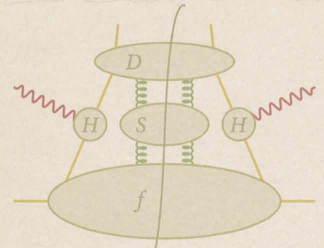
Feynman diagrams Lagrangians and stuff



igure 8.11: Factorisation in SIDIS: the bull diagram. All IR divergences are absorbed in the soft factor S, that hence only interacts with the TMD and FF. Note that

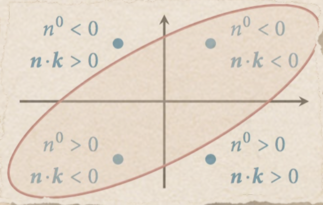


Figure 2.1: As a parallel transporter transforms in function of its path endpoints only, all paths shown will give rise to equivalent $\mathcal{U}_{(y;x)}$'s, shifting a field at x to a field at y.

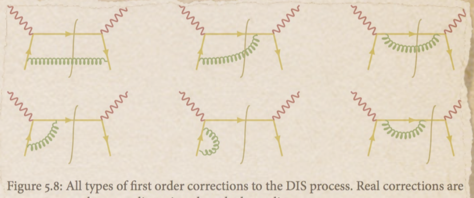


Figure 5.8: All types of first order corrections to the DIS process. Real corrections are on the upper line; virtual on the lower line.

The Mikowskian loop integrals are then the same as the Euclidian ones, up to a possible sign difference:

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{1}{(k^2-\Delta)^n} = i \frac{(-)^n}{(4\pi)^{\frac{\omega}{2}}} \frac{\Gamma\left(n-\frac{\omega}{2}\right)}{\Gamma(n)} \Delta^{\frac{\omega}{2}-n}, \tag{B.25a}$$

$$\begin{pmatrix} d \geq 2n \\ d \text{ even} \end{pmatrix} = i \frac{\Delta^{\frac{d}{2}-n}}{(4\pi)^{\frac{d}{2}}} \frac{(-)^{\frac{d}{2}}}{(n-1)!\left(\frac{d}{2}-n\right)!} \left(\frac{1}{\epsilon} - \gamma_E + \sum^{\frac{d}{2}-n} \frac{1}{j} + \ln 4\pi - \ln \Delta\right),$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{k^2}{(k^2-\Delta)^n} = i \frac{(-)^{n+1}}{(4\pi)^{\frac{\omega}{2}}} \frac{\omega}{2} \frac{\Gamma\left(n-\frac{\omega}{2}-1\right)}{\Gamma(n)} \Delta^{\frac{\omega}{2}+1-n}, \tag{B.25b}$$

$$\begin{pmatrix} d \geq 2n-2 \\ d \text{ even} \end{pmatrix} = i \frac{\Delta^{\frac{d}{2}+1-n}}{(4\pi)^{\frac{d}{2}}} \frac{\omega}{2} \frac{(-)^{\frac{d}{2}}}{(n-1)!\left(\frac{d}{2}+1-n\right)!} \left(\frac{1}{\epsilon} - \gamma_E + \sum^{\frac{d}{2}+1-n} \frac{1}{j} + \ln 4\pi - \ln \Delta\right)$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{k^4}{(k^2-\Delta)^n} = i \frac{(-)^n}{(4\pi)^{\frac{\omega}{2}}} \frac{\omega(\omega+2)}{4} \frac{\Gamma\left(n-\frac{\omega}{2}-2\right)}{\Gamma(n)} \Delta^{\frac{\omega}{2}+2-n}, \tag{B.25c}$$

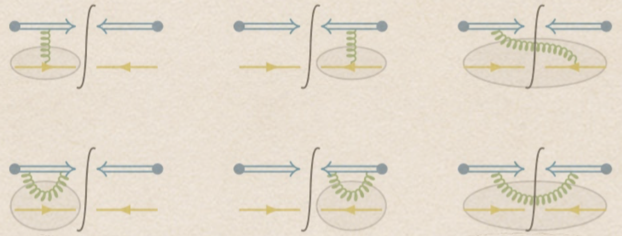
$$\begin{pmatrix} d \geq 2n-4 \\ d \text{ even} \end{pmatrix} = i \frac{\Delta^{\frac{d}{2}+2-n}}{(4\pi)^{\frac{d}{2}}} \frac{\omega(\omega+2)}{4} \frac{(-)^{\frac{d}{2}}}{(n-1)!\left(\frac{d}{2}+2-n\right)!} \left(\frac{1}{\epsilon} - \gamma_E + \sum^{\frac{d}{2}+2-n} \frac{1}{j} + \ln 4\pi - \ln$$

We list some other common Minkowskian integrals:

$$\int \frac{d^\omega k}{(2\pi)^\omega} \ln(k^2-a) = -\frac{i}{(4\pi)^{\frac{\omega}{2}}} \Gamma\left(-\frac{\omega}{2}\right) a^{\frac{\omega}{2}}, \tag{B.26a}$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} e^{ak^2 - ib\cdot k} = \frac{i}{(4\pi)^{\frac{\omega}{2}}} a^{-\frac{\omega}{2}} e^{\frac{b^2}{4a}}, \tag{B.26b}$$

$$\int \frac{d^\omega k}{(2\pi)^\omega} \frac{1}{(-k^2)^\alpha} e^{-ib\cdot k} = \frac{i}{4^\alpha \pi^{\frac{\omega}{2}}} \frac{\Gamma\left(\frac{\omega}{2}-\alpha\right)}{\Gamma(\alpha)} \frac{1}{(-b^2)^{\frac{\omega}{2}-\alpha}}. \tag{B.26c}$$



$$\oint_C dx \cdot A = \int_\Sigma d\sigma \cdot (\partial \wedge A)$$

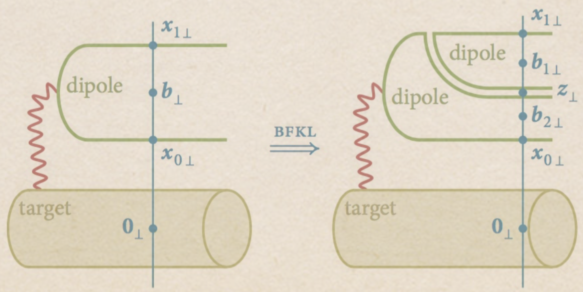


Figure 9.6: In the dipole picture, the BFKL evolution is an evolution in dipoles, i.e. new dipoles are created during the evolution. A gluon that is radiated from the dipole can be represented as two fundamental lines (see Equation 10.13). This essentially splits the dipole in two at the point $z_\perp$, as is illustrated in the second diagram.

$$\mathrm{tr}(t^a t^x t^b t^x) = -\frac{1}{4N_c}\delta^{ab},$$

$$\mathrm{tr}(t^b t^x t^y) f^{ayx} = -i\frac{N_c}{4}\delta^{ab},$$

$$\mathrm{tr}(t^y t^z) f^{axy} f^{bzx} = -\frac{N_c}{2}\delta^{ab},$$

$$f^{xay} f^{ycz} f^{zbw} f^{wcx} = \frac{N_c^2}{2}\delta^{ab},$$

$$f^{avw} f^{xby} f^{ywz} f^{zvx} = \frac{N_c^2}{2}\delta^{ab},$$

$$f^{awv} f^{bzw} f^{xzy} f^{yvx} = N_c^2 \delta^{ab},$$

$$f^{xay} f^{ycz} f^{zbw} f^{wcx} = \frac{N_c^2}{2}\delta^{ab},$$

$$f^{vaw} f^{wbz} f^{xzy} f^{yvx} = N_c^2 \delta^{ab},$$

and similarly for the seven remaining diagrams.

Goals

¿ Lagrangian ?

¿ Symmetries ?

¿ Feynman diagrams ?

¿ Standard Model ?

Goals

This is not a regular lecture

→ feel free to interrupt
& ask questions

Feynman Diagrams

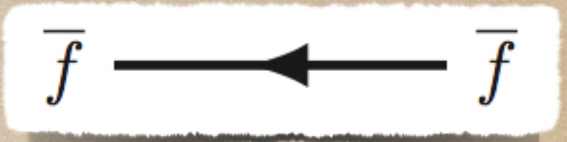
- Follow from perturbative expansion (see later)
- Particles are drawn on space-time plane
- Are an easy visual way to calculate elementary processes
- Order of diagram = number of vertices / 2
- Use Feynman rules to get mathematical expression

Feynman Diagrams

Different lines for different particle types:

- fermion (matter particle)

- antifermion (antimatter particle)



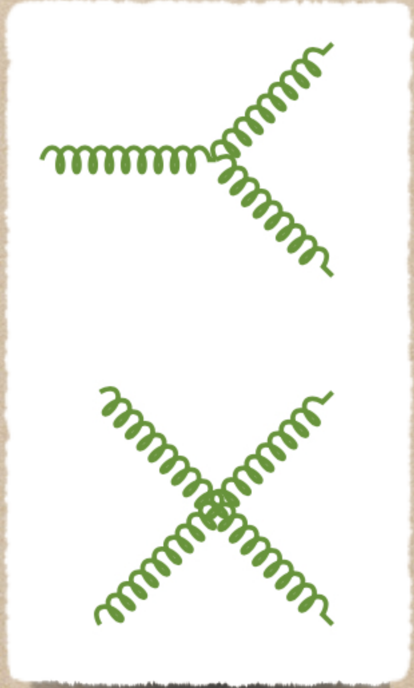
- photon

- gluon

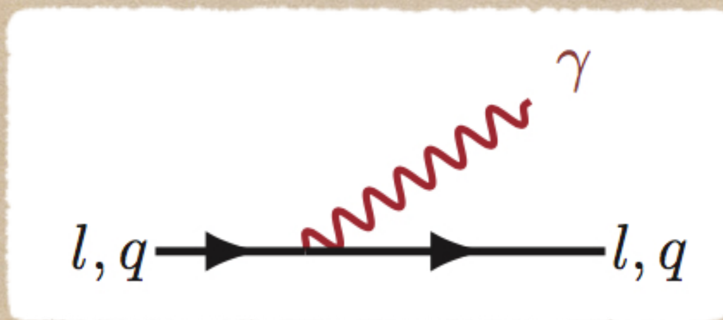
- weak boson

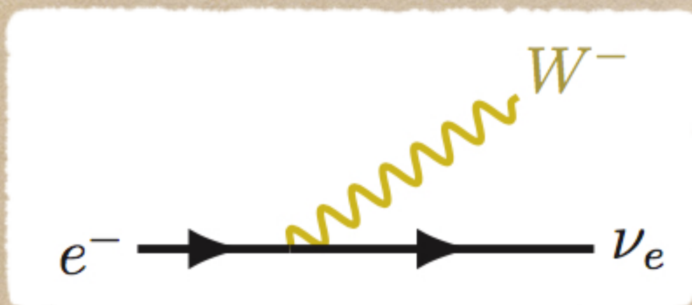
Feynman Diagrams

gluon self-interactions



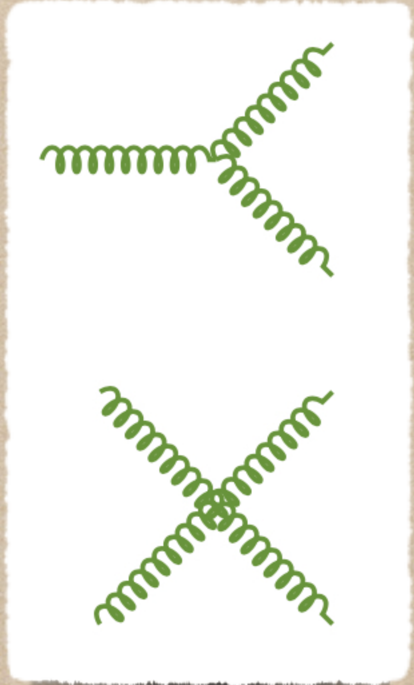
boson - fermion interactions



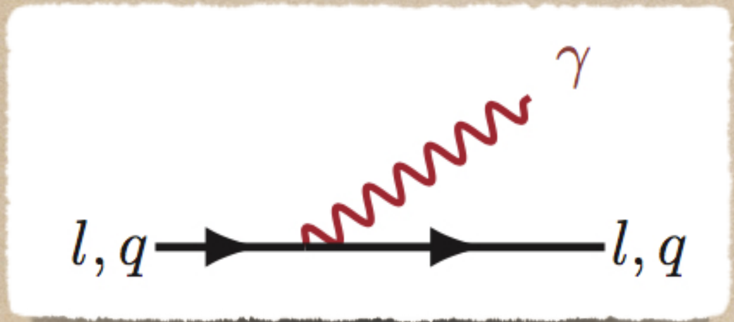


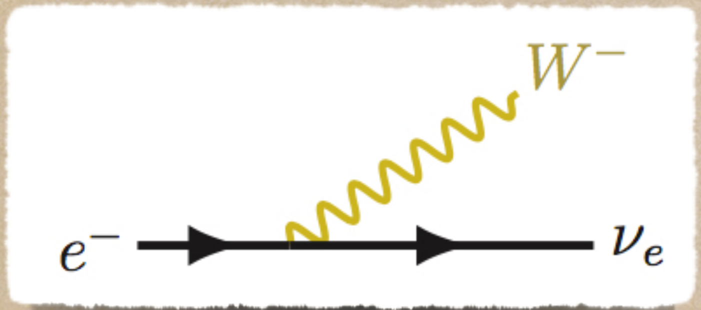
Feynman Diagrams

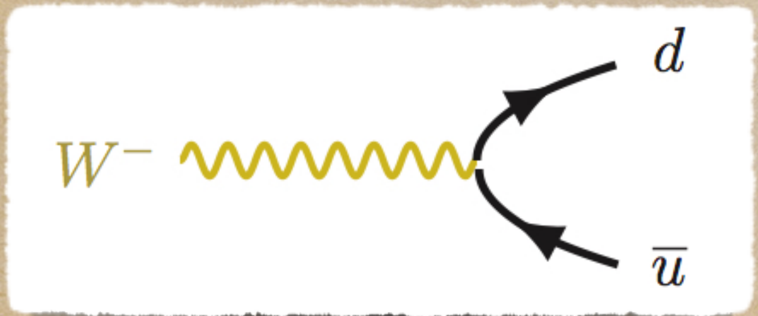
gluon self-interactions



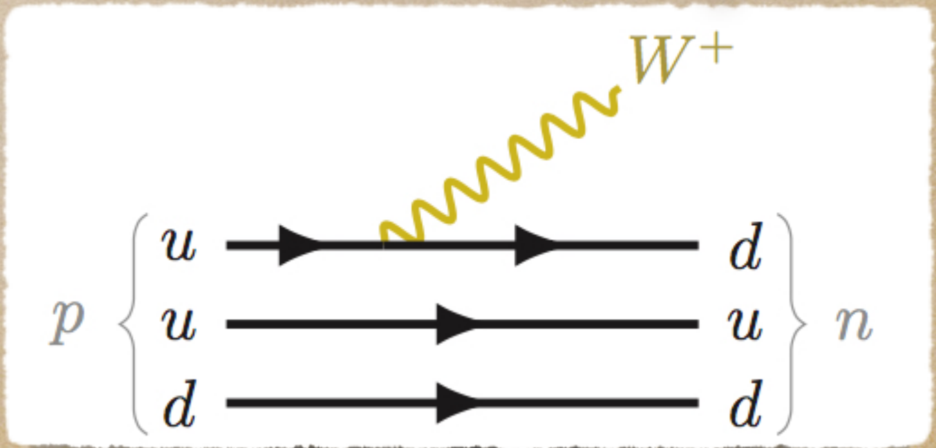
boson - fermion interactions







proton -> neutron decay



That was nice and stuff, but let's try to be rigorous and build from first principles...

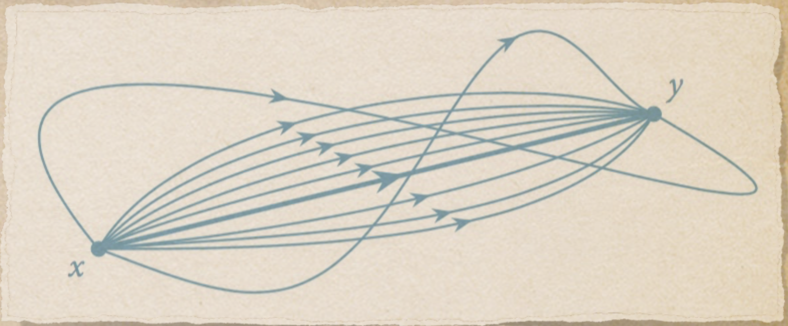
Symmetries:

Building the Lagrangian

Basics

- basic components are **fields**
  - just mathematical tools
  - will give rise to particles

- principal quantity is the **action**, which is the integral of the Lagrangian:



$$S = \int d^4x \, \mathcal{L}(x, \phi, \partial\phi)$$

  - all paths possible (simultaneous), but path with **least action** is favoured
  - minimising action leads to **equations of motion**

Lagrangian?

- is kinetic energy minus potential energy

$$\mathcal{L} = T - V$$

- classical example: spring

$$\mathcal{L} = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 - \frac{1}{2}kx^2 \quad \Rightarrow \quad x = x_0 \cos\sqrt{\frac{k}{m}}t$$

- field example: free electron field

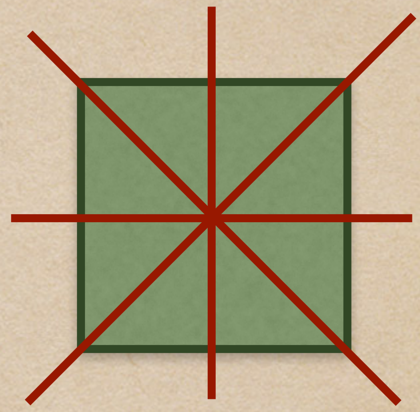
$$\mathcal{L} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi \quad \Rightarrow \quad \left(i\not{\partial} - m\right)\psi = 0 \quad \text{(QM)}$$

Lagrangian!

- kinetic terms are quadratic and have derivatives $\bar{\psi}\partial\!\!\!/\psi, \;\; \partial_\mu\phi\partial^\mu\phi, \;\; \ldots$

- potential terms are what is left
  - special type: mass terms: $m\bar{\psi}\psi, \;\; m^2|\phi|^2, \;\; \ldots$ quadratic without derivatives
  - others are interaction terms $\bar{\psi}A\!\!\!/\psi, \;\; \ldots$

Symmetries



- leave theory unchanged
- symmetry => conservation

  Emmy Noether

- homogeneity of space

  => translational invariance

  => momentum conservation
- isotropy of space

  => rotational invariance

  => angular momentum conservation

Symmetries

- in quantum mechanics, ψ is an amplitude
  - not physical
  - $|\psi|^2$ is probability, physical

- phase is undetermined, because we can scale $\psi \to e^{i\alpha}\psi$, then $\bar{\psi} \to e^{-i\alpha}\bar{\psi}$, such that $|\psi|^2 \to |\psi|^2$

=> invariant!

Symmetries

- similar in quantum field theories:

$$i\bar{\psi}\partial\!\!\!/\psi \to i\bar{\psi}\partial\!\!\!/\psi \qquad m\bar{\psi}\psi \to m\bar{\psi}\psi$$

in other words $\mathcal{L} = \mathcal{L}'$

- free electron field

=> conservation of electric charge

- BUT... phase α can depend on spacetime coordinates: $\alpha = \alpha(x)$

$$i\bar{\psi}\partial\!\!\!/\psi \to i\bar{\psi}\partial\!\!\!/\psi - \bar{\psi}(\partial\!\!\!/\alpha)\psi \qquad \mathcal{L}' = \mathcal{L} - \bar{\psi}(\partial\!\!\!/\alpha)\psi$$

=> no longer invariant!

Lagrangian

- add term $g\bar{\psi}\not{A}\psi$ to the Lagrangian
    - with property $A_\mu \to A_\mu + {}^1\!/_g\, \partial_\mu a$
    - because then $g\bar{\psi}\not{A}\psi \to g\bar{\psi}\not{A}\psi + \bar{\psi}(\not{\partial} a)\psi$
    - invariant !
- but new field also needs kinetic terms
    - symmetry => conserved tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

    - its square will be kinetic term:

$$-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

QED:

From the Lagrangian to a full theory

QED

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{QED} = \underbrace{i\bar{\psi}\partial\!\!\!/\psi}_{\text{electron kinetic term}} - \underbrace{m\bar{\psi}\psi}_{\text{electron mass term}} + \underbrace{g\bar{\psi}A\!\!\!/\psi}_{\text{electron-photon interaction term}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{photon kinetic term}}$$

QED

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{QED} = i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi + g\bar{\psi}A\!\!\!/\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

'free' theory

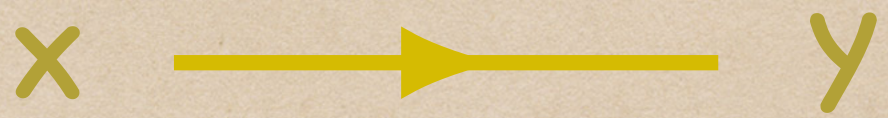
'interaction' theory

$$\mathcal{L}_{QED} = \mathcal{L}_0 + \mathcal{L}_I$$

Keep $\mathcal{L}_0$ exact, but expand $\mathcal{L}_I$ => perturbation theory

Free Theory

Propagation of an electron from x to y:



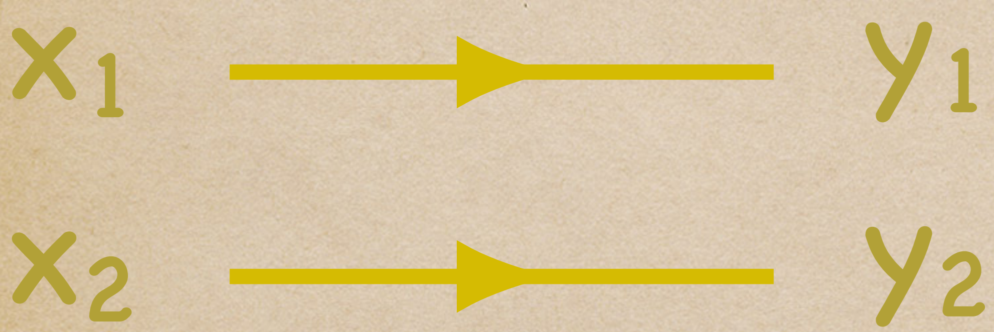
$$\langle 0|\psi(y)\bar{\psi}(x)|0\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \;\; \psi(y)\bar{\psi}(x)\, e^{iS_0}$$

Propagation of a photon from x to y:

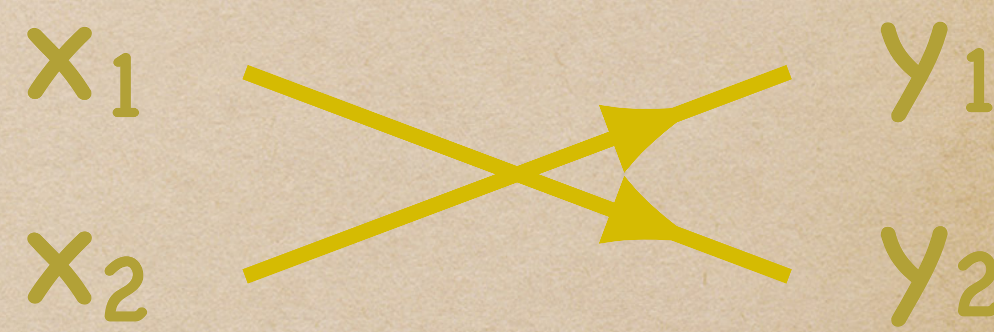
$$\langle 0|A_\mu(x)A_\mu(y)|0\rangle = \int \mathcal{D}A_\mu \;\; A_\mu(x)A_\mu(y)\, e^{iS_0}$$

Free Theory

Easily generalised to more points:



and



$$\langle 0|\psi_{y_1}\psi_{y_2}\bar{\psi}_{x_1}\bar{\psi}_{x_2}|0\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi \;\; \psi_{y_1}\psi_{y_2}\bar{\psi}_{x_1}\bar{\psi}_{x_2}\, e^{S_0}$$

Interaction

- Add interaction part from action (Lagrangian) to the exponential $e^{S_0+S_I}$
- Equations are not solvable anymore

=> expand interaction part:

$$e^{S_I} \approx 1 + S_I + \frac{1}{2}S_I^2 + \dots$$

- Propagation of electron from x to y is now:

$$\langle \psi(y)\bar{\psi}(x)\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu\, \psi(y)\bar{\psi}(x)\, e^{S_0}\left(1+S_I+\frac{1}{2}S_I^2+\dots\right)$$

Interaction

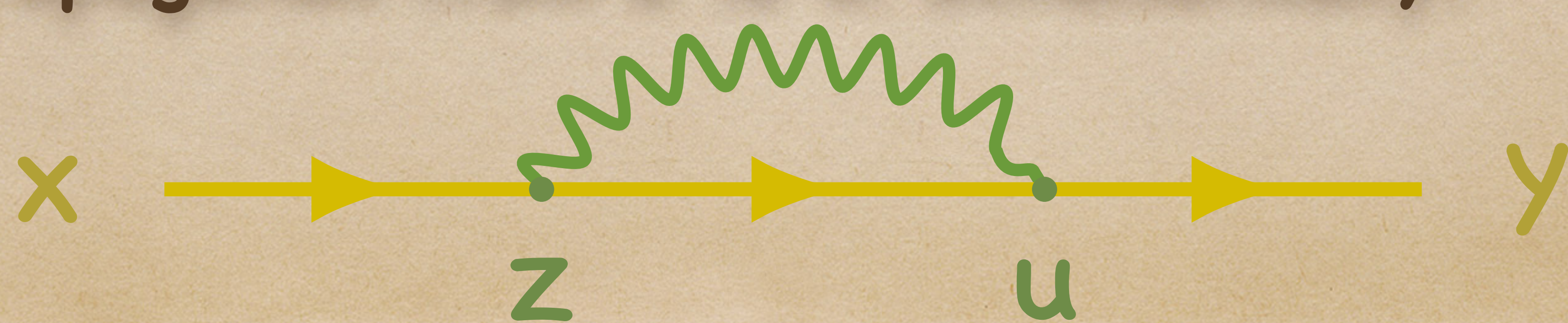
- Propagation of electron from x to y is now:

$$\langle\psi(y)\bar{\psi}(x)\rangle = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\mathcal{D}A_\mu \; \psi(y)\bar{\psi}(x)\, e^{S_0}\left(1 + S_I + \frac{1}{2}S_I^2 + \ldots\right)$$

- Take the second order as example:

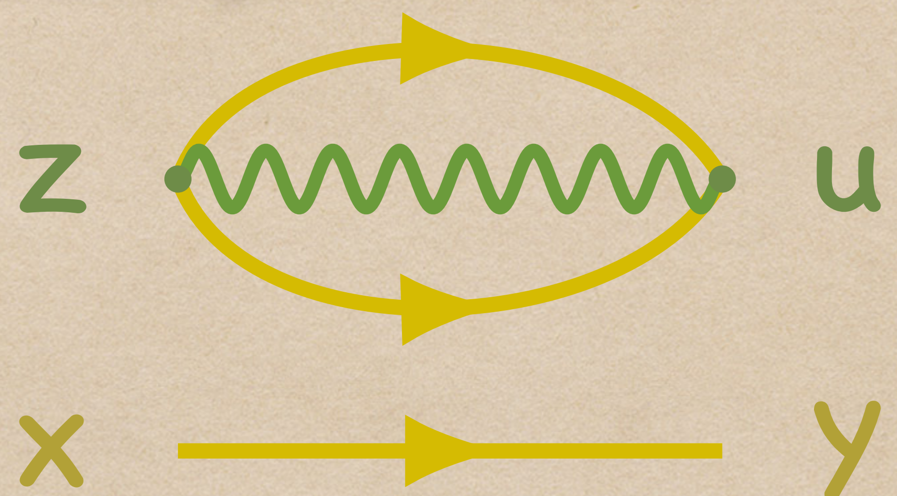
$$S_I^2 = g^2 \int dz\, du\, \left(\bar{\psi}\not{A}\psi\right)_z \left(\bar{\psi}\not{A}\psi\right)_u$$

- So we have electron propagation from x to z, from z to u, and from u to y. We also have photon propagation from z to u. Schematically:



Interaction

- Other possibility:



- Vacuum diagrams

  => These are unwanted and have to be cancelled:

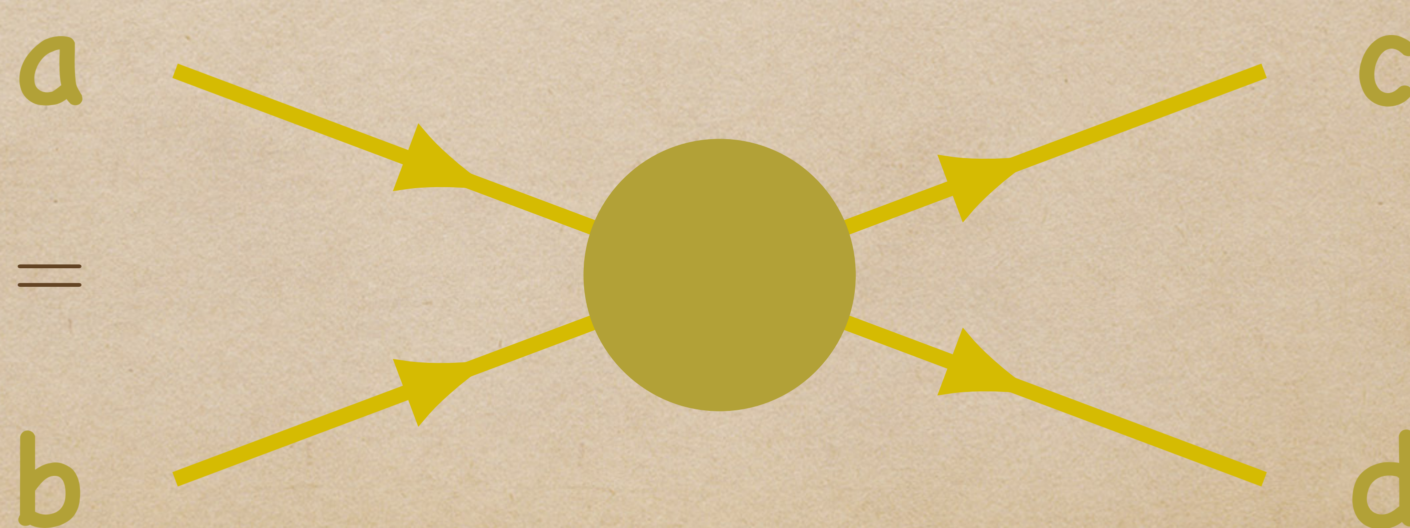
$$\langle \psi(y)\bar{\psi}(x)\rangle = \frac{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \;\; \psi(y)\bar{\psi}(x)\; e^{S}}{\int \mathcal{D}\bar{\psi}\mathcal{D}\psi \; e^{S}}$$

Interaction

- So the full propagator is:

- This can be extended to any number of particles, i.e. electron-electron collision:

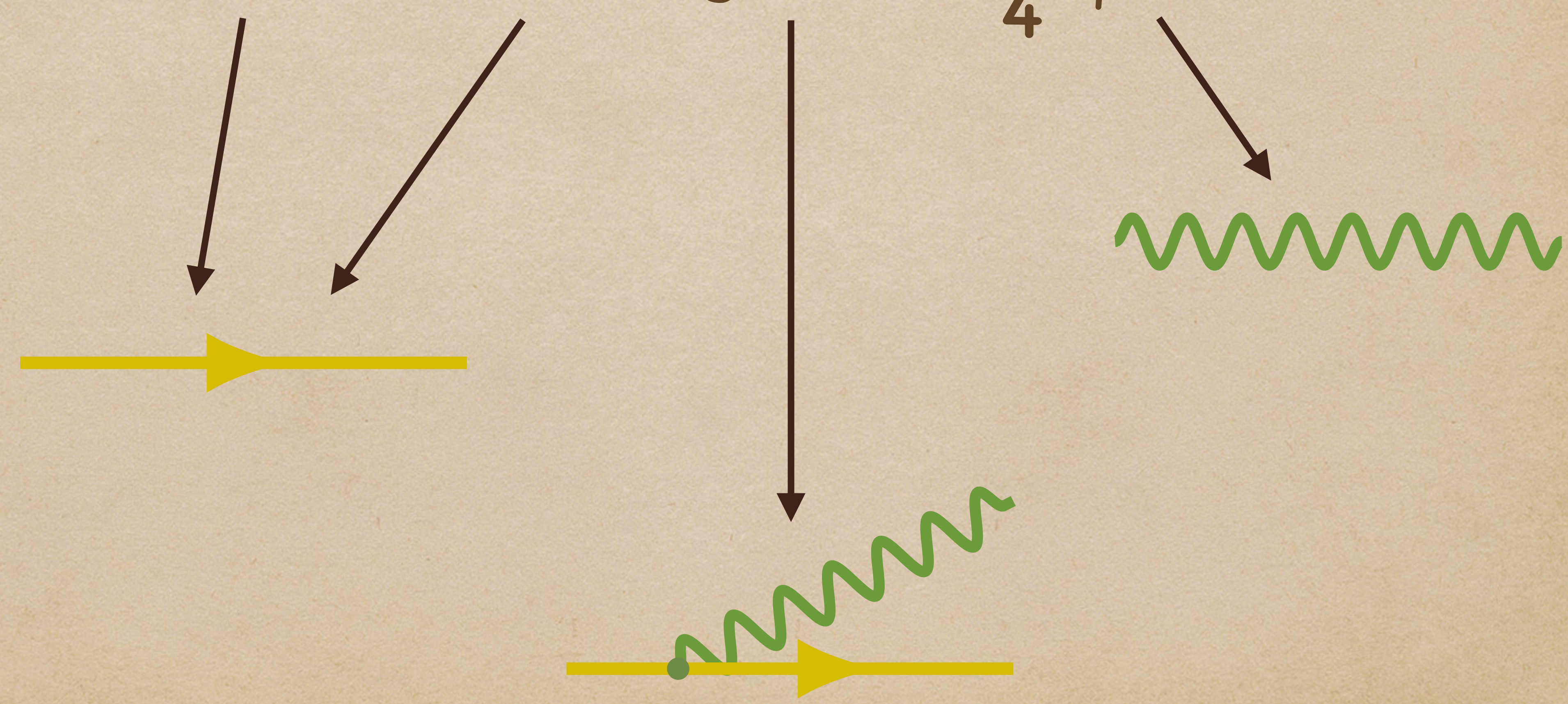
$$\langle \psi_c \psi_d \bar{\psi}_a \bar{\psi}_b \rangle =$$



Feynman Rules

Full Lagrangian for Quantum Electro Dynamics:

$$\mathcal{L}_{QED} = i\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi + g\bar{\psi}\not{A}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$



SM:

The Standard Model of particle physics

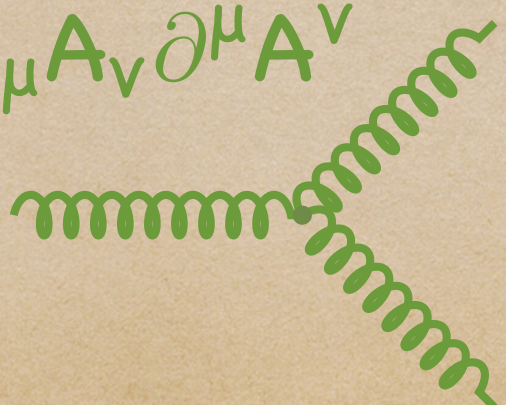
Lagrangian

- We can easily extend our theory by adding new parts to our Lagrangian:

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \mathcal{L}_{\text{QFD}} + \mathcal{L}_{\text{QCD}} + \ldots$$

- QFD (weak force) and QCD (strong force) are very similar to QED. They only add two different types of interaction terms:

$A_\mu A_\nu \partial^\mu A^\nu$



$A_\mu A_\nu A^\mu A^\nu$

Lagrangian

interaction particles

hot coffee (can be omitted)

interactions between matter particles

$$
\begin{aligned}
\mathscr{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
& + i\overline{\psi}\not{D}\psi + h.c. \\
& + \psi_i y_{ij}\psi_j\phi + h.c. \\
& + \left|D_\mu\phi\right|^2 - V(\phi)
\end{aligned}
$$

mass for antimatter particles

mass for matter particles

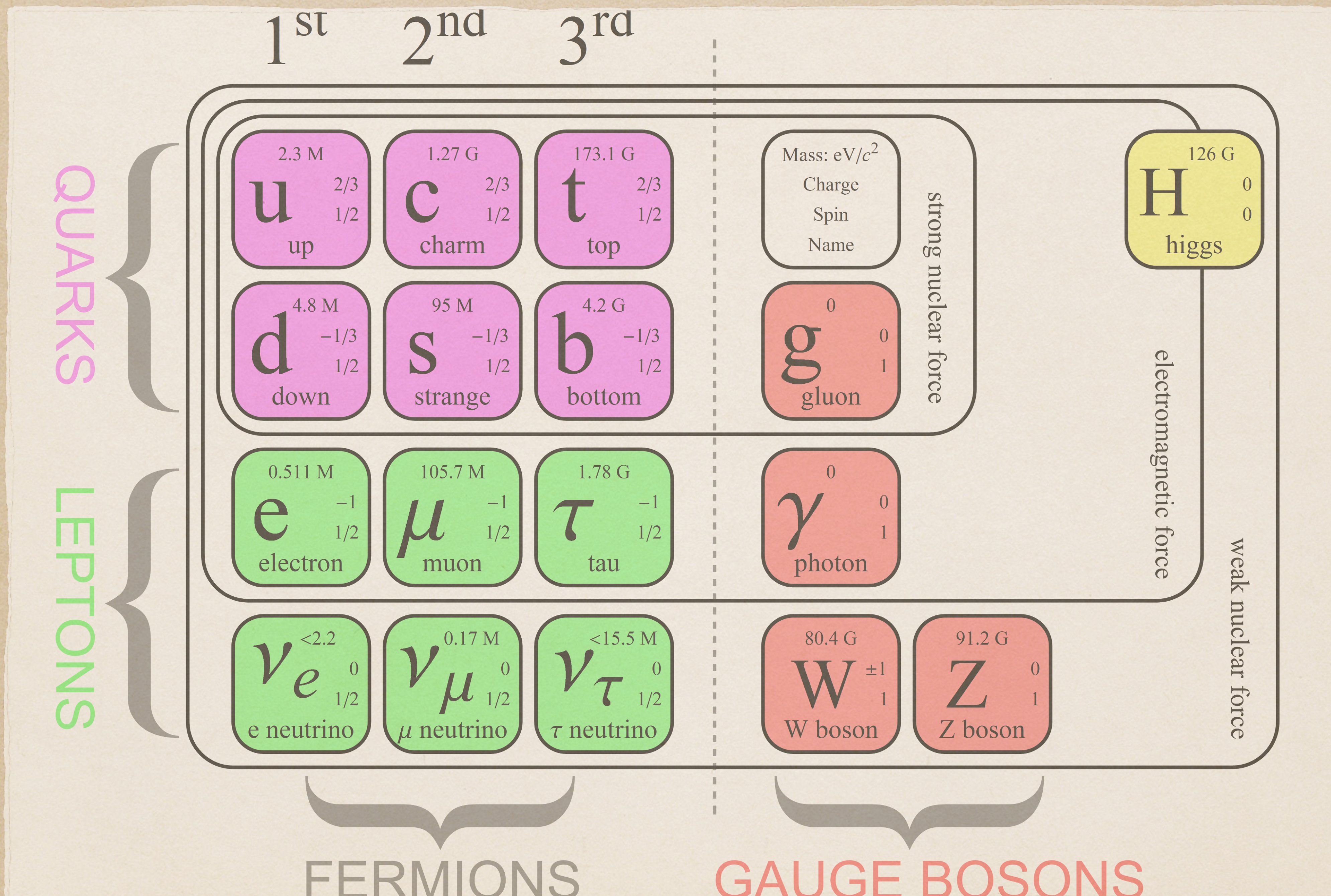
mass for interaction particles

Higgs self-interactions

Let's have a coffee with the Standard Model of particle physics!

J Woithe, GJ Wiener, FF VdV, Phys Educ 52 (2017) no 3, 034001

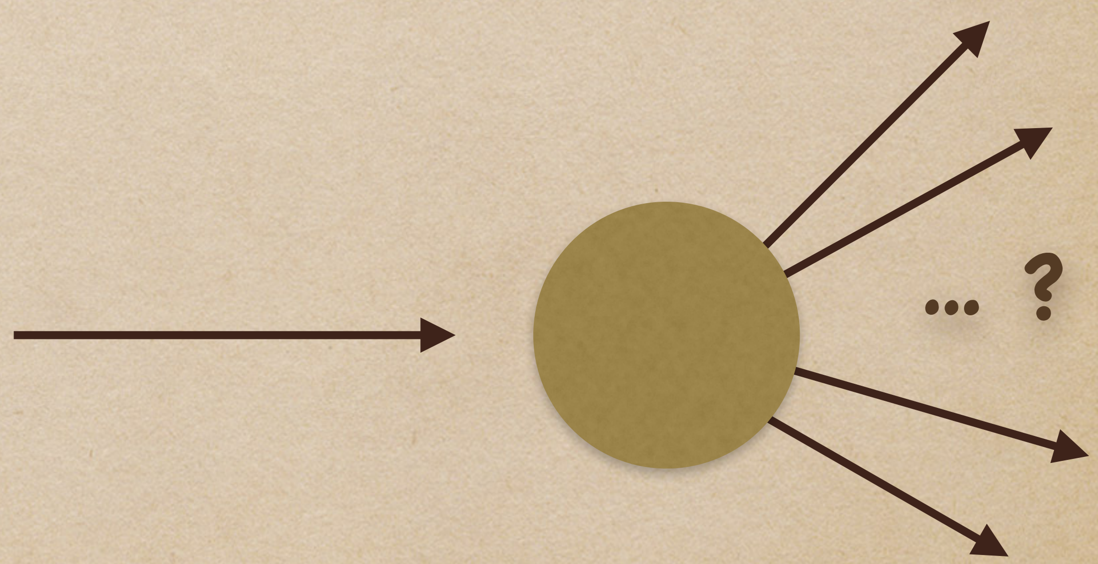
Particle Zoo



source: http://www.physik.uzh.ch/groups/serra/StandardModel.html

Symmetries

- Electric charge, hypercharge, colour charge
- 4-momentum (restriction on possible total mass)
- Lepton number (total and individual)
- Baryon number
- NOT meson number
- Isospin
- ....



- Example: Muon decay

Interactivity

Did you pay attention?

Exercises

- Draw Feynman diagrams for the following processes using the weak interaction:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\Lambda \rightarrow p + e^- + \bar{\nu}_e$$

$$K^0 \rightarrow \pi^+ + \pi^-$$

$$\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$$

- Draw Feynman diagrams for the following processes using the strong interaction:

$$\omega^0 \rightarrow \pi^+ + \pi^- + \pi^0$$

$$\rho^0 \rightarrow \pi^+ + \pi^-$$

$$\Delta^{++} \rightarrow p + \pi^+$$

Challenge Exercise

A proton target is hit by a proton beam with momentum |p|=12GeV/c. In one specific event, 6 tracks are observed. Two of these point to the interaction point and from their curvature we know these are positively charged particles. The other tracks form two pair of opposite charge. Both pairs are visible only a few cm past the interaction point. It is hence clear that two neutral particles were produced that later decayed into charged particles.

1. Make a sketch of this event
2. Discuss which mesons and baryons would be possible candidates for these decays (use the particle data - mass and lifetime - from the PDG booklet. Look for decay channels into two charged particles)
3. The measured momenta for the two pairs are:
   a. $|p_+| = 0.68$ GeV/c $|p_-| = 0.27$ GeV/c $\theta_{+-} = 11°$
   b. $|p_+| = 0.25$ GeV/c $|p_-| = 2.16$ GeV/c $\theta_{+-} = 16°$

   with a measurement error of 5%. Calculate the total energy to decide with hypothesis from 2. agrees with these measurements
4. Use these results to draw a Feynman diagram. Is this the only possible solution?

Questions?

→ frederik@cern.ch*

*reply guaranteed within the decade

C'est tout

Thanks for your attention!