Towards extracting γ from $B \to DK$ without binning

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Extracting γ from $B \to DK$

[Some history: 2006.12404]

- $B^\pm \to DK^\pm$: underlying quark decays $b \to c\bar{u}s$ and $b \to u\bar{c}s$. [Bigi Sanda 1988]
- Interference of diagrams gives access to γ.
- Current world average: $\gamma = (65.9^{+3.3}_{-3.5})^\circ$ [PDG]
- Ultimate theory error very small. [Brod Zupan 2014]
- Interference of decay modes through D decay into common final state:

$$B^\pm \to D^0 K^\pm\,, \quad B^\pm \to \overline{D}^0 K^\pm\,, \quad B^\pm \to D_{CP} K^\pm$$

- Neglecting D^0–$\overline{D}^0$ mixing: $D_{CP} = \frac{1}{\sqrt{2}}\left(D^0 + \overline{D^0}\right)$.

CP Violation is an interference effect

- Interference for example through:

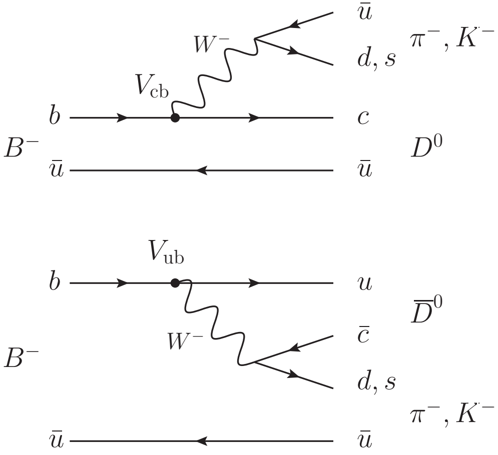
$$B^+ \to \bar{D}^0 K^+ \to K^+ K^- K^+ \, ,$$
$$B^+ \to D^0 K^+ \to K^+ K^- K^+ \, .$$

- Relative weak phase:

$$\arg\left(\frac{V_{ub}^* V_{us}}{V_{cb}^* V_{cs}}\right) \approx -\gamma + \mathcal{O}(\lambda^4)$$

- Challenge for any method using $B \to DK$:

$$r_B = \frac{|\mathcal{A}(B^- \to \overline{D}^0 K^-)|}{|\mathcal{A}(B^- \to D^0 K^-)|} \approx 0.1 \, .$$



[LHCb 2110.02350]

Classification of different methods

GLW method

[Gronau London 1991, Gronau Wyler 1991]

- D decay to CP eigenstate, such as K^+K^-, $\pi^+\pi^-$, $K_S^0\pi^0$.

ADS method

[Atwood Dunietz Soni 1996]

- D decay to non-CP eigenstates: $D \to K^\mp\pi^\pm$.

BPGGSZ method

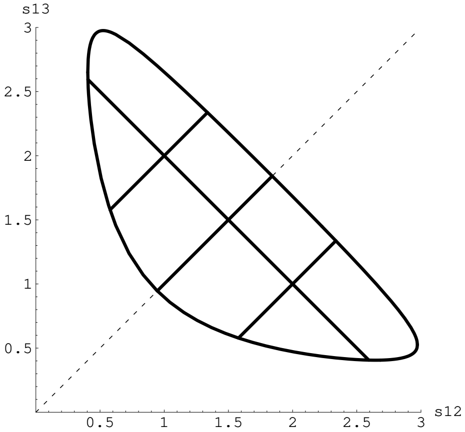
[Bondar 2002, Giri Grossman Soffer Zupan 2003, Poluektov & Belle 2004]

- D decays into CP self-conjugate three-body final state, like $K_S\pi^-\pi^+$

BPGGSZ

[Bondar 2002, Giri Grossman Soffer Zupan 2003, Poluektov & Belle 2004]

- Exploit that D^0 and $\overline{D^0}$ may have large $\mathcal{B}$ to CP self-conjugate 3-body states.
- Analysis can be optimized by examining Dalitz plot dependence of interference.



[GGSZ hep-ph/0303187]

Going further

- Issue: BPGGSZ requires binning of D Dalitz plot.
- Therefore: Dependence on bin-averaged D decay amplitude/phase.
- Amplitude/phase vary within bin, even with optimized bins.
- Worth to investigate if procedure without binning could work even better.

Unbinned γ is exciting! Development of new methodologies

- Replacing binning by Fourier transform of phase space variables. [Poluektov 2017]
- γ as parameter that brings two functions of empirical cumulative probability distributions into agreement across whole Dalitz plot. [Backus Freytsis Grossman StS Zupan 2022]
- Quasi model independent: Correct phase of amplitude model in unbinned, model-ind. way. [Lane, E. Gersabeck, Rademacker 2023]

Towards an unbinned method for γ [Backus Freytsis Grossman StS Zupan 2022]

- Like BPGGSZ, use $B^\pm \to DK^\pm \to (K_S\pi^-\pi^+)_D K^\pm$.
- Key idea: Build optimized observables by (anti-)symmetrizing reduced decay width.
  - With respect to $s_{12} = s_{13}$ axis of Dalitz plot:

$$d\Sigma_\pm(s_{12}, s_{13}),\ d\Delta_\pm(s_{12}, s_{13}) \equiv \frac{d\widehat{\Gamma}_\pm(s_{12}, s_{13}) \pm d\widehat{\Gamma}_\pm(s_{13}, s_{12})}{2}\,.$$

  - And, additionally, with respect to B meson charge:

$$d\Sigma_{S,A}(s_{12}, s_{13}) \equiv \frac{d\Sigma_+(s_{12}, s_{13}) \pm d\Sigma_-(s_{12}, s_{13})}{2}\,,$$

$$d\Delta_{S,A}(s_{12}, s_{13}) \equiv \frac{d\Delta_+(s_{12}, s_{13}) \pm d\Delta_-(s_{12}, s_{13})}{2}\,.$$

Smart combinations of (anti-)symmetrized observables

- Smart ratios depend on r_B, δ_B and γ and are constant across the Dalitz plot.

$$\left.\frac{\mathrm{d}\Sigma_S}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}}\right|_{\mathrm{sub}} \Big/ \frac{\mathrm{d}\Sigma_A}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}} = -\cot\delta_B\cot\gamma\,,$$

$$\frac{\mathrm{d}\Delta_A}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}}_{\mathrm{sub}} \Big/ \frac{\mathrm{d}\Delta_S}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}} = \tan\delta_B\cot\gamma\,,$$

$$\left(\left.\frac{\mathrm{d}\Sigma_S}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}}\right|_{\mathrm{sub}} \left.\frac{\mathrm{d}\Delta_A}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}}\right|_{\mathrm{sub}}\right) \Big/ \left(\frac{\mathrm{d}\Sigma_A}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}}\frac{\mathrm{d}\Delta_S}{\mathrm{d}s_{12}\,\mathrm{d}s_{13}}\right) = -\cot^2\gamma\,.$$

➥ Basis for unbinned method of extracting γ.

In practice, need cumulative reduced partial decay widths

- Example: $R_\pm(s_{12}, s_{13}) \equiv \int_0^{s_{12}} \mathrm{d}s'_{12} \int_0^{s_{13}} \mathrm{d}s'_{13} \frac{\mathrm{d}\hat{\Gamma}_\pm}{\mathrm{d}s'_{12}\,\mathrm{d}s'_{13}}$.
- In practice, construct R simply by counting events.
- Linearity of integration: Pointwise relations apply also to cumulative observables:

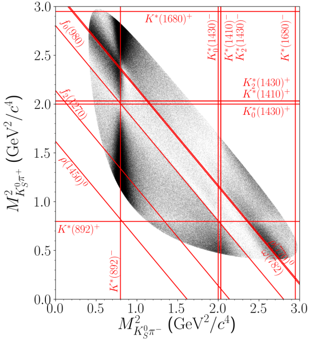
$$\frac{R_{\Sigma S}|_{\mathrm{sub}}}{R_{\Sigma A}} = -\cot\delta_B \cot\gamma \qquad \frac{R_{\Delta A}|_{\mathrm{sub}}}{R_{\Delta S}} = \tan\delta_B \cot\gamma,$$

$$\left(\frac{R_{\Sigma S}|_{\mathrm{sub}}}{R_{\Sigma A}}\right)\left(\frac{R_{\Delta A}|_{\mathrm{sub}}}{R_{\Delta S}}\right) = -\cot^2\gamma.$$

- Recast problem of measuring γ as finding optimal values for γ, δ_B, r_B such that above relations between distribution holds across the Dalitz plot.
- At optimal point, $R_{\Delta A}|_{\mathrm{sub}}$ and $-R_{\Delta S}\tan\delta_B\cot\gamma$ should be same distribution.

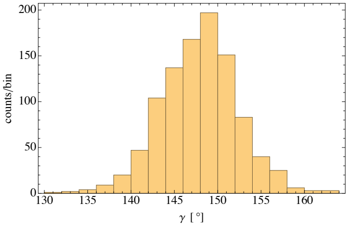
Implementation in toy study

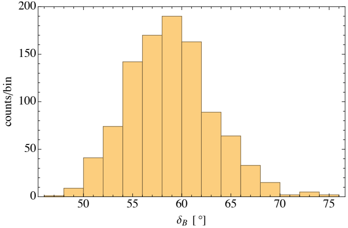
- Implementation using measure adapted from Kolmogorov-Smirnov test.
- Optimization of binning corresponds to choosing optimal test statistics.
  E.g.: Different ways of integrating over Dalitz plot for defining cumulative distribution.
- Generation of toy Monte Carlo Dalitz plots based on amplitude model for $D \to K_S \pi^- \pi^+$ from [BaBar & Belle: 1804.06153].



Proof of Principle

- Check input = output for toy example, based on generated Monte Carlo points.
- Unbinned strategy not yet optimized, but has potential to be competitive with BPGGSZ.
- Presently unclear if our method can provide a superior statistical error.
- Each approach requires different kind of statistical optimization.
- Here: Required optimization = choice of test statistic: Variants of cumulative distribution.





Variable	Input	Output
γ	$150°$	$(148^{+3}_{-5})°$
r_B	0.9	$0.90^{+0.01}_{-0.01}$
δ_B	$60°$	$(59^{+4}_{-4})°$

Conclusions

- New, model-independent, unbinned method for γ from $B^\pm \to DK^\pm \to (K_S\pi^-\pi^+)_D K^\pm$.
- No optimization of auxiliary variables that specify the analysis (such as shapes of bins).
- Effectiveness of binned methods depends on choice of the binning.
- Effectiveness of unbinned method depends on choice of integration ordering for cumulative functions.
- Not clear if binned or unbinned methods will ultimately give most competitive results.
- Proof of principle for toy data demonstrated.
- Future work is needed to optimize test statistic.
- Competitiveness of the unbinned method could be greatly enhanced in the future by extending this approach to include data from correlated charm decays.