



Baryon number fluctuations at large baryon chemical potentials

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Based on:

[1] Wei-jie Fu, Xiaofeng Luo, Jan M. Pawłowski, Fabian Rennecke, Rui Wen, SY ; Phys. Rev. D 104 (2021), 094047.

[2] Wei-jie Fu, Xiaofeng Luo, Jan M. Pawłowski, Fabian Rennecke, Rui Wen, SY ; in preparation.

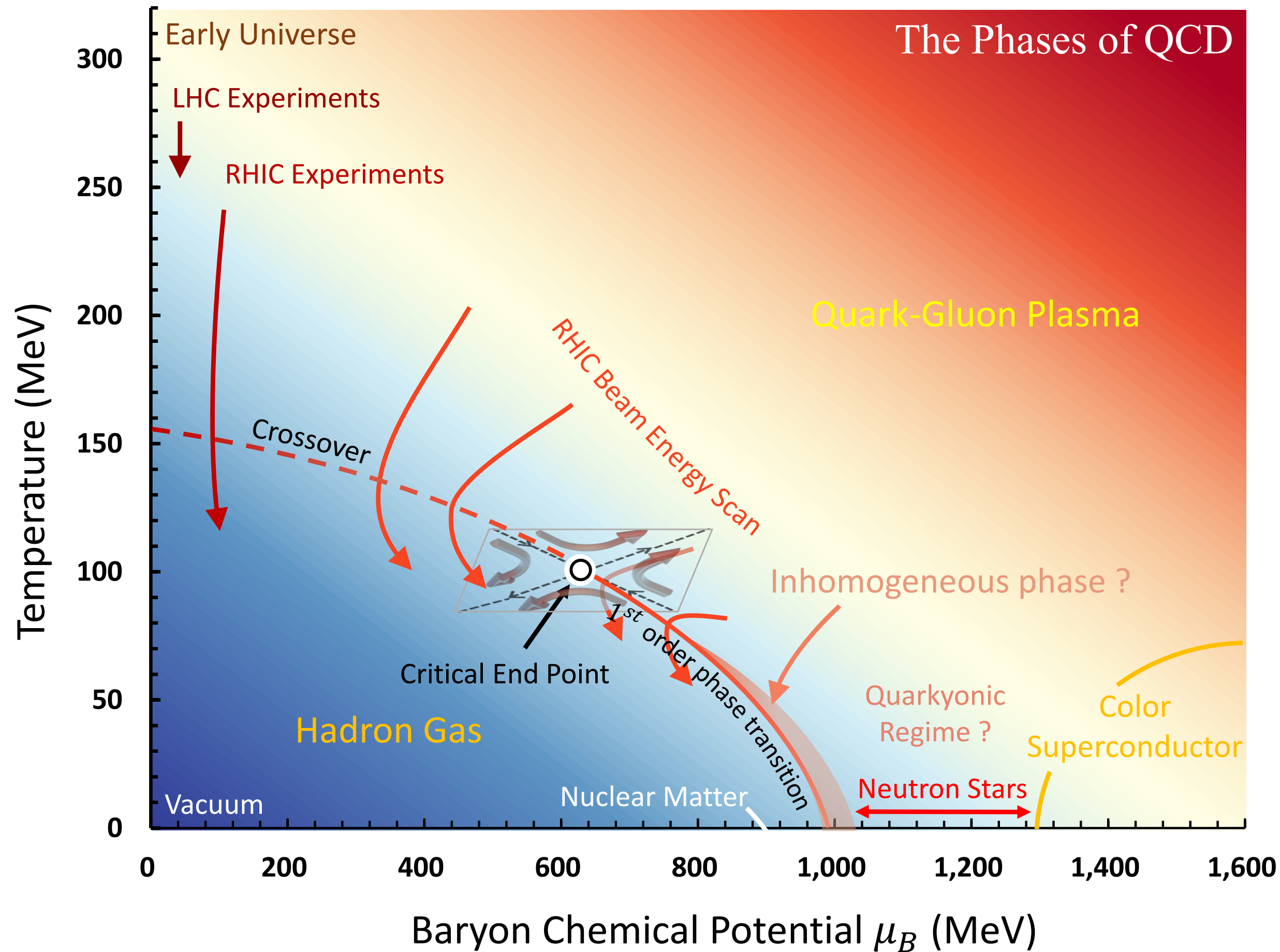
fQCD collaboration:

J. Braun, Y.-r. Chen, W.-j. Fu, F. Gao, Geissel, J. Horak, C. Huang, F. Ihssen, J. M. Pawłowski, F. Rennecke, F. Sattler, B. Schallmo, Y.-y. Tan, S. Töpfel, R. Wen, J. Wessely, N. Wink, S. Yin

Outline

- ▶ Introduction
- ▶ QCD-assisted LEFT within fRG
- ▶ Baryon number fluctuations on freeze-out curves
- ▶ Error estimates of theoretical predictions
- ▶ Summary and outlook

QCD phase diagram and CEP



Scaling of baryon number fluctuations

Baryon number fluctuations of the n -th order:

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n} \frac{p}{T^4}$$

relation to the cumulant:

$$M = VT^3 \chi_1^B, \quad \sigma^2 = VT^3 \chi_2^B,$$

$$S = \chi_3^B / (\chi_2^B \sigma), \quad \kappa = \chi_4^B / (\chi_2^B \sigma^2)$$

Scaling of fluctuations:

$$\chi_2^B \sim \xi^2, \quad \chi_3^B \sim \xi^{4.5}, \quad \chi_4^B \sim \xi^7$$

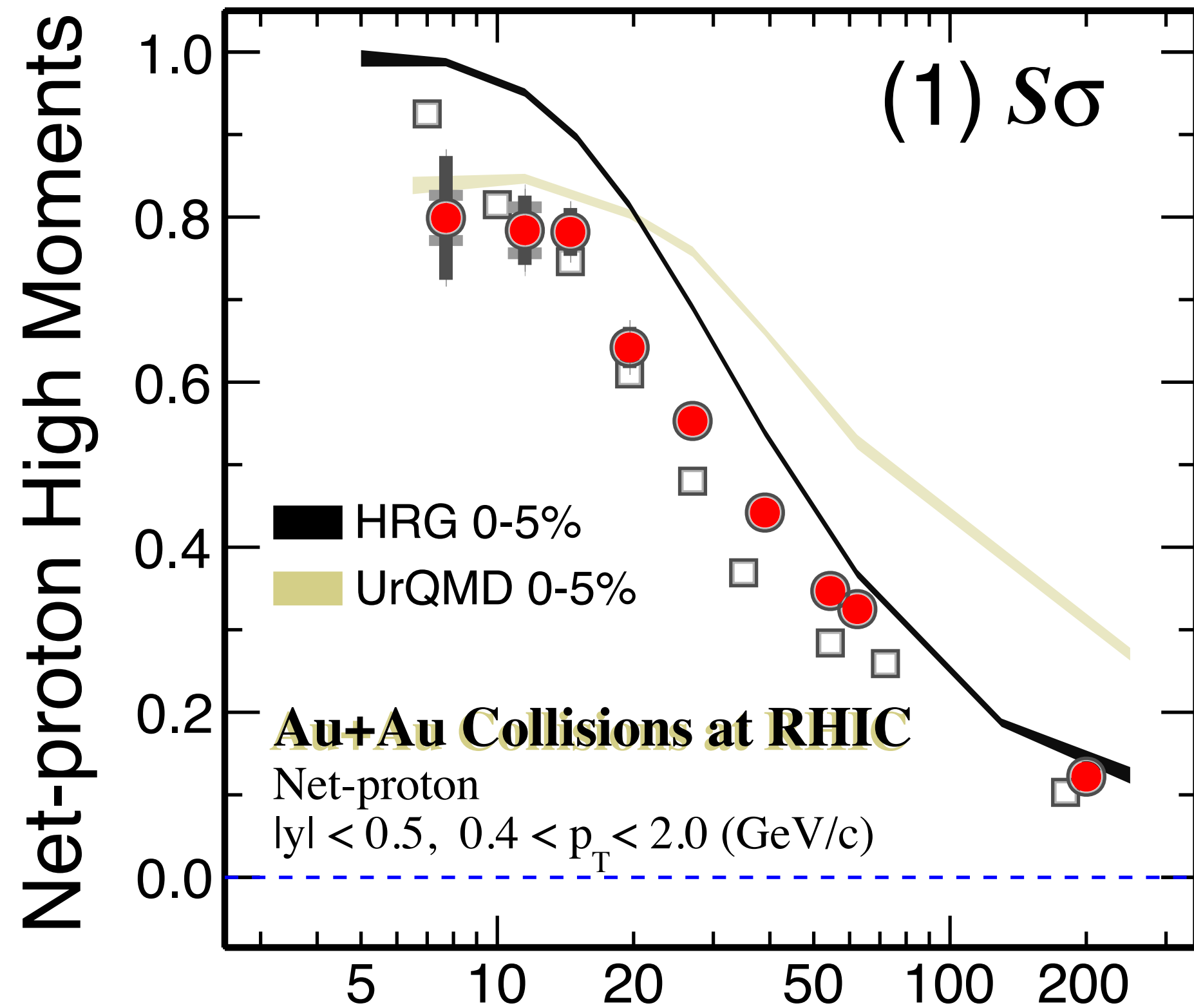
ξ : correlation length

Calls for:

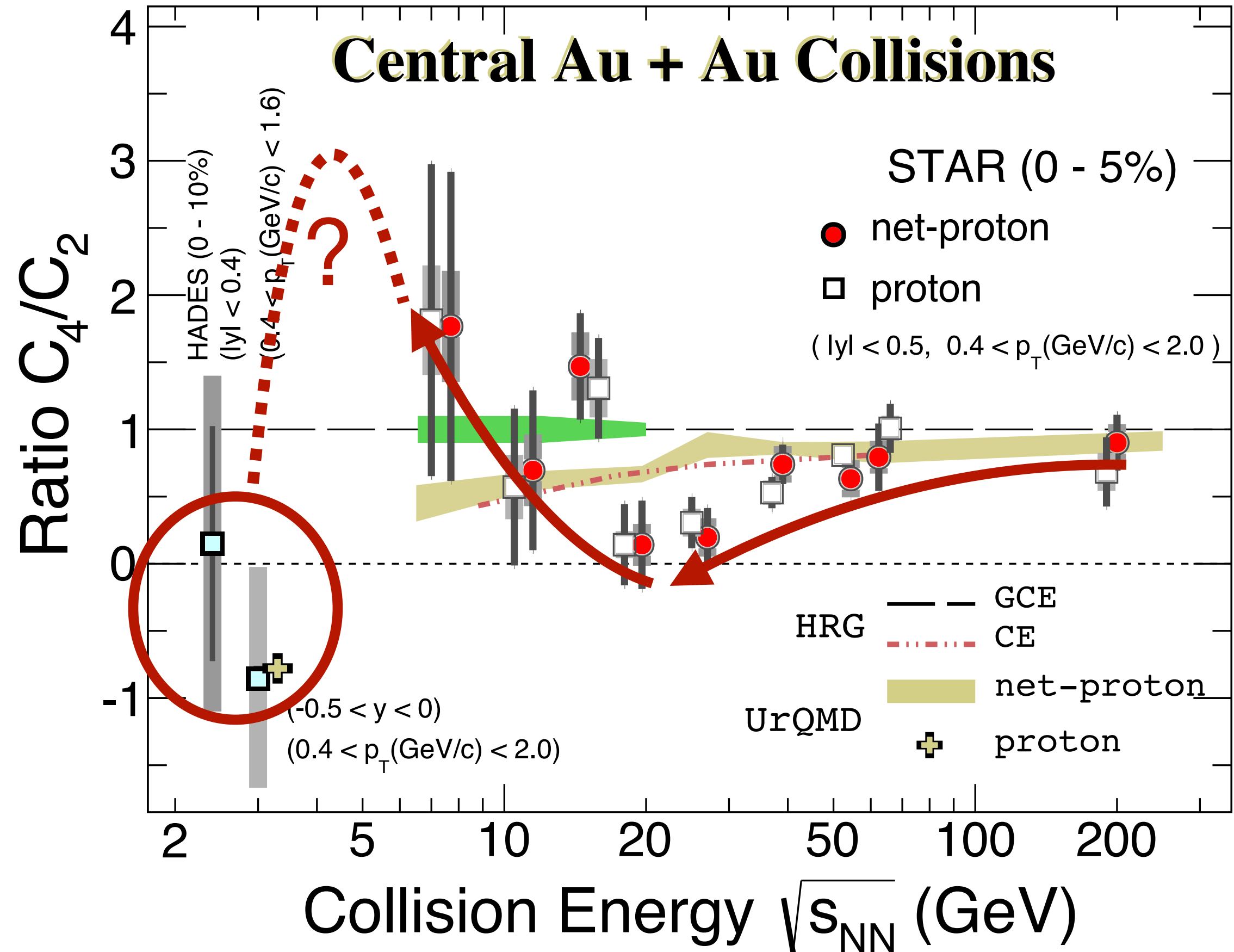
- Nonperturbative approach of QCD.
- Error-controllable calculations at finite densities.
- Real-time description of strongly interacting systems.

Stephanov, *PRL* 102 (2009), 032301

Recent experimental results at RHIC



J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301



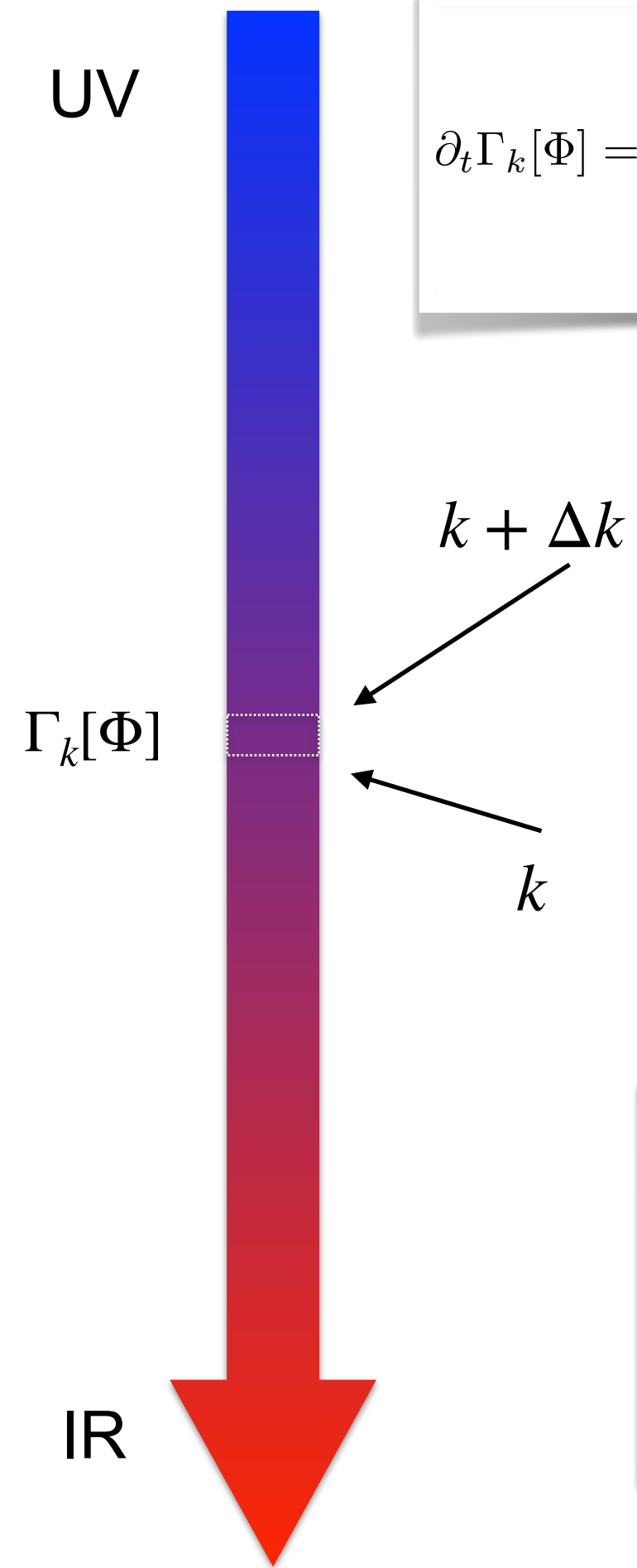
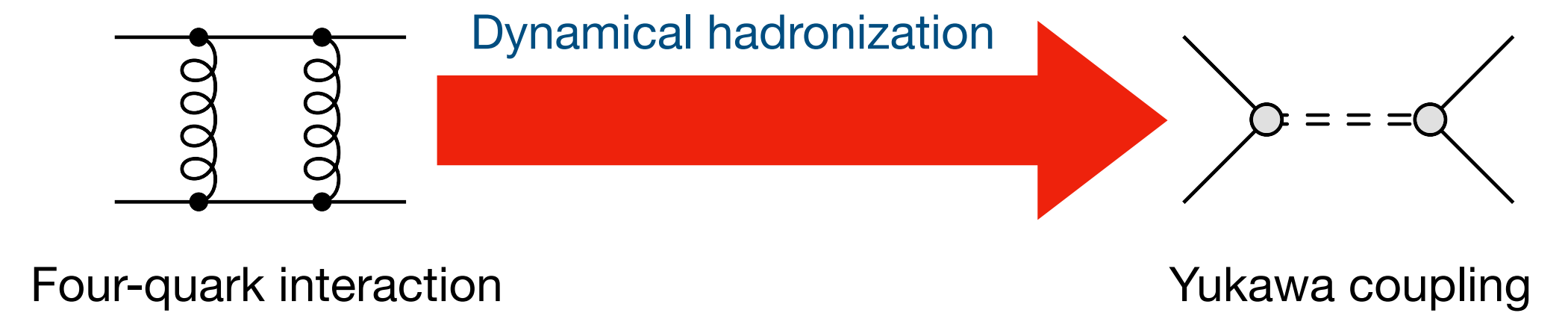
M. Abdallah *et al.* (STAR), *PRL* 128 (2022) 20, 202303

- ★ The non-monotonicity of the kurtosis is observed with 3.1σ significance.
- ★ Is there a “peak” structure in the regime of low colliding energy?

QCD-assisted LEFT within fRG

First Principle QCD flow equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \text{[Orange loop]} - \text{[Dotted loop]} - \text{[Black loop]} + \frac{1}{2} \text{[Blue loop]}$$

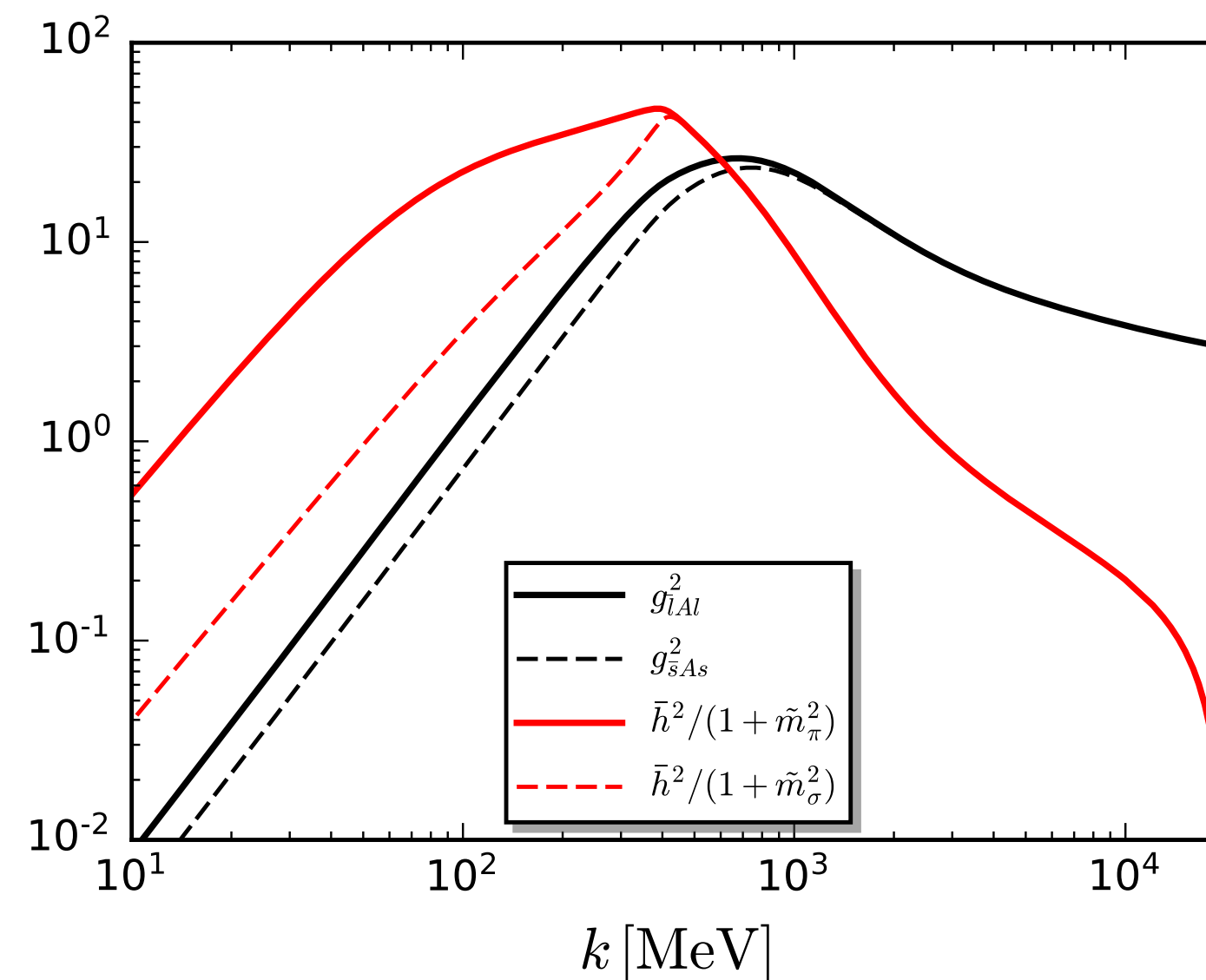


LEFT flow equation:

$$\partial_t \Gamma_k[\Phi] = - \text{[Black loop]} + \frac{1}{2} \text{[Blue loop]}$$

quark

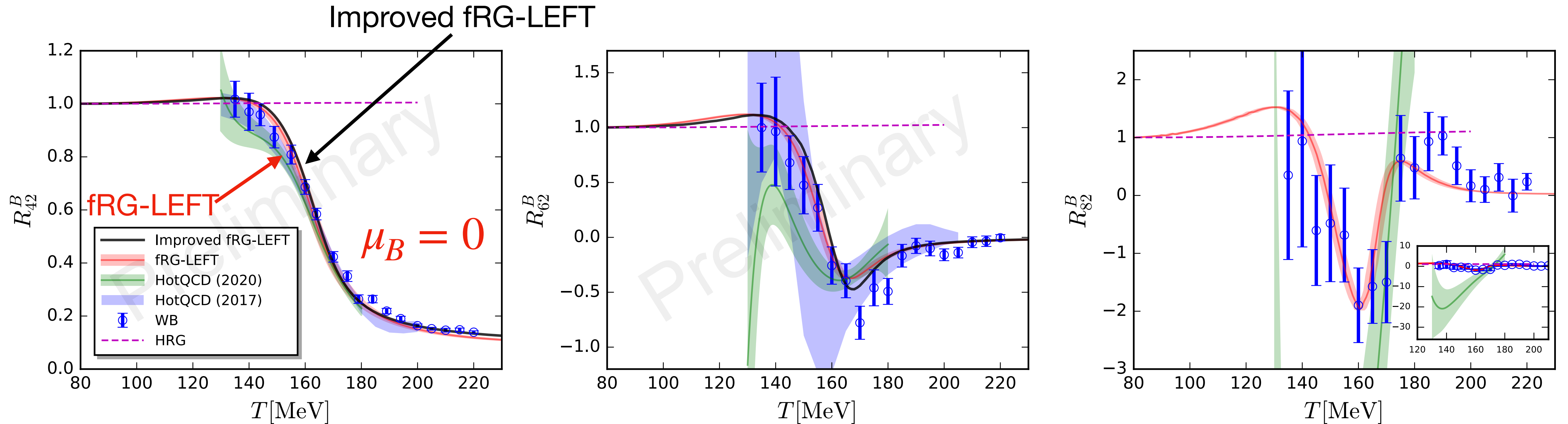
meson



Fu, Pawłowski,
Rennecke, *PRD* 101
(2020), 054032

- ✳ fRG-LEFT have been improved.
- ✳ Dynamics of first-principle fRG-QCD has been encoded in the LEFT via the Yukawa coupling.

Update: Baryon number fluctuations at $\mu_B = 0$



Improved fRG-LEFT: Fu, Luo, Pawłowski, Rennecke, Wen, SY; in preparation

fRG-LEFT: Fu, Luo, Pawłowski, Rennecke, Wen, SY, *PRD* 104 (2021) ,094047

HotQCD: A. Bazavov *et al.*, *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

WB: S. Borsanyi *et al.*, *JHEP* 10 (2018) 205

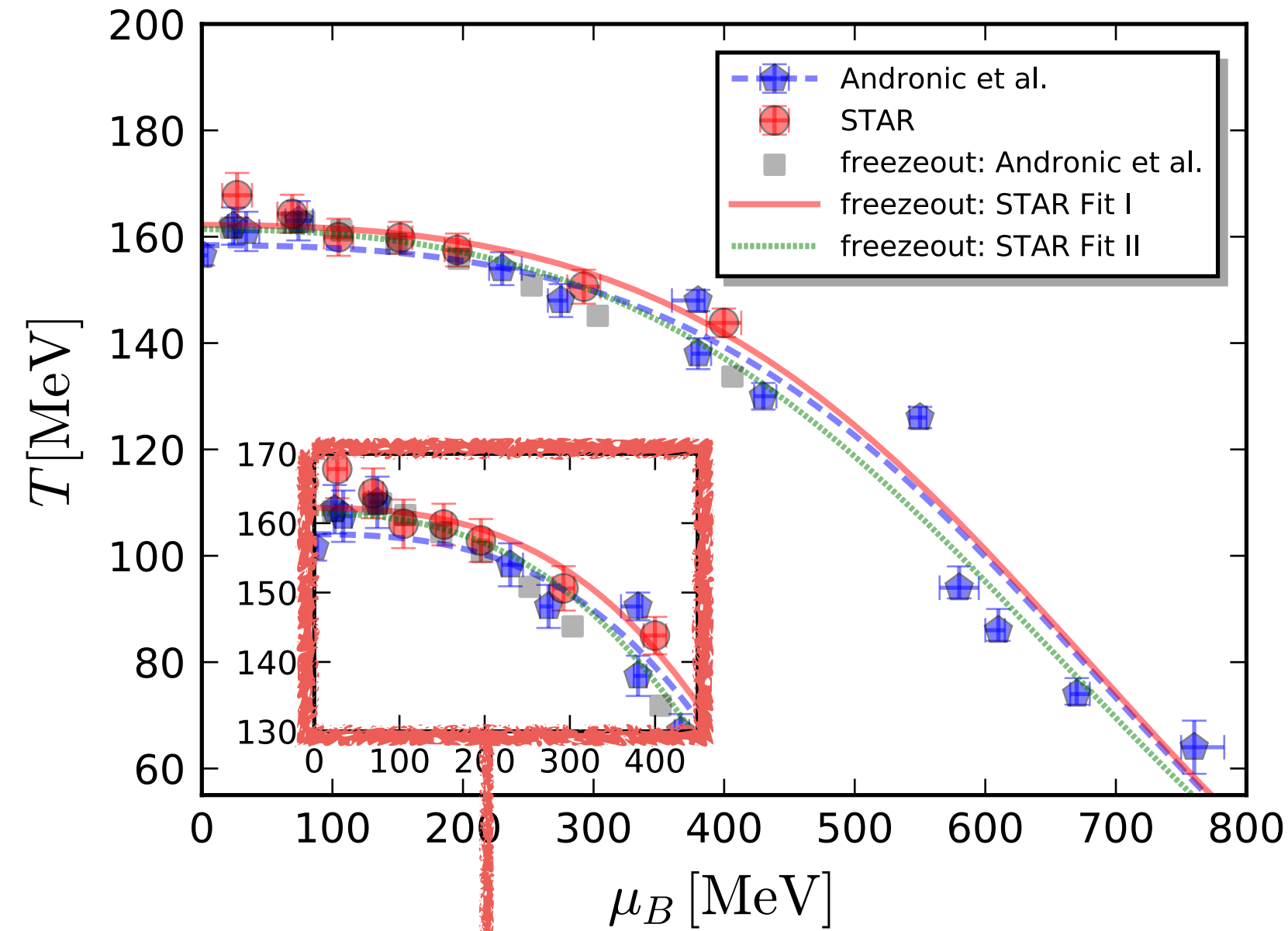
baryon number fluctuations

$$\chi_n^B = \frac{\partial^n p}{\partial (\mu_B/T)^n T^4}$$

$$R_{nm}^B = \frac{\chi_n^B}{\chi_m^B}$$

★ In comparison to our former results and lattice results, the improved results of baryon number fluctuations at vanishing chemical potential are **convergent** and **consistent**.

Determination of the freeze-out curve



three freeze-out curves

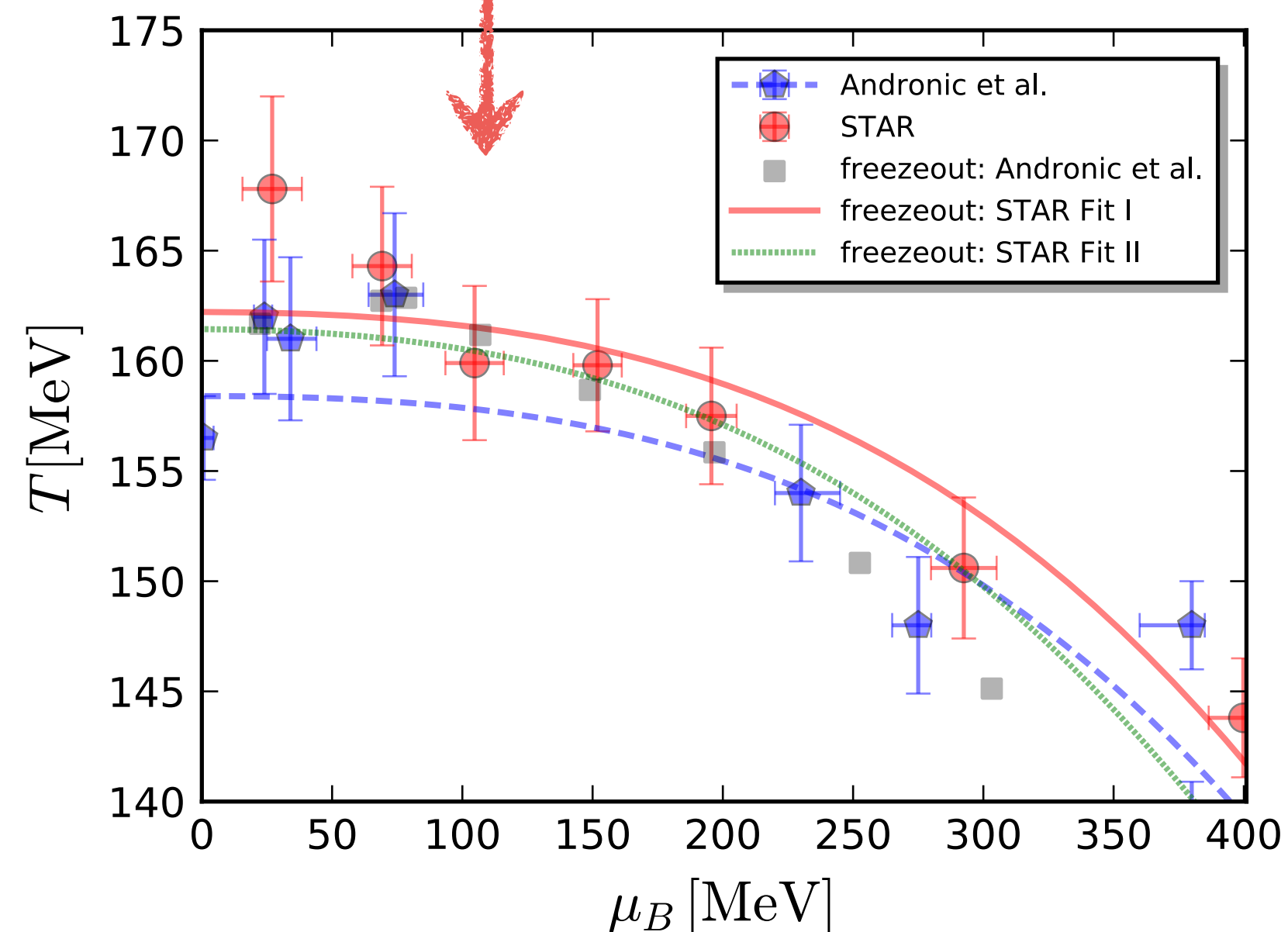
1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich,
Nature 561 (2018) 7723, 321

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

3. freeze-out: STAR Fit II



$$\mu_{B_{CF}} = \frac{a}{1 + 0.288\sqrt{s_{NN}}},$$

$$T_{CF} = \frac{T_{CF}^{(0)}}{1 + \exp(2.60 - \ln(\sqrt{s_{NN}})/0.45)}$$

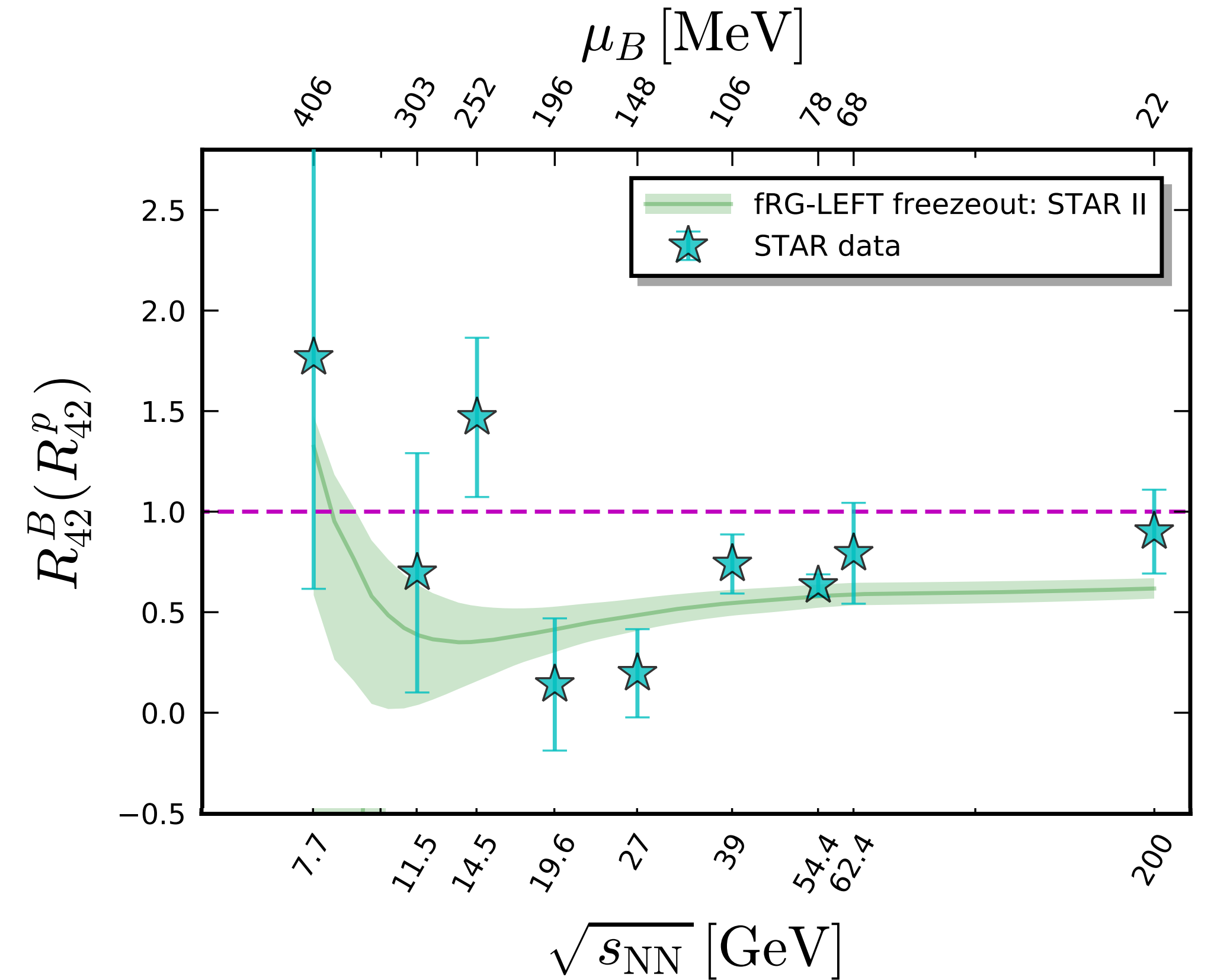
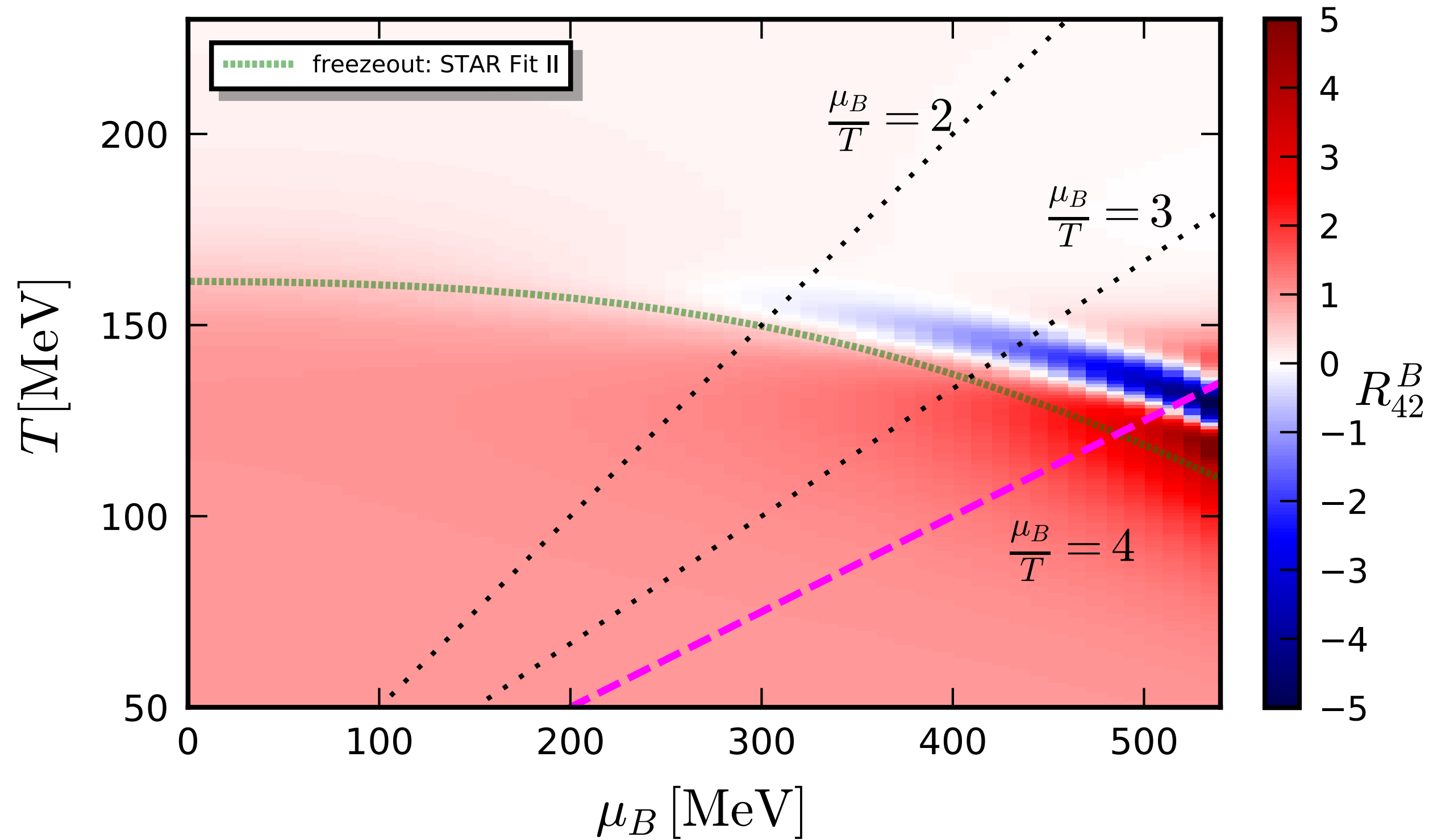
all data points

neglecting first two at low μ_B
and the last one

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

Fluctuations on the freeze-out curve

fRG-LEFT



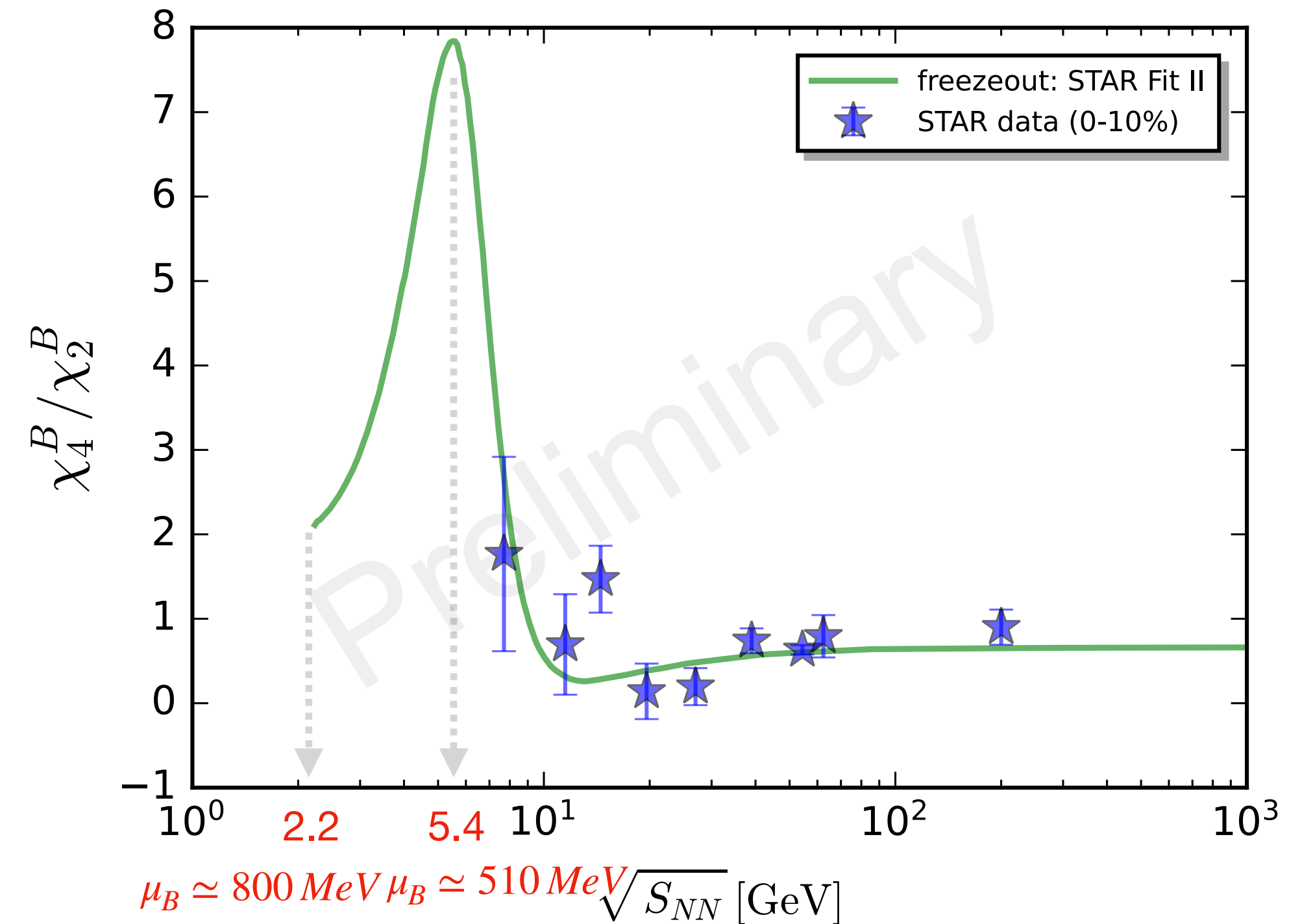
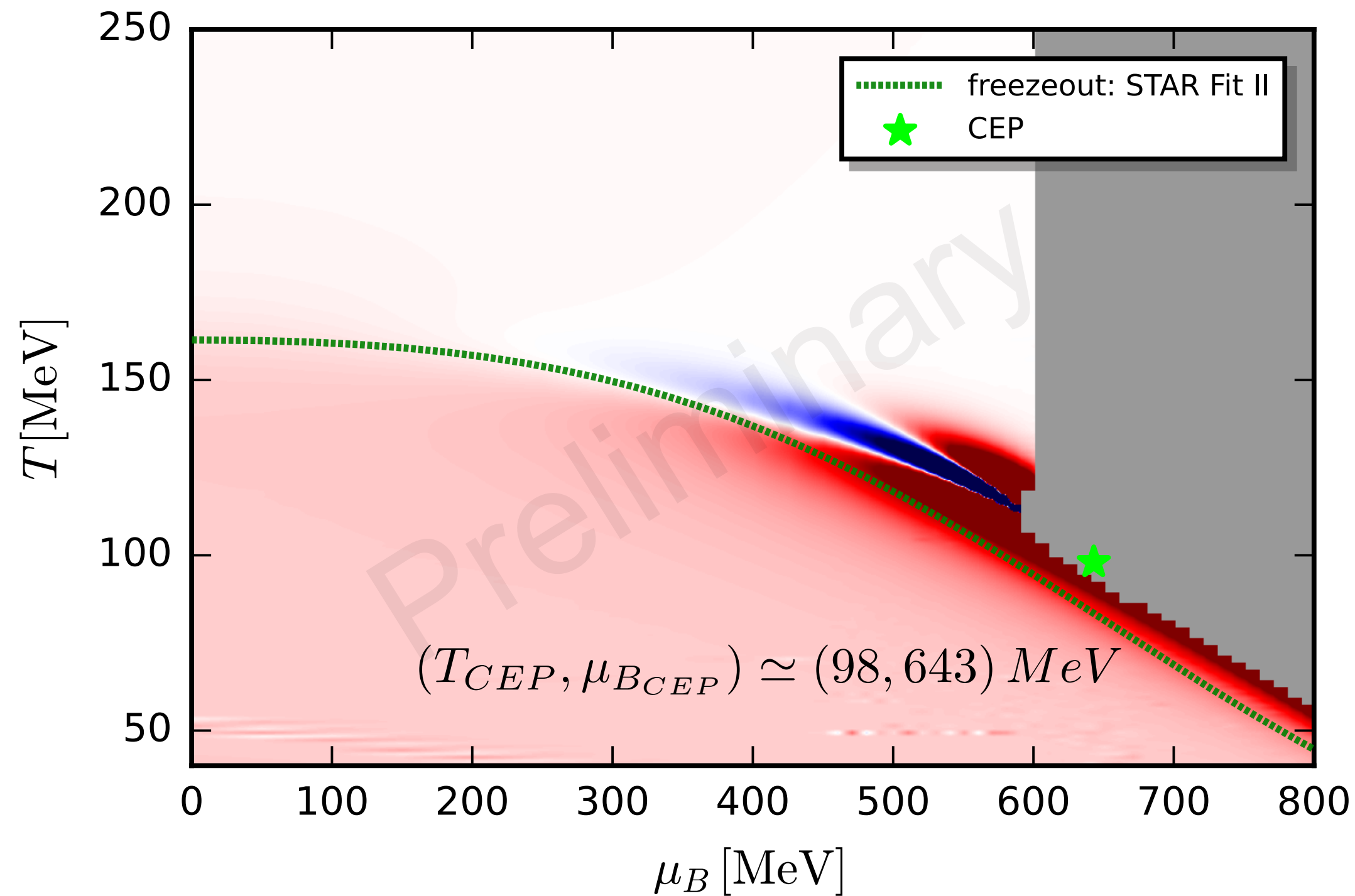
Fu, Luo, Pawłowski, Rennecke, Wen, SY, *PRD* 104 (2021) ,094047

Caveat !

- ★ The calculation here is based on the **grand canonical ensemble**. The global charge conservation effect is not taken into account.
- ★ The current results are obtained in the **equilibrium** system.

Update: Fluctuations on the freeze-out curve

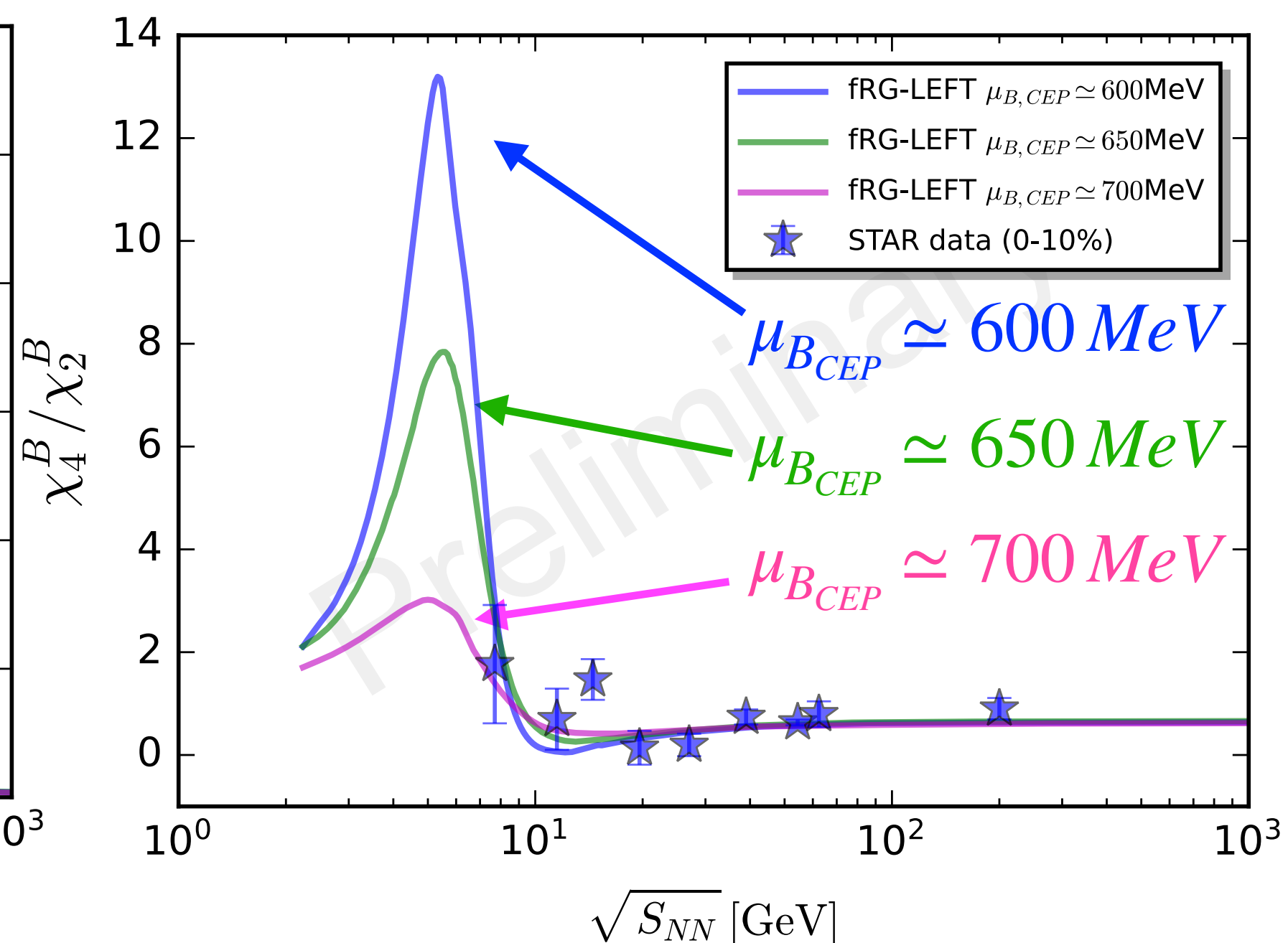
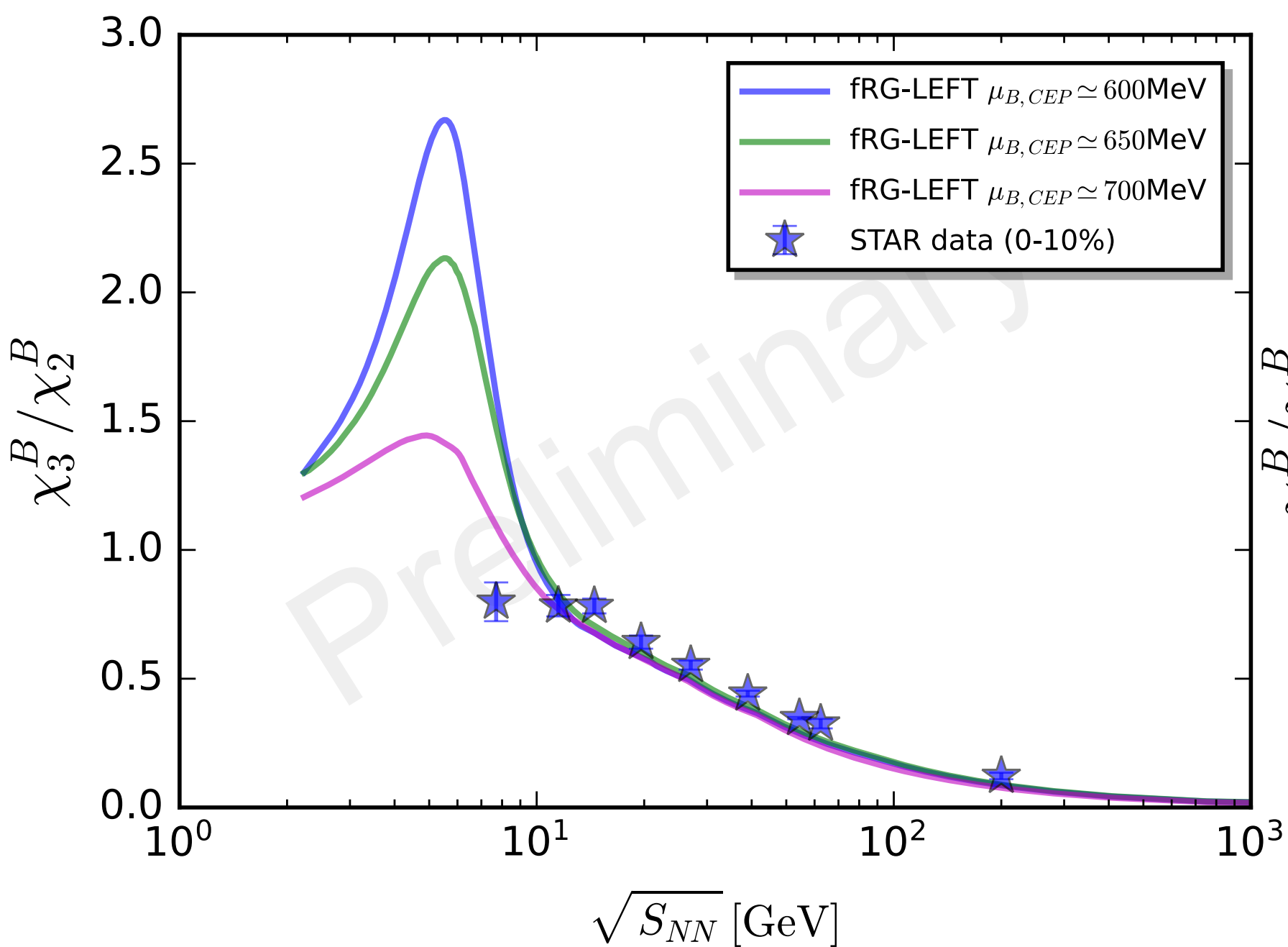
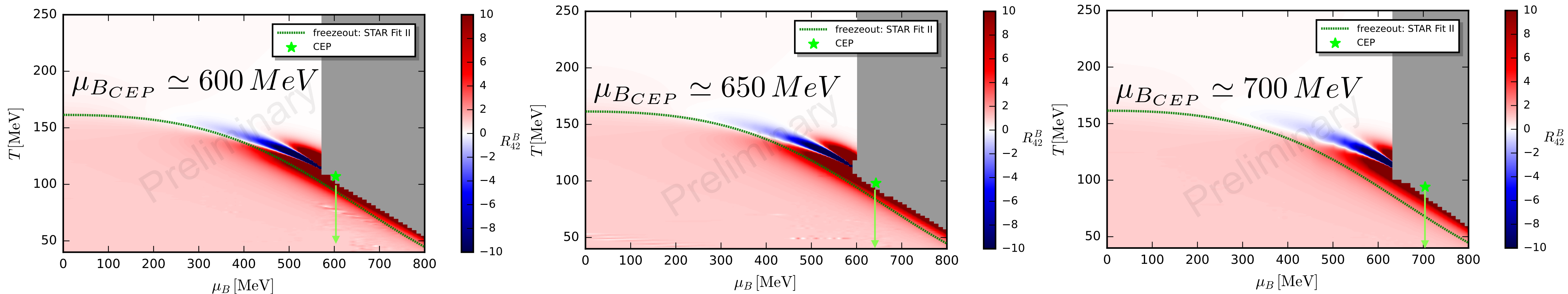
Improved fRG-LEFT



Fu, Luo, Pawłowski, Rennecke, Wen, SY; in preparation

- ★ Calculations have been extended from $\mu_B \sim 500$ to 800 MeV in the improved fRG-LEFT.
- ★ A “peak” structure is found in the regime of low collision energy.

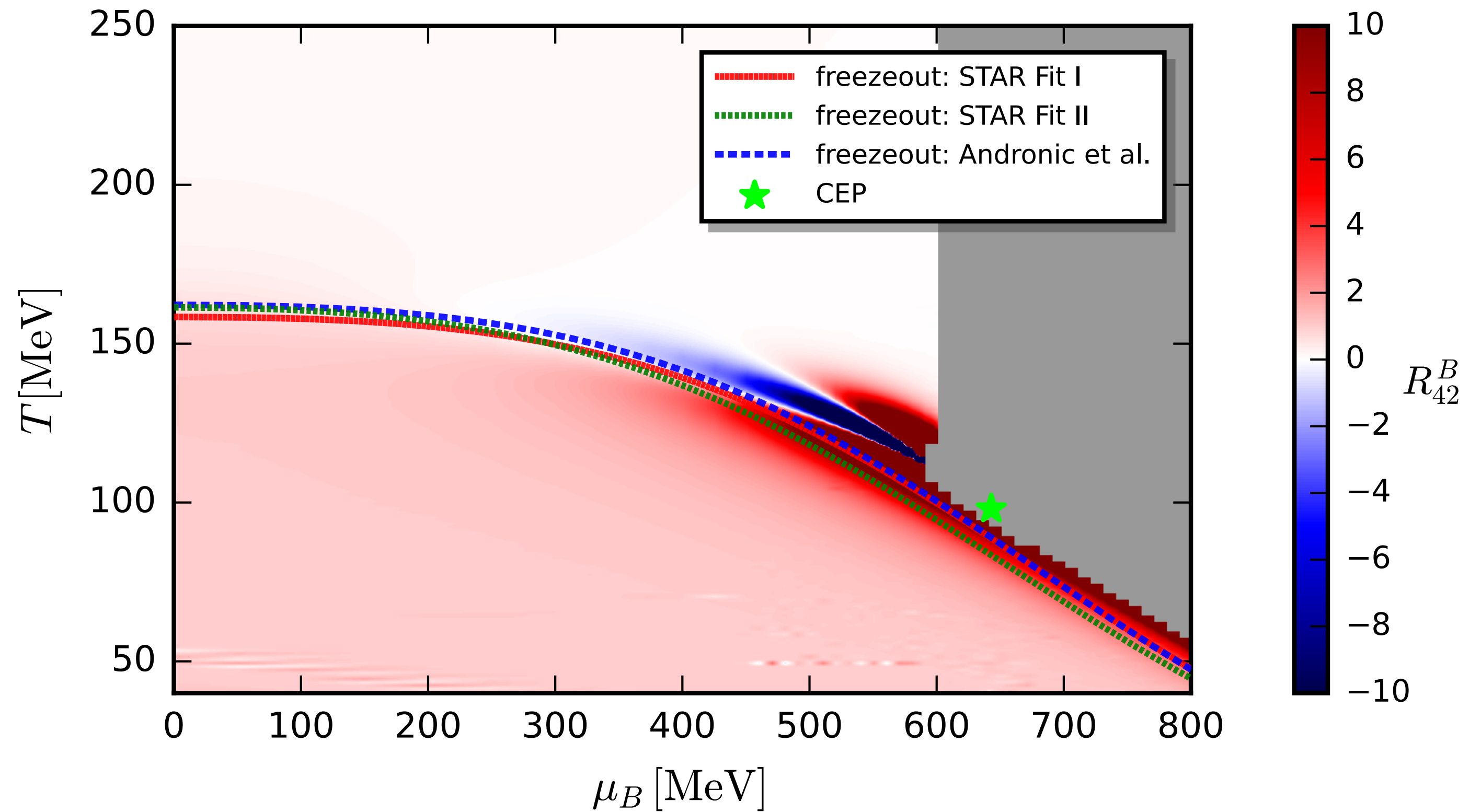
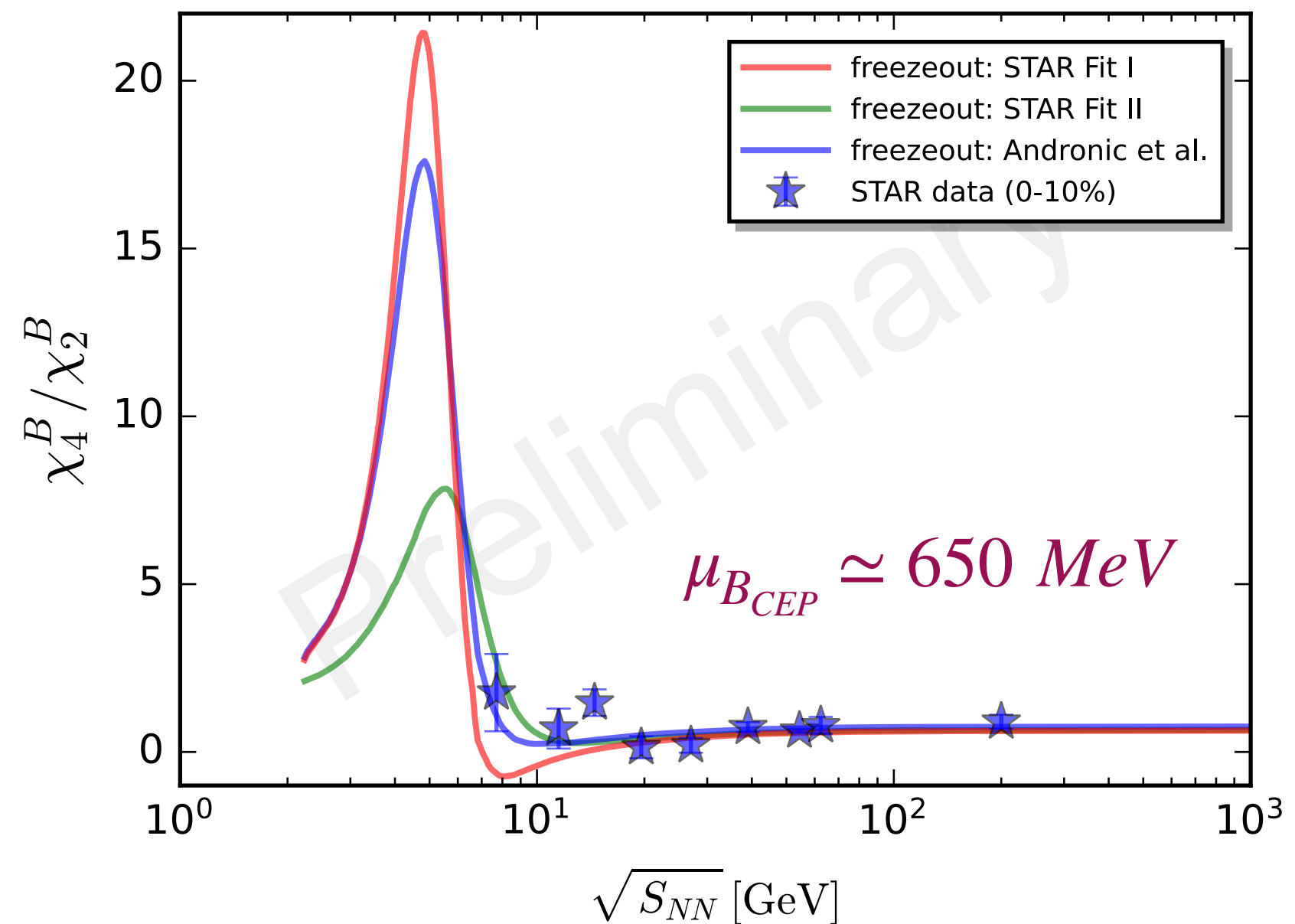
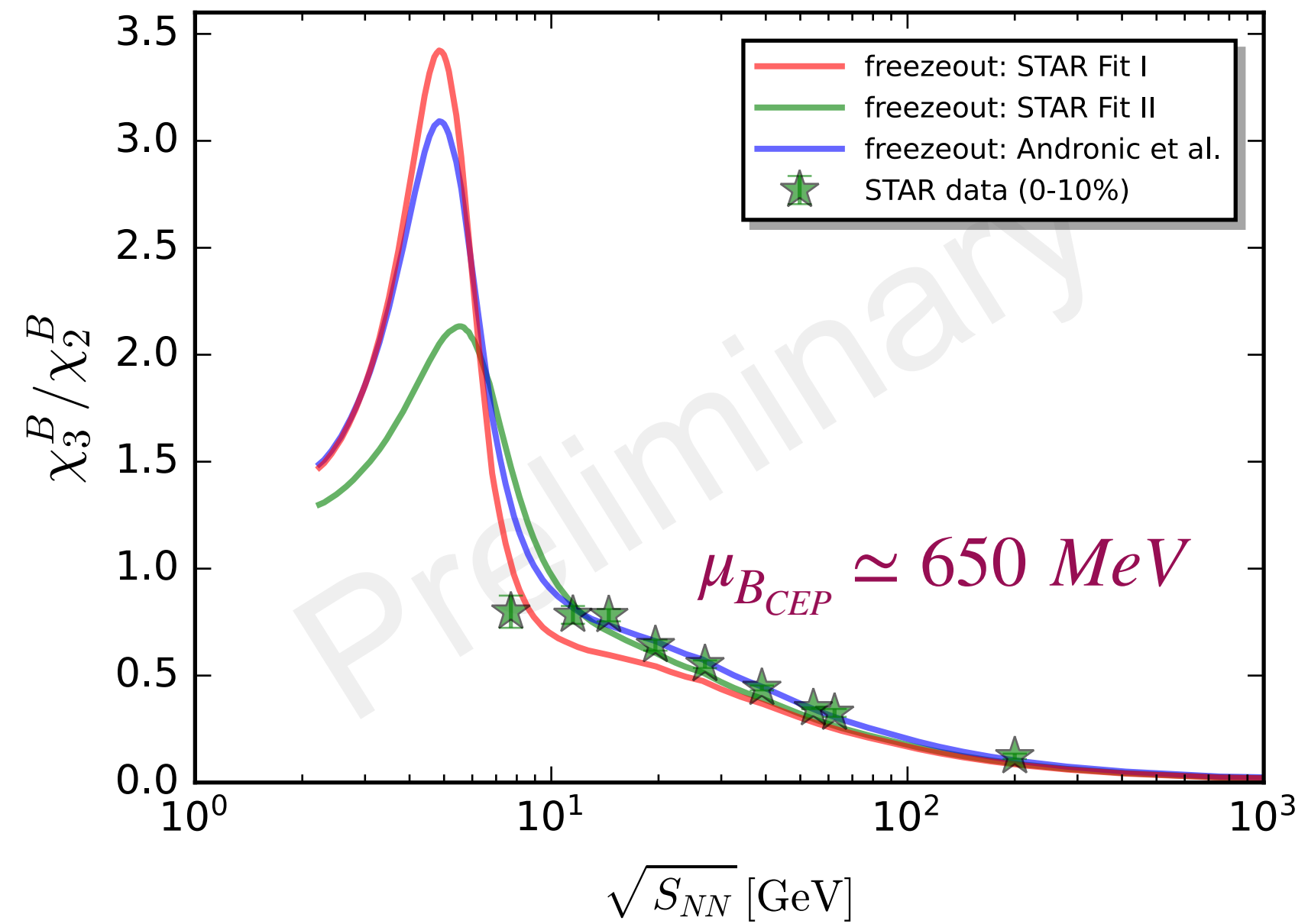
Error estimate: Location of CEP



★ **Location** of the peak, i.e., the corresponding collision energy shows very **mild** dependence on the location of CEP in the range $\mu_{B,CEP} \simeq (600 \sim 700)$ MeV.

★ While the **height** of the peak shows **significant** dependence.

Error estimate: Different freeze-out curves



★ Height and location of the peak are influenced by different freeze-out curves, especially for the height.

★ Closer the freeze-out curve is to the phase boundary, higher the peak.

Summary and outlook

- ▶ A **peak** structure of the kurtosis of the baryon number fluctuations as a function the collision energy is found in the regime of low collision energy, where the computation is done in a QCD assisted LEFT within the fRG approach.
- ▶ The **height of peak** is influenced by the location of **CEP** and **freeze-out curves** in the phase diagram.
- ▶ The **location of the peak** shows **very mild** dependence on the location of CEP, and **weak** dependence on freeze-out curves.
- ◆ First-principle QCD calculations at large baryon chemical potential to pin down the location of CEP and more experimental data to determine the freeze-out curves are highly required.
- ◆ Important effect in the region of large baryon chemical potentials, such as the global baryon number conservation, should be taken into account.

Thanks for your attention !

Back up

Considering Global charge conservation effect

$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1 - \alpha) \frac{\chi_2^B}{\chi_1^B},$$

$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1 - 2\alpha) \frac{\chi_3^B}{\chi_2^B},$$

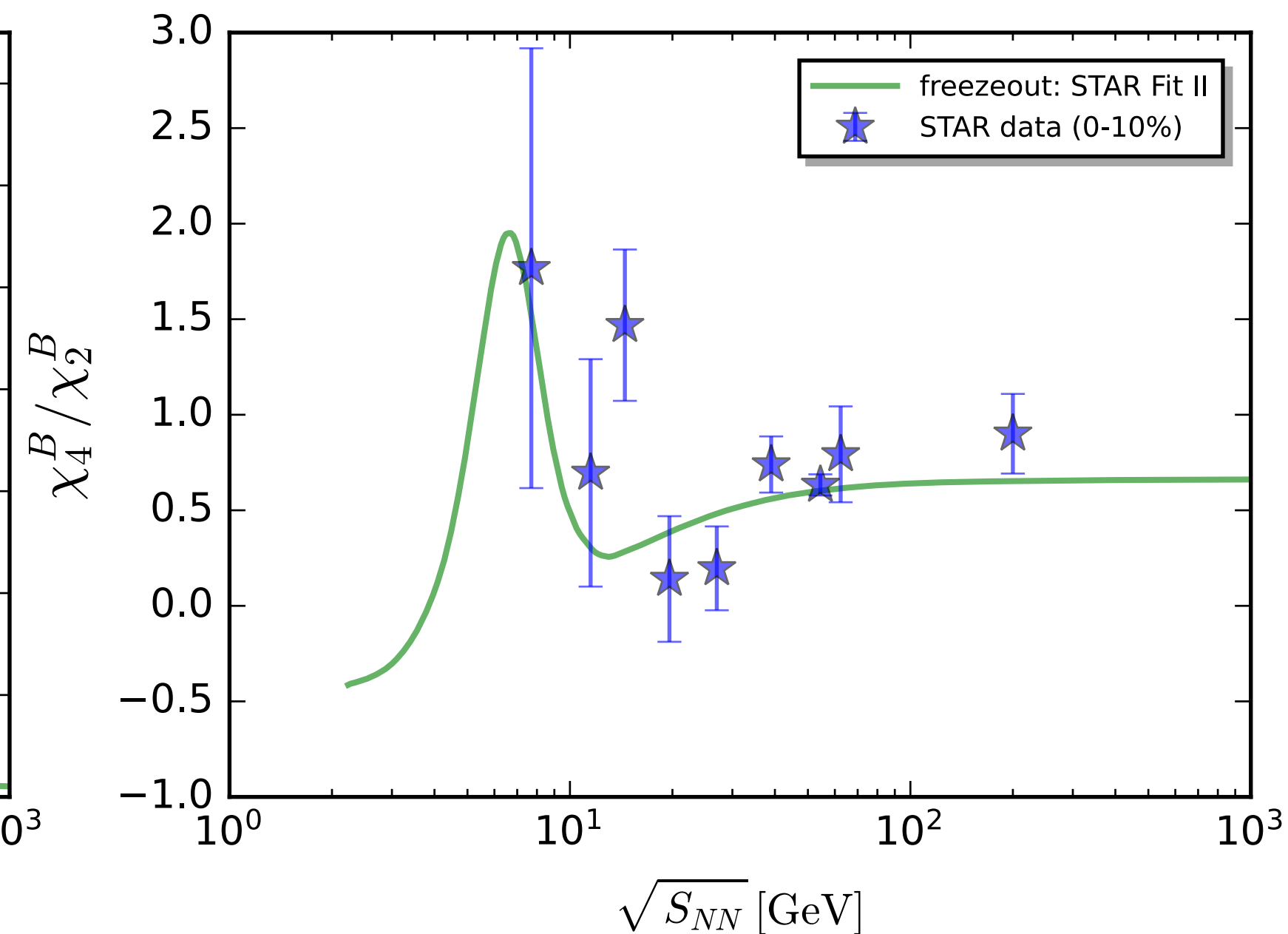
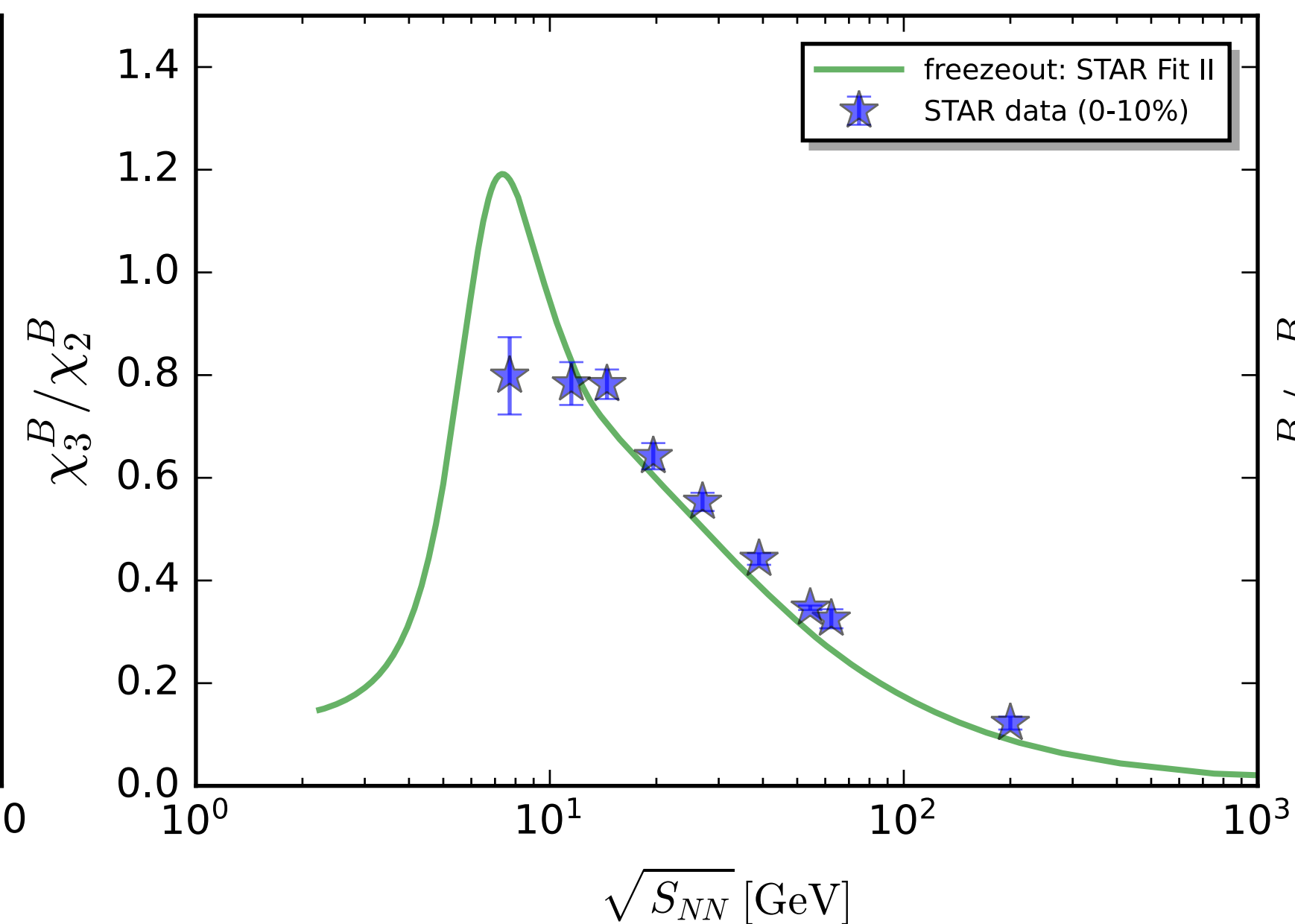
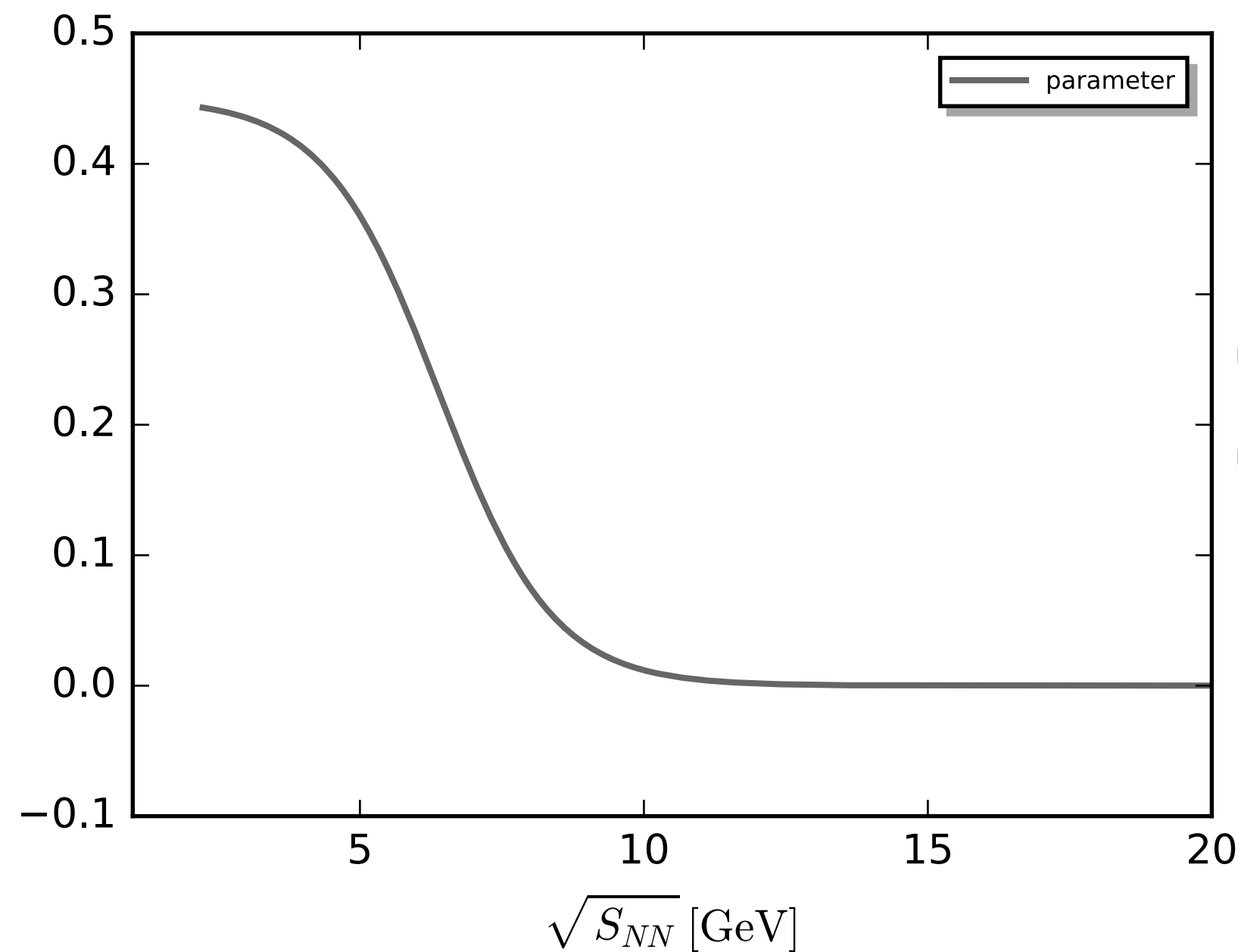
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1 - 3\alpha\beta) \frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta \left(\frac{\chi_3^B}{\chi_2^B} \right)^2$$

★ Considering conserved charge measured in a subvolume of a thermal system.

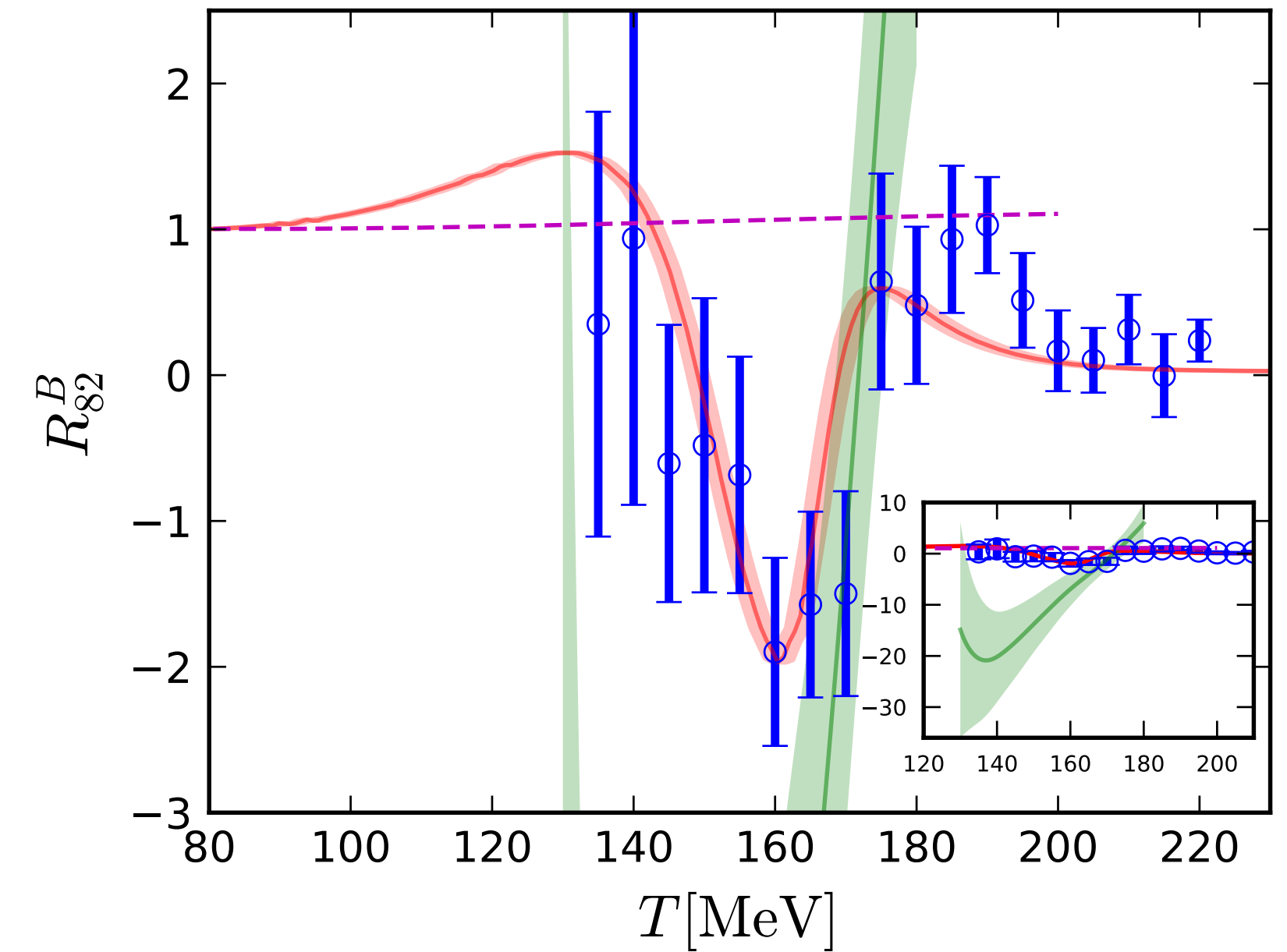
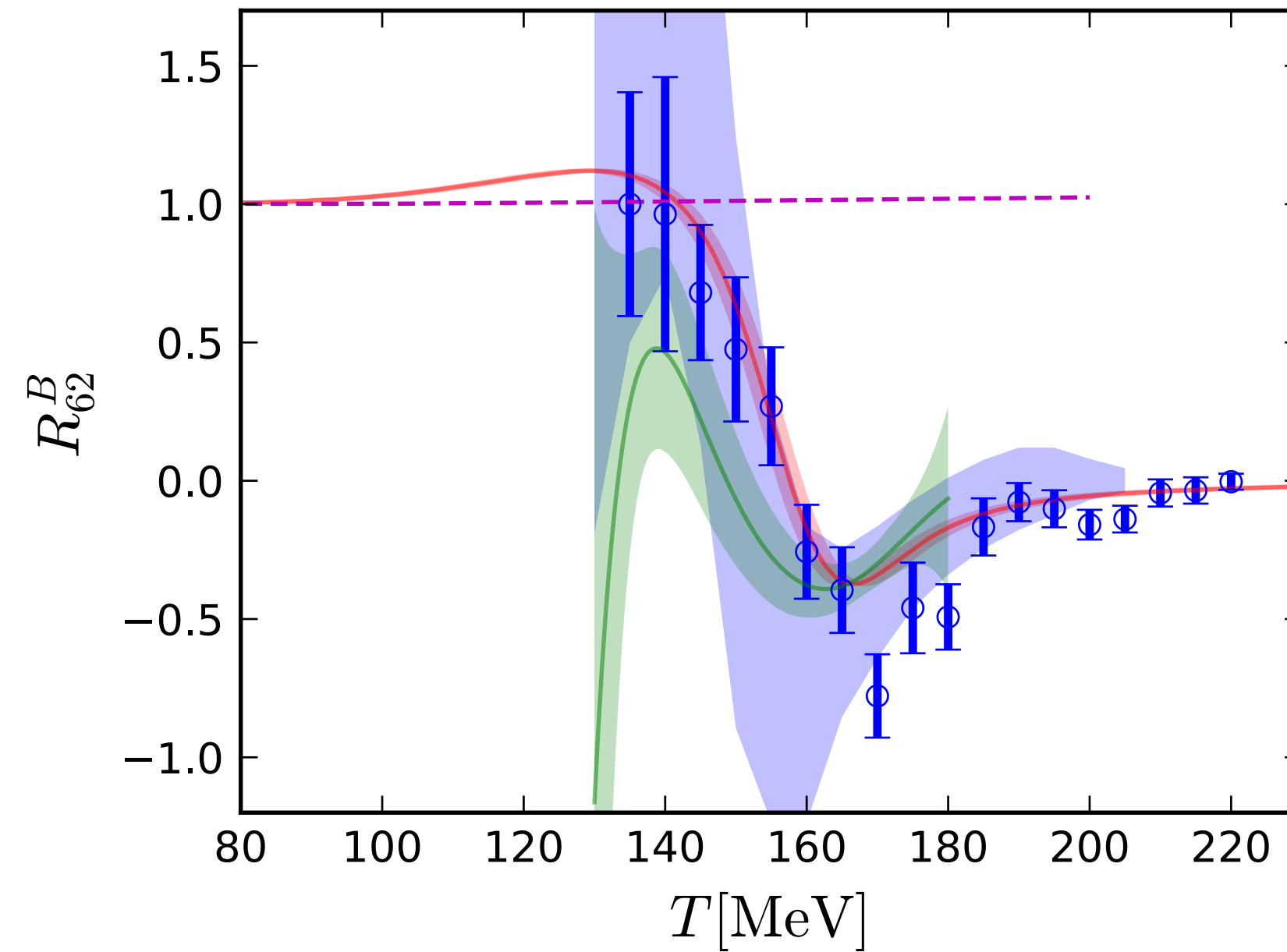
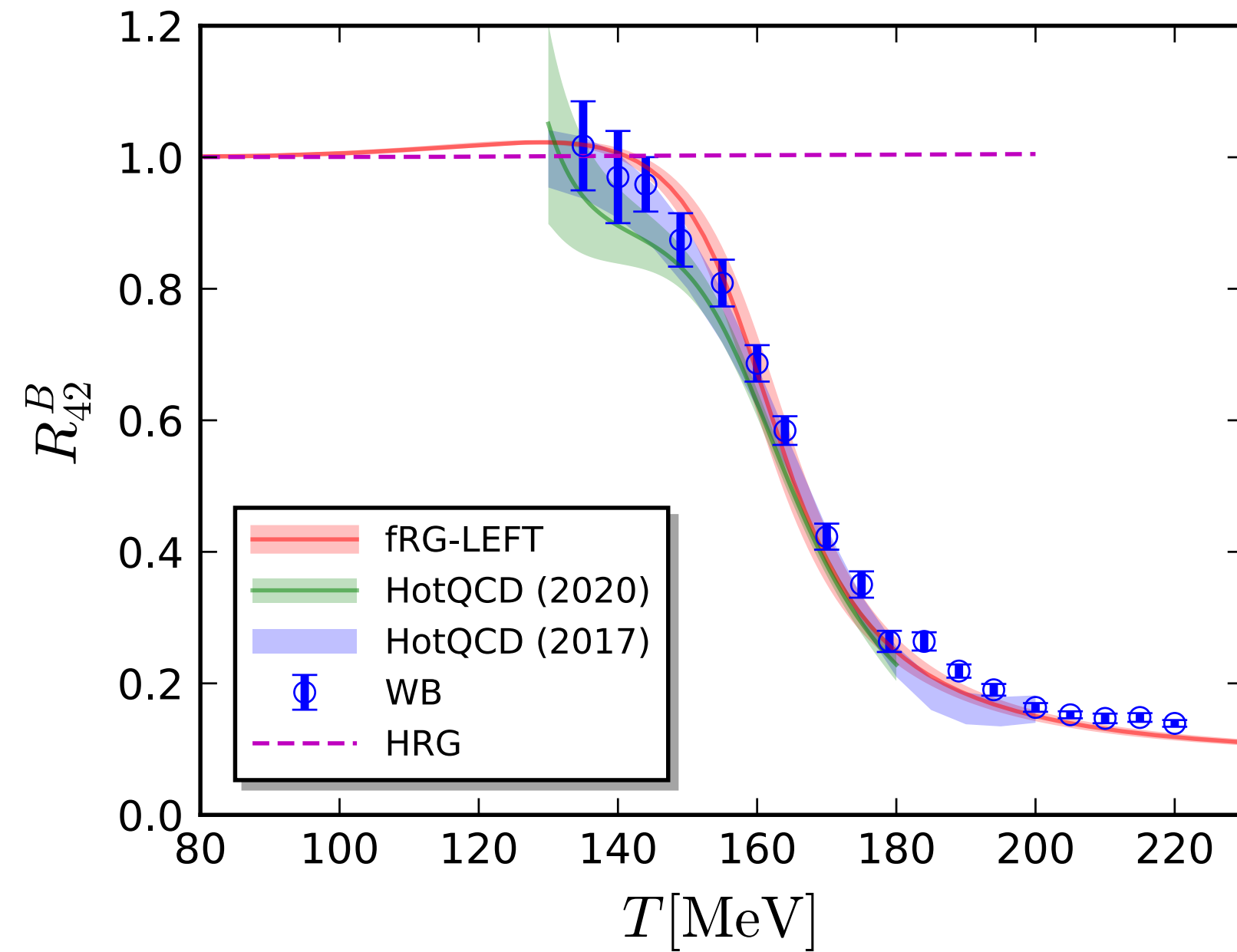
$V_1 = \alpha V$ α is the proportion of the subvolume to the total volume.

$\beta = 1 - \alpha$ β is the proportion of the remaining volume.

V. Vovchenko et al., *Phys.Lett.B* 811 (2020) 135868



QCD-assisted LEFT within fRG



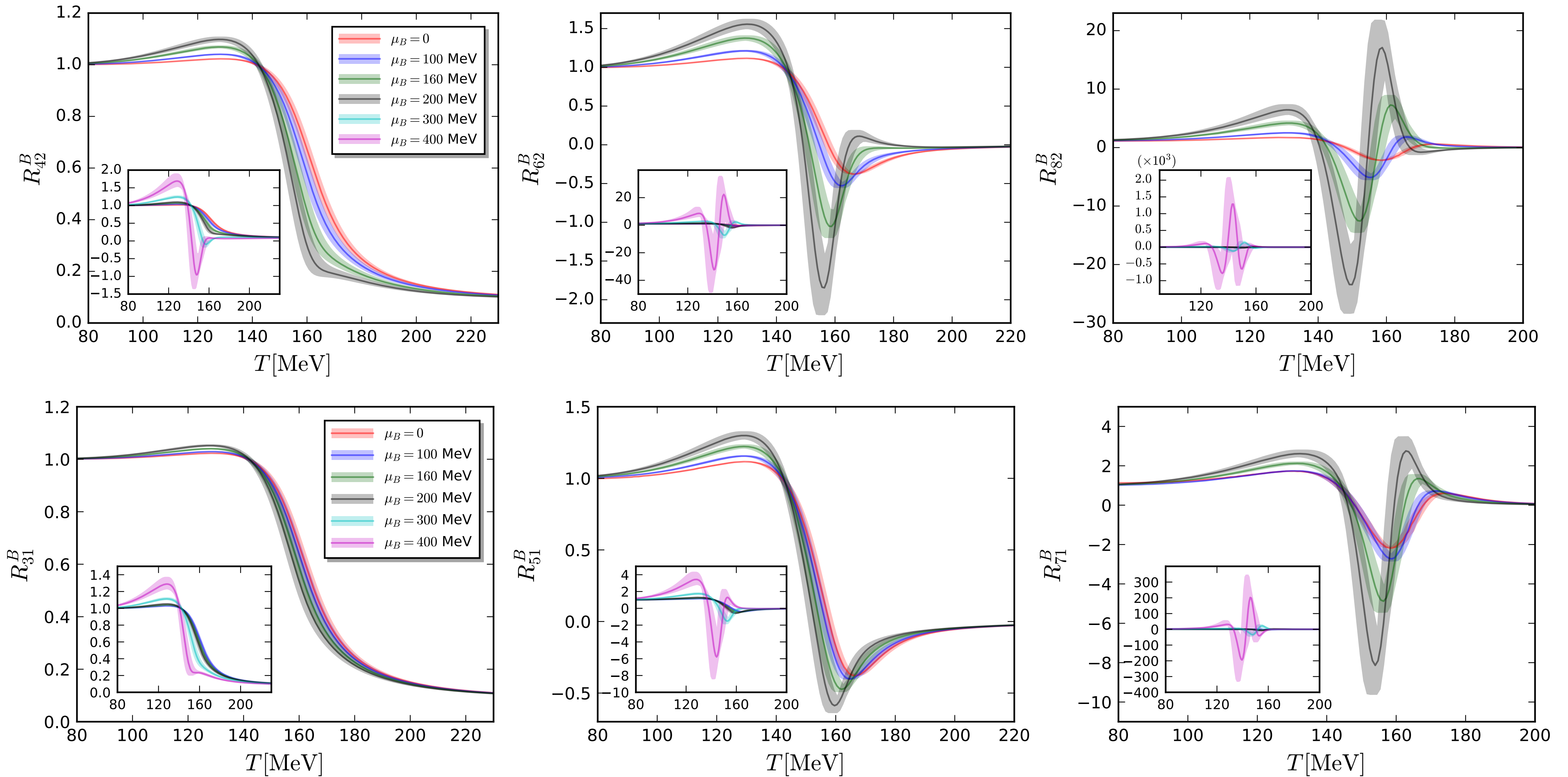
Rescale coefficients

$$T_{LEFT} = c_T T, \quad \mu_{B_{LEFT}} = c_\mu \mu_B$$

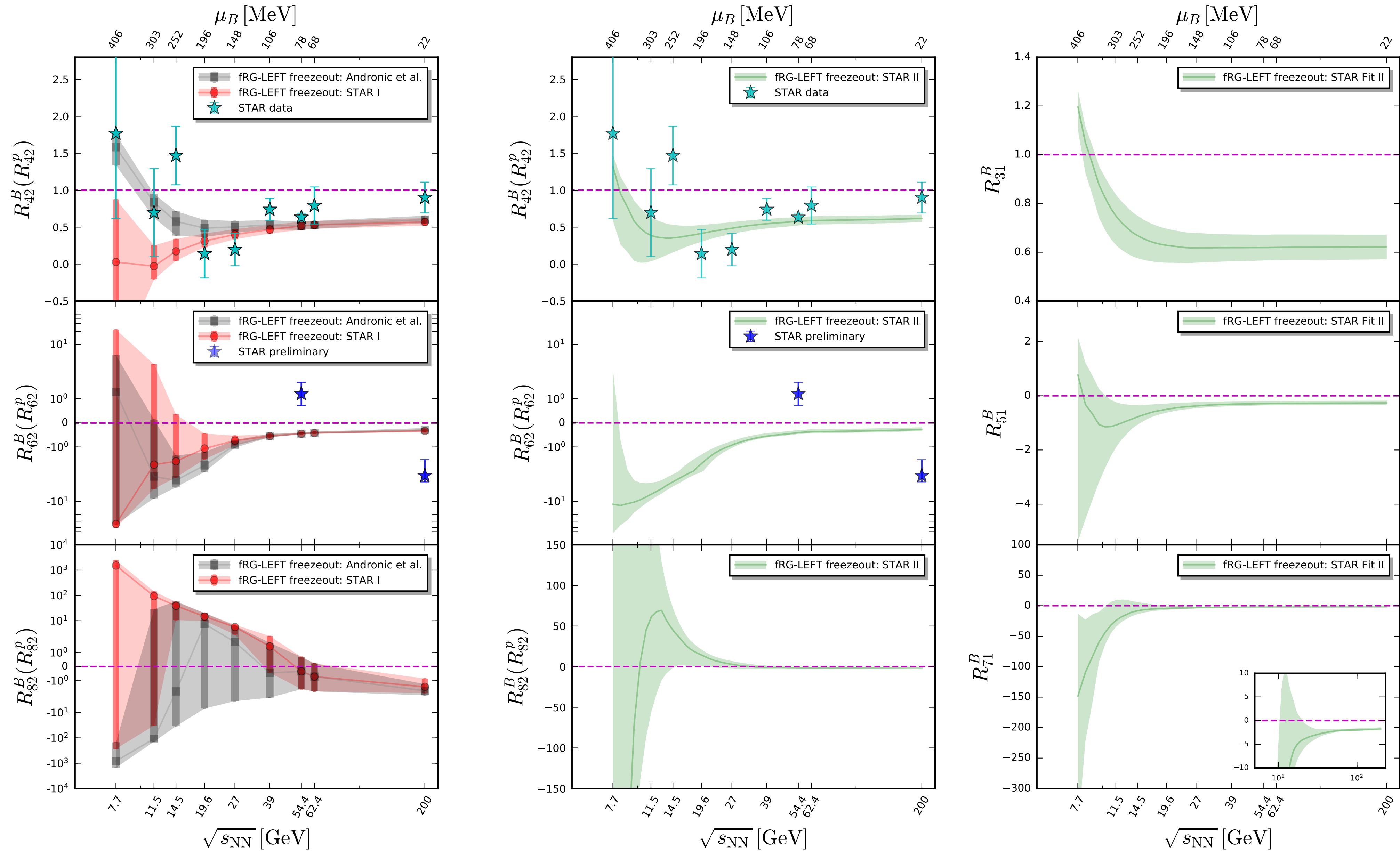
$$c_T = \frac{T_{c,LEFT}}{T_{c,QCD}} \simeq 1.247(12) \quad c_{\mu_B} = \frac{\kappa_{LEFT}}{\kappa_{QCD}} \simeq 1.110(66)$$

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c} \right)^2 + \lambda \left(\frac{\mu_B}{T_c} \right)^4 + \dots$$

Baryon number fluctuations at finite density



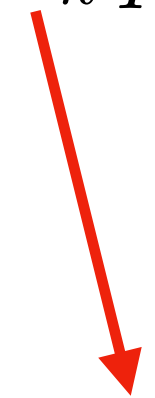
Baryon number fluctuations on the freeze-out curve



QCD-assisted LEFT within fRG

Effective action:

$$\Gamma_k = \int_x \left\{ Z_{q,k} \bar{q} (\gamma_\mu \partial_\mu - \gamma_0 (\mu + igA_0)) q + \frac{1}{2} Z_{\phi_k} (\partial_\mu \phi)^2 + h_k \bar{q} (T^0 \sigma + i\gamma_5 \mathbf{T} \cdot \boldsymbol{\pi}) q + V_k(\rho) - c\sigma \right\}$$



h_k is the Yukawa coupling

★ The flow of Yukawa coupling is taken from the fRG-QCD computation.

Fu, Pawłowski, Rennecke, *PRD* 101 (2020), 054032

LEFT flow equation:

$$\partial_t \Gamma_k[\Phi] = - \text{quark} + \frac{1}{2} \text{meson}$$

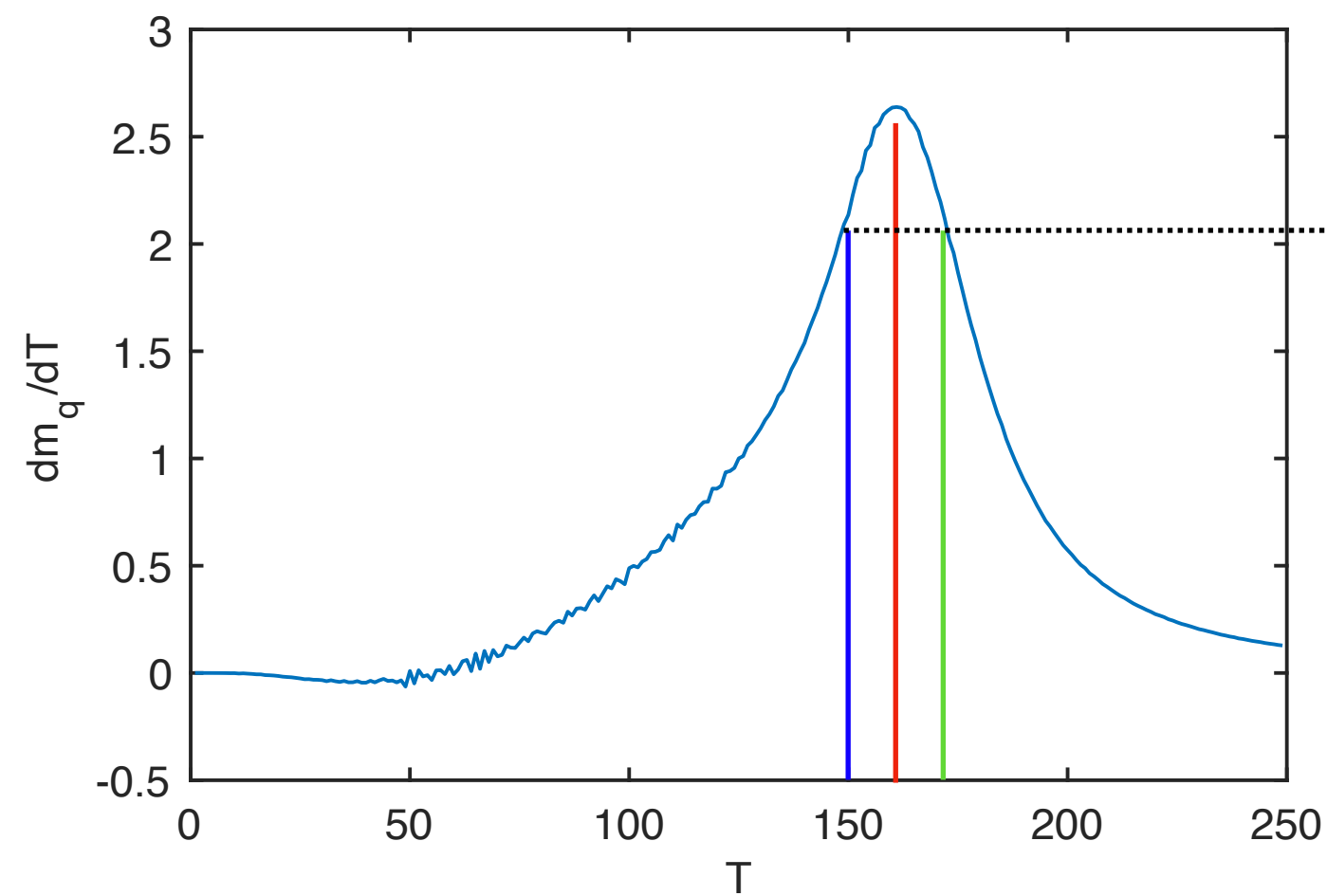
Flow equations of two-point functions:

$$\partial_t \longrightarrow^{-1} = \tilde{\partial}_t \left(\text{quark self-energy} \right)$$

$$\partial_t \equiv\equiv^{-1} = \tilde{\partial}_t \left(\text{meson loop} + \text{meson self-energy} - \frac{1}{2} \text{meson tadpole} \right)$$

Determination of Critical point

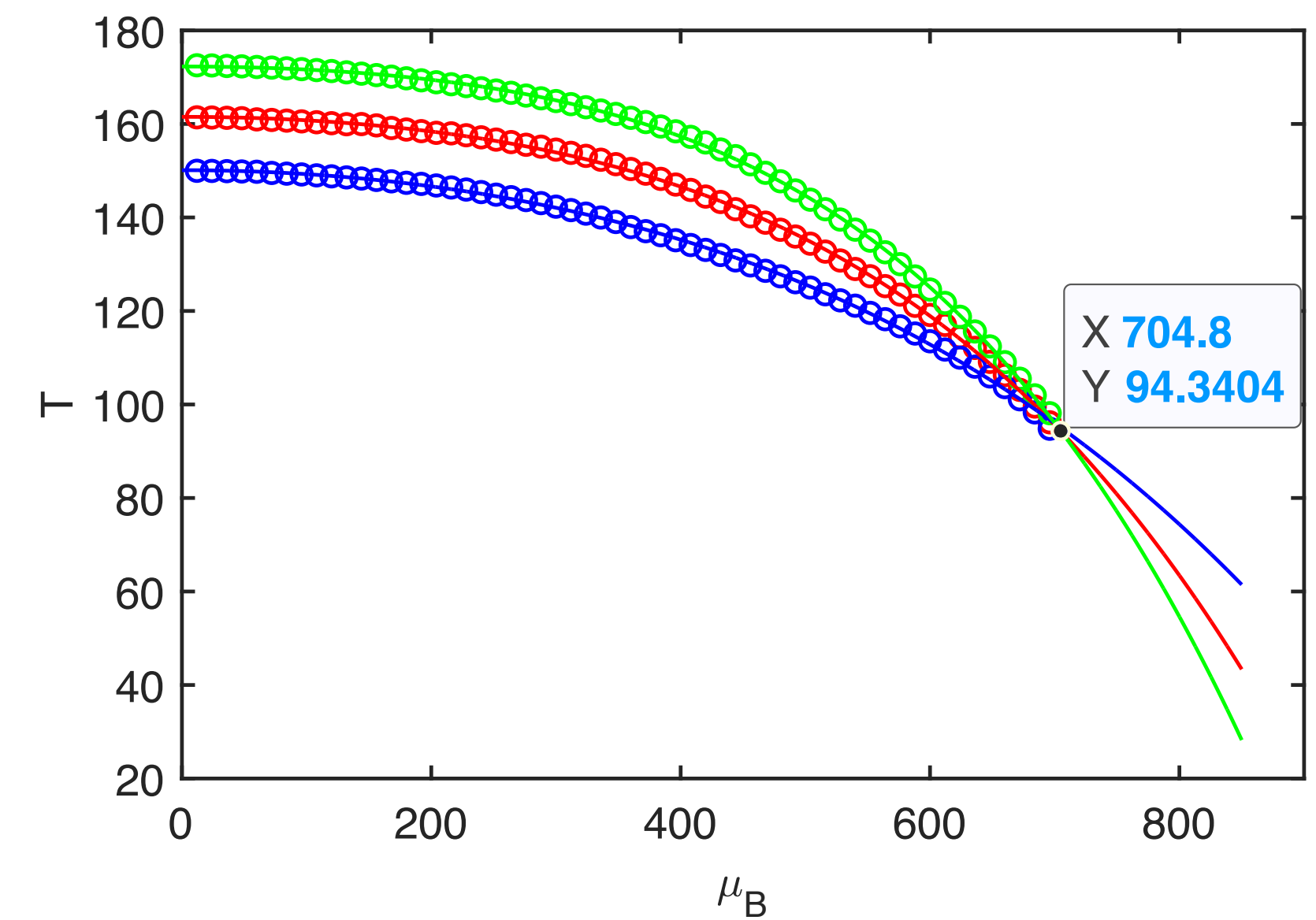
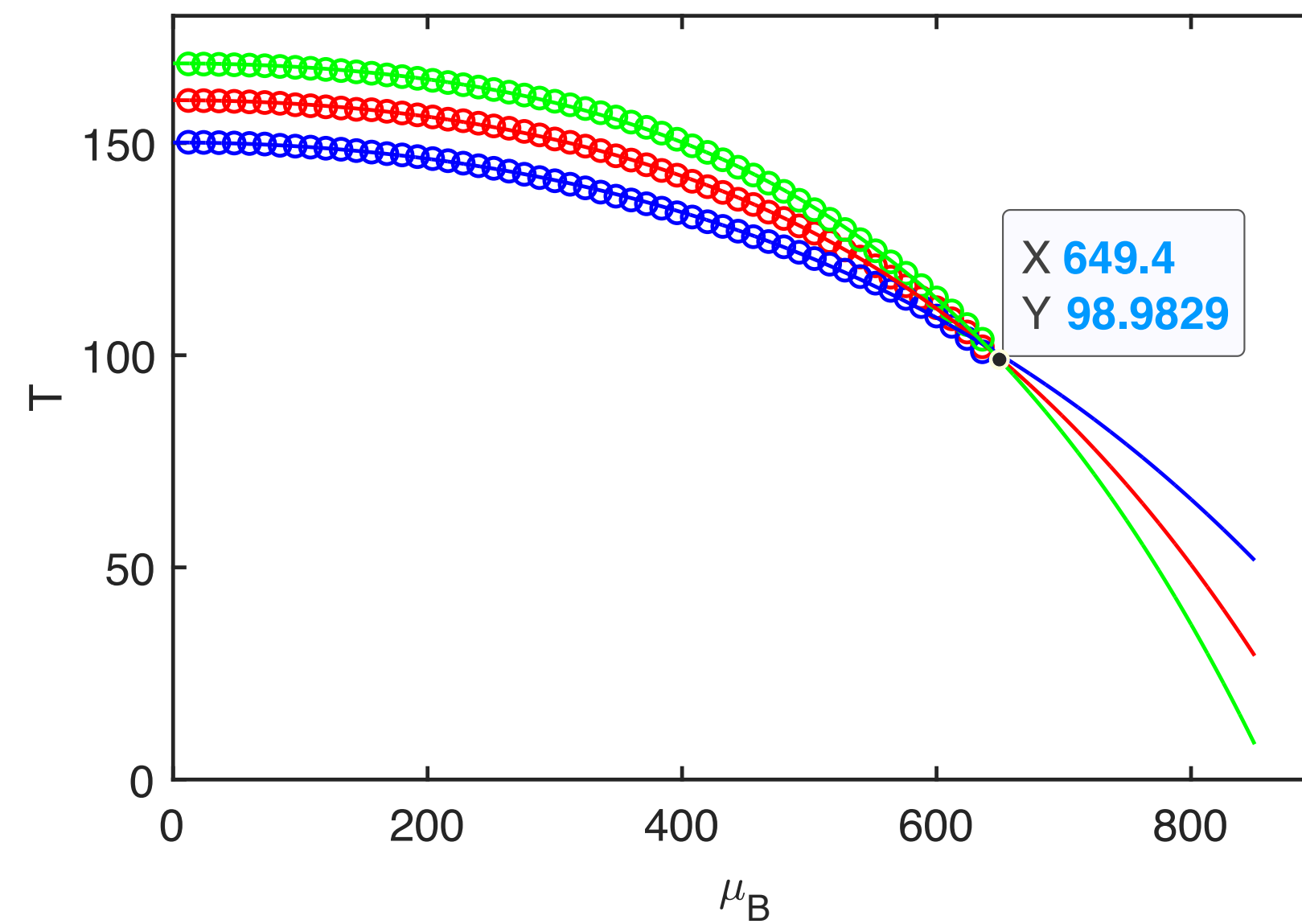
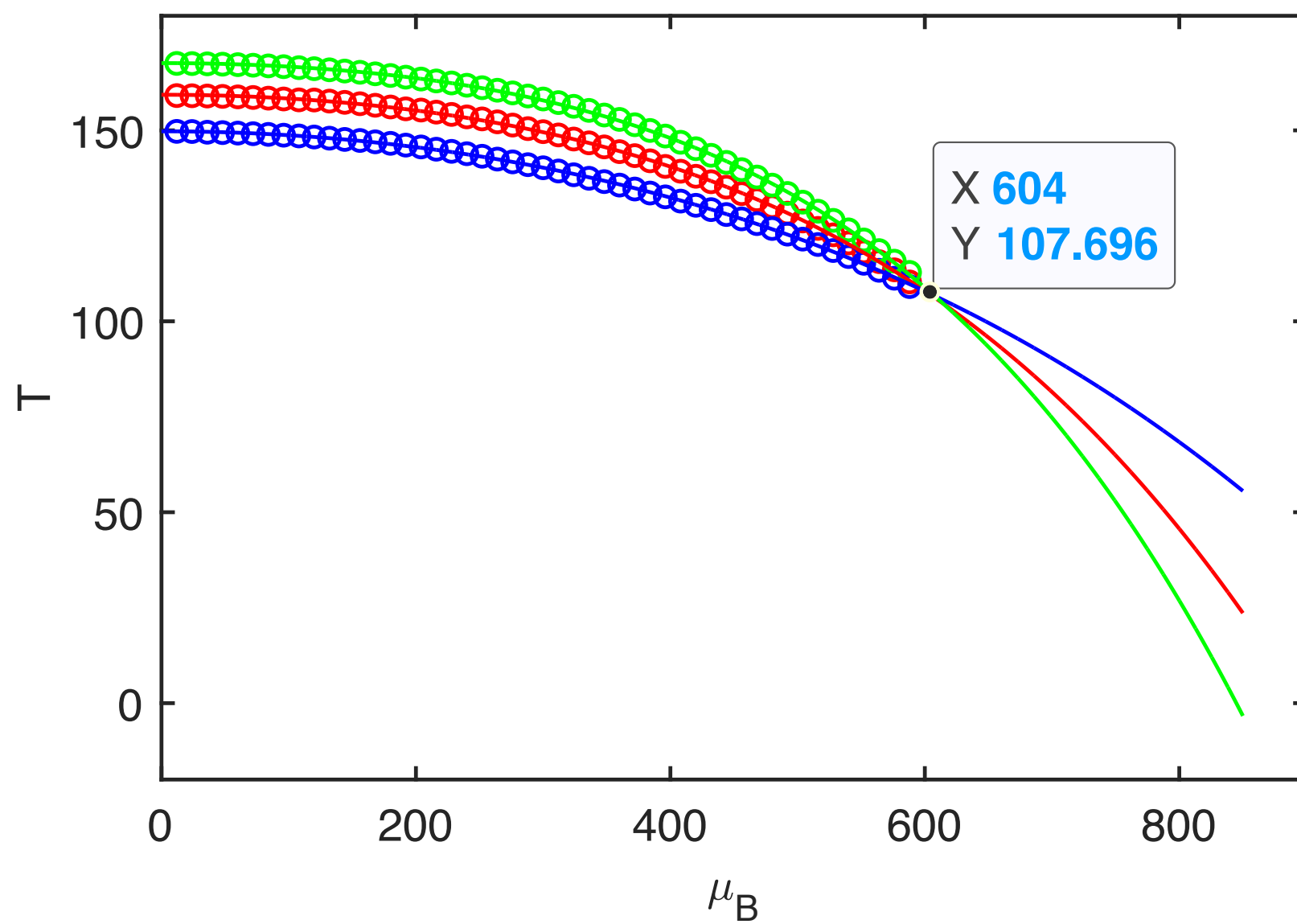
Here we use the thermal derivative of the quark mass to determine the phase boundary and critical point.



$$80\% \times \left. \frac{\partial m_q(T)}{\partial T} \right|_{T=T_c}$$

★ Red lines are the temperature of the peak.

★ Blue and green lines are temperature of 80% peak height.



Error estimate: freeze-out curve

