Isentropic equation of state and speed of sound of (2+1)-flavor QCD from Lattice QCD calculations

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Acknowledgement: To all members of the HotQCD collaboration
QCD phase diagram

Bigger Picture: Understand the thermodynamics at the QCD crossover, study the QCD phase diagram, indication of the location of the critical point.....
Outline and motivation

- **Bulk observables**: Pressure, energy and entropy density at finite chemical potential.

- EoS is crucial input to the interpretation of heavy ion data at freeze-out and the hydrodynamic modelling of matter created in HIC

  - In addition it finds application in the analysis of the “cosmic trajectory” at high T and small $\mu$. And, at low T and high $\mu$ are relevant for Eos of neutron stars.

- **Fluctuation observables**: Specific heat and speed of sound at finite chemical potential.

  - Mainly relevant for understanding the critical behaviour.

  - At vanishing chemical potential, $c_s^2 = s/C_V$, hence, at second order phase transition (not O(n)!!), $C_V \to \infty$, $c_s^2 \to 0$.

Lattice QCD has very good control on these observables at zero chemical potential.

We use HISQ/tree lattice action with temporal extent $N_{\tau} = [6,8,12,16]$ and $(2 + 1)$–flavor to evaluate expansion coefficients at vanishing chemical potentials.

We have approximately 1.5 mil., 300 k and 22 k configs. for $N_{\tau} = 8,12$ and 16 respectively in the temperature range $T \in [135 : 175]$ MeV.

We also included a new temperature point $T = 125$ MeV and currently we are also focussing on increasing the statistics.

$p, n_B, \epsilon$ and $s$ at non-zero temperature and baryon chemical potential can be constructed using these expansion coefficients.
Taylor expansion of Bulk Thermodynamic observables

Expansion of the QCD pressure:

\[
\frac{P(T, \overrightarrow{\mu})}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi^{BQS}_{ijk} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k, \hat{\mu} = \mu/T,
\]

\[
\chi^{BQS}_{ijk}(T,0) = \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_X^i \hat{\mu}_X^j \hat{\mu}_X^k} \bigg|_{\mu_X=0}, \quad X = B, Q, S
\]

Parametrization of \(\mu_Q, \mu_S\) \([n_Q/n_B = r, n_S = 0]\):

\[
\mu_S = s_1 \mu_B + s_3 \mu_B^3 + \ldots
\]

\[
\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + \ldots
\]

Condition in HIC, \(n_S = 0, \quad n_Q/n_B = 0.4, \quad \Rightarrow \mu_B, \mu_S \neq 0, \mu_Q \ll \mu_B\)

Isospin symmetric case, \(n_S = 0, \quad n_Q/n_B = 0.5, \quad \Rightarrow \mu_B, \mu_S \neq 0, \mu_Q = 0,\)
Expansion coefficients show pronounced structure only at $T < 200$ MeV.

Hence, higher order expansion coefficients will only be important for Eos at $T < 200$ MeV.
There is good agreement between the eighth-order, sixth-order and fourth-order Taylor series up to $\mu_B/T = 2.5$ for $T \leq 175$ and $\mu_B/T \simeq 3$ for $T \geq 175$.

Sixth order Taylor series and $[2,4]$—Padé approximant agree with each other up to $\mu_B/T \leq 2.5$ for $T \leq 175$ and $\mu_B/T \simeq 3$ for $T \geq 175$.

Reliability of pressure: $\mu_B/T \leq 2.5$ for $T \leq 175$ and $\mu_B/T \leq 3$ for $T \geq 175$. 
Taylor expansion of $\epsilon$ and $s$

Reliability of energy and entropy density: $2 \leq \mu_B/T \leq 2.5$ for $T \leq 175$ and $\mu_B/T \leq 3$ for $T \geq 175$.

At large $\mu_B$, increasing order make the “wiggles” less pronounced.

Padé based resummation is able to remove the “wiggles” of $\epsilon$ and $s$ at $\mu_B/T > 2.0$ : Work in progress [HotQCD]
EoS of (2+1)-flavor QCD

This is an update on the EoS [ Taylor exp.]:
Phys.Rev.D 95 (2017) 5, 054504
Consistent with [Img. mu]:
Phys. Rev. D 105, 114504

- $P_0$, $\epsilon_0$ and $s_0$ taken from Phys.Rev.D 90 (2014) 094503.

The isentropic equation of state of 2-flavor QCD

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(Dated: November 6, 2018)

In a heavy ion collision a dense medium is created which after thermalization is expected to expand without further generation of entropy ($S$) and with fixed baryon number ($N_B$) or, equivalently, with fixed quark number $N_q = 3N_B$. During the isentropic expansion the ratio $S/N_B$ thus remains constant$^1$. The cooling of the expanding system then is controlled by the equation of state on lines of constant $S/N_B$. From a knowledge of the energy density and pressure at non-vanishing quark chemical potential we can calculate trajectories in the $\mu_q$-$T$ phase diagram of QCD that correspond to constant $S/N_B$ and can determine the isentropic equation of state on these trajectories.
**Baryon chemical potentials in the constant $s/n_B$ trajectories**

\[
\hat{s}/\hat{n}_B = (\hat{P} + \hat{\epsilon})/\hat{n}_B - \hat{\mu}_B - \hat{\mu}_Q\hat{n}_Q/\hat{n}_B
\]

- $\mu_B/T$ is almost constant at high temperature for $\hat{s}/\hat{n}_B \geq 100$.

- $\mu_B/T$ slightly increases with decreasing temperature at $T \leq 160$ MeV, that is in accordance with the HRG models.

- At, smaller $\hat{s}/\hat{n}_B$, $\mu_B/T$ increases both at high and low temperatures. The deviation from the Ideal gas in all cases is about 20% at the highest temperature.
Comparison of $s/n_B$ QCD trajectories with RHIC Freezeout

![Graph showing comparison of $s/n_B$ QCD trajectories with RHIC Freezeout.](chart)


Isentropic Eos in (2+1)-flavor is well described down to the smallest beam energy of BESII in collider mode.

Determination of $\mu_B$ at freeze out from HRG model motivated relations maybe problematic for smaller beam energies.
Specific heat and speed of sound in \((2 + 1)-\)flavor QCD

\[ c_s^2 = \frac{4\hat{p} + T \frac{\partial \hat{p}}{\partial T}}{4\hat{\epsilon} + T \frac{\partial \hat{\epsilon}}{\partial T}} \bigg|_{s/n_B, n_Q/n_B, n_S=0} \]

\[ c_V / T^3 = 3\hat{s} + T \frac{\partial \hat{s}}{\partial T} \bigg|_{s/n_B, n_Q/n_B, n_S=0} \]

HotQCD in preparation, D. Clarke Lattice 2022.
Specific heat and speed of sound in (2 + 1)–flavor QCD

- $c_s^2$ and $c_V/T^3$ increases rapidly close to the $T_{pc}$ and then slowly increase.

- $c_s^2$ develop a dip at lower temperature and we see a hint of an $s/n_B$ ordering change at low temperatures.

- This is consistent with model calculations.

Wei-bo He et. al, Phys. Rev. D 105, 094024
Conclusions

★ Reliable Eos of \((2 + 1)\)-flavor QCD for \(2 \leq \mu_B/T \leq 2.5\) for \(T < 175\) and \(\mu_B/T \simeq 3\) for \(T > 175\).

★ This range covers the beam energy, \(\sqrt{s} \geq 7.7\) GeV.
Isentropic EoS of (2+1)-flavor QCD: relevant for HIC

\[ \hat{s}/\hat{n}_B = (\hat{P} + \hat{\epsilon})/\hat{n}_B - \hat{\mu}_B - \hat{\mu}_Q\hat{n}_Q/\hat{n}_B \]

\[ \hat{\mu}_Q = q_1\hat{\mu}_B + q_3\hat{\mu}_B^3 + \ldots, \hat{n}_Q/\hat{n}_B = 0.4 \]

\[ \hat{\mu}_B = \sum_{n=1}^{\infty} h_{2n-1} \left( \frac{\hat{s}}{\hat{n}_B} \right)^{1-2n} \]