Update on $SU(3)$-breaking ratios and bag parameters for $B_{(s)}$ mesons

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RBC/UKQCD and JLQCD

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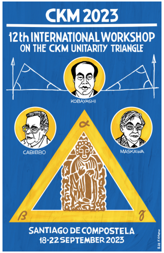
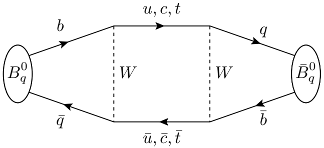


B_q-MESON MIXING

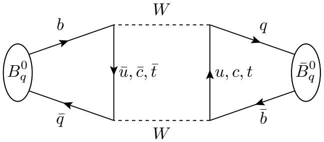
B-mesons B_d, B_s have mass eigenstates

$$|B^0_{qL}\rangle = p_q|B^0_q\rangle + q_q|\bar{B}^0_q\rangle$$
$$|B^0_{qH}\rangle = p_q|B^0_q\rangle - q_q|\bar{B}^0_q\rangle$$



with mass m_{qL} and total decay width Γ_{qL} for the lighter eigenstate. Splittings:

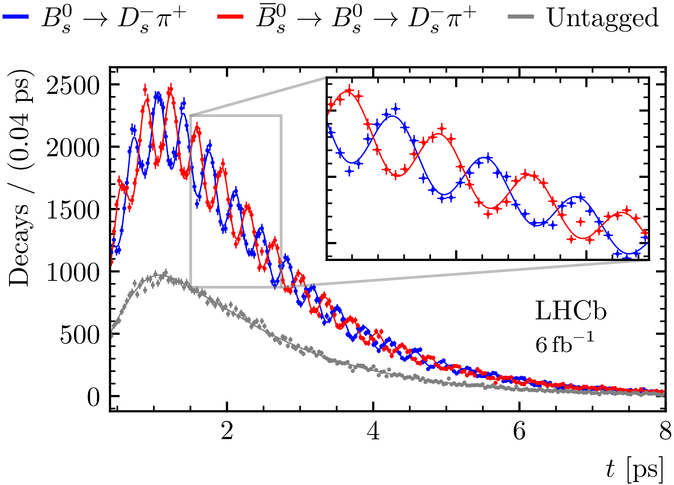
$$\Delta m_q = m_{qH} - m_{qL}$$
$$\Delta\Gamma_q = \Gamma_{qL} - \Gamma_{qH}$$



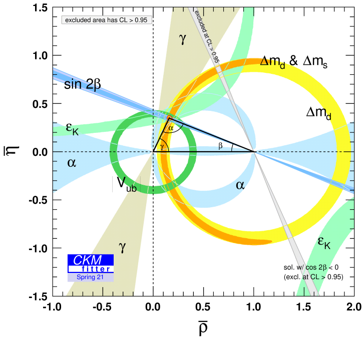
Experimentally, time dependent probabilities give access to the splittings, e.g.

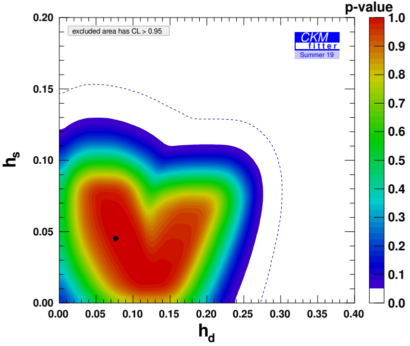
$$\mathcal{P}(B^0_q \to \bar{B}^0_q) = \frac{1}{2}e^{-\Gamma_q t}[\cosh(\frac{1}{2}\Delta\Gamma_q t) - \cos(\Delta m_q t)]|q_q/p_q|^2$$

LHCb 2021 measurement [Nature Phys. 18 (2022) 1, 1-5]



B_q MESON MIXING





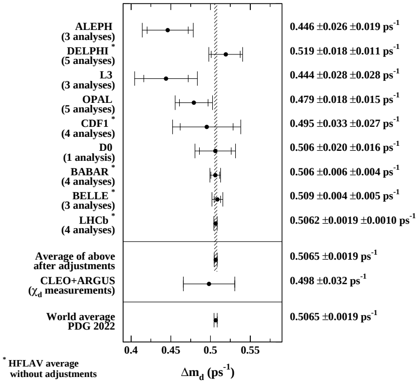
- CKM unitarity triangle [CKMfitter Spring 2021]
- see also UTfit [UTfit Summer 2022]
- B_q mixing constrains $\Delta m_d, \Delta m_s, \sin 2\beta$

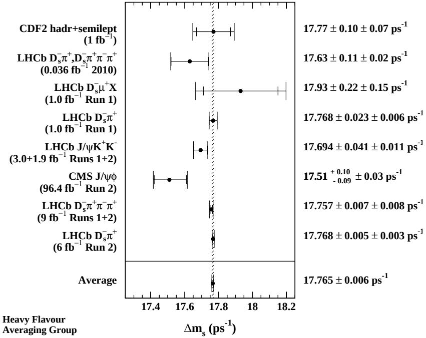
- NP fit agrees with SM at $\sim 1\sigma$ level

  [Charles et al., Phys.Rev.D 102 (2020) 5, 056023]

- See talk by Luiz Vale Silva [Mon 18/9, 9:00]

Experimental results, HFLAV 2021 [Phys.Rev.D 107 (2023) 5]



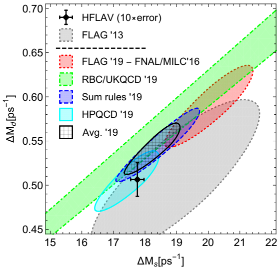


$$\Delta m_d = 0.5065(19)\mathrm{ps}^{-1}$$

$$\Delta m_s = 17.765(6)\mathrm{ps}^{-1}$$

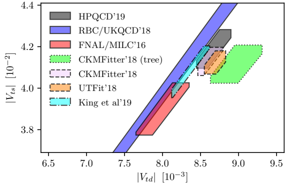
B_q MIXING - LATTICE

- current tension between Δm_d, Δm_s lattice determinations
  - FNAL/MILC '16 is in tension with experiment
  - HPQCD '19 is compatible with experiment
  - RBC/UKQCD '18 result still missing renormalization factors
  - theory uncertainty dominates experimental one



[Di Luzio et al., JHEP 12 (2019) 009]

- similar picture in $|V_{td}|$, $|V_{ts}|$
  - lattice results in slight tension, but all compatible with sum-rules (King et al. '19)
  - unitarity-triangle fits favour HPQCD '19 result



[HPQCD 19, Phys. Rev. D 100, 094508]
[King et al. 19, JHEP 05 (2019) 034]

$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle = \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} + \langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{LD}$$
$$= \langle B_q^0|\mathcal{H}_W^{\Delta B=2}|\bar{B}_q^0\rangle + \sum_n \frac{\langle B_q^0|\mathcal{H}_W^{\Delta B=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta B=1}|\bar{B}_q^0\rangle}{M_{B_q} - E_n}$$

short-distance contribution:

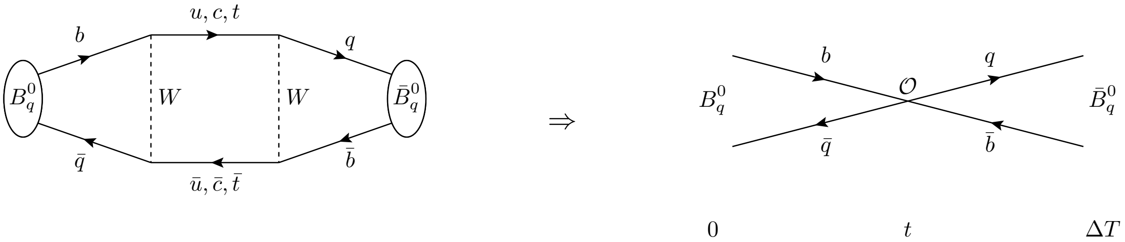
- t-loop enhancement
- additional CKM hierachy enhancement

$$\langle B_q^0|\mathcal{H}_W^{eff}|\bar{B}_q^0\rangle_{SD} \sim \left(\sum_{q'=u,c,t} V_{q'q}^* V_{q'b} S_0(m_{q'}^2/M_W^2)\right)^2$$

long-distance contribution:

- CKM-suppressed

B_q-mixing dominated by short-distance contribution



- $\Delta B = 2$ process
- enhanced by top quark $\Rightarrow$ short-distance dominated
- OPE shrinks box diagram to local four-quark operator

$$\langle \bar{B}_q^0 | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_q^0 \rangle \sim \langle \bar{B}_q^0 | \mathcal{O}_i | B_q^0 \rangle$$

- 5 parity-even, dimension 6, $\Delta B = 2$ operators $\mathcal{O}_i$

- bag parameters $\mathcal{B}$ give access to mass splittings Δm

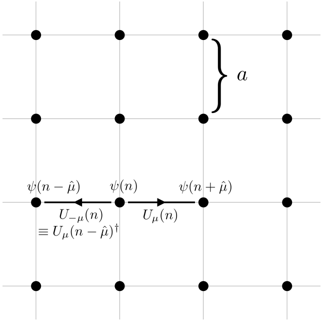
$$\mathcal{B}_{B_q}^{[i]} = \frac{\langle \bar{B}_q^0 | \mathcal{O}_i | B_q \rangle}{\langle \bar{B}_q^0 | \mathcal{O}_i | B_q \rangle_{\text{VSA}}}$$

$$\Delta m_q = |V_{td} V_{tq}^*|^2 \mathcal{K} M_{B_q} f_{B_q}^2 \mathcal{B}_{B_q}^{[1]}$$

- $\mathcal{K}$ known (perturbative)
- $M_{B_q}, f_{B_q}, \mathcal{B}_{B_q}^{[i]}$ non-perturbatively from lattice QCD
- Δm_q as input $\Rightarrow |V_{tq}|$ (or other way round)
- Additional $\mathcal{B}_{B_q}^{[i]}$ give access to $\Delta\Gamma_q$ and constrain various BSM models

Lattice QCD: method to compute correlation functions non-perturbatively and from first principles

- Discrete, finite Euclidean space-time grid
  - quark fields ψ on sites n
  - gluons U_μ as gauge links
  - finite lattice spacing a (UV regulator)
  - finite volume L, T (IR regulator)
- Path integral



$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int dU d\psi d\bar{\psi}\, \mathcal{O}[U, \psi, \bar{\psi}] e^{-S[U,\psi,\bar{\psi}]}$$

- even relatively small grids have size $\Lambda = (L/a) \times (T/a) = 24^3 \times 48$
  - exact evaluation prohibitively expensive

$\Rightarrow$ stochastic sampling of ensembles

We need to control on each ensemble

- light-quark discretisation effects $\Rightarrow M_\pi L \gtrsim 4$
- heavy-quark discretisation effects am_h

Two approaches for heavy quark:

effective theories

- allow expansion in $1/am_b$
- truncation at some order
- not easily improvable

method:

- Relativistic action (HQET, RHQ, Fermilab method)
- Nonrelativistic QCD (NRQCD)

fully relativistic

- $am_h \ll 1$ needed

$\Rightarrow$ fine lattice spacing for am_b^{phys}

- improvable with finer, larger boxes

method:

- extrapolation $am_h \to am_b$ for multiple $am_h < am_b$
- today impossible to reach am_l^{phys}, am_b^{phys} simultaneously

Full recipe

2pt-functions

$$\langle B_q(t)B_q^\dagger(0)\rangle_{L,a,m_l,m_h} \Rightarrow M_{B_q}(L,a,m_l,m_h), f_{B_q}(L,a,m_l,m_h)$$

3pt-functions

$$\langle B_q(\Delta T)\mathcal{O}_i(t)B_q^\dagger(0)\rangle_{L,a,m_l,m_h} \Rightarrow \mathcal{B}_{B_q}^{[i]}(L,a,m_l,m_h)$$

Leading to

$$\Delta m_q = |V_{td}V_{tq}^*|^2 \mathcal{K} \lim_{a\to 0}\lim_{L\to\infty}\lim_{m_l\to m_l^p}\lim_{m_h\to m_h^p}(M_{B_q}f_{B_q}^2\mathcal{B}_{B_q}^{[1]})(L,a,m_l,m_h)$$

or more precise results for

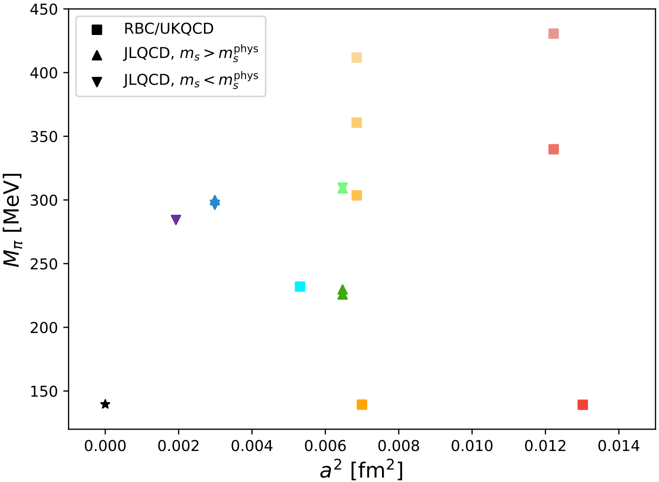
$$\xi^2 = \frac{f_{B_s}^2\mathcal{B}_{B_s}^{[1]}}{f_{B_d}^2\mathcal{B}_{B_d}^{[1]}} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{M_{B_d}}{M_{B_s}}\frac{\Delta m_s}{\Delta m_d}$$

RBC/UKQCD:

- 8 ensembles
- 3 lattice spacings $a = 0.073 - 0.11\text{fm}$
- two ensembles at physical point M_π^{phys}

JLQCD:

- 7 ensembles
- 3 lattice spacings $a = 0.044 - 0.081\text{fm}$
- one pair of ensembles with $M_\pi L \sim 3$ and $M_\pi L \sim 4$



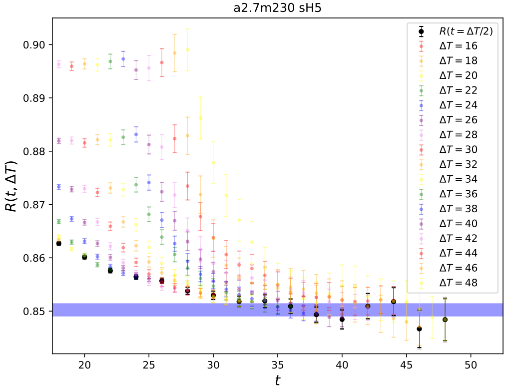
[PoS LATTICE2021 (2022) 224]

Fits to lattice correlation functions

- this projects includes $B_{(s)}$ mixing, leading to:
- 15 ensembles
- 5 operators
- 4-6 heavy-quark masses per ensemble
- heavy-light and heavy-strange sector

$\Rightarrow$ over 700 combined fits

- multiple values for ΔT to control fits better
- two independent analyses by FE and J.T. Tsang

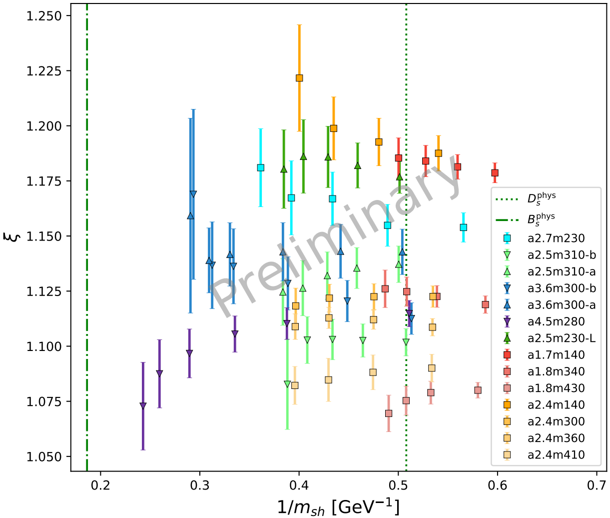


- Example of combined correlated fit to heaviest heavy-strange meson on ”a2.7m230” ensemble

Mixing ratios ξ

- update of RBC/UKQCD work

  [Boyle et al., arxiv 1812.08791]
- includes JLQCD ensembles
- completely new, fully correlated fitting strategy
- cancellation of renormalisation constants
- relatively flat $1/m_{sh}$ dependence with improved reach towards m_b^{phys}
- we are currently investigating various global fits on the data

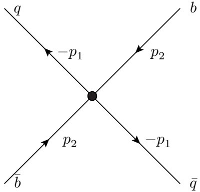


$$\langle \mathcal{O} \rangle_i^{S}(\mu) = \lim_{a^2 \to 0} \sum_{j=1}^{5} [Z_{\mathcal{O}}^{S}(a,\mu)]_{ij} \langle \mathcal{O} \rangle_j^{\text{bare}}(a)$$

for some regularisation independent scheme S at mass scale μ. Continuum perturbation theory can then match

$$\langle \mathcal{O} \rangle_i^{\overline{\text{MS}}}(\mu) = R^{\overline{\text{MS}} \leftarrow S} \langle \mathcal{O} \rangle_i^{S}(\mu)$$

We use the "RI-SMOM" scheme. Requires computation of four-quark vertices for $(\bar{b}q) \to (\bar{q}b)$. [Boyle et al., JHEP 10 (2017) 054]



DOMAIN-WALL FERMIONS

- we use "Domain-Wall Fermions"
  - automatic $O(a)$ improvement in absence of odd powers in a
  - $\Rightarrow$ reduced discretisation effects
  - chirally symmetric formulation
  - $\Rightarrow$ leads to simple mixing pattern of operators $\mathcal{O}_i$

$$\mathcal{O}_1 = \mathcal{O}^{VV+AA}$$
$$\mathcal{O}_2 = \mathcal{O}^{VV-AA}$$
$$\mathcal{O}_3 = \mathcal{O}^{SS-PP}$$
$$\mathcal{O}_4 = \mathcal{O}^{SS+PP}$$
$$\mathcal{O}_5 = \mathcal{O}^{TT}$$

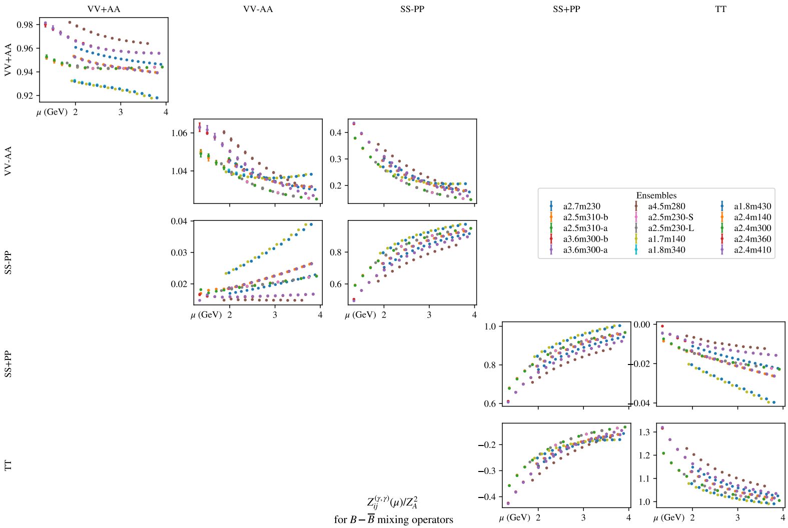
$$\begin{pmatrix} \mathcal{O}_1 & 0 & 0 \\ 0 & \begin{pmatrix} \mathcal{O}_{2/2} & \mathcal{O}_{2/3} \\ \mathcal{O}_{3/2} & \mathcal{O}_{3/3} \end{pmatrix} & 0 \\ 0 & 0 & \begin{pmatrix} \mathcal{O}_{4/4} & \mathcal{O}_{4/5} \\ \mathcal{O}_{5/4} & \mathcal{O}_{5/5} \end{pmatrix} \end{pmatrix}$$

Block-structure:

- $\mathcal{O}_2, \mathcal{O}_3$ as well as $\mathcal{O}_4, \mathcal{O}_5$ mix
- linearly independent from each other and from $\mathcal{O}_1$
- more complicated mixing pattern for other lattice fermions

[Boyle et al., JHEP 10 (2017) 054]

Non-perturbative renormalisation - full matrix



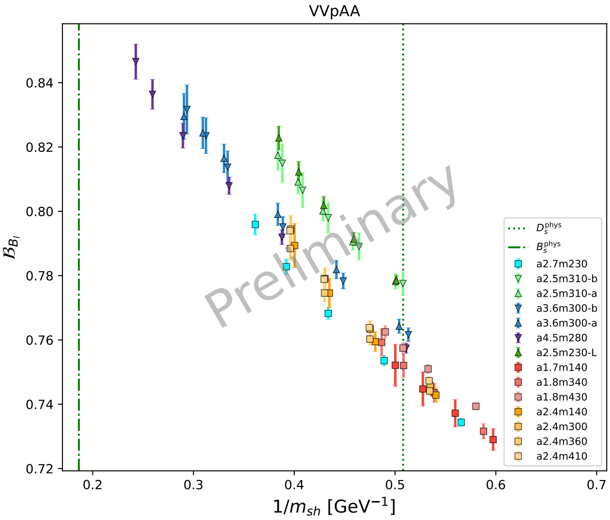
plot and work by Rajnandini Mukherjee (University of Southampton)

Bag parameter $\mathcal{B}_{hl}$ - $VV + AA$

- heavy-light bag parameters, renormalised at mass scale μ

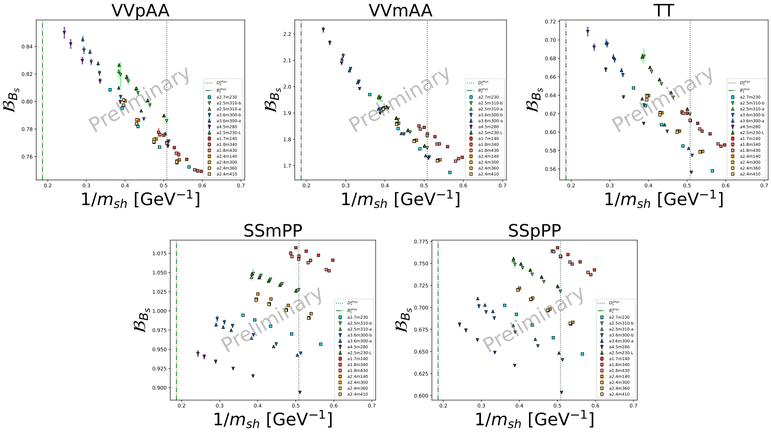
$\Rightarrow$ matching to continuum scheme still to do!

- discretisation effects for O_1 are small
- global fits also to renormalised bag parameters are investigated



Bag parameter $\mathcal{B}_{hs}$ - all 5 operators

- heavy-strange bag parameters, renormalised at mass scale μ
- O_1, O_2: mild a^2 dependence
- O_3, O_4: strong a^2 dependence
- O_5: medium a^2 dependence and curvature in $1/m_{sh}$
- very similar for heavy-light sector



OUTLOOK: FIT STRATEGY

We are exploring various parametrisations for a global fit to:

$$B = B(0)\left[1 + f_J^{disc}(a^2) + f_{R/U}^{disc}(a^2) + f^{chir}(M_\pi^2) + f^s(2M_K^2 - M_\pi^2) + f^b(1/M_{hs})\right]$$

with

- $f_J^{disc}(a^2)$: discretisation $a^2 \to 0$, separate trajectories for RBC/UKQCD and JLQCD ensembles
- $f^{chir}(M_\pi^2)$: chiral extrapolation $M_\pi^2 \to (M_\pi^{phys})^2$
- $f^s(2M_K^2 - M_\pi^2)$: strange-quark extrapolation to physical $(2M_K^2 - M_\pi^2)^{phys}$
- $f^b(1/M_{hs})$: heavy-quark extrapolation $1/M_{hs} \to 1/M_{B_s}$
- additional terms? higher powers?

$\Rightarrow$ current investigation

Conclusions

- B_q-mixing $\Delta B = 2$ bag parameters with fully relativistic heavy-quark action
- data for full 5-operator basis available
- 15 ensembles, 6 lattice spacings from 2 collaborations, including two ensembles at M_π^{phys}
- global fits are being worked on
- simple renormalisation for chiral Domain-Wall Fermions
- fully relativistic treatment of heavy-quark
- very fine lattice spacings
- large variety of ensembles to control relevant limits
- programme extends to D-mixing and K-mixing

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Backup

LATTICE SETUP

- RBC-UKQCD's 2+1 flavour domain wall fermions [Blum et al. Phys.Rev.D 93 (2016) 7]
  - pion masses from $M_\pi = 139$ MeV to $M_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5m_b$, using a stout-smeared action ($\rho = 0.1, N = 3$) with $M_5 = 1.0$, $L_s = 12$ and Möbius-scale $= 2$ [Boyle et al. arxiv:1812.08791]
  - light and strange quarks: sign function approximated via:
    - Shamir approximation for heavier pion masses
    - Möbius approximation at M_π^{phys} and on the finest ensemble
- JLQCD's 2+1 flavour domain wall fermions [Kaneko et al. EPJ Web Conf. 175 (2018) 13007]
  - pion masses from $M_\pi = 226$ MeV to $M_\pi = 310$ MeV
  - heavy-quark masses from m_c nearly up to m_b, using the same stout-smeared action.
  - light and strange quarks use the same action as the heavy quarks.

LATTICE SETUP

	L/a	T/a	a^{-1} [GeV]	M_π [MeV]	$M_\pi L$	hits $\times N_{conf}$	collaboration id
a1.7m140	48	96	1.730(4)	139.2	3.9	48 × 90	R/U C0
a1.8m340	24	64	1.785(5)	339.8	4.6	32 × 100	R/U C1
a1.8m430	24	64	1.785(5)	430.6	5.8	32 × 101	R/U C2
a2.4m140	64	128	2.359(7)	139.3	3.8	64 × 82	R/U M0
a2.4m300	32	64	2.383(9)	303.6	4.1	32 × 83	R/U M1
a2.4m360	32	64	2.383(9)	360.7	4.8	32 × 76	R/U M2
a2.4m410	32	64	2.383(9)	411.8	5.5	32 × 81	R/U M3
a2.5m230-L	48	96	2.453(4)	225.8	4.4	24 × 100	J C-ud2-sa-L
a2.5m230-S	32	64	2.453(4)	229.7	3.0	16 × 100	J C-ud2-sa
a2.5m310-a	32	64	2.453(4)	309.1	4.0	16 × 100	J C-ud3-sa
a2.5m310-b	32	64	2.453(4)	309.7	4.0	16 × 100	J C-ud3-sb
a2.7m230	48	96	2.708(10)	232.0	4.1	48 × 72	R/U F1M
a3.6m300-a	48	96	3.610(9)	299.9	3.9	24 × 50	J M-ud3-sa
a3.6m300-b	48	96	3.610(9)	296.2	3.9	24 × 50	J M-ud3-sb
a4.5m280	64	128	4.496(9)	284.3	4.0	32 × 50	J F-ud3-sa

List of ensembles used in this work. For consistency of naming conventions in our set of ensembles from two collaborations, we introduce a shorthand notation in the first column which is used throughout this work. The last

For other neutral mesons $M^0 \in \{K, D, B_q\}$

$$\begin{aligned}\langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle &= \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{SD} + \langle M^0|\mathcal{H}_W^{eff}|\bar{M}^0\rangle_{LD} \\ &= \langle M^0|\mathcal{H}_W^{\Delta F=2}|\bar{M}^0\rangle + \sum_n \frac{\langle M^0|\mathcal{H}_W^{\Delta F=1}|n\rangle\langle n|\mathcal{H}_W^{\Delta F=1}|\bar{M}^0\rangle}{M_M - E_n}\end{aligned}$$

short-distance contribution:

- t enhancement for K, $B_{(s)}$
- additional CKM hierachy enhancement for $B_{(s)}$
- sub-dominant for D, but ok to describe CP-violating contributions

long-distance contribution:

- relevant but smaller than short-distance for K
- dominant for D
- CKM-suppressed for $B_{(s)}$