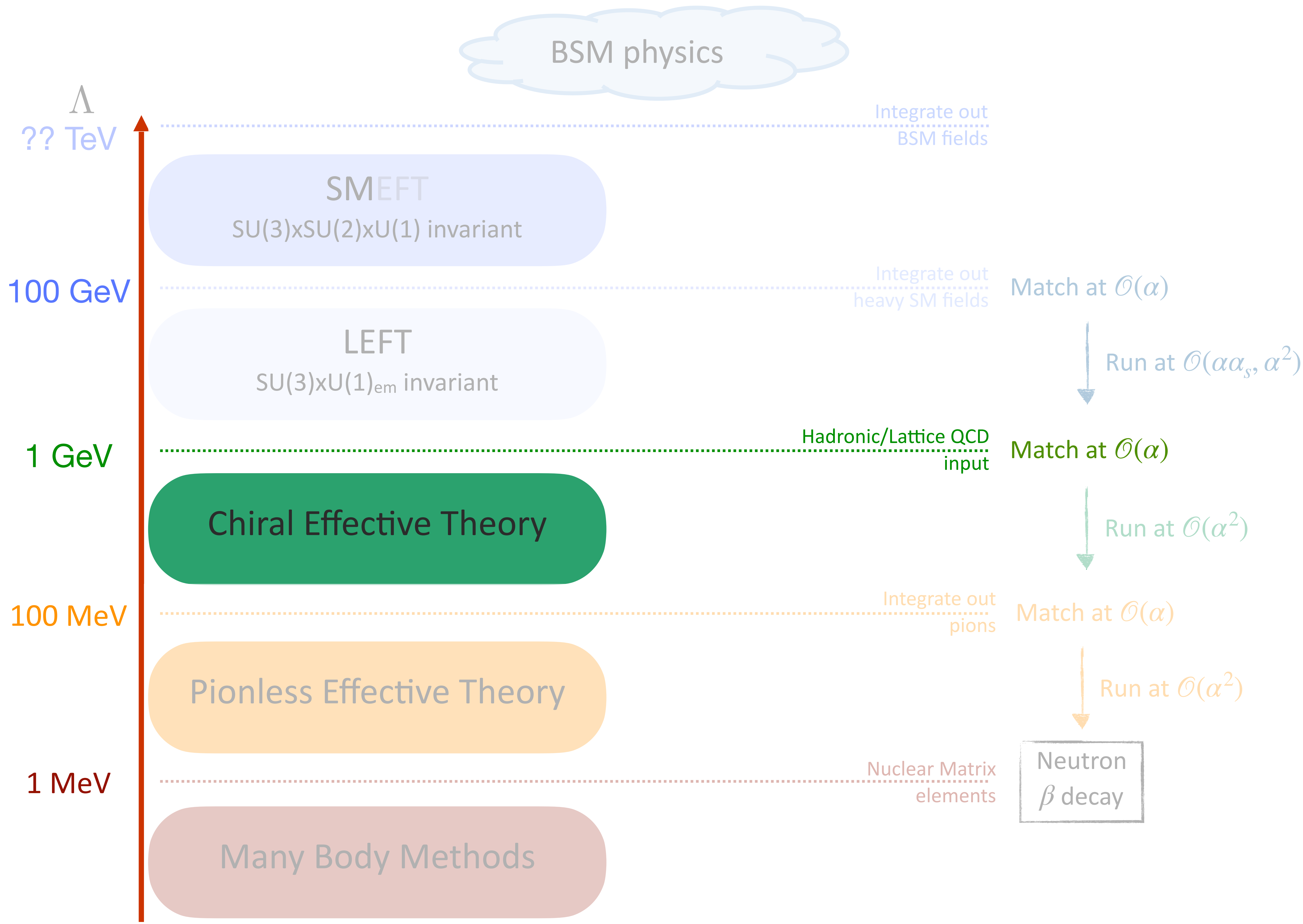
Use determination of Seng et. al. '18

- In agreement with first Lattice determination

Ma et al. '23

Seng, Gorchtein, Ramsey-Musolf, '19




BSM physics
Λ
?? TeV
Integrate out
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SU(3)xSU(2)xU(1) invariant
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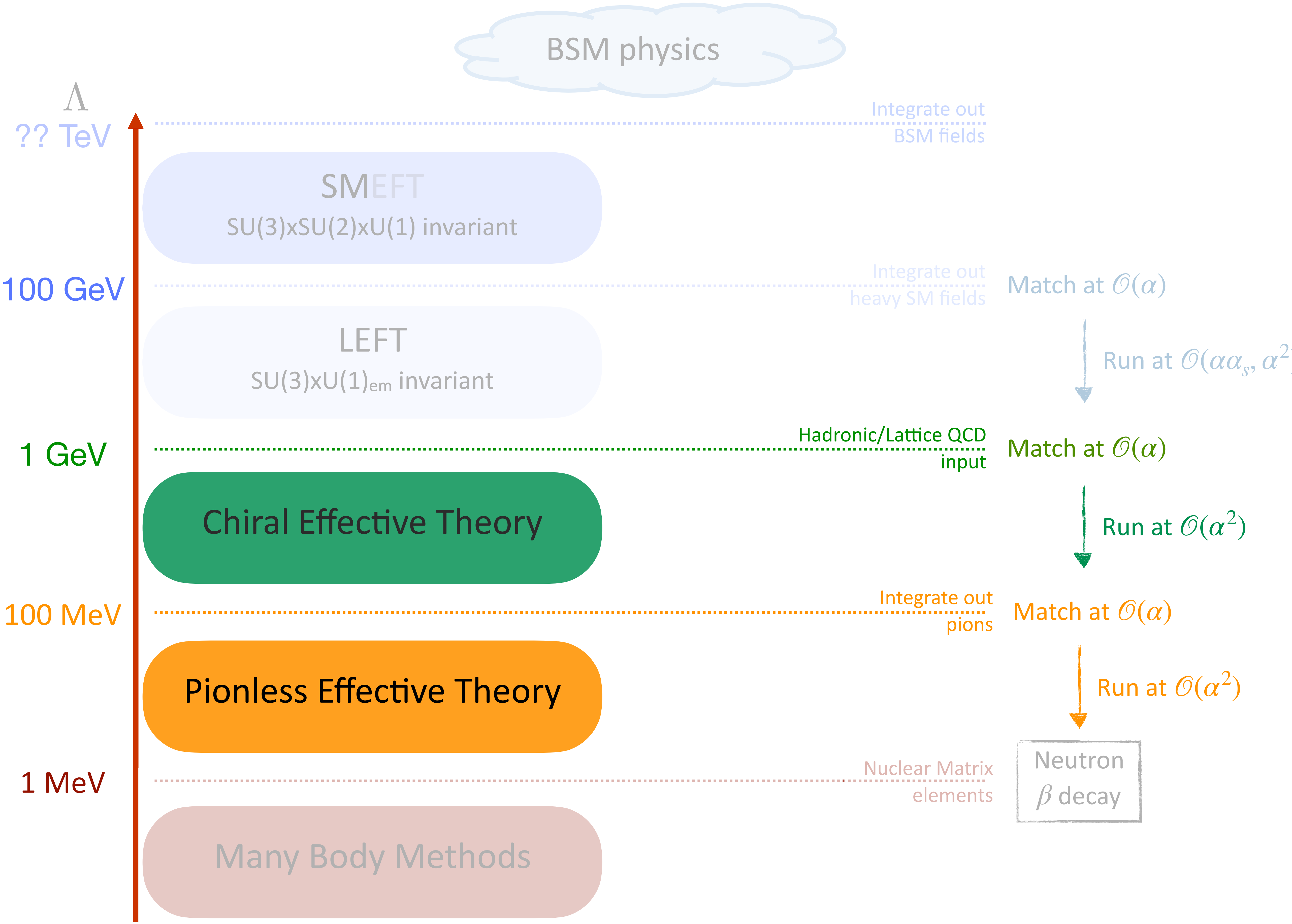

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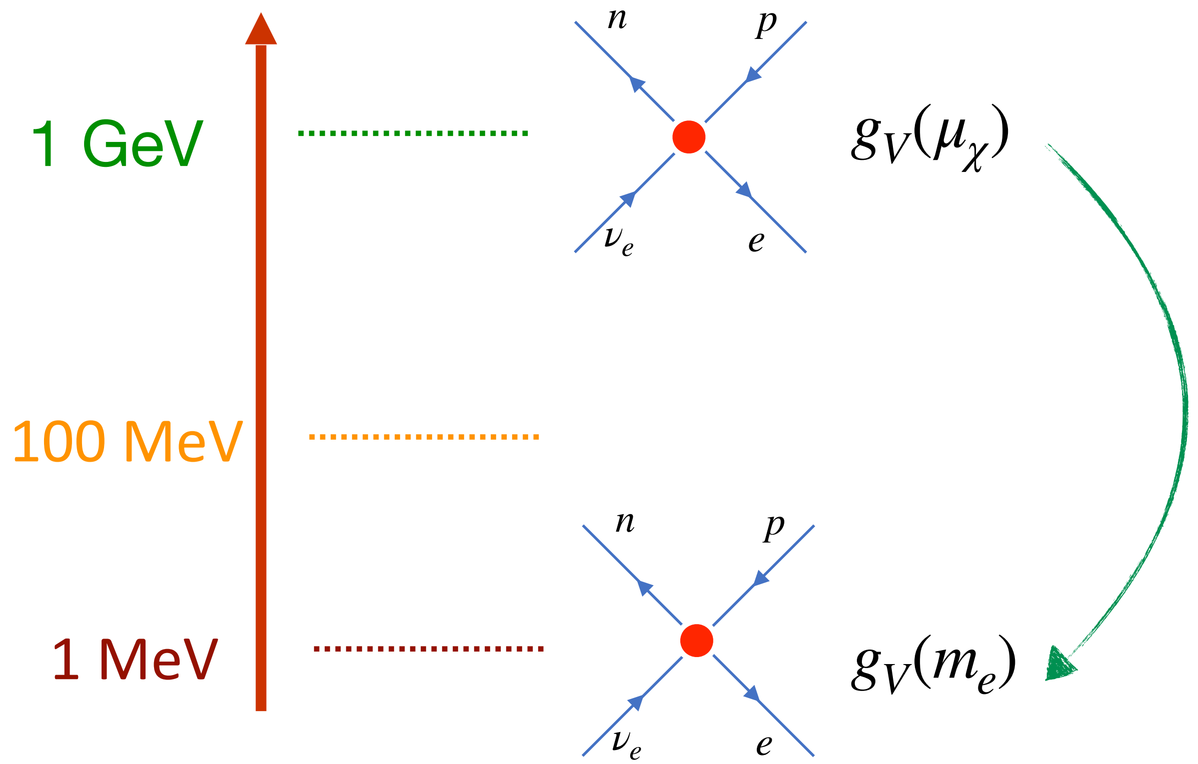
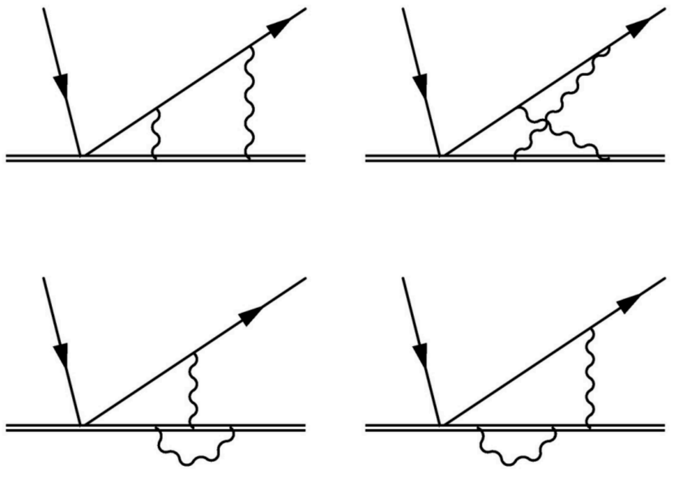
Running

ChPT/pion-less EFT





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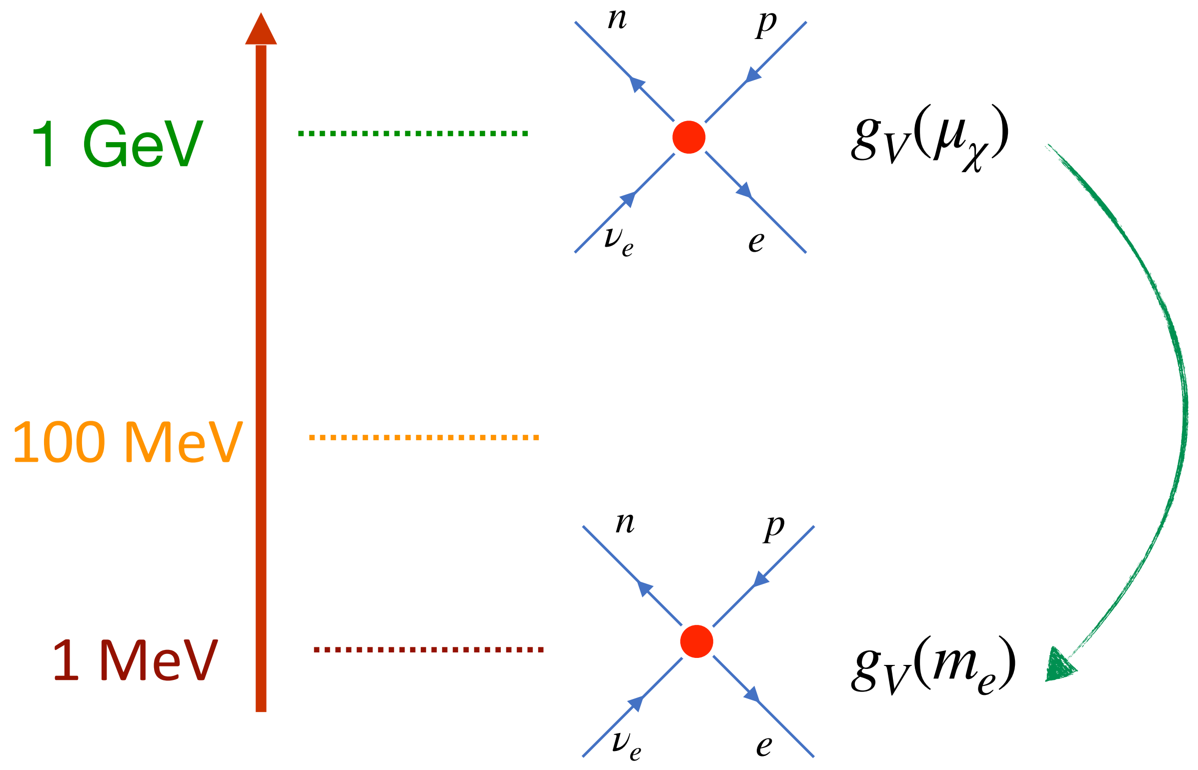

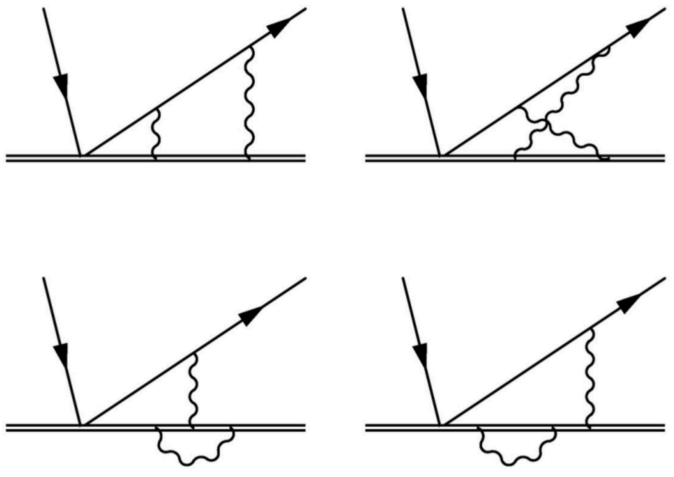

Cirigliano, de Vries, Mereghetti, Walker-Loud '22

- No matching contributions to g_V at $\mu = m_\pi$

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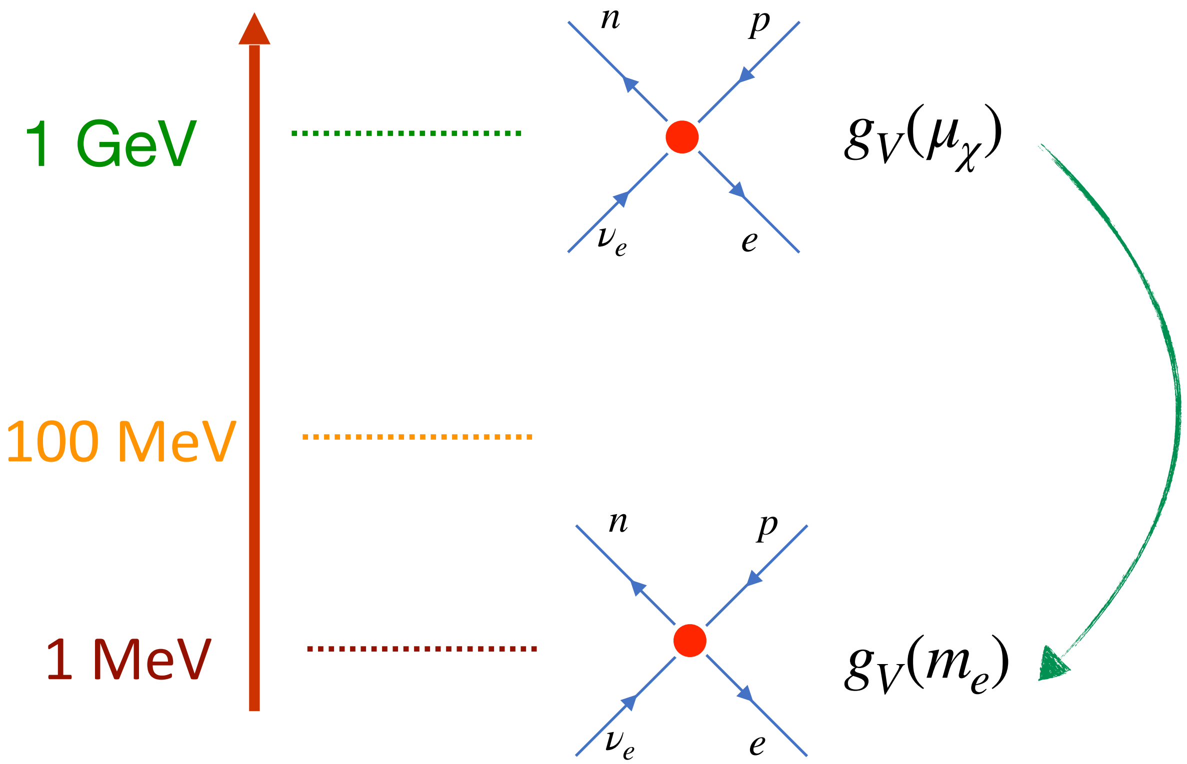

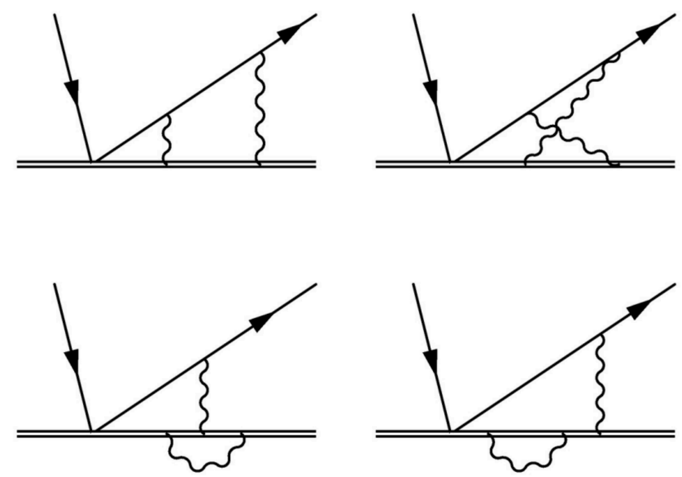

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Giménez '92; Broadhurst et al. '91; Hoang '97;

$$g_V(m_e) = \left[1 + \mathcal{O}\left(\alpha^n \ln^n \frac{m_N}{m_e}\right) + \mathcal{O}\left(\alpha^{n+1} \ln^n \frac{m_N}{m_e}\right)\right] g_V(m_N)$$

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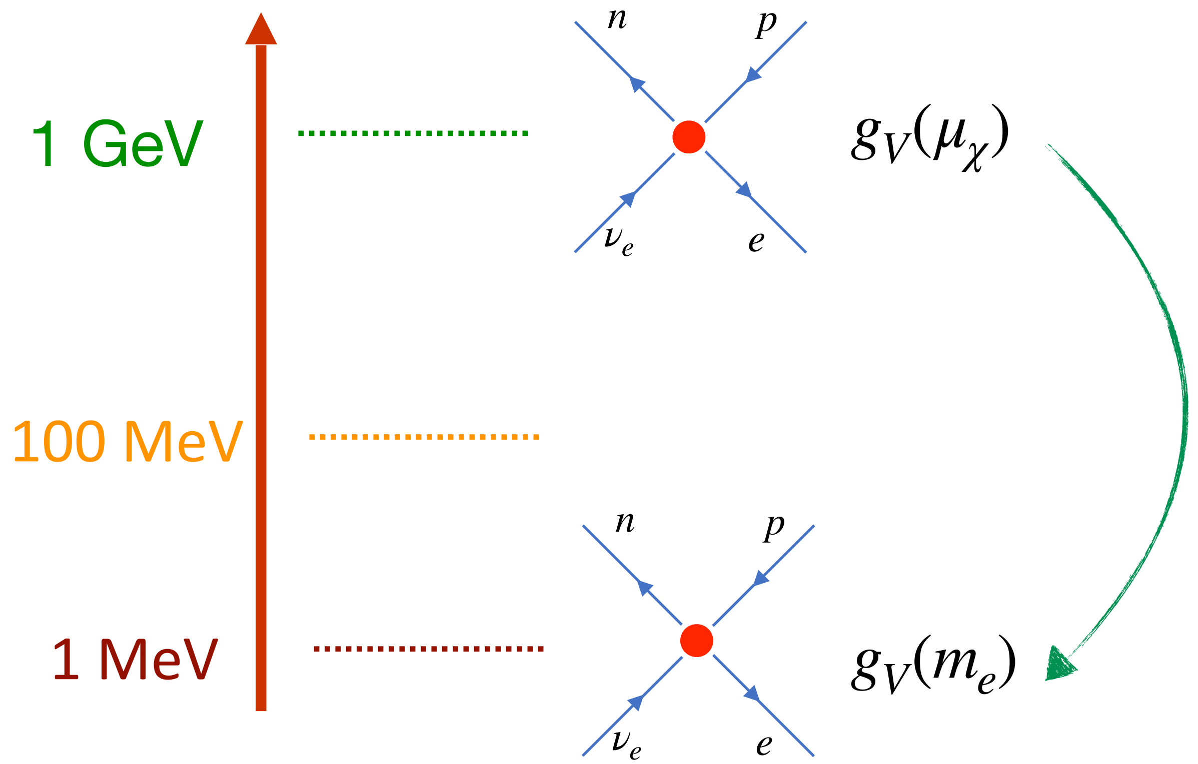

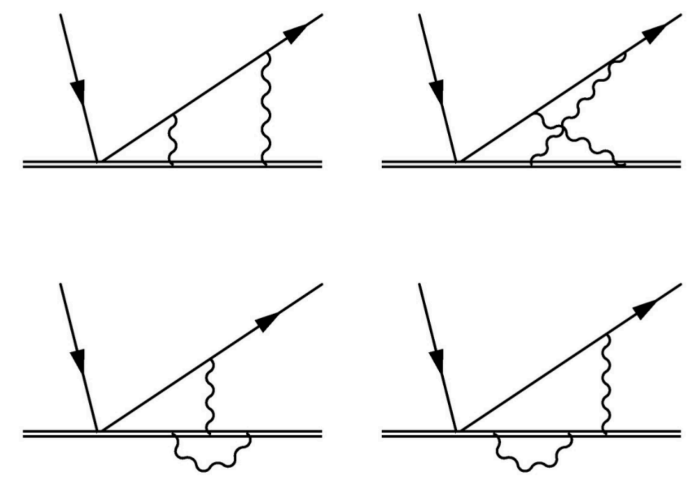

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$\tilde{\gamma}_0$

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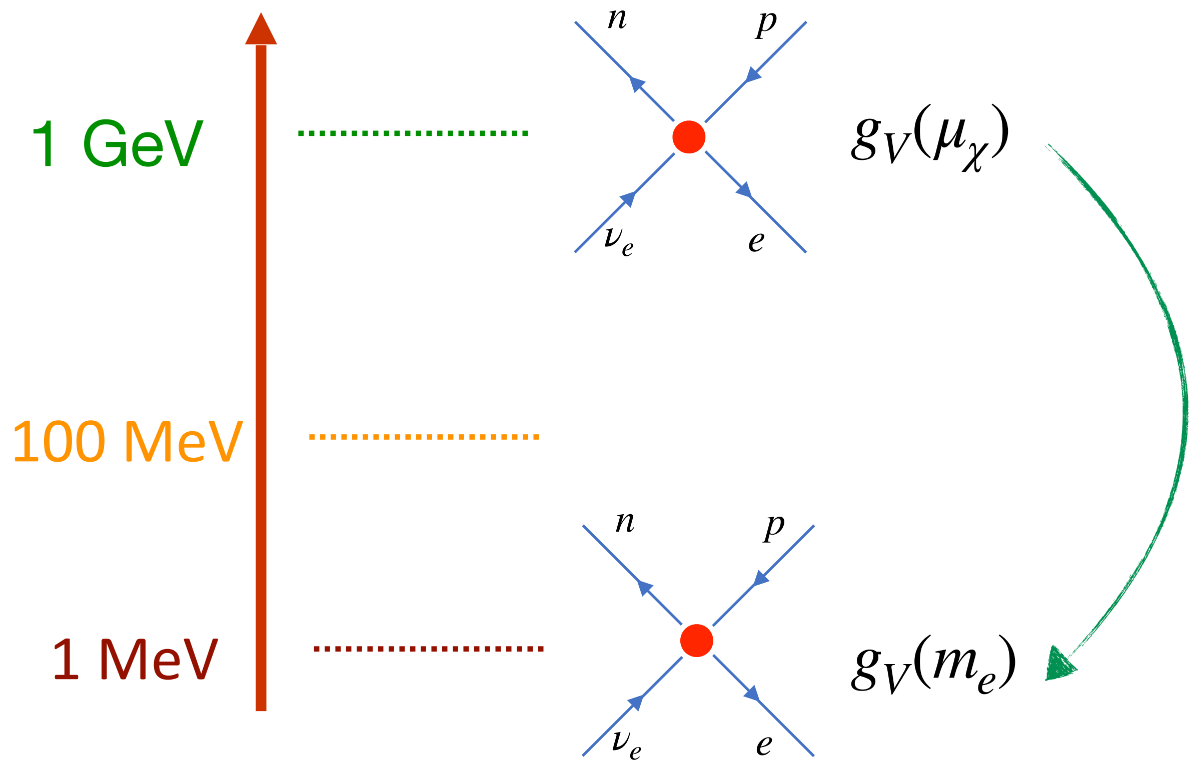

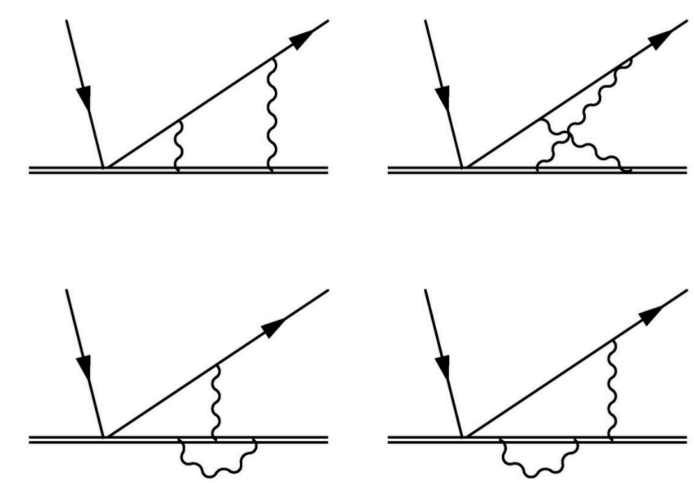

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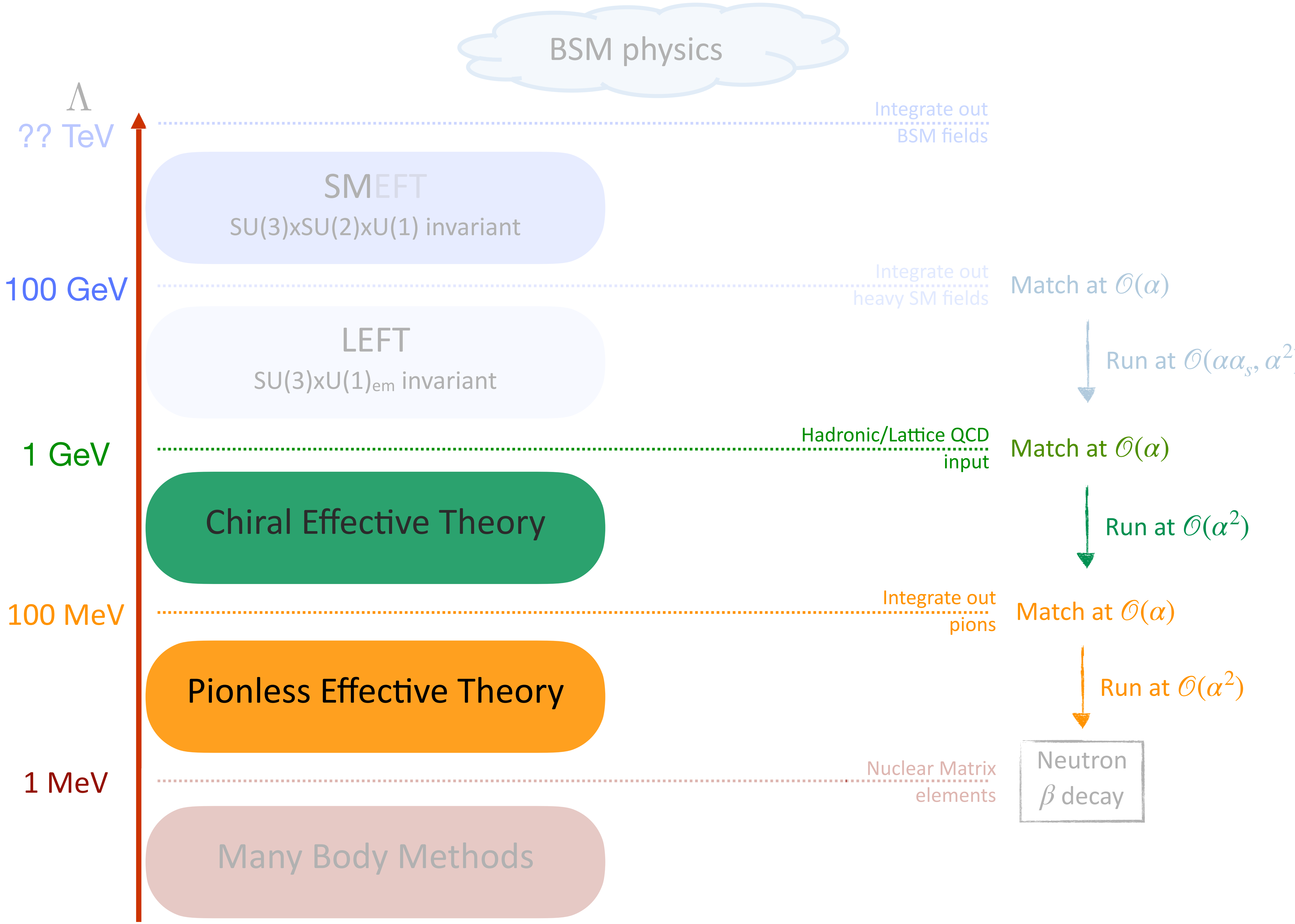
$\tilde{\gamma}_0$ $\tilde{\gamma}_1$

- Discrepancy with $\mathcal{O}(\alpha^2 \ln(m_N/m_e))$ terms in literature

Jaus, Rasche ’97; Sirlin, Zucchini ‘86



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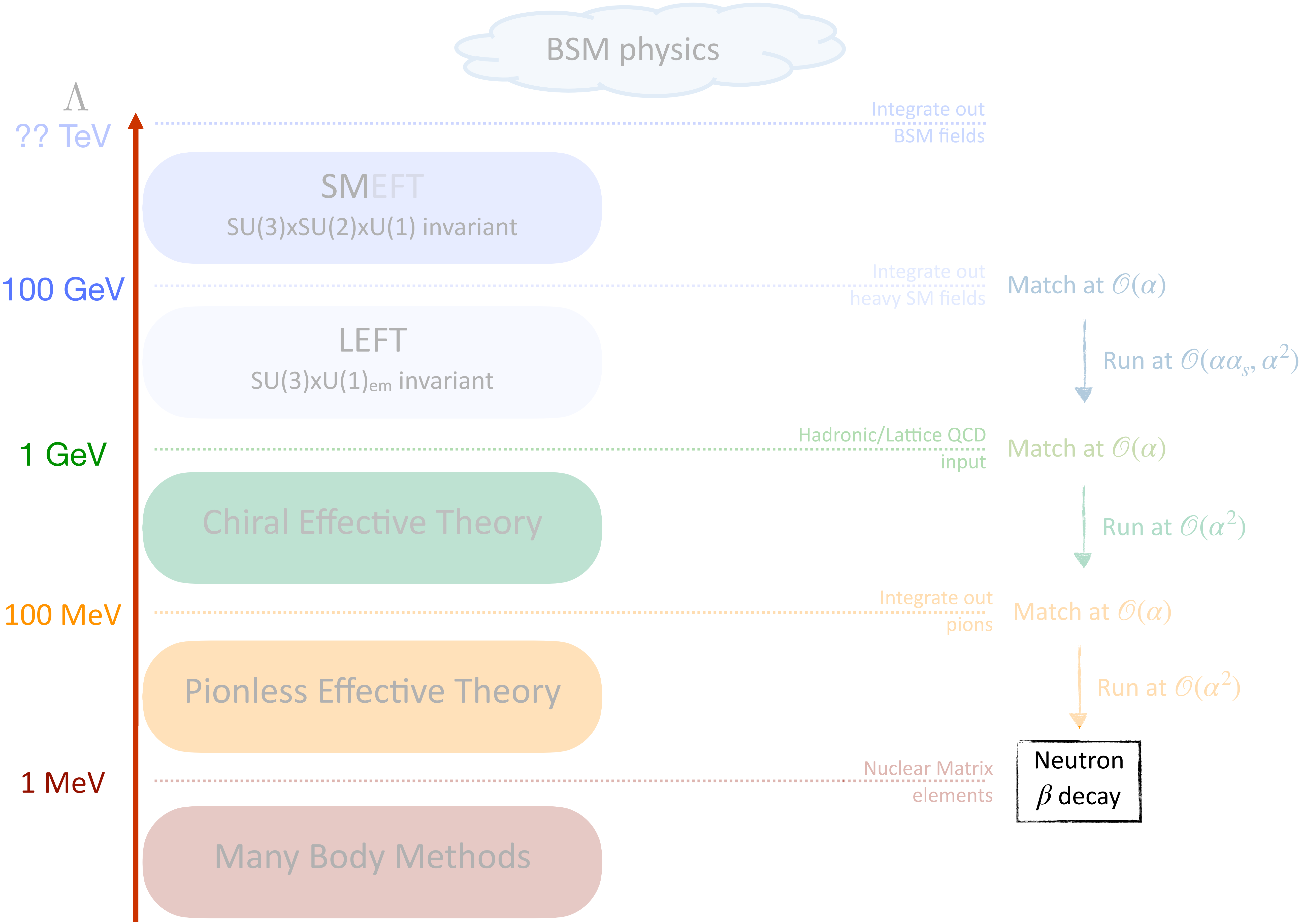

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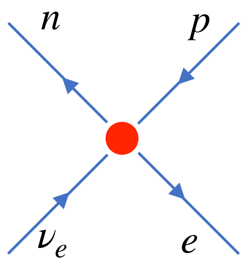
Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2} G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



- Use pion-less interactions at $\mu \simeq m_e$ to compute Γ_n
    - Need amplitude $n \to pe\nu$ at $\mathcal{O}(\alpha)$: loops, real radiation

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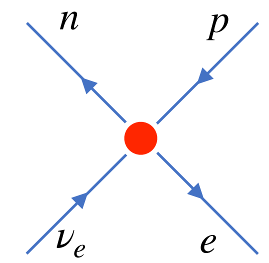

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- Complication, Coulomb terms scale as $\sim \left(\frac{\pi\alpha}{\beta}\right)^n, \quad \beta = |\boldsymbol{p}_e|/E_e$
    - Enhanced compared to naive expectation $\left(\frac{\alpha}{\pi}\right)^n$
    - Need some higher-order terms

Matrix element

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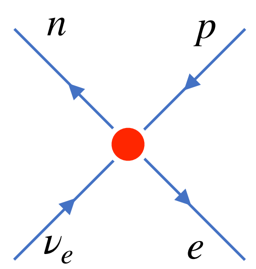
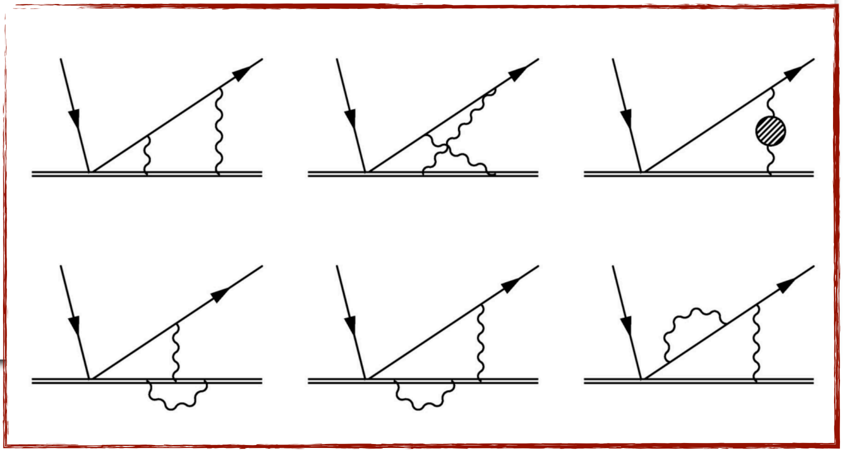


- Use pion-less interactions at $\mu \simeq m_e$ to compute Γ_n
    - Need amplitude $n \to pe\nu$ at $\mathcal{O}(\alpha)$: loops, real radiation

- Complication, Coulomb terms scale as $\sim \left(\frac{\pi\alpha}{\beta}\right)^n, \qquad \beta = |\boldsymbol{p}_e|/E_e$
    - Enhanced compared to naive expectation $\left(\frac{\alpha}{\pi}\right)^n$
    - Need some higher-order terms



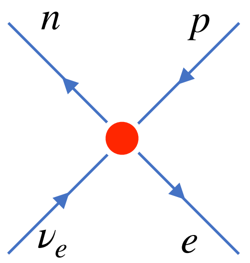
- Known from NRQCD literature $\Longrightarrow$ non-relativistic Fermi function

Hoang '97; Czarnecki, Melnikov '98; Beneke et al. '99;

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2}G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



$$\frac{\mathrm{d}\Gamma_n}{\mathrm{d}E_e} = \frac{G_F^2\,|V_{ud}|^2}{(2\pi)^5}\,(1+3\lambda^2)\;p_e E_e (E_0 - E_e)^2\;[g_V(\mu_\chi)]^2\;F_{NR}(\beta)\Big(1+\delta_{\mathrm{RC}}(E_e,\mu_\chi)\Big)\Big(1+\delta_{\mathrm{recoil}}(E_e)\Big).$$

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1 MeV

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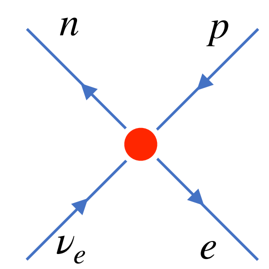

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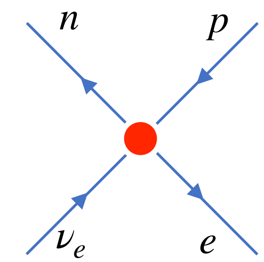
$\lambda = g_A / g_V$
Extract from Experiment

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2} G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



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$\lambda = g_A/g_V$
Extract from Experiment

vector coupling

Matrix element

1 MeV

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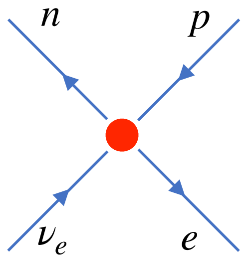

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$\lambda = g_A/g_V$
Extract from Experiment

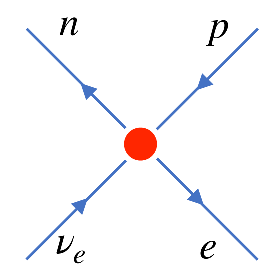
vector coupling

$\pi^2, 1/\beta$
Enhanced

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2}G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



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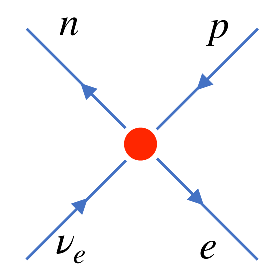
$\pi^2, 1/\beta$
Enhanced

$\mathcal{O}(\alpha)$
[no logs]

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2}\,G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



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$\lambda = g_A/g_V$
Extract from Experiment

vector coupling

$\pi^2, 1/\beta$
Enhanced

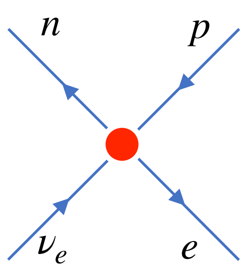
$\mathcal{O}(\alpha)$
[no logs]

$\mathcal{O}(m_e/m_N)$

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2} G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L \, \bar{N} v_\mu \tau^+ N$$



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Error budget g_V:

$$g_V(m_e) = 1 + \left(2.430 \pm 0.003_\chi \pm 0.012_{\mathrm{non-pert.}} \pm 0.004_{\mathrm{LEFT}}\right) \cdot 10^{-2}$$

Matrix element

1 MeV

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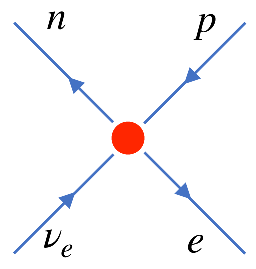

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Error budget g_V:

ChPT running

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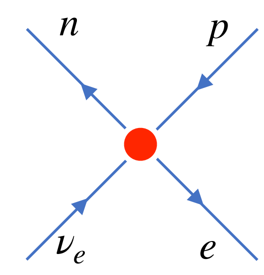

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Error budget g_V:

ChPT running

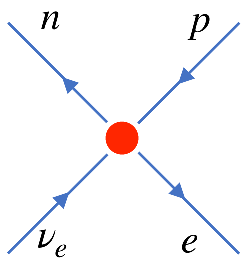
LEFT-ChPT Matching

$$g_V(m_e) = 1 + \Big(2.430 \pm 0.003_\chi \pm 0.012_{\mathrm{non-pert.}} \pm 0.004_{\mathrm{LEFT}}\Big) \cdot 10^{-2}$$

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2} G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



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Error budget g_V:

ChPT running

LEFT-ChPT Matching

LEFT running

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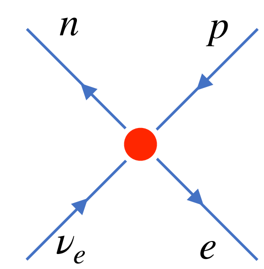
Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2} G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



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$$g_V(m_e) = 1 + \Big(2.430 \pm 0.003_\chi \pm 0.012_{\mathrm{non-pert.}} \pm 0.004_{\mathrm{LEFT}}\Big) \cdot 10^{-2}$$

Difference with literature:

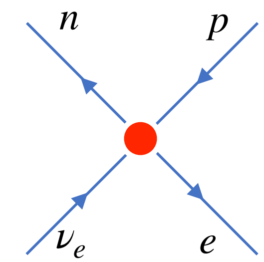
$\alpha^2 \ln(m_N/m_e)$ & $\alpha^2 \ln(m_W/m_N)$:

+0.061%

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2} G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



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$$g_V(m_e) = 1 + \left(2.430 \pm 0.003_\chi \pm 0.012_{\mathrm{non-pert.}} \pm 0.004_{\mathrm{LEFT}}\right)\cdot 10^{-2}$$

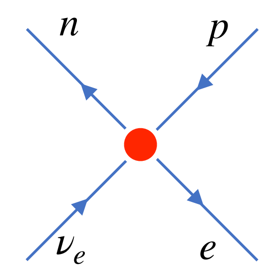
Difference with literature:

$\alpha^2 \ln(m_N/m_e)$ & $\alpha^2 \ln(m_W/m_N)$: +0.061%	NR vs relativistic Fermi function -0.035%

Matrix element

1 MeV

$$\mathscr{L} = -\sqrt{2} G_F V_{ud}\, g_V(m_e)\, \bar{e}_L \gamma^\mu \nu_L\, \bar{N} v_\mu \tau^+ N$$



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Error budget g_V:

$$g_V(m_e) = 1 + \left(2.430 \pm 0.003_\chi \pm 0.012_{\mathrm{non-pert.}} \pm 0.004_{\mathrm{LEFT}}\right) \cdot 10^{-2}$$

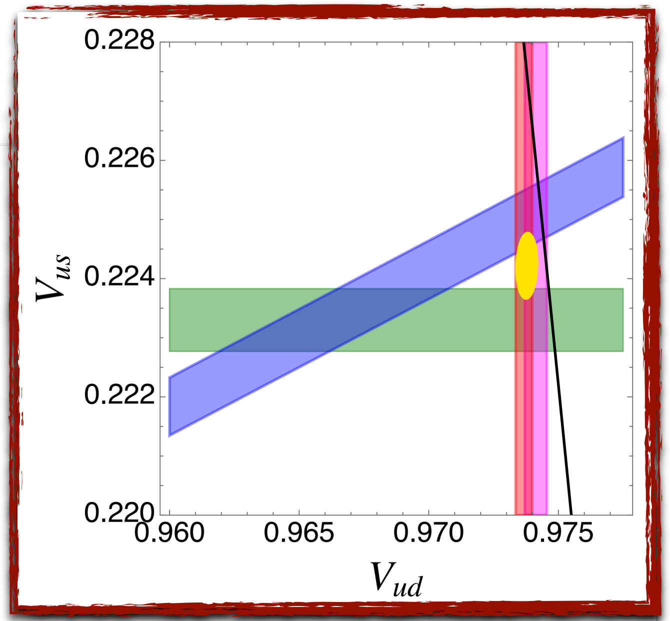
In total:

$$V_{ud}^{\mathrm{n,\ best}} = 0.97402(2)_{\Delta_f}(13)_{\Delta_R}(35)_\lambda(20)_{\tau_n}[42]_{\mathrm{total}},$$

- About 1σ below recent literature determination

Cirigliano, Crivellin, Moulson, Hoferichter ‘23

Summary



- Neutron β decay, $0^+ \to 0^+$ sensitive probes of (B)SM physics
  - V_{ud} determination
  - CKM unitarity test
  - Reach of scales $\Lambda \sim \mathcal{O}(10\,\mathrm{TeV})$
- Requires control of SM uncertainties

Summary

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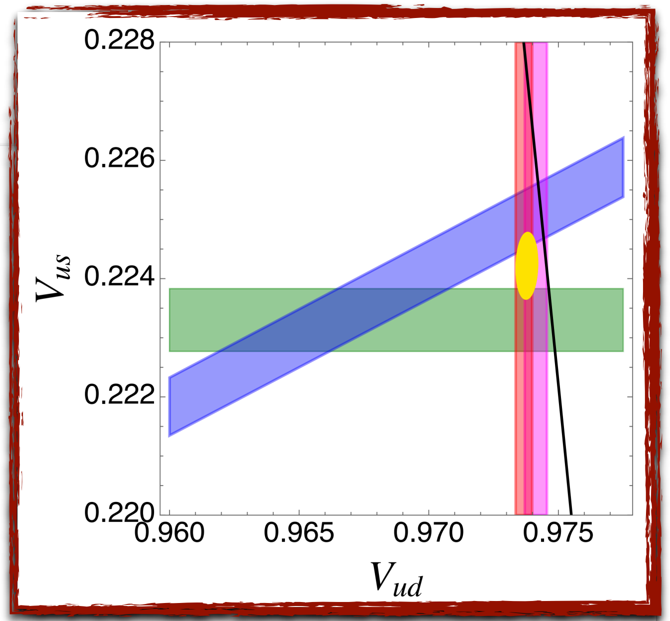
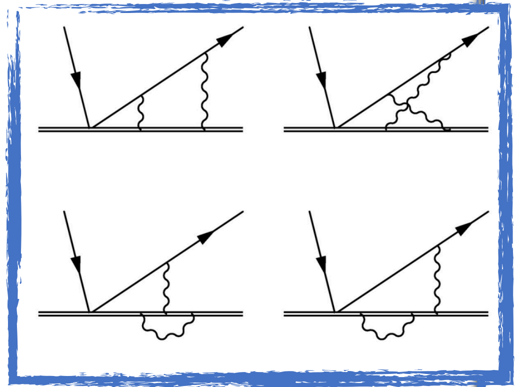
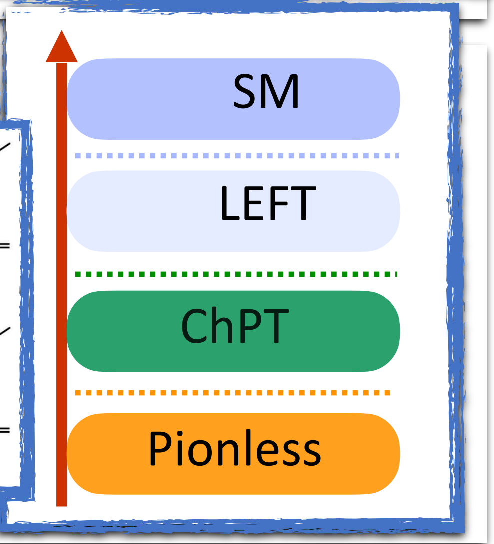
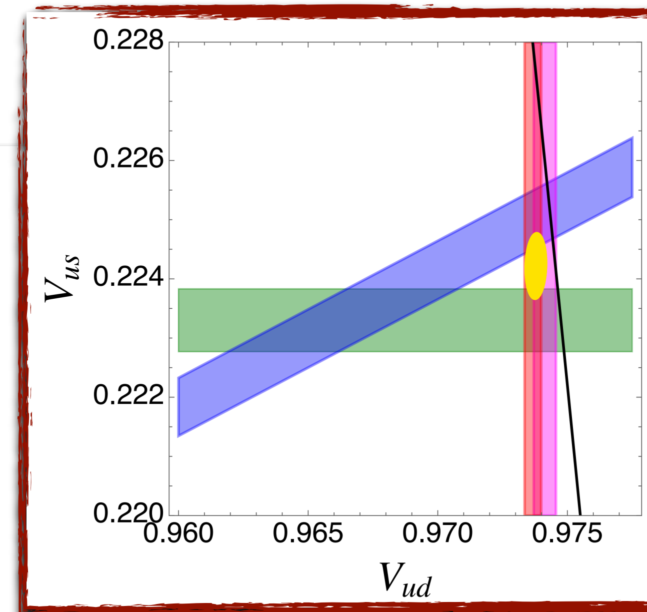


- Effective Field Theory framework
  - Explicit separation of scales
  - Resum logs
  - Systematically improvable
  - Several differences with the literature





Summary



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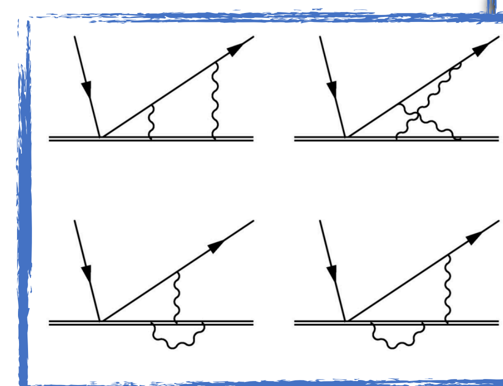
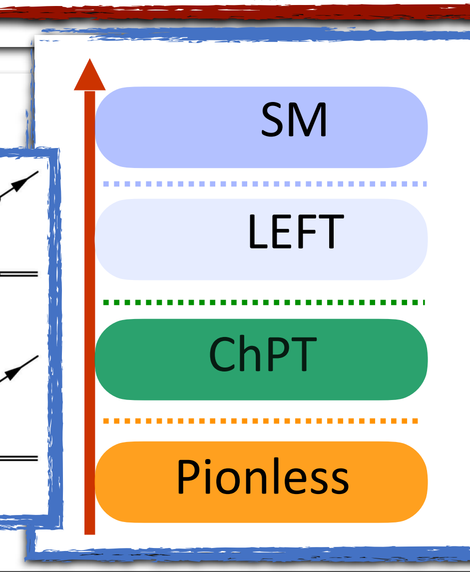


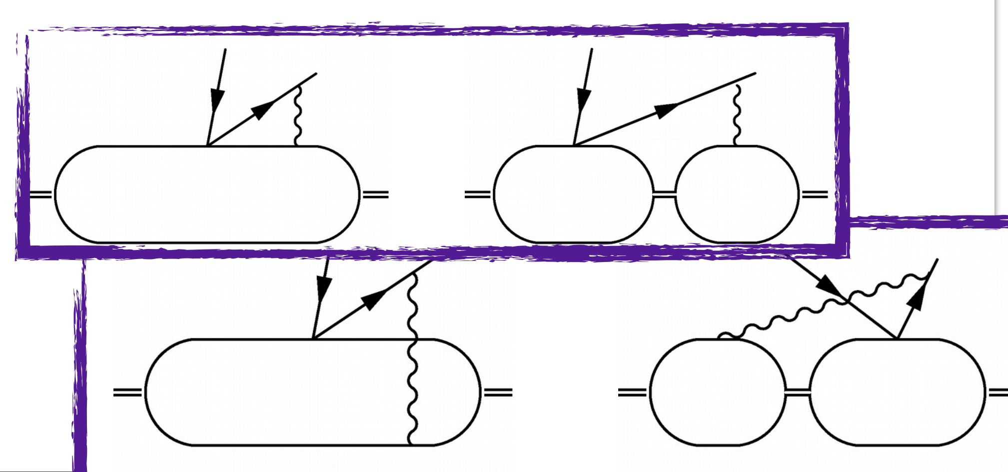


Outlook

- Axial current, g_A
- Superallowed nuclear decays $(A > 1)$
  - Nuclear-structure dependence
  - Z dependence/Fermi function

Hill, Plestid, '23

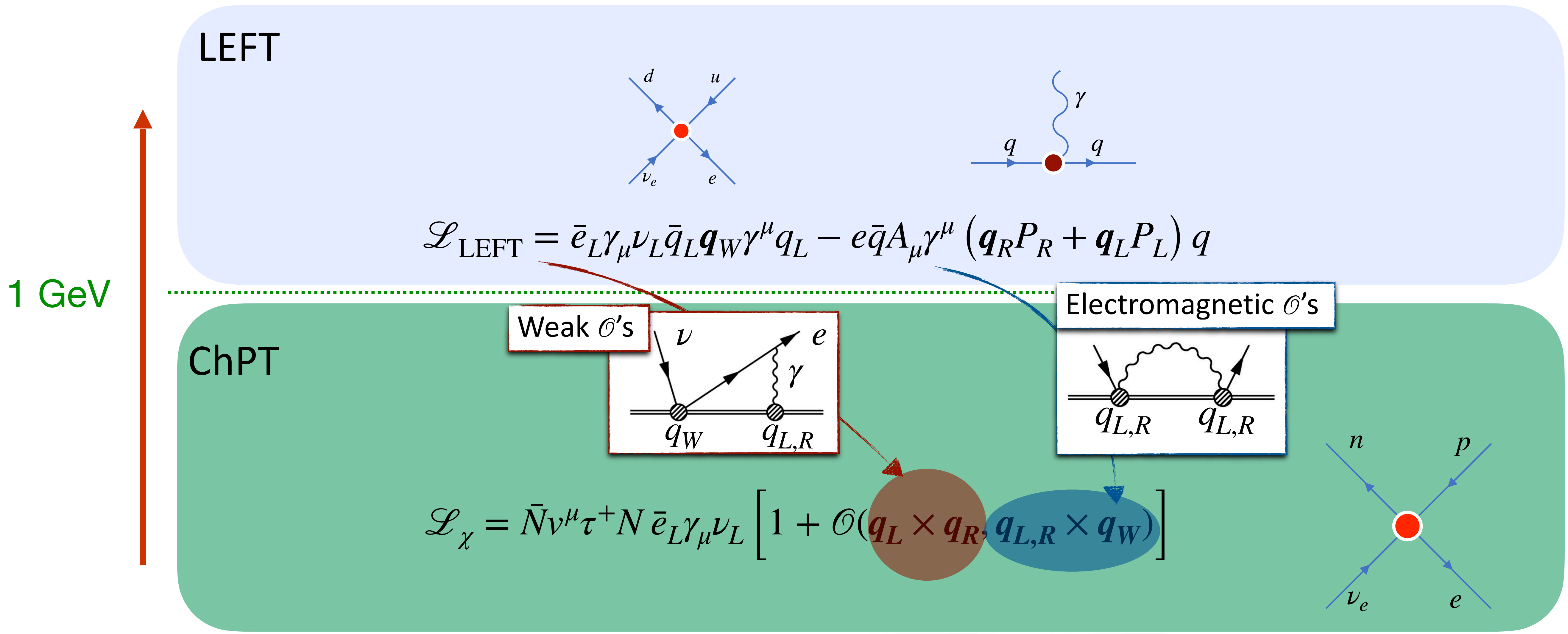
See M. Gorchtein's talk tomorrow



Back up slides

Matching

$\mathscr{O}(e^2)$ Operators



Weak operators:

$$\mathcal{L}^{e^2p}_{\pi N\ell} = e^2 \sum_{i=1} \bar{e}_L\gamma_\rho\nu_{eL}\bar{N}_v \left(V_i v^\rho\right) \mathrm{O}_i N_v + \text{h.c.},$$

$$\mathrm{O}_1 = [\mathcal{Q}_L, \mathcal{Q}_L^W], \qquad \mathrm{O}_2 = [\mathcal{Q}_R, \mathcal{Q}_L^W],$$

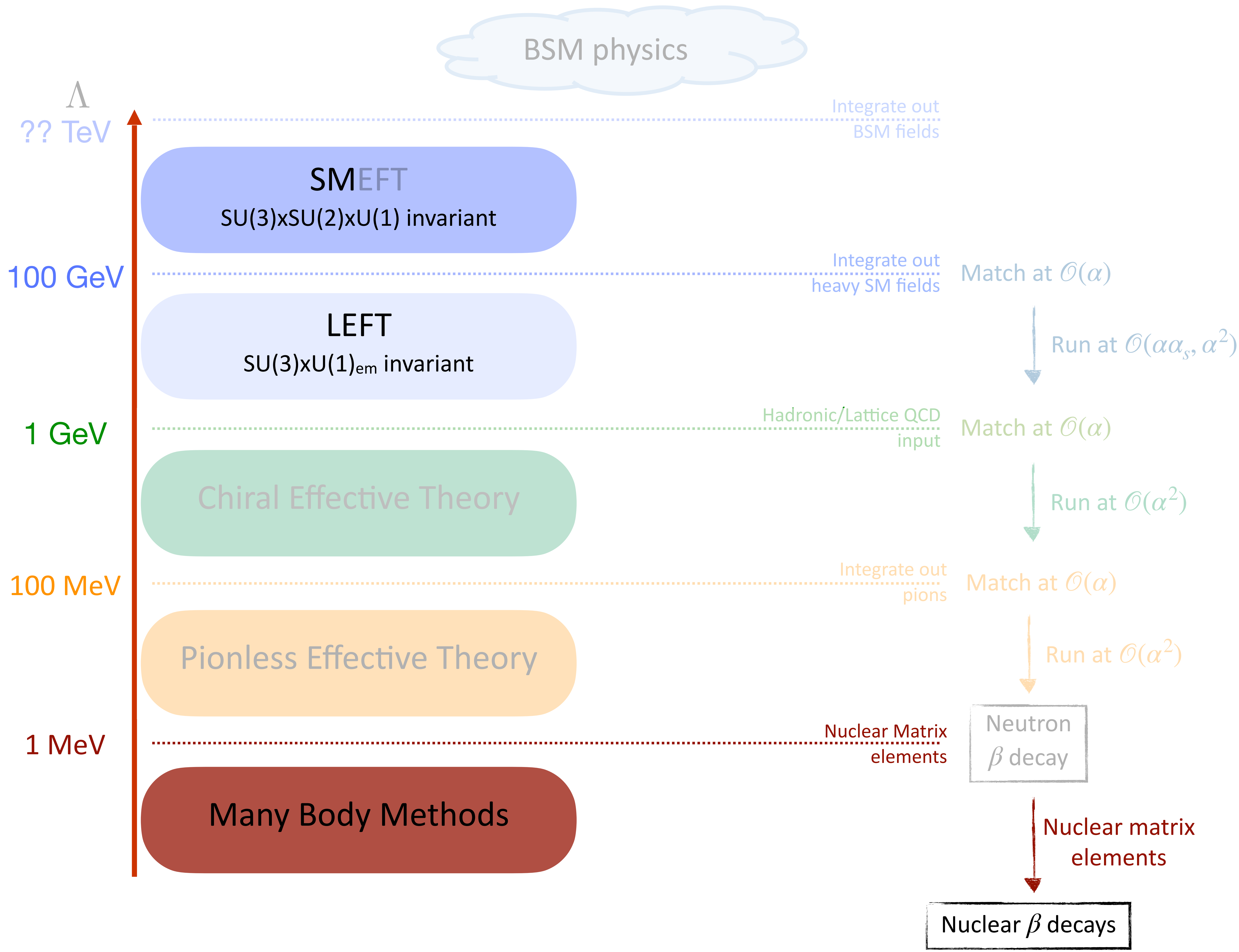
$$\mathrm{O}_3 = \{\mathcal{Q}_L, \mathcal{Q}_L^W\}, \qquad \mathrm{O}_4 = \{\mathcal{Q}_R, \mathcal{Q}_L^W\},$$

EM operators:

$$\mathcal{L}^{e^2p}_{\pi N} = e^2 g_9 \bar{N}_v \left(\frac{i}{2}\left[\mathcal{Q}_+, v\cdot c^+\right] + \text{h.c.}\right) N_v.$$

$$c^\pm_\rho = \frac{1}{2}\left(u\left(D_\rho\mathbf{q}_L\right)u^\dagger \pm u^\dagger\left(D_\rho\mathbf{q}_R\right)u\right),$$

$$D^\rho\mathbf{q}_L \equiv \partial^\rho\mathbf{q}_L - i\left[l^\rho, \mathbf{q}_L\right], \quad D^\rho\mathbf{q}_R \equiv \partial^\rho\mathbf{q}_R - i\left[r^\rho, \mathbf{q}_R\right],$$


BSM physics
Λ
?? TeV
Integrate out
BSM fields
SMEFT
SU(3)xSU(2)xU(1) invariant
100 GeV
Integrate out
heavy SM fields
Match at $\mathcal{O}(\alpha)$
LEFT
$SU(3)xU(1)_{em}$ invariant
Run at $\mathcal{O}(\alpha\alpha_s, \alpha^2)$
1 GeV
Hadronic/Lattice QCD
input
Match at $\mathcal{O}(\alpha)$
Chiral Effective Theory
Run at $\mathcal{O}(\alpha^2)$
100 MeV
Integrate out
pions
Match at $\mathcal{O}(\alpha)$
Pionless Effective Theory
Run at $\mathcal{O}(\alpha^2)$
1 MeV
Nuclear Matrix
elements
Neutron
β decay
Many Body Methods
Nuclear matrix
elements
Nuclear β decays

Application to $0^+ \to 0^+$

- Nuclear decay rate:

$$\Gamma \sim G_F^2 |V_{ud}|^2 (1 + \Delta_R^V) F (1 + \delta_R' + \delta_{NS} - \delta_C)$$

Fermi constant From μ decay

CKM element

Short-distance corrections $\mathcal{O}(\alpha)$

Phase space factor

Transition dependent Corrections

- Usual approach:

$$\Delta_R^V\big|_{\text{Traditional}} = [g_V(m_N)]^2 \left(1 + \frac{5\alpha(m_N)}{8\pi}\right) - 1 = 2.471(25)\%,$$

- In the EFT

$$\Delta_R^V\big|_{\text{EFT}} = [g_V(m_e)]^2 \left(1 + \frac{5\alpha(m_e)}{8\pi}\right) - 1,$$

- Needs to be combined with matrix element computed in the same scheme
- Requires EFT versions of
  - Fermi function
  - δ_R'
  - Nuclear-structure dependence δ_{NS}