NNLO QCD corrections to $\Delta\Gamma_{(s)}$ in the $B_{(s)}$-$\bar{B}_{(s)}$ system

12th International Workshop on the CKM Unitarity Triangle

Pascal Reeck | Santiago de Compostela, CKM 2023

INSTITUTE FOR THEORETICAL PARTICLE PHYSICS, KARLSRUHE INSTITUTE OF TECHNOLOGY

Outline

Motivation

Quantum mechanics of B mesons

We will focus on B_s mesons with quark content $\bar{b}s$.

Wigner-Weisskopf-approximation

Relation between self-energy and scattering matrix elements (Nierste 2009; Weisskopf and Wigner 1930; Lee, Oehme, and Yang 1957):

$$-i(2\pi)^4\delta^{(4)}(p_i - p_j)\Sigma_{ij} = \frac{1}{2M_B}\langle B_i|S|B_j\rangle \tag{1}$$

$\Sigma = M - \frac{i}{2}\Gamma$ appears in the Schrödinger equation:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|B(t)\rangle \\ |\bar{B}(t)\rangle\end{pmatrix} = \Sigma\begin{pmatrix}|B(t)\rangle \\ |\bar{B}(t)\rangle\end{pmatrix}. \tag{2}$$

Mass vs flavour eigenstates

- Diagonalising $\Sigma \longrightarrow$ eigenstates B_L and B_H
- $\Delta M = M_H - M_L$ and $\Delta\Gamma = \Gamma_L - \Gamma_H$ related to off-diagonal elements

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Introduction to B meson observables

Mass difference

- Off-diagonal matrix element $\longrightarrow$ mass difference

$$\begin{aligned}\Delta M &\equiv M_H - M_L \\ &= 2|M_{12}| + \mathcal{O}\left(\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\right)\end{aligned} \tag{3}$$

- Dispersive part of self-energy $\longrightarrow$ off-diagonal matrix element

$$M_{12} = \frac{\Sigma_{12} + \Sigma_{21}^*}{2} \tag{4}$$

Lifetime difference

- Off-diagonal matrix element $\longrightarrow$ width difference

$$\begin{aligned}\Delta\Gamma &\equiv \Gamma_L - \Gamma_H \\ &= -2|\Gamma_{12}|\cos(\phi_\Gamma - \phi_M) + \mathcal{O}\left(\frac{|\Gamma_{12}|^2}{|M_{12}|^2}\right)\end{aligned} \tag{5}$$

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Previous calculations

Comparison of theory and experiment from 2020

Experimental results ((HFLAV) 2020; Aad et al. 2021; Sirunyan et al. 2021) vs prior calculations (Beneke, Buchalla, Greub, et al. 1999; Ciuchini, Franco, Lubicz, and Mescia 2002; Ciuchini, Franco, Lubicz, Mescia, and Tarantino 2003; Lenz and Nierste 2007; Asatrian, Asatryan, et al. 2020; Asatrian, Hovhannisyan, et al. 2017; Aaij et al. 2019; Aaltonen et al. 2012; Abazov et al. 2012):

$$(\Delta\Gamma)^{\text{exp}} = (0.085 \pm 0.005)\,\text{ps}^{-1} \tag{7}$$

$$(\Delta\Gamma)^{\text{th}} = \left(0.077 \pm 0.015_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.017_{1/m_b}\right)\text{ps}^{-1} \quad (\text{pole}) \tag{8}$$

$$(\Delta\Gamma)^{\text{th}} = \left(0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{1/m_b}\right)\text{ps}^{-1} \quad (\overline{\text{MS}}) \tag{9}$$

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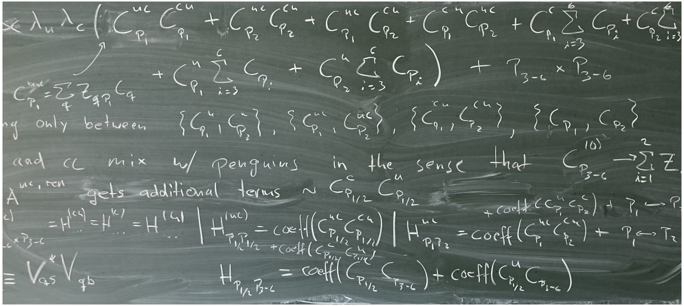
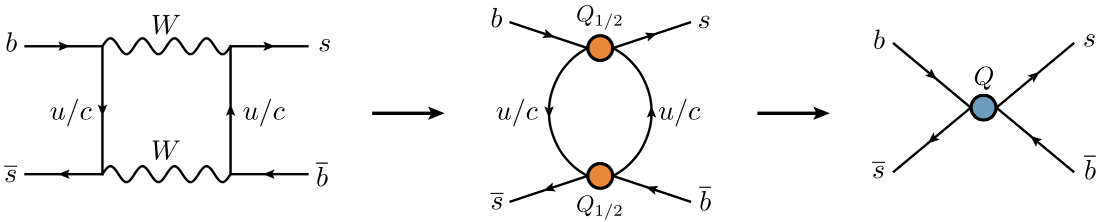
Calculation



Operator product expansion

$$\mathcal{O}_1(x)\mathcal{O}_2(0) \to \sum_n C_{12}^n(x)\mathcal{O}_n(0) \quad (10)$$

B mixing to leading order:



Effective field theory (I)

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Integrate out heavy W boson first:

$|\Delta B| = 1$ effective Hamiltonian

$$\mathcal{H}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \sum_{j=1}^{2} C_j \left(V_{cb} V_{cs}^* P_j^{cc} + V_{cb} V_{us}^* P_j^{cu} + V_{ub} V_{cs}^* P_j^{uc} + V_{ub} V_{us}^* P_j^{uu} \right)$$
$$- \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\sum_{j=3}^{6} C_j P_j + C_8 P_8 \right) + \sum C_j E_j + \text{h.c.} \tag{11}$$

Operator definitions

We work in the Chetyrkin-Misiak-Münz (CMM) (Chetyrkin, Misiak, and Münz 1998) basis:

- Current-current operators

$$P_1^{qp} = (\bar{b}_i \gamma^\mu P_L T^a_{ij} p_j)(\bar{q}_k \gamma_\mu P_L T^a_{kl} s_l), \qquad P_2^{qp} = (\bar{b}_i \gamma^\mu P_L p_i)(\bar{q}_j \gamma_\mu P_L s_j). \tag{12}$$

- Penguin operators, e.g.

$$P_3 = (\bar{b}_i \gamma^\mu P_L s_i) \sum_q (\bar{q}_j \gamma_\mu q_j). \tag{13}$$

- Chromomagnetic operator

$$P_8 = \frac{g_s}{16\pi^2} m_b (\bar{b}_i \sigma^{\mu\nu} P_L T^a_{ij} s_j) G^a_{\mu\nu}. \tag{14}$$

- Evanescent operators E_j vanish in 4 dimensions, see later

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Effective field theory (II)

Evaluating the absorptive part of the matrix element:

$$\sum_{\alpha,\beta} \lambda_\alpha \lambda_\beta \mathrm{Im}(\mathcal{M}_{\alpha\beta}) = -\frac{1}{2}\tau \sum_{\alpha,\beta} \lambda_\alpha \lambda_\beta \left[H^{\alpha\beta} \langle B|Q|\bar{B}\rangle + \tilde{H}_S^{\alpha\beta} \langle B|\tilde{Q}_S|\bar{B}\rangle \right] + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right) \quad (15)$$

$|\Delta B| = 2$ transition operator

With Heavy Quark Expansion (HQE) (Khoze and Shifman 1983; Khoze, Shifman, et al. 1987; Blok et al. 1994; Lenz 2014):

$$\begin{aligned} \mathcal{T}^{|\Delta B|=2} &= \mathrm{Abs}\, i \int \mathrm{d}^4x\, T\, \mathcal{H}^{|\Delta B|=1}(x) \mathcal{H}^{|\Delta B|=1}(0). \\ &= \frac{G_F^2 m_b^2}{12\pi^2} \left[HQ + \tilde{H}_S \tilde{Q}_S + \sum_i H_{E_i} E_i \right] + \mathrm{h.c.} + \mathcal{T}_{1/m_b}^{|\Delta B|=2}. \end{aligned} \quad (16)$$

$$\lambda_\alpha \equiv V_{\alpha s}^* V_{\alpha b} \quad (17)$$

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Obtaining Γ_{12}

CKM elements factorise:

$$\Gamma_{12} = -(\lambda_c^2 \Gamma_{12}^{cc} + 2\lambda_u\lambda_c \Gamma_{12}^{uc} + \lambda_u^2 \Gamma_{12}^{uu}) \tag{18}$$

$$H_{12} = -(\lambda_c^2 H_{12}^{cc} + 2\lambda_u\lambda_c H_{12}^{uc} + \lambda_u^2 H_{12}^{uu}) \tag{19}$$

Γ_{12} in terms of low-energy matrix elements

$$\Gamma_{12} = -\sum_{\alpha,\beta} \lambda_\alpha \lambda_\beta \Gamma_{12}^{\alpha\beta} = -\sum_{\alpha,\beta} \lambda_\alpha \lambda_\beta \frac{G_F^2 m_b^2}{24\pi M_B} \left[H^{\alpha\beta} \langle B|Q|\bar{B}\rangle + \tilde{H}_S^{\alpha\beta} \langle B|\tilde{Q}_S|\bar{B}\rangle \right] + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right) \tag{20}$$

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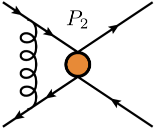
Evanescent operators

- Relevant from NLO upwards
- Vanish with 4-dimensional Fierz identities
- Mix with physical operators under renormalisation
- Physical quantities independent of C_{E_i}



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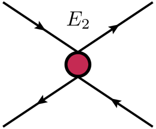

(ii)

Example for $|\Delta B| = 1$ theory

$$E_2^{(1)} = (\bar{b}_i \gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} P_L T^a_{ij} c_j)(\bar{c}_k \gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} P_L T^a_{kl} s_l) - 16 P_2 \quad (21)$$

Example for $|\Delta B| = 2$ theory

$$E_2^{(1)} = (\bar{b}_i \gamma^{\mu}\gamma^{\nu}\gamma^{\rho} P_L s_j)(\bar{b}_j \gamma_{\mu}\gamma_{\nu}\gamma_{\rho} P_L s_i) - (16 - 4\varepsilon - 4\varepsilon^2)\tilde{Q} \quad (22)$$

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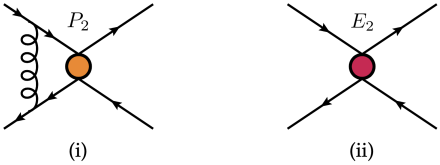

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Physical operators and R_0 in the $|\Delta B| = 2$ theory

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Physical operators

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$$\tilde{Q}_S = (\bar{b}_i P_L s_j)(\bar{b}_j P_L s_i) \quad (26)$$

However, we have the following linear relations:

$$E_1^{(1)} = \tilde{Q} - Q \quad (27)$$

$$= \frac{1}{2} Q + Q_S + \tilde{Q}_S \quad (28)$$

The R_0 operator

- Physical operator with Λ_{QCD}/m_b suppression
- Contains unsuppressed evanescent part

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Toolchain

- `QGRAF` for diagram generation (Nogueira 1993)
- `TAPIR` to insert Feynman rules and identify topologies (Gerlach, Herren, and Lang 2023)
- Amplitude evaluation with in-house tool called `calc` written in `FORM` (Ruijl, Ueda, and Vermaseren 2017)
- `Kira` for IBP reductions (Klappert et al. 2021)
- Master integrals evaluated with `HyperInt` (Panzer 2015) $\longrightarrow$ only imaginary parts

Results

Contributions to Γ_{12}

NB: $z \equiv \frac{m_c^2}{m_b^2}$

Contribution	Previous results	(Gerlach, Nierste, Shtabovenko, Steinhauser 2022)
$P_{1,2} \times P_{3-6}$	2 loops, z-exact, n_f-part only[1]	2 loops, $\mathcal{O}(z)$, full
$P_{1,2} \times P_8$	2 loops, z-exact, n_f-part only[1]	2 loops, $\mathcal{O}(z)$, full
$P_{3-6} \times P_{3-6}$	1 loop, z-exact, full[2]	2 loops, $\mathcal{O}(z)$, full
$P_{3-6} \times P_8$	1 loop, z-exact, n_f-part only[1]	2 loops, $\mathcal{O}(z)$, full
$P_8 \times P_8$	1 loop, z-exact, n_f-part only[1]	2 loops, $\mathcal{O}(z)$, full
$P_{1,2} \times P_{1,2}$	3 loops, $\mathcal{O}(\sqrt{z})$, n_f-part only[1]	3 loops, $\mathcal{O}(z)$, full

[1] (Asatrian, Asatryan, et al. 2020)
[2] (Beneke, Buchalla, and Dunietz 1996)

Improving numerics

Recall,

$$\Gamma_{12} = -\lambda_t^2 \left[\Gamma_{12}^{cc} + 2\frac{\lambda_u}{\lambda_t}(\Gamma_{12}^{cc} - \Gamma_{12}^{uc}) + \left(\frac{\lambda_u}{\lambda_t}\right)^2 (\Gamma_{12}^{uu} + \Gamma_{12}^{cc} - 2\Gamma_{12}^{uc}) \right] \tag{29}$$

$$M_{12} \propto \lambda_t^2 M_B f_B^2 B_Q \tag{30}$$

Moreover,

$$\langle B | Q_i | \bar{B} \rangle \propto M_B^2 f_B^2 B_i \tag{31}$$

$\Delta\Gamma$ without knowledge of $|V_{cb}|$

$$\Delta\Gamma = \left(\frac{\Delta\Gamma}{\Delta M}\right)^{\text{th}} (\Delta M)^{\text{exp}} \tag{32}$$

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Numerical results

$\Delta\Gamma$ to NNLO (Gerlach, Nierste, Shtabovenko, Steinhauser 2022)

$$\left.\frac{\Delta\Gamma_s}{\Delta M_s}\right|_{\text{pole}} = \left(3.79\,^{+0.53}_{-0.58}{}_{\text{scale}}\,^{+0.09}_{-0.19}{}_{\text{scale},1/m_b} \pm 0.11_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}\right) \times 10^{-3}, \tag{33}$$

$$\left.\frac{\Delta\Gamma_s}{\Delta M_s}\right|_{\overline{\text{MS}}} = \left(4.33\,^{+0.23}_{-0.44}{}_{\text{scale}}\,^{+0.09}_{-0.19}{}_{\text{scale},1/m_b} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}\right) \times 10^{-3}, \tag{34}$$

$$\left.\frac{\Delta\Gamma_s}{\Delta M_s}\right|_{\text{PS}} = \left(4.20\,^{+0.36}_{-0.39}{}_{\text{scale}}\,^{+0.09}_{-0.19}{}_{\text{scale},1/m_b} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}\right) \times 10^{-3}. \tag{35}$$

Overall result:

$$\Delta\Gamma^{th} = (0.076 \pm 0.017)\,\text{ps}^{-1} \tag{36}$$

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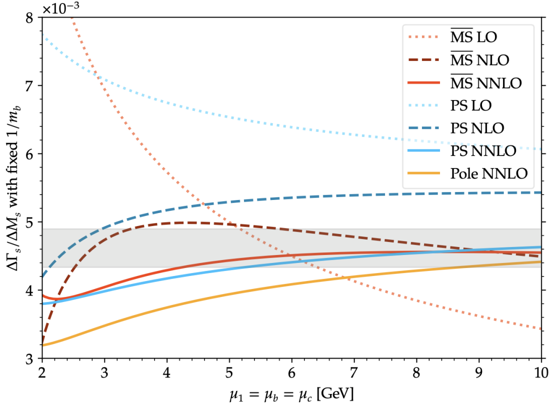
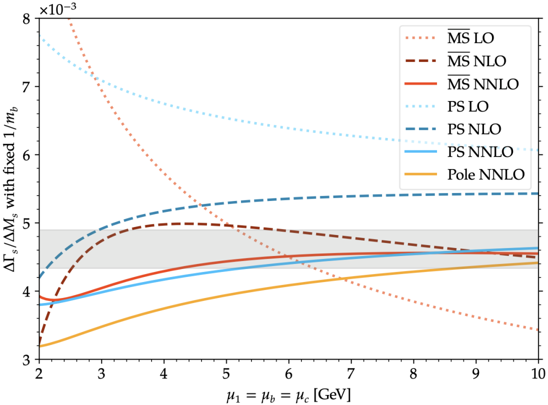
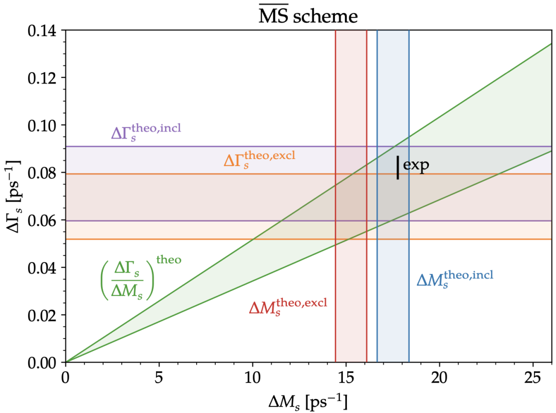
Visualisation of results



Visualisation of results





Outlook

Contribution	(Gerlach, Nierste, Shtabovenko, Steinhauser 2022)	WIP (Chen, Nierste, Reeck, Shtabovenko, Steinhauser)
$P_{1,2} \times P_{3-6}$	2 loops, $\mathcal{O}(z)$	3 loops, $\mathcal{O}(z^{10})$
$P_{1,2} \times P_8$	2 loops, $\mathcal{O}(z)$	3 loops, $\mathcal{O}(z^{10})$
$P_{3-6} \times P_{3-6}$	2 loops, $\mathcal{O}(z)$	3 loops, $\mathcal{O}(z^{10})$
$P_{3-6} \times P_8$	2 loops, $\mathcal{O}(z)$	3 loops, $\mathcal{O}(z^{10})$
$P_8 \times P_8$	2 loops, $\mathcal{O}(z)$	3 loops, $\mathcal{O}(z^{10})$
$P_{1,2} \times P_{1,2}$	3 loops, $\mathcal{O}(z)$	3 loops, $\mathcal{O}(z^{10})$

Thank you for your attention!

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