Cabibbo unitarity status: superallowed nuclear and Kl3 decays

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Neutron beta decay review: MG, Seng, Universe **2023**, 9(9), 422, arXiv:**2307.01145**
Nuclear beta decay review: MG, Seng (for Annual Reviews Part. Nucl. Sci. - deadline Nov 2)

XII CKM Unitarity Triangle Workshop, Santiago de Compostela, September 18-22, 2023

Outline

Status of Cabibbo unitarity

Superallowed nuclear decays

RC to β-decays: overall setup, scale separation, δ_{NS}

Dispersion theory of nuclear-structure RC δ_{NS}

Nuclear inputs in ft-values

Isospin-symmetry breaking correction δ_C

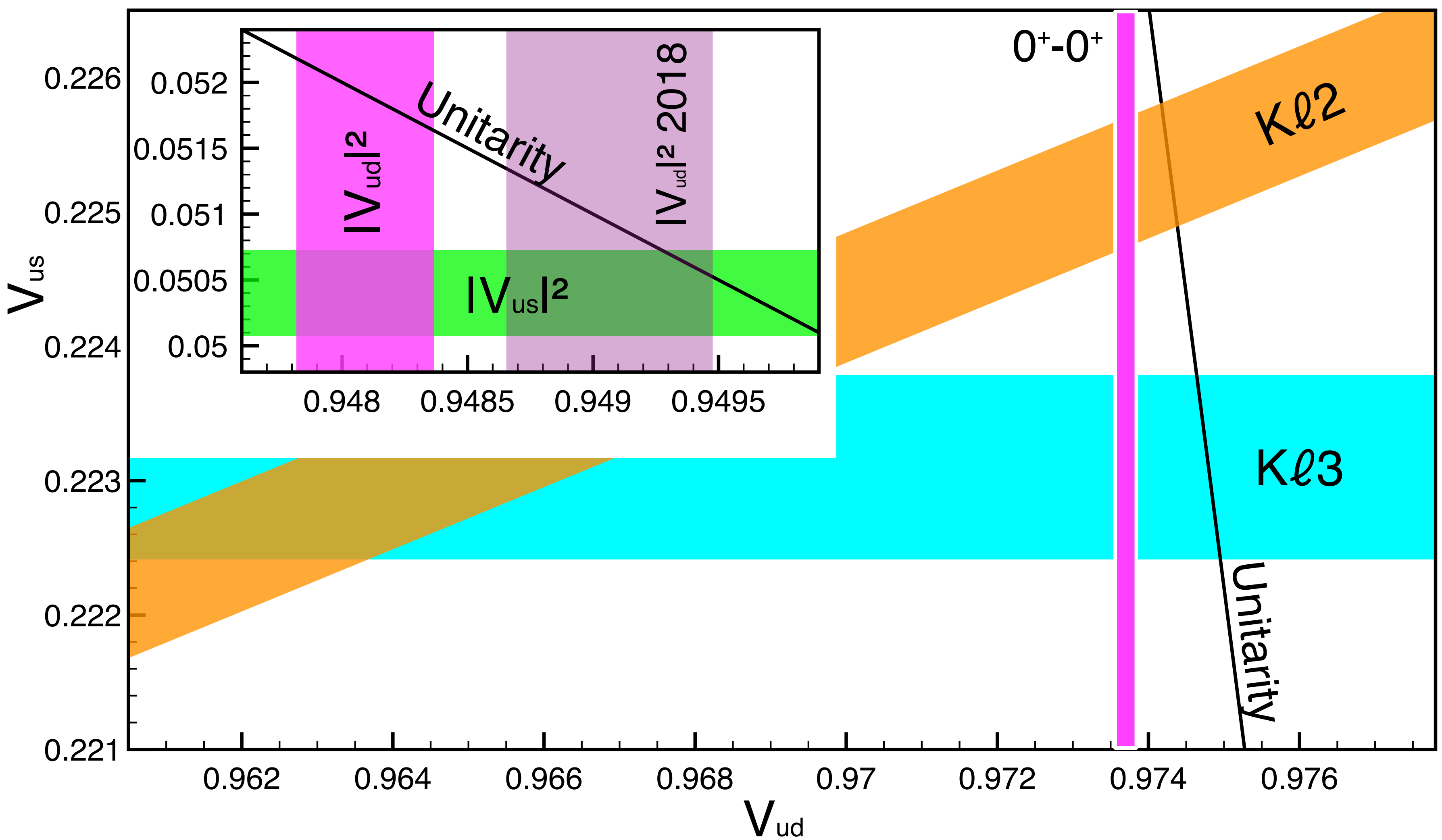
RC for $K\ell 3$ and V_{us}

Summary & Outlook

Status of Cabibbo unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

~ 0.95 ~ 0.05 $\sim 10^{-5}$



Talks at this workshop:
Wouter Deckens
Martin Hoferichter
Bastian Märkisch
Matthew Moulson
Ulrich Schmidt
Luiz Vale Silva

Inconsistencies between measurements of V_{ud} and V_{us} and SM predictions

Main reason for Cabibbo-angle anomaly: shift in V_{ud} (and small uncertainties?)

Status of V_{ud}

Theory: Major reduction of uncertainties in the past few years

Universal correction Δ_R^V to free and bound neutron decay

Identified 40 years ago as the bottleneck for precision improvement

Novel approach dispersion relations + experimental data + EFT + lattice QCD

Δ_R^V uncertainty: factor 2 reduction

δ_{NS} uncertainty: factor 3 increase!!!

C-Y Seng et al., Phys.Rev.Lett. 121 (2018) 24, 241804;
C-Y Seng, MG, M.J. Ramsey-Musolf, Phys.Rev. D 100 (2019) 1, 013001;
MG, Phys.Rev.Lett. 123 (2019) 4, 042503;
C-Y Seng, X. Feng, MG, L-C Jin, Phys.Rev. D 101 (2020) 11, 111301;
A. Czarnecki, B. Marciano, A. Sirlin, Phys.Rev. D 100 (2019) 7, 073008
C-Y Seng, X. Feng, MG, L-C Jin, 2308.16755;

RC to semileptonic pion decay

δ Factor 3 reduction

X. Feng, MG, L-C Jin, P-X Ma, C-Y Seng, Phys.Rev.Lett. 124 (2020) 19, 192002
Yoo,J.S.;Bhattacharya,T.;Gupta,R.;Mondal,S.;Yoon,B.. 2305.03198

Experiment

$g_A = -1.27641(56)$

3.4σ — Factor 4 reduction

$g_A = -1.2677(28)$

PERKEO-III *B. Märkisch et al, Phys.Rev.Lett. 122 (2019) 24, 242501*

aSPECT *M. Beck et al, Phys. Rev. C101 (2020) 5, 055506; 2308.16170*

$\tau_n = 877.75(28)^{+16}_{-12}$

4σ — Factor 2-3 reduction

$\tau_n = 887.7(2.3)$

UCNτ *F. M. Gonzalez et al. Phys. Rev. Lett. 127 (2021) 162501*

BL1 (NIST) *Yue et al, PRL 111 (2013) 222501*

Status of V_{ud}

0^+-0^+ nuclear decays: long-standing champion

$$|V_{ud}|^2 = \frac{2984.43s}{\mathcal{F}t(1+\Delta_R^V)}$$

$$|V_{ud}^{0^+-0^+}| = 0.97370\,(1)_{exp,\,nucl}\,(3)_{NS}\,(1)_{RC}[3]_{total}$$

Nuclear uncertainty x 3

Neutron decay: discrepancies in lifetime τ_n and axial charge g_A; competitive!

$$|V_{ud}|^2 = \frac{5024.7\text{ s}}{\tau_n(1+3g_A{}^2)(1+\Delta_R)}$$

Single best measurements only

$$|V_{ud}^{\text{free n}}| = 0.9733\,(2)_{\tau_n}\,(3)_{g_A}\,(1)_{RC}[4]_{total}$$

PDG average

$$|V_{ud}^{\text{free n}}| = 0.9733\,(3)_{\tau_n}\,(8)_{g_A}\,(1)_{RC}[9]_{total}$$

RC not a limiting factor: more precise experiments a-coming

Bastian and Ulrich’s talks

Pion decay $\pi^+ \to \pi^0 e^+ \nu_e$: theoretically cleanest, experimentally tough

$$|V_{ud}|^2 = \frac{0.9799}{(1+\delta)}\frac{\Gamma_{\pi\ell 3}}{0.3988(23)\text{ s}^{-1}}$$

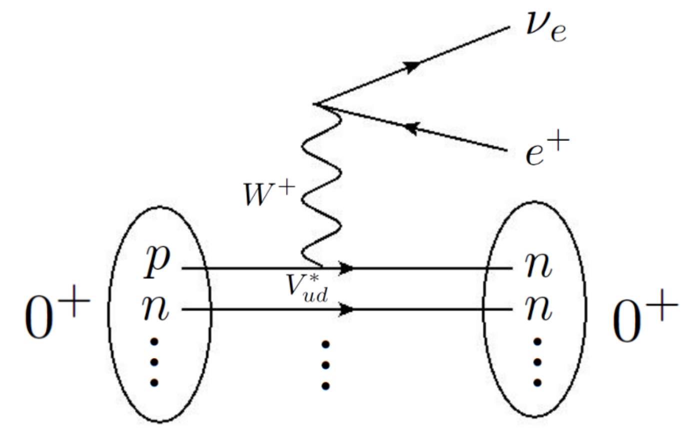
$$|V_{ud}^{\pi\ell 3}| = 0.9739\,(27)_{exp}\,(1)_{RC}$$

Future exp: 1 o.o.m. (PIONEER)

Martin’s talk

Superallowed nuclear decays

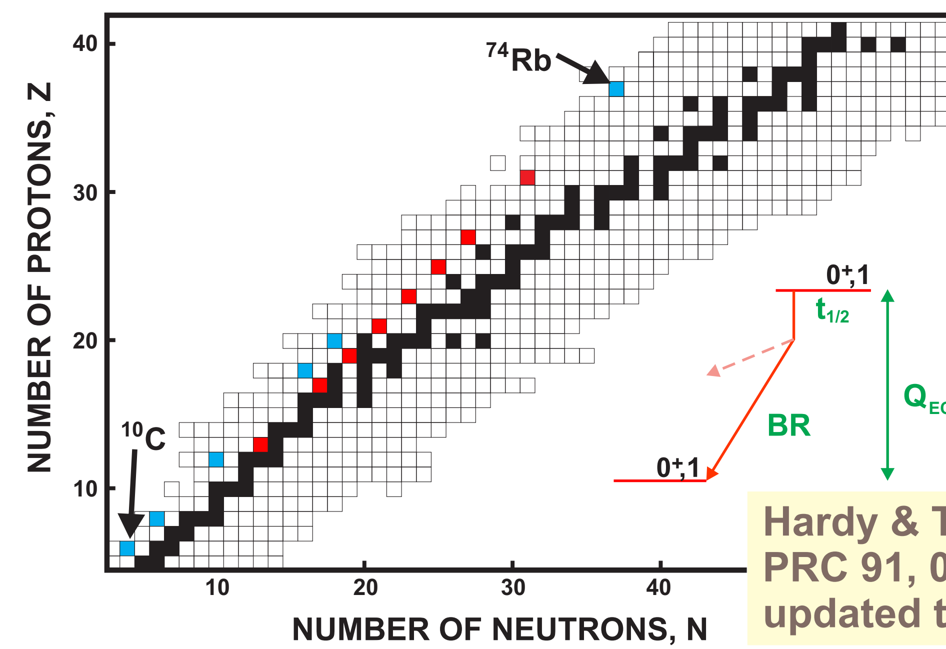
Precise V_{ud} from superallowed decays



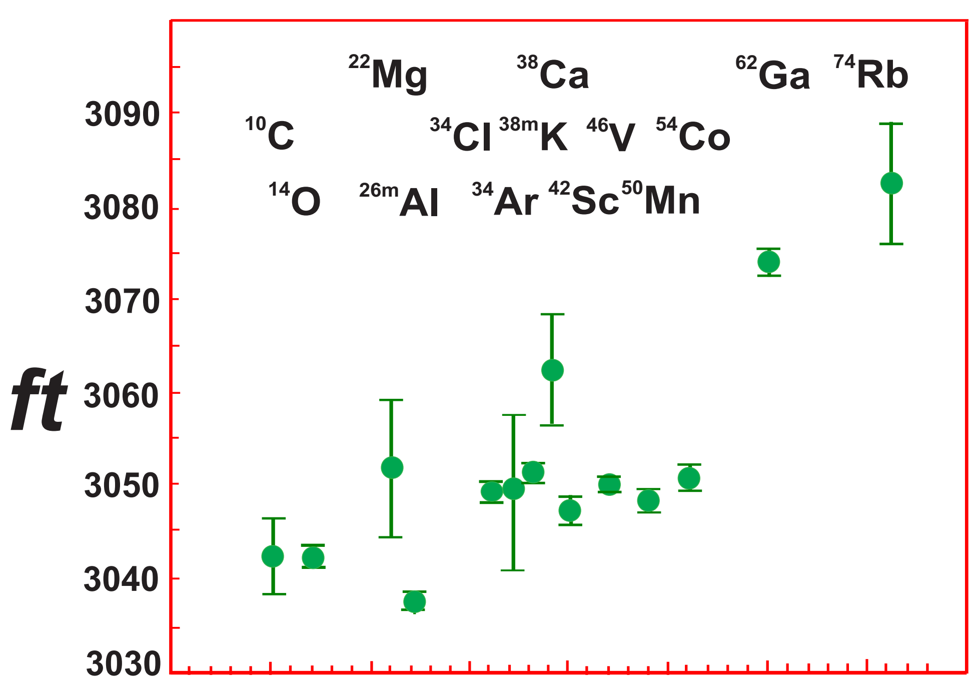
Superallowed 0^+-0^+ nuclear decays:
- only conserved vector current
- many decays
- all rates equal modulo phase space

Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- **8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**



Hardy & Towner
PRC 91, 025501 (2015)
updated to 2017



ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken
a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
(proton and neutron distribution not the same)

V_{ud} extraction: Universal RC and Universal Ft

To obtain Vud —> absorb all decay-specific corrections into universal **Ft**

$$ft(1 + \mathrm{RC} + \mathrm{ISB}) = \mathscr{F}\mathrm{t}(1 + \Delta_R^V) = \mathrm{ft}(1 + \delta_R')(1 - \delta_C + \delta_{NS})(1 + \Delta_R^V)$$

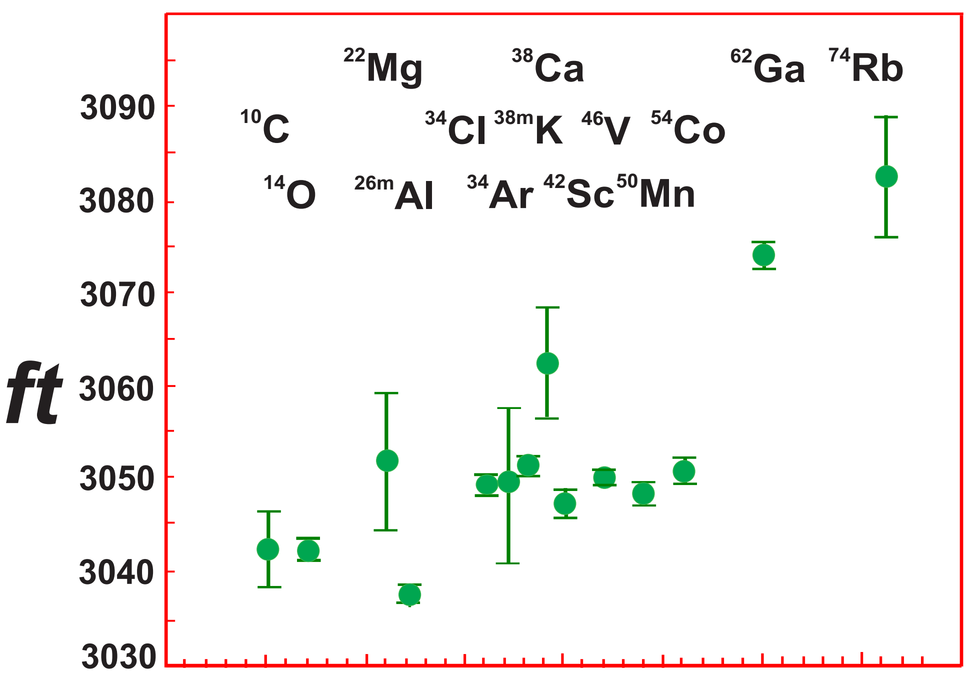
~ Measured

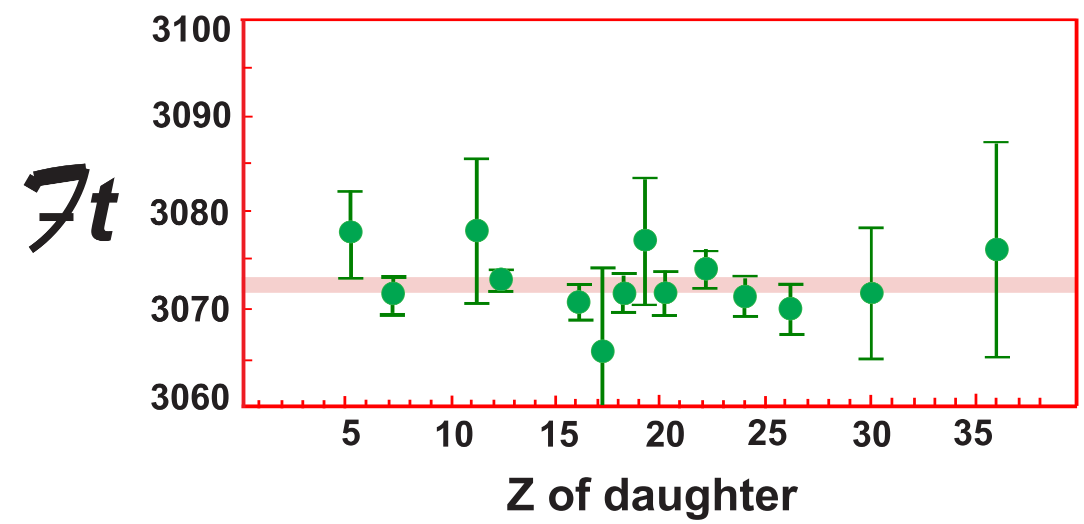
Outer: QED

Isospin-breaking

Nuclear structure

Universal inner





$$|V_{ud}|^2 = \frac{2984.43s}{\mathscr{F}t(1+\Delta_R^V)}$$

Average of 14 decays - 0.02%

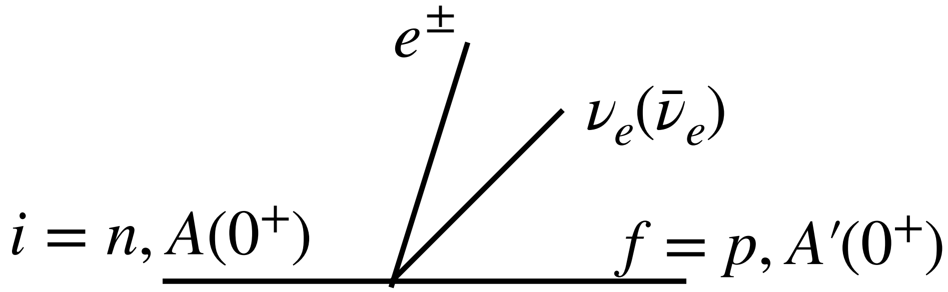
$$\overline{\mathcal{F}t} = 3072.1 \pm 0.7$$

Hardy, Towner 1972 - 2020

RC to nuclear beta decay: overall setup

RC to nuclear beta decay: overall setup

Tree-level amplitude



$\sim V_{ud}$

Radiative corrections to tree-level amplitude $\sim \alpha/2\pi \approx 10^{-3}$

Precision goal for V_{ud} extraction 1×10^{-4}

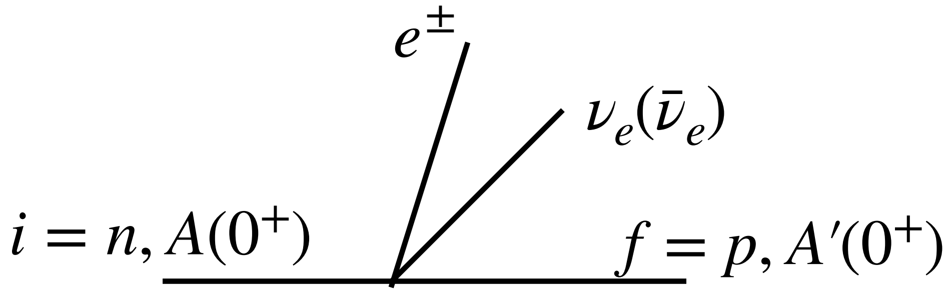
Electron carries away energy E < Q-value of a decay

E-dep RC: $\frac{\alpha}{2\pi}\left(\frac{E}{\Lambda}, \ln\frac{E}{\Lambda}, \ldots\right)$

Energy scales Λ

RC to nuclear beta decay: overall setup

Tree-level amplitude



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Energy scales Λ

Λ

Weak boson scale
$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale
$\Lambda_{\mathrm{had}} = 300\,\mathrm{MeV}$

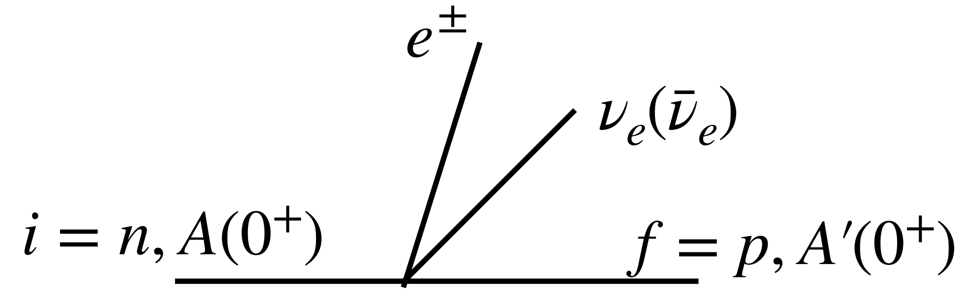
Nuclear scale
$\Lambda_{\mathrm{nuc}} = 10 - 30\,\mathrm{MeV}$

Decay Q-value (endpoint energy)
$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Electron mass
$m_e \approx 0.5\,\mathrm{MeV}$

RC to nuclear beta decay: overall setup

Tree-level amplitude



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Energy scales Λ

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Weak boson scale

$M_Z, M_W \sim 90\,\mathrm{GeV}$

Hadronic scale

$\Lambda_{\rm had} = 300\,\mathrm{MeV}$

Universal

Nuclear scale

$\Lambda_{\rm nuc} = 10 - 30\,\mathrm{MeV}$

Nuclear structure dependent (QCD)

Decay Q-value (endpoint energy)

$Q_{if} = M_i - M_f = 1 - 10\,\mathrm{MeV}$

Nucleus-specific

Electron mass

$m_e \approx 0.5\,\mathrm{MeV}$

Nuclear structure independent (QED)

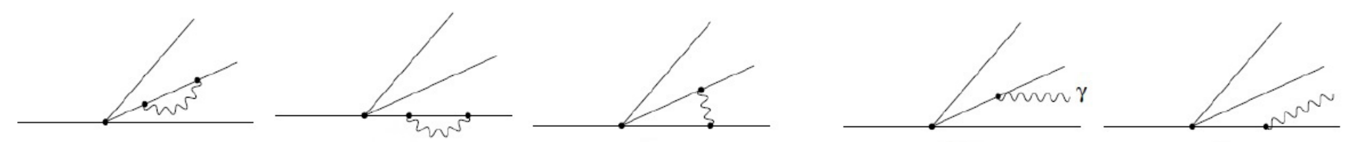
RC to beta decay: overall setup

Generically: only IR and UV extremes feature large logarithms!
Works by Sirlin (1930-2022) and collaborators: all large logs under control

IR: Fermi function + Sirlin function

Fermi function: resummation of $(Z\alpha)^n$ —> Dirac - Coulomb problem

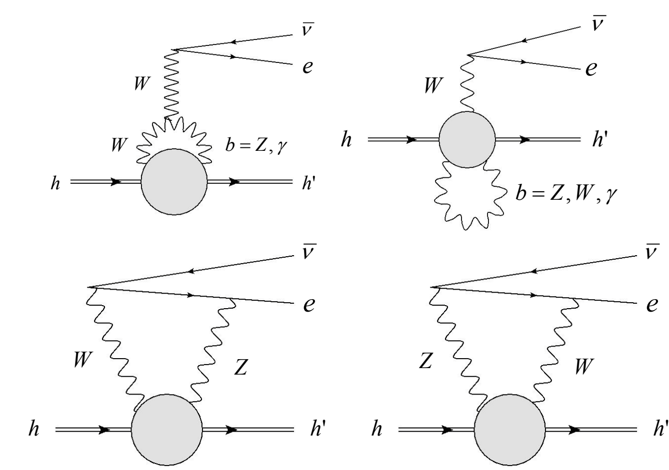
Sirlin function (outer correction):
All IR-div. pieces beyond Coulomb distortion



UV: large EW logs + pQCD corrections

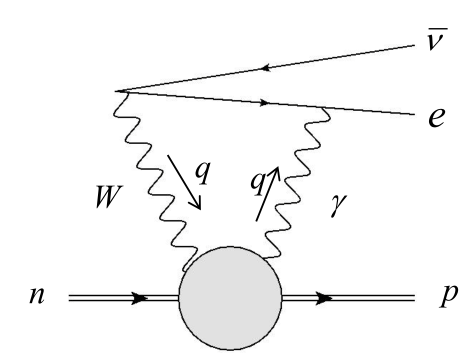
Inner RC:
energy- and model-independent

W,Z - loops
UV structure of SM



***γW*-box: sensitive to all scales**

New method for computing EW boxes: dispersion theory
Combine exp. data with pQCD, lattice, EFT, ab-initio nuclear



RC to β decay - scale separation: history

Fermi function (pure Coulomb + nuclear size & recoil + atomic) —> phase-space **f**

Fermi, Behrens-Bühring, Wilkinson…

Soft Bremsstrahlung: universal Sirlin's function + nucleus specific corrections —> δ_R'

All IR-sensitive pieces: recent review Hayen et al RMP 2018

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All IR-sensitive pieces: recent review

Hayen et al RMP 2018

UV-sensitive RC on free neutron Δ_R^V: Sirlin, Marciano, Czarnecki 1967 - 2006

$$g_V^2 = |V_{ud}|^2 \left[1 + \frac{\alpha}{2\pi} \left\{ 3 \ln \frac{M_Z}{M_p} + \ln \frac{M_Z}{M_W} + \tilde{a}_g \right\} + \delta_{\text{QED}}^{HO} + 2 \, \Box_{\gamma W} \right]$$

Wouter's talk

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Wouter's talk

All scales are assumed to be perfectly separated!

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Wouter's talk

All scales are assumed to be perfectly separated!

Isospin breaking (non-RC): Coulomb repulsion b. protons —> δ_C

MacDonald 1958

Nuclear structure δ_{NS} —> only since1990

Jaus, Rasche 1990

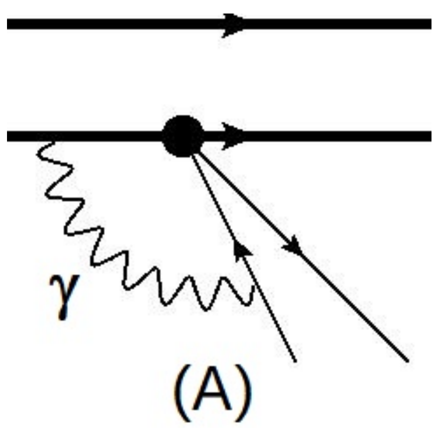
Hardy, Towner 1992-2020

$$ft(1 + \text{RC} + \text{ISB}) = \mathscr{F}\text{t}(1 + \Delta_\text{R}^\text{V}) = \text{ft}(1 + \delta'_\text{R})(1 - \delta_\text{C} + \delta_\text{NS})(1 + \Delta_\text{R}^\text{V})$$

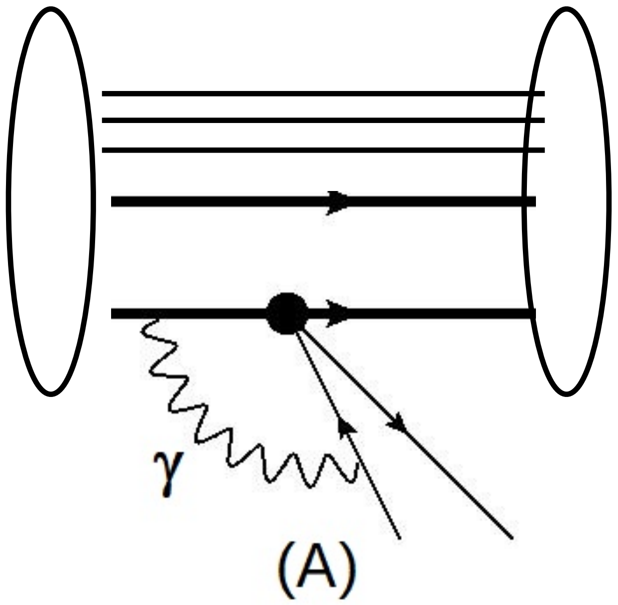
Nuclear-structure RC δ_{NS}: history

Jaus, Rasche 1990

γ and W on same nucleon —> already in Δ_R^V: drop!



Nuclear-structure RC δ_{NS}: history



Jaus, Rasche 1990

γ and W on same nucleon —> already in Δ_R^V: drop!

Towner 1994

Nucleons are bound — free-nucleon RC modified: δ_{NS}^A

Nuclear WF — filter 0+ states (nuclear shell model)

Nuclear-structure RC δ_{NS}: history

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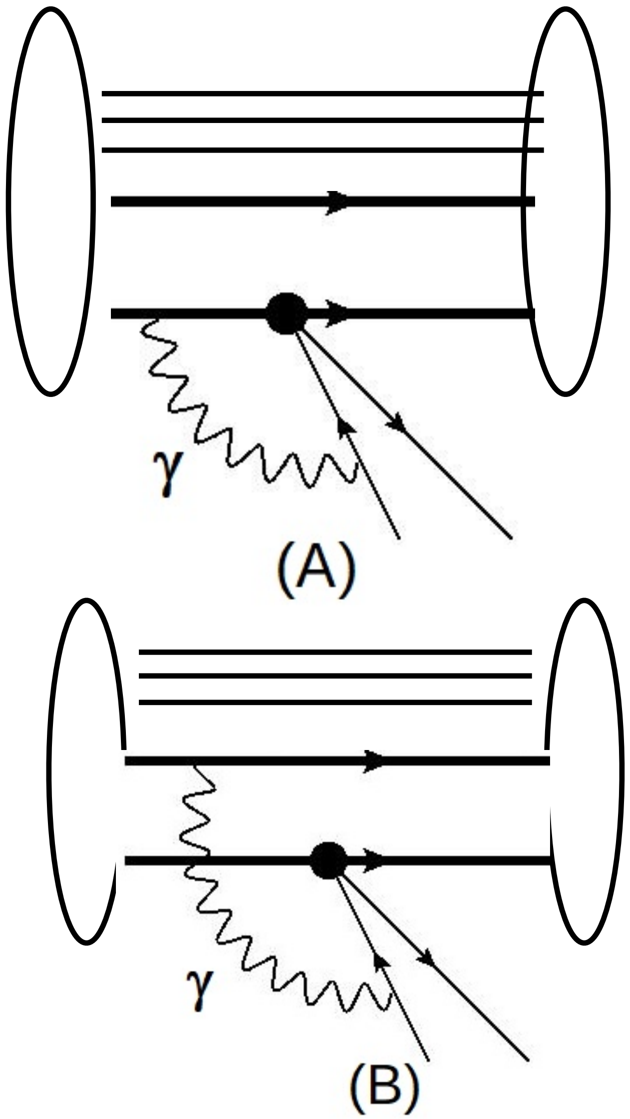
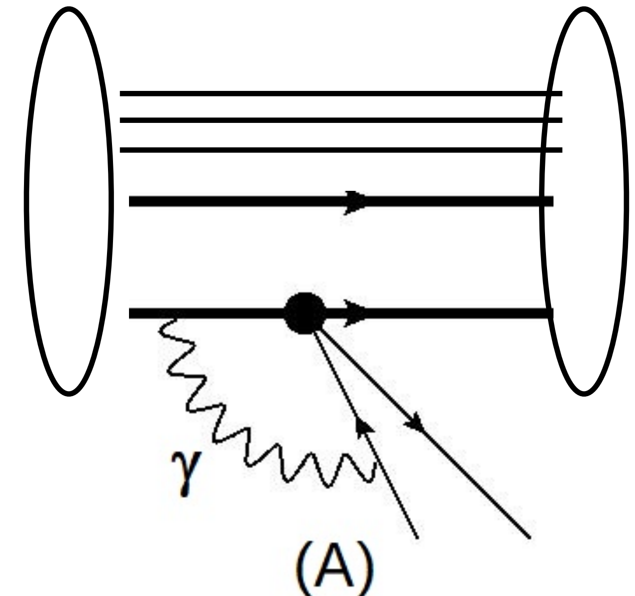
Jaus, Rasche 1990; Hardy, Towner 1992-2020

γ and W on distinct nucleons —> only in nuclei: δ_{NS}^B



Nuclear-structure RC δ_{NS}: history



Jaus, Rasche 1990

γ and W on same nucleon —> already in Δ_R^V: drop!

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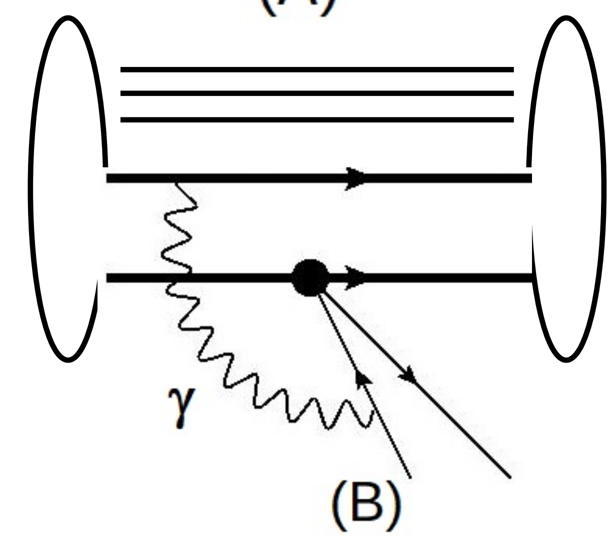


Jaus, Rasche 1990; Hardy, Towner 1992-2020

γ and W on distinct nucleons —> only in nuclei: δ_{NS}^B

Implementation $\delta_{NS} \sim \int d^4x e^{iqx} \langle \phi_{0^+}(P_f) | T\{J_W^{\nu\dagger}(x) J_\gamma^\mu(0)\} | \phi_{0^+}(P_i) \rangle$

One-body nucleon currents
(Only axial and magnetic needed)

$$J_A^\nu(q) \to G_A(q^2)\bar{u}(p_1+q)\gamma^\mu\gamma_5 u(p_1)$$

$$J_A^\nu(q) \to G_M(q^2)\bar{u}(p_1+q)\frac{F_{\mu\nu}\sigma^{\mu\nu}}{4M}u(p_1)$$

Nuclear-structure RC δ_{NS}: history

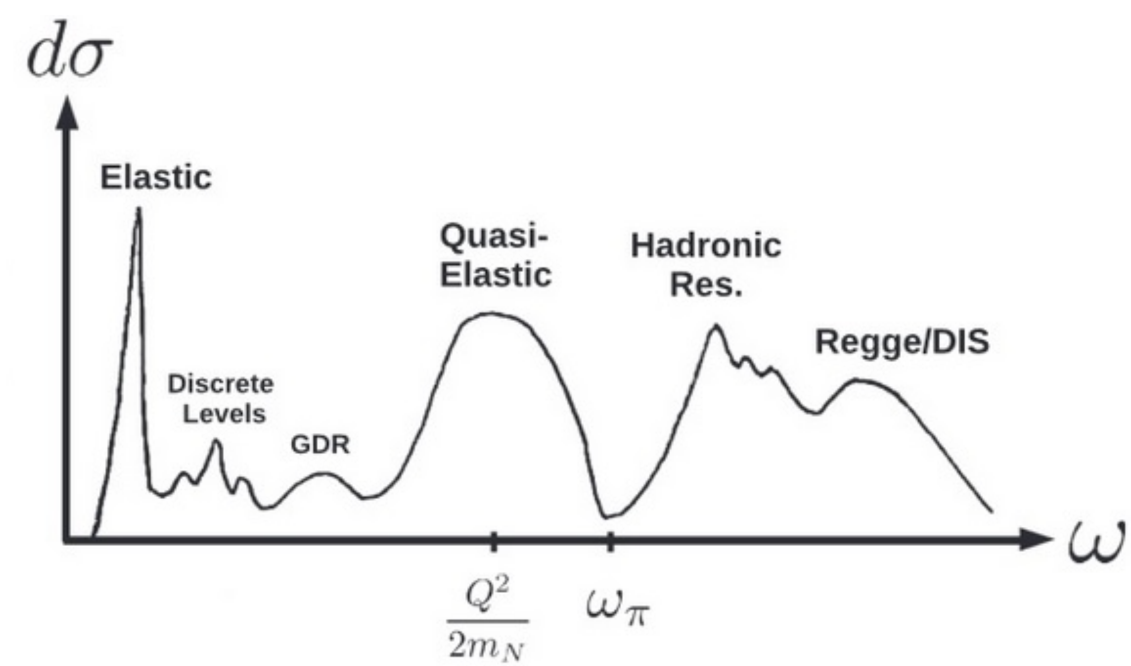
However, this implementation is flawed!

$$\int d^4x e^{iqx} \langle \phi_{0^+}(\vec{0}) | T\{J_W^{\nu\dagger}(x) J_\gamma^\mu(0)\} | \phi_{0^+}(\vec{0}) \rangle = \sum_X \left[\frac{\langle \phi_{0^+}(\vec{0}) | J_W^{\nu\dagger} | X \rangle \langle X | J_\gamma^\mu | \phi_{0^+}(\vec{0}) \rangle}{\nu - \nu_X + i\epsilon} + \frac{\langle \phi_{0^+}(\vec{0}) | J_\gamma^\mu | X \rangle \langle X | J_W^{\nu\dagger} | \phi_{0^+}(\vec{0}) \rangle}{\nu + \nu_X + i\epsilon} \right]$$

—> Nuclear Green's function G — complete information about a nuclear system

G encodes all possible intermediate states

Importantly: nuclear photoabsorption
features low-lying discrete states, QE peak,
and is not limited to low energies (shadowing etc.)



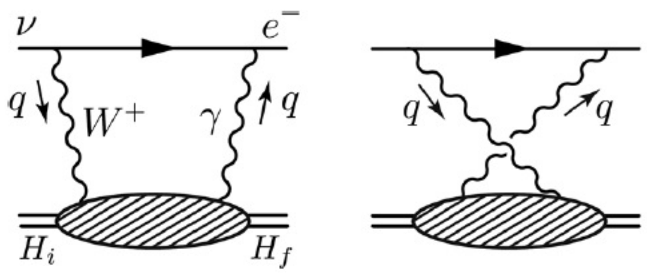
Since 2018 we have a new tool: Dispersion Relations
DR can naturally be used to test all assumptions:

1B currents; nuclear resonances; scale separation; nuclear effects at high energies; …

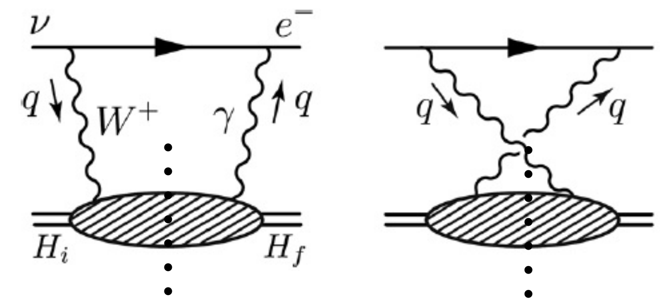
Dispersion Formalism for γW-box

γW-box from dispersion relations

Model-dependent part or RC: γW-box



Generalized Compton tensor
time-ordered product — complicated!



Commutator (Im part) - only on-shell
hadronic states — related to data

$$\int dx e^{iqx} \langle H_f(p) | T\{J^\mu_{em}(x) J^{\nu,\pm}_W(0)\} | H_i(p)\rangle$$

$$\int dx e^{iqx} \langle H_f(p) | [J^\mu_{em}(x), J^{\nu,\pm}_W(0)] | H_i(p)\rangle$$

Generalized (non-diagonal) Compton amplitudes

Interference structure functions

Interference γW structure functions

$$\mathrm{Im} T^{\mu\nu}_{\gamma W} = \ldots + \frac{i\varepsilon^{\mu\nu\alpha\beta} p_\alpha q_\beta}{2(pq)} F_3^{\gamma W}(x, Q^2)$$

Long- and intermediate-range part of the box ~ hadronic/nuclear **polarizabilities**
Polarizabilities related to the excitation spectrum via dispersion relation

γW-box from dispersion relations

After some algebra
(isospin decomposition, loop integration)

$$T_{3,\pm}(\nu,Q^2) \equiv \frac{1}{2}\left[T_3(\nu,Q^2) \pm T_3(-\nu,Q^2)\right]$$

$$\Box^{b,\mathrm{e}}_{\gamma W}(E_e) = \frac{\alpha}{\pi}\int_0^\infty dQ^2 \frac{M_W^2}{M_W^2+Q^2}\int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+2\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^2}\frac{F_{3,-}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^2)$$

$$\Box^{b,\mathrm{o}}_{\gamma W}(E_e) = \frac{2\alpha E_e}{3\pi}\int_0^\infty dQ^2 \int_{\nu_{\mathrm{thr}}}^\infty \frac{d\nu'}{\nu'}\frac{\nu'+3\sqrt{\nu'^2+Q^2}}{(\nu'+\sqrt{\nu'^2+Q^2})^3}\frac{F_{3,+}(\nu',Q^2)}{Mf_+(0)} + \mathcal{O}(E_e^3)$$

Same formulas for free neutron and nuclei;

NS correction reflects extraction of the free box $\delta_{\mathrm{NS}} = 2[\,\Box^{\mathrm{VA,\,nucl}}_{\gamma W} - \Box^{\mathrm{VA,\,free\,n}}_{\gamma W}\,]$

RC on a free neutron $\Delta_R^V \propto F_3^{\mathrm{free\,n}} \propto \int dx e^{iqx}\sum_X \langle p\,|\,J_{em}^{\mu,(0)}(x)\,|\,X\rangle\langle X\,|\,J_W^{\nu,+}(0)\,|\,n\rangle$

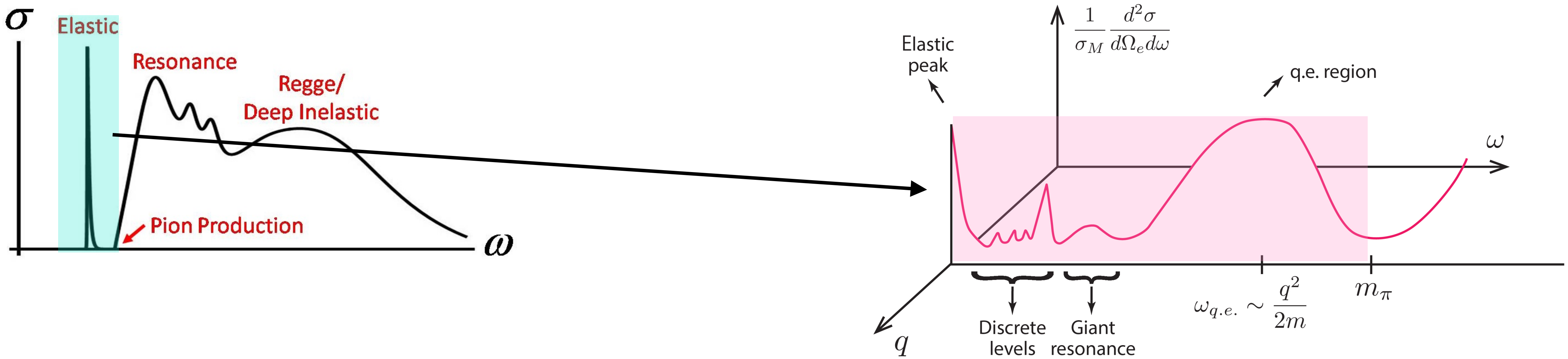
RC on a nucleus $\Delta_R^V + \delta_{NS} \propto F_3^{\mathrm{Nucl.}} \propto \int dx e^{iqx}\sum_{X'} \langle A'\,|\,J_{em}^{\mu,(0)}(x)\,|\,X'\rangle\langle X'\,|\,J_W^{\nu,+}(0)\,|\,A\rangle$

Splitting the γW-box into Universal and Nuclear Parts

Can already test some assumptions:
extraction of a free-nucleon RC; energy independence

Input in the DR for the universal RC

Input in the DR for the RC on a nucleus



δ_{NS} from DR with energy dependence averaged over the β spectrum

$$\delta_{NS} = \frac{2\alpha}{\pi M}\int_0^{\text{few GeV}^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu}\left[\frac{\nu+2q}{(\nu+q)^2}\left(F_3^{(0)\,Nucl.} - F_3^{(0),\,B}\right) + \frac{2\langle E\rangle}{3}\frac{\nu+3q}{(\nu+q)^3}F_3^{(-)\,Nucl.}\right]$$

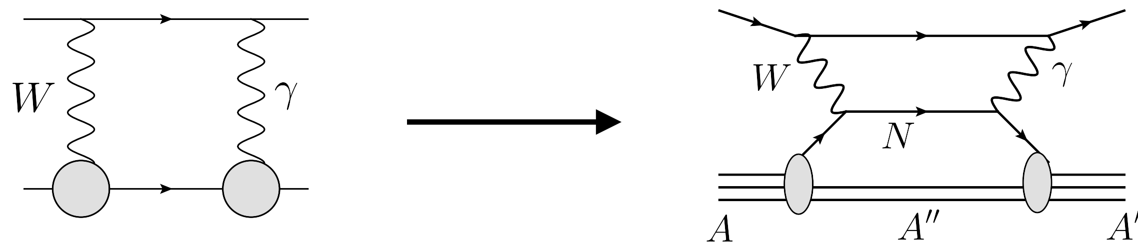
Splitting the γW-box into Universal and Nuclear Parts

δ^A_{NS} from DR with energy dependence

C-Y Seng, MG, M J Ramsey-Musolf 1812.03352

MG 1812.04229

Elastic nucleon box —> single N QE knockout



QE contribution from DR: $\delta^{\rm QE}_{\rm NS} = \delta^{\rm QE,\,0}_{\rm NS} + \langle E\rangle \delta^{\rm QE,\,1}_{\rm NS}$

$$\delta^A_{NS} = \frac{2\alpha}{\pi NM}\int_0^{\rm few\,GeV^2} dQ^2 \int_{\nu_{thr}}^{\nu_\pi} \frac{d\nu}{\nu}\left[\frac{\nu+2q}{(\nu+q)^2}\left(F_3^{(0)\,QE} - F_3^{(0),\,B}\right) + \frac{2\langle E\rangle}{3}\frac{\nu+3q}{(\nu+q)^3}F_3^{(-)\,QE}\right]$$

HT value 2018:

$\mathscr{F}t = 3072.1(7)s$

Old estimate: $\delta\mathscr{F}t = -\,(1.8 \pm 0.4)s \quad + \quad (0 \pm 0)s$

New estimate: $\delta\mathscr{F}t = -\,(3.5 \pm 1.0)s \quad + \quad (1.6 \pm 0.5)s$

Nuclear structure uncertainty tripled!

$\mathscr{F}t = (3072 \pm 2)s$

Next step: ab-Initio $\delta_{\rm NS}$

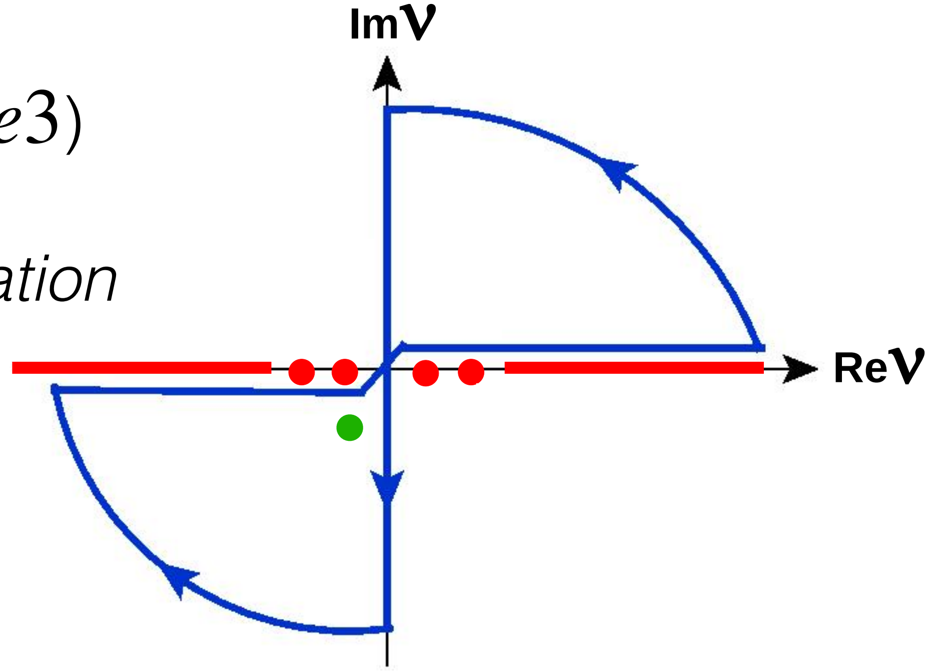
Only a warm-up calculation — ab-initio $\delta_{\rm NS}$ necessary!

Dispersion theory of $\delta_{\rm NS}$: isospin structure + multipole expansion **Seng, MG 2211.10214**

Interesting effects detected for the first time:

Mixed isospin *structure due to 2B currents* (absent for n, $\pi e3$)

Anomalous threshold *possible —> residue upon Wick rotation*



Currently, effort on light systems C-10, O-14

No-Core Shell Model **Michael Gennari, Petr Navratil, Mehdi Drissy**
Green's Function MC **Garrett King, Saori Pastore**
Coupled Clusters **Sonia Bacca, Asia Sobczyk, Gaute Hagen**

Important cross checks should become possible very soon: stay tuned!

Nuclear beta decay review: MG, Seng (Annual Review of Nucl. Part. Sci. - deadline Nov 2)

Nuclear inputs in ft

Nuclear structure in ft

Differential decay spectrum:

$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3}\ F_0(Z,W)\ L_0(Z,W)\ U(Z,W)\ D_{\mathrm{FS}}(Z,W,\beta_2)\ R(W,W_0)\ R_N(W,W_0,M)$$
$$\times\ Q(Z,W)\ S(Z,W)\ X(Z,W)\ r(Z,W)\ C(Z,W)\ D_C(Z,W,\beta_2)\ pW(W_0-W)^2\ dW$$

Nuclear structure in ft

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RC + Recoil QED

Atomic effects: QED

Nuclear structure in ft

Differential decay spectrum:

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Fermi Fn: daughter **Charge FF** $F_{Ch}(q^2)$

RC + Recoil QED

Atomic effects: QED

Nuclear structure in ft

Differential decay spectrum:

$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} F_0(Z,W)\ L_0(Z,W)\ U(Z,W)\ D_{\rm FS}(Z,W,\beta_2)\ R(W,W_0)\ R_N(W,W_0,M)$$
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Fermi Fn: daughter **Charge FF** $F_{Ch}(q^2)$

RC + Recoil QED

Atomic effects: QED

Shape factor:
weak CC transition FF $F_{CW}(q^2)$

Nuclear structure in ft

Differential decay spectrum:

$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} F_0(Z,W)\ L_0(Z,W)\ U(Z,W)\ D_{\rm FS}(Z,W,\beta_2)\ R(W,W_0)\ R_N(W,W_0,M)$$
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Fermi Fn: daughter **Charge FF** $F_{Ch}(q^2)$

RC + Recoil QED

Atomic effects: QED

Shape factor:
weak CC transition FF $F_{CW}(q^2)$

ft-values obtained as $ft = \int_{m_e}^{W_{max}} N(W)dW\, t_{1/2}$

Nuclear structure in ft

Differential decay spectrum:

$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} F_0(Z,W)\ L_0(Z,W)\ U(Z,W)\ D_{\mathrm{FS}}(Z,W,\beta_2)\ R(W,W_0)\ R_N(W,W_0,M)$$
$$\times\ Q(Z,W)\ S(Z,W)\ X(Z,W)\ r(Z,W)\ C(Z,W)\ D_C(Z,W,\beta_2)\ pW(W_0-W)^2\ dW$$

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Charge form factors: combination of e-scattering, X-ray/laser/optical atom spectroscopy
Not all radii are known
Even if known: inherent nuclear uncertainty: nuclear polarization contribution

Nuclear structure in ft

Differential decay spectrum:

$$N(W)dW = \frac{G_V^2 V_{ud}^2}{2\pi^3} F_0(Z,W)\ L_0(Z,W)\ U(Z,W)\ D_{\rm FS}(Z,W,\beta_2)\ R(W,W_0)\ R_N(W,W_0,M)$$
$$\times\ Q(Z,W)\ S(Z,W)\ X(Z,W)\ r(Z,W)\ C(Z,W)\ D_C(Z,W,\beta_2)\ pW(W_0-W)^2\ dW$$

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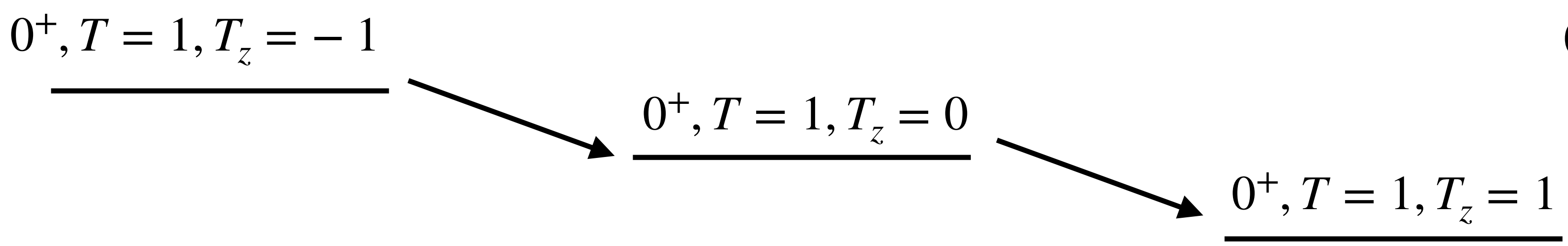
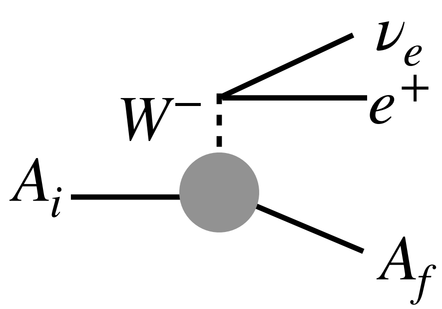
Charge form factors: combination of e-scattering, X-ray/laser/optical atom spectroscopy
Not all radii are known
Even if known: inherent nuclear uncertainty: nuclear polarization contribution

Charged-current weak transition form factors: only accessible with the decay itself (tough);
Historically estimated in nuclear shell model with 1B current (Wilkinson; Hardy & Towner; …)
Typical result: very similar to charge FF

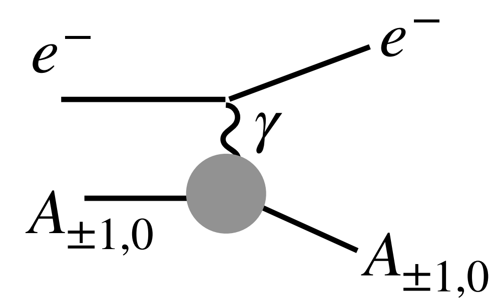
Isospin symmetry + Charge Radii in $T = 1, O^{+}$isotriplet

CY Seng, 2212.02681





$$F_{CW}(Q^2) = 1 - R_{CW}^2 Q^2/6 + \ldots$$



$$F_{Ch}(Q^2) = 1 - R_{Ch}^2 Q^2/6 + \ldots$$

Isospin symmetry: CW <—> charge radii
Remove the symmetry energy (energy cost to add a neutron to symmetric nucleus)
Superallowed isotriplet ≈ closed symmetric core + (pp - np - nn)

$$R^2_{\rm CW} = R^2_{\rm Ch,1} + Z_0(R^2_{\rm Ch,0} - R^2_{\rm Ch,1}) = R^2_{\rm Ch,1} + \frac{Z_{-1}}{2}(R^2_{\rm Ch,-1} - R^2_{\rm Ch,1})$$

Large factors ~Z multiply small radii differences

Isospin symmetry + Charge Radii in $T=1, O^{+}$isotriplet

A	$R_{\mathrm{Ch},-1}$ (fm)	$R_{\mathrm{Ch},0}$ (fm)	$R_{\mathrm{Ch},1}$ (fm)	$R^2_{\mathrm{Ch},1}$ (fm^2)	R^2_{CW} (fm^2)
10	${}^{10}_{6}$C	${}^{10}_{5}$B(ex)	${}^{10}_{4}$Be: 2.3550(170)[a]	5.546(80)	N/A
14	${}^{14}_{8}$O	${}^{14}_{7}$N(ex)	${}^{14}_{6}$C: 2.5025(87)[a]	6.263(44)	N/A
18	${}^{18}_{10}$Ne: 2.9714(76)[a]	${}^{18}_{9}$F(ex)	${}^{18}_{8}$O: 2.7726(56)[a]	7.687(31)	13.40(53)
22	${}^{22}_{12}$Mg: 3.0691(89)[b]	${}^{22}_{11}$Na(ex)	${}^{22}_{10}$Ne: 2.9525(40)[a]	8.717(24)	12.93(71)
26	${}^{26}_{14}$Si	${}^{26m}_{13}$Al	${}^{26}_{12}$Mg: 3.0337(18)[a]	9.203(11)	N/A
30	${}^{30}_{16}$S	${}^{30}_{15}$P(ex)	${}^{30}_{14}$Si: 3.1336(40)[a]	9.819(25)	N/A
34	${}^{34}_{18}$Ar: 3.3654(40)[a]	${}^{34}_{17}$Cl	${}^{34}_{16}$S: 3.2847(21)[a]	10.789(14)	15.62(54)
38	${}^{38}_{20}$Ca: 3.467(1)[c]	${}^{38m}_{19}$K: 3.437(4)[d]	${}^{38}_{18}$Ar: 3.4028(19)[a]	11.579(13)	15.99(28)
42	${}^{42}_{22}$Ti	${}^{42}_{21}$Sc: 3.5702(238)[a]	${}^{42}_{20}$Ca: 3.5081(21)[a]	12.307(15)	21.5(3.6)
46	${}^{46}_{24}$Cr	${}^{46}_{23}$V	${}^{46}_{22}$Ti: 3.6070(22)[a]	13.010(16)	N/A
50	${}^{50}_{26}$Fe	${}^{50}_{25}$Mn: 3.7120(196)[a]	${}^{50}_{24}$Cr: 3.6588(65)[a]	13.387(48)	23.2(3.8)
54	${}^{54}_{28}$Ni: 3.738(4)[e]	${}^{54}_{27}$Co	${}^{54}_{26}$Fe: 3.6933(19)[a]	13.640(14)	18.29(92)
62	${}^{62}_{32}$Ge	${}^{62}_{31}$Ga	${}^{62}_{30}$Zn: 3.9031(69)[b]	15.234(54)	N/A
66	${}^{66}_{34}$Se	${}^{66}_{33}$As	${}^{66}_{32}$Ge	N/A	N/A
70	${}^{70}_{36}$Kr	${}^{70}_{35}$Br	${}^{70}_{34}$Se	N/A	N/A
74	${}^{74}_{38}$Sr	${}^{74}_{37}$Rb: 4.1935(172)[b]	${}^{74}_{36}$Kr: 4.1870(41)[a]	17.531(34)	19.5(5.5)

CY Seng, 2212.02681

Photon probes the entire nuclear charge
Only the outer protons can decay: all neutron states in the core occupied

ft values update — work in progress; more and more precise charge radii necessary!
Working closely with exp. (PSI, FRIB, ISOLDE)
Nuclear polarization (EM analog of δ_{NS}) crucial for improved radius extraction

Isospin breaking in nuclear WF: δ_C

Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f | \tau^+ | i \rangle$$

τ^+ — Isospin operator

$|i\rangle, |f\rangle$ — members of T=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states (e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

$$|M_F|^2 = |M_0|^2(1 - \delta_C)$$

MacDonald 1958

ISB correction is crucial for V_{ud} extraction

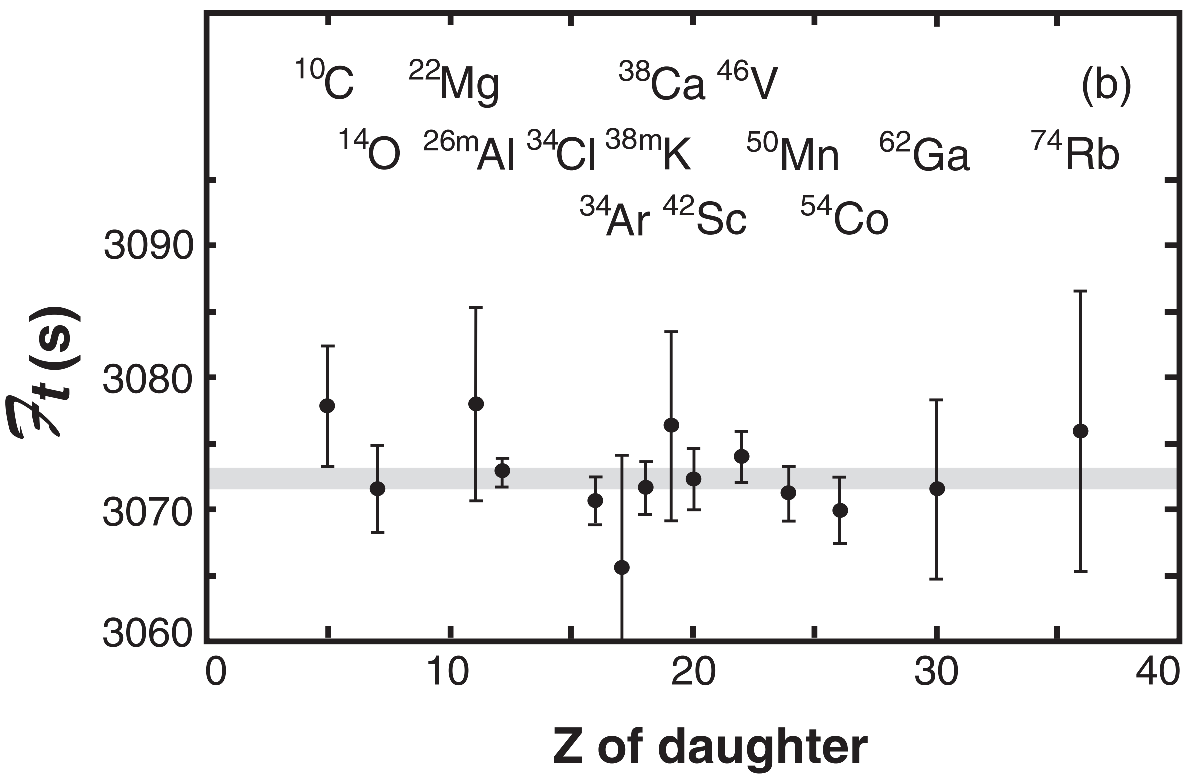
TABLE X. Corrections δ'_R, δ_{NS}, and δ_C that are applied to experimental ft values to obtain $\mathcal{F}t$ values.

Parent nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	δ_C (%)
$T_z = -1$					
^{10}C	1.679	−0.345(35)	0.010(10)	0.165(15)	0.175(18)
^{14}O	1.543	−0.245(50)	0.055(20)	0.275(15)	0.330(25)
^{18}Ne	1.506	−0.290(35)	0.155(30)	0.405(25)	0.560(39)
^{22}Mg	1.466	−0.225(20)	0.010(10)	0.370(20)	0.380(22)
^{26}Si	1.439	−0.215(20)	0.030(10)	0.405(25)	0.435(27)
^{30}S	1.423	−0.185(15)	0.155(20)	0.700(20)	0.855(28)
^{34}Ar	1.412	−0.180(15)	0.030(10)	0.665(55)	0.695(56)
^{38}Ca	1.414	−0.175(15)	0.020(10)	0.745(70)	0.765(71)
^{42}Ti	1.427	−0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0$					
^{26m}Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
^{34}Cl	1.443	−0.085(15)	0.100(10)	0.550(45)	0.650(46)
^{38m}K	1.440	−0.100(15)	0.105(20)	0.565(50)	0.670(54)
^{42}Sc	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
^{46}V	1.445	−0.035(10)	0.075(30)	0.545(55)	0.620(63)
^{50}Mn	1.444	−0.040(10)	0.035(20)	0.610(50)	0.645(54)
^{54}Co	1.443	−0.035(10)	0.050(30)	0.720(60)	0.770(67)
^{62}Ga	1.459	−0.045(20)	0.275(55)	1.20(20)	1.48(21)
^{66}As	1.468	−0.060(20)	0.195(45)	1.35(40)	1.55(40)
^{70}Br	1.486	−0.085(25)	0.445(40)	1.25(25)	1.70(25)
^{74}Rb	1.499	−0.075(30)	0.115(60)	1.50(26)	1.62(27)

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

$$\delta_C \sim 0.17\% - 1.6\%\,!$$

Nuclear Corrections vs. scalar BSM



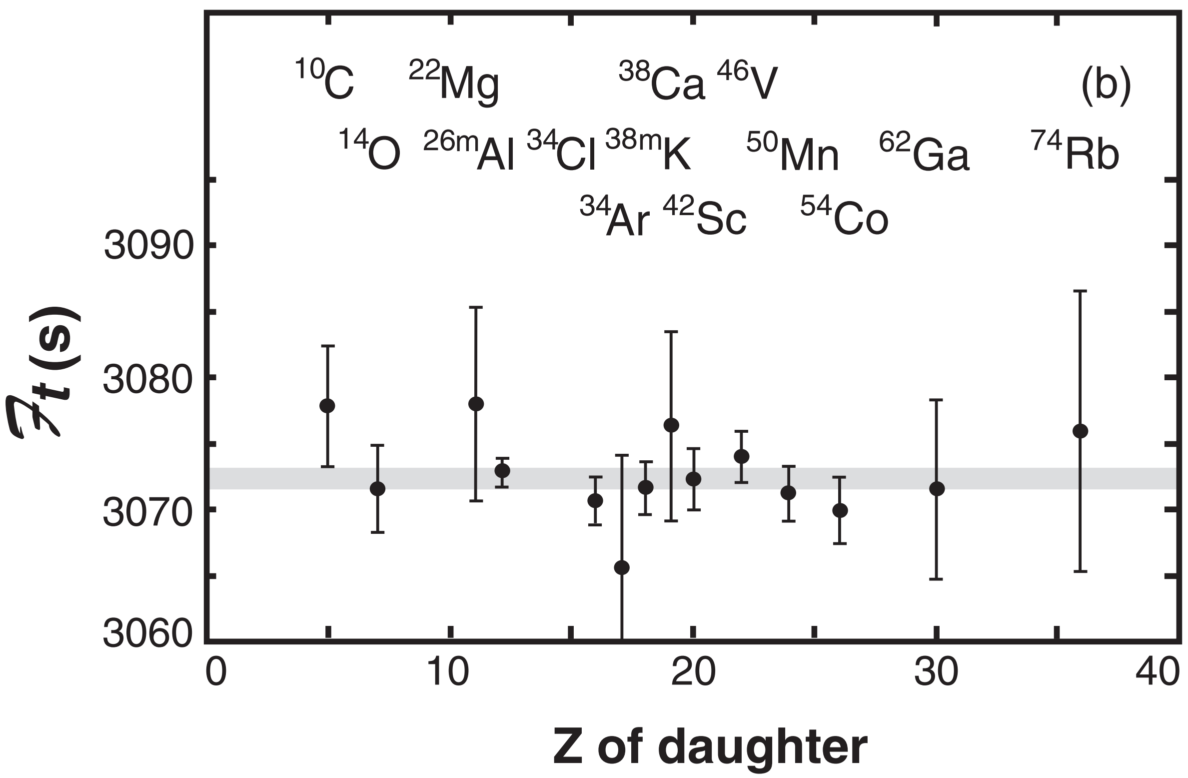
Once all corrections are included:
CVC —> Ft constant

δ_C particularly important for alignment!

Fit to 14 transitions:
Ft constant within 0.02%

Hardy, Towner 2020

Nuclear Corrections vs. scalar BSM



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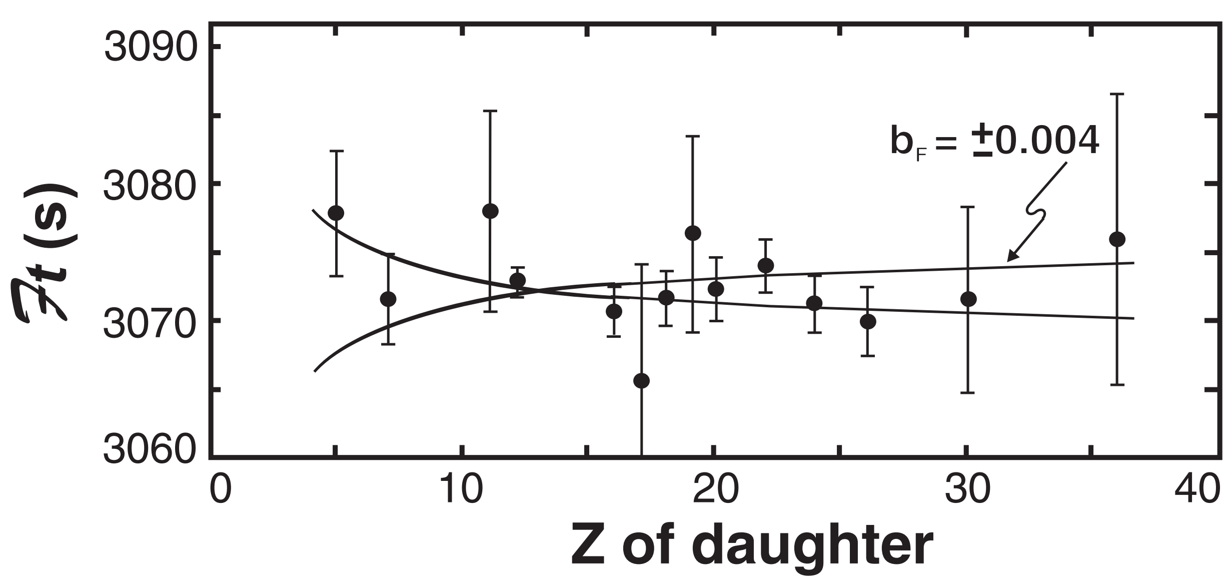
Hardy, Towner 2020



If BSM scalar currents present: Fierz interference b_F

$$\mathscr{F}t^{SM} \rightarrow \mathscr{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

$Q_{EC} \uparrow$ with Z —> effect of $b_F \downarrow$ with Z
Introduces nonlinearity in the Ft plot

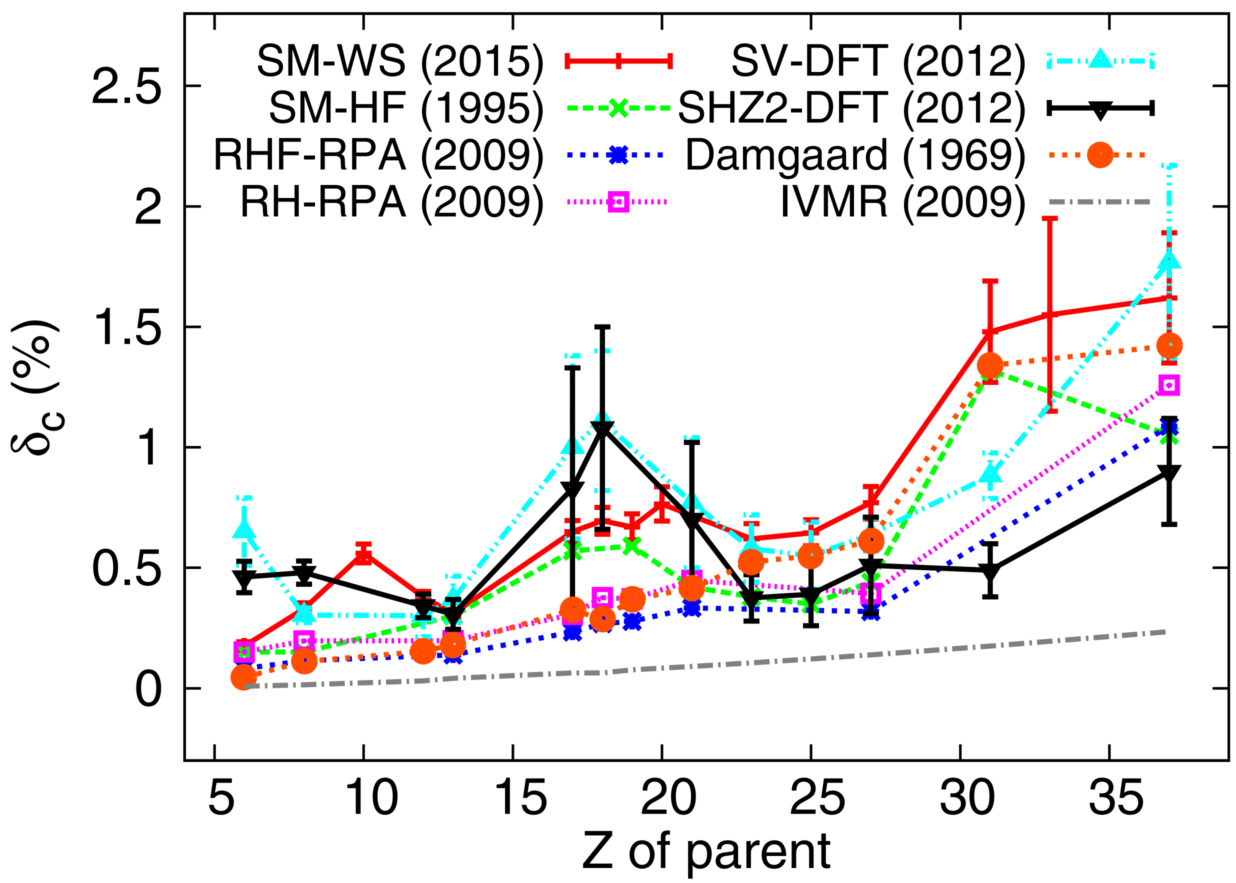
$b_F = -0.0028(26)$ ~ consistent with 0

Nuclear model comparison for δ_C

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

	SM-WS	SM-HF	RPA PKO1	RPA DD-ME2	RPA PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



HT: χ^2 as criterion to prefer SM-WS; V_{ud} and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio δ_C calculations (NCSM, GFMC, CC, IMSRG)
Especially interesting for light nuclei accessible to different techniques!

Constraints on δ_C from nuclear radii

$0^+, T=1, T_z=-1$

$0^+, T=1, T_z=0$

$0^+, T=1, T_z=1$

Auerbach 0811.4742; 2101.06199;
Seng, MG 2208.03037; 2304.03800; 2212.02681

ISB-sensitive combinations of radii: Wigner-Eckart theorem

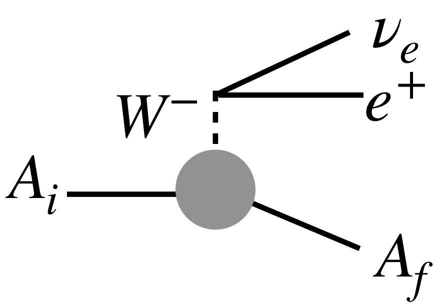
$$\Delta M_A^{(1)} \equiv \langle f | M_{+1}^{(1)} | i \rangle + \langle f | M_0^{(1)} | f \rangle$$

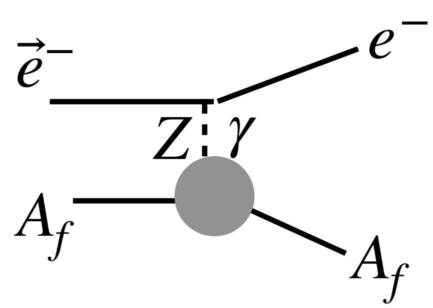
$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2 \right) - Z_0 R_{p,0}^2$$

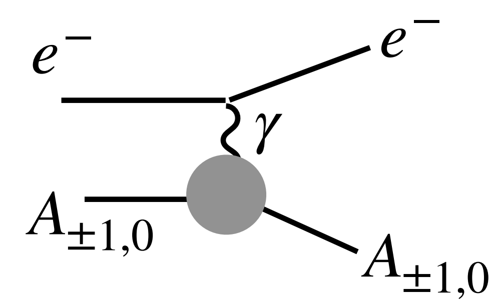
Transition radius From β spectrum

Neutron skin From PVES

Charge radii from atomic spectra and electron scattering







$$F_{CW}(Q^2) = 1 - R_{CW}^2 Q^2/6 + \ldots$$

$$A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \frac{Q_W}{Z} \frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$$F_{Ch}(Q^2) = 1 - R_{Ch}^2 Q^2/6 + \ldots$$

Since N ≠ Z for $T_z = \pm 1$ factors $Z_{\pm 1,0}$ remove the symmetry energy to isolate ISB
(Usually PVES —> neutron skins —> symmetry energy —> nuclear EOS —> nuclear astrophysics)

Electroweak radii constrain ISB in superallowed β-decay

For numerical analysis: lowest isovector monopole resonance dominates

One ISB matrix element, one energy splitting

Model for δ_C → prediction for $\Delta M^{(1)}_{A,B}$

Seng, MG 2208.03037; 2304.03800

Transitions	δ_C (%)					$\Delta M_A^{(1)}$ (fm^2)					$\Delta M_B^{(1)}$ (fm^2)				
	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro
$^{26m}\mathrm{Al} \to {}^{26}\mathrm{Mg}$	0.310	0.329	0.30	0.139	0.08	-2.2	-2.3	-2.1	-1.0	-0.6	-0.12	-0.12	-0.11	-0.05	-0.03
$^{34}\mathrm{Cl} \to {}^{34}\mathrm{S}$	0.613	0.75	0.57	0.234	0.13	-5.0	-6.1	-4.6	-1.9	-1.0	-0.17	-0.21	-0.16	-0.06	-0.04
$^{38m}\mathrm{K} \to {}^{38}\mathrm{Ar}$	0.628	1.7	0.59	0.278	0.15	-5.4	-14.6	-5.1	-2.4	-1.3	-0.15	-0.42	-0.15	-0.07	-0.04
$^{42}\mathrm{Sc} \to {}^{42}\mathrm{Ca}$	0.690	0.77	0.42	0.333	0.18	-6.2	-6.9	-3.8	-3.0	-1.6	-0.15	-0.17	-0.09	-0.07	-0.04
$^{46}\mathrm{V} \to {}^{46}\mathrm{Ti}$	0.620	0.563	0.38	/	0.21	-5.8	-5.3	-3.6	/	-2.0	-0.12	-0.11	-0.08	/	-0.04
$^{50}\mathrm{Mn} \to {}^{50}\mathrm{Cr}$	0.660	0.476	0.35	/	0.24	-6.4	-4.6	-3.4	/	-2.4	-0.12	-0.09	-0.06	/	-0.04
$^{54}\mathrm{Co} \to {}^{54}\mathrm{Fe}$	0.770	0.586	0.44	0.319	0.28	-7.8	-5.9	-4.4	-3.2	-2.8	-0.13	-0.10	-0.07	-0.05	-0.05

Can discriminate models if independent information on nuclear radii is available

ΔM_A from measured radii —> test models for δ_C

Working closely with PVES exp. in Mainz: neutron skins of stable daughters can be measured!

Summary on V_{ud} from superallowed nuclear decays

- Superallowed nuclear decays are a powerful tool to extract V_{ud}
- New method to compute nuclear-structure correction developed
- Dispersion relations allow to study the scale separation explicitly, combine inputs from exp, ab-initio etc
- Modern nuclear theory being applied to selected transitions
- TRIUMF group (Gennari, Drissy, Navratil): NCSM for δ_{NS} in $^{10}C \to {}^{10}B$
- Work on δ_{NS} and δ_C by other groups under way!
- Nuclear charge radii help constraining ISB corrections
- Motivates a dedicated experimental program on more and more precise nuclear radii at PSI, FRIB, ISOLDE, …
- A global program towards a complete update of all nuclear effects (δ_{NS}, δ_C and f) has commenced!

Improved RC to $K\ell 3$ decays

V_{us} from $K\ell 3$

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

$S_{\rm EW}$ Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

Inputs from theory:

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

$\Delta_{K\ell}^{\rm EM}$ Form-factor correction for long-distance EM effects

2.5σ discrepancy between leptonic and semileptonic modes

$|V_{us}^{K\ell 3}| = 0.2231(6)$ $|V_{us}^{K\mu 2}| = 0.2252(5)$

Large missing contribution to RC for $K\ell 3$ was long considered viable option

RC to $K\ell 3$

Until 2021: best way to compute long-distance EM RC was with ChPT

	$I^{(0)}_{K\ell}(\lambda_i)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_3)(\%)$	$\delta^{K\ell}_{\rm EM}(\mathcal{D}_{4-3})(\%)$	$\delta^{K\ell}_{\rm EM}(\%)$
K^0_{e3}	0.103070	0.50	0.49	0.99 ± 0.30
$K^\pm_{e3}$	0.105972	-0.35	0.45	0.10 ± 0.30
$K^0_{\mu 3}$	0.068467	1.38	0.02	1.40 ± 0.30
$K^\pm_{\mu 3}$	0.070324	0.007	0.009	0.016 ± 0.30

Cirigliano, Gianotti, Neufeld 0807.4507

A series of works reformulated the problem as a hybrid of Sirlin's representation and ChPT, plus input from lattice QCD calculations of γW-box for $\pi e3$ and $K\ell 3$

	$\delta^{K\ell}_{\rm EM}$ $[10^{-3}]$	ChPT
$K^0 e$	$11.6(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{e^2p^4}$	$9.9(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ e$	$2.1(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(1)_{e^2p^4}$	$1.0(1.9)_{e^2p^4}(1.6)_{\rm LEC}$
$K^0 \mu$	$15.4(2)_{\rm inel}(1)_{\rm lat}(1)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$14.0(1.9)_{e^2p^4}(1.1)_{\rm LEC}$
$K^+ \mu$	$0.5(2)_{\rm inel}(1)_{\rm lat}(4)_{\rm NF}(2)_{\rm LEC}(2)_{e^2p^4}$	$0.2(1.9)_{e^2p^4}(1.6)_{\rm LEC}$

Seng, Galviz, Meißner 1910.13208
Seng, Galviz, MG, Meißner 2103.04843
Seng, Galviz, MG, Meißner 2203.05217

Feng, MG, Jin, Ma, Seng 2003.09798
Ma, Feng, MG, Jin, Seng 2102.12048

Uncertainties reduced by an o.o.m.
Long-distance EM RC not responsible for the $K\ell 2$-$K\ell 3$ discrepancy!