Isentropic equation of state and speed of sound of (2+1)-flavor QCD from Lattice QCD calculations

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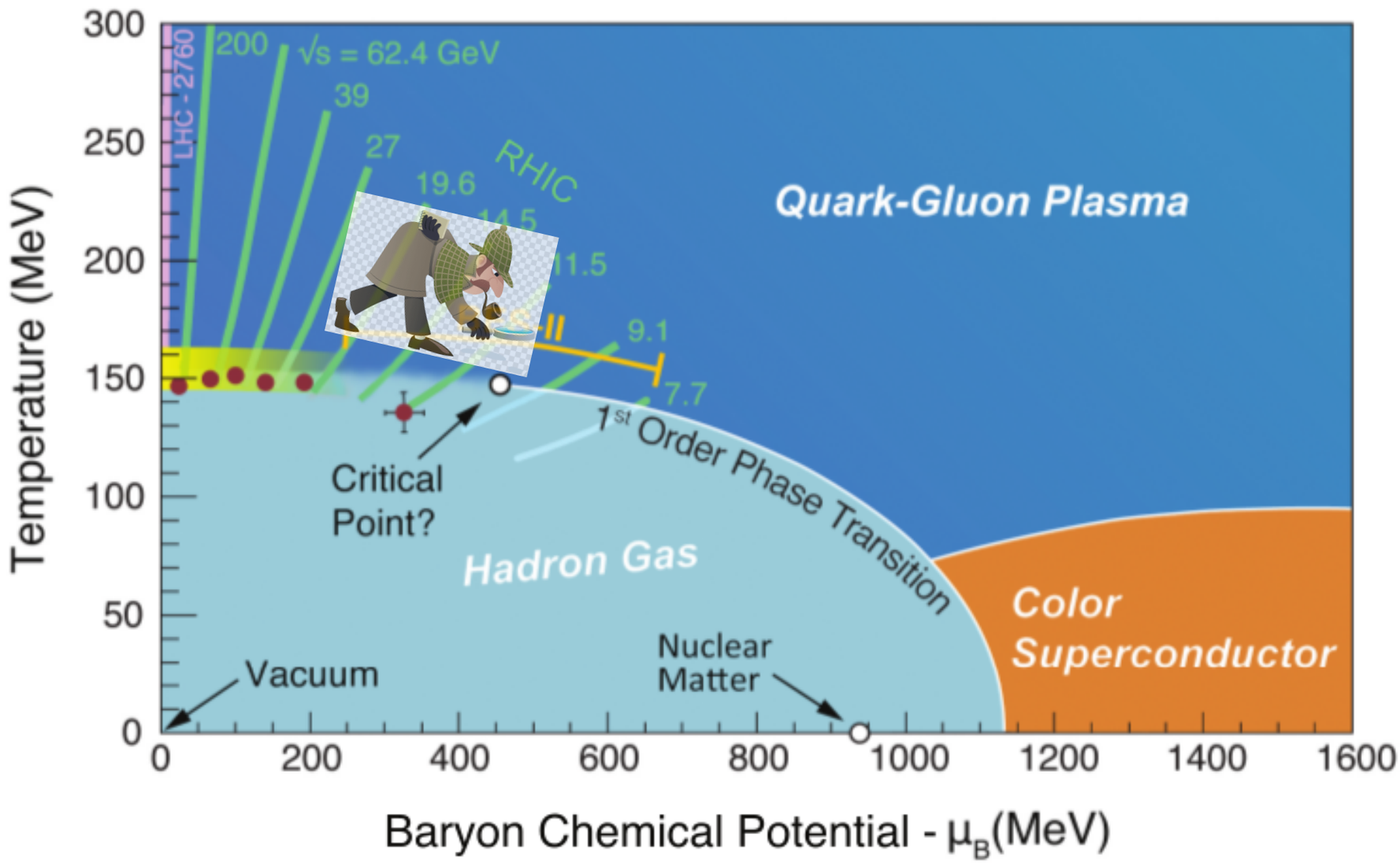
Acknowledgement: To all members of the HotQCD collaboration

CPOD2022 - Workshop on Critical Point and Onset of Deconfinement

QCD phase diagram

Bigger Picture : Understand the thermodynamics at the QCD crossover, study the QCD phase diagram, indication of the location of the critical point.....



Outline and motivation

- **Bulk observables** : Pressure, energy and entropy density at finite chemical potential.

- EoS is crucial input to the interpretation of heavy ion data at freeze-out and the hydrodynamic modelling of matter created in HIC

  - In addition it finds application in the analysis of the “cosmic trajectory” at high T and small μ. And, at low T and high μ are relevant for Eos of neutron stars.

- **Fluctuation observables:** Specific heat and speed of sound at finite chemical potential.

  - Mainly relevant for understanding the critical behaviour.

  - At vanishing chemical potential, $c_s^2 = s/C_V$, hence, at second order phase transition (not O(n) !!) , $C_V \to \infty$, $c_s^2 \to 0$.

Lattice QCD has very good control on these observables at zero chemical potential.

Stout : Phys.Lett. B370, 99 (2014), 1309.5258 , HISQ : *Phys.Rev.D* 90 (2014) 094503

Thermodynamics using Lattice QCD

- We use HISQ/tree lattice action with temporal extent $N_\tau = [6,8,12,16]$ and $(2+1)-$flavor to evaluate expansion coefficients at vanishing chemical potentials.

- We have approximately 1.5 mil., 300 k and 22 k configs. for $N_\tau = 8,12$ and 16 respectively in the temperature range $T \in [135 : 175]$ MeV.

- We also included a new temperature point $T = 125$ MeV and currently we are also focussing on increasing the statistics.

- p, n_B, ϵ and s at non-zero temperature and baryon chemical potential can be constructed using these expansion coefficients.

Taylor expansion of Bulk Thermodynamic observables

Expansion of the QCD pressure:

$$\frac{P(T,\vec{\mu})}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k \ , \ \hat{\mu} = \mu/T,$$

$$\chi_{ijk}^{BQS}(T,0) = \left. \frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_X^{i,j,k}} \right|_{\mu_X=0} \ , \ X = B, Q, S$$

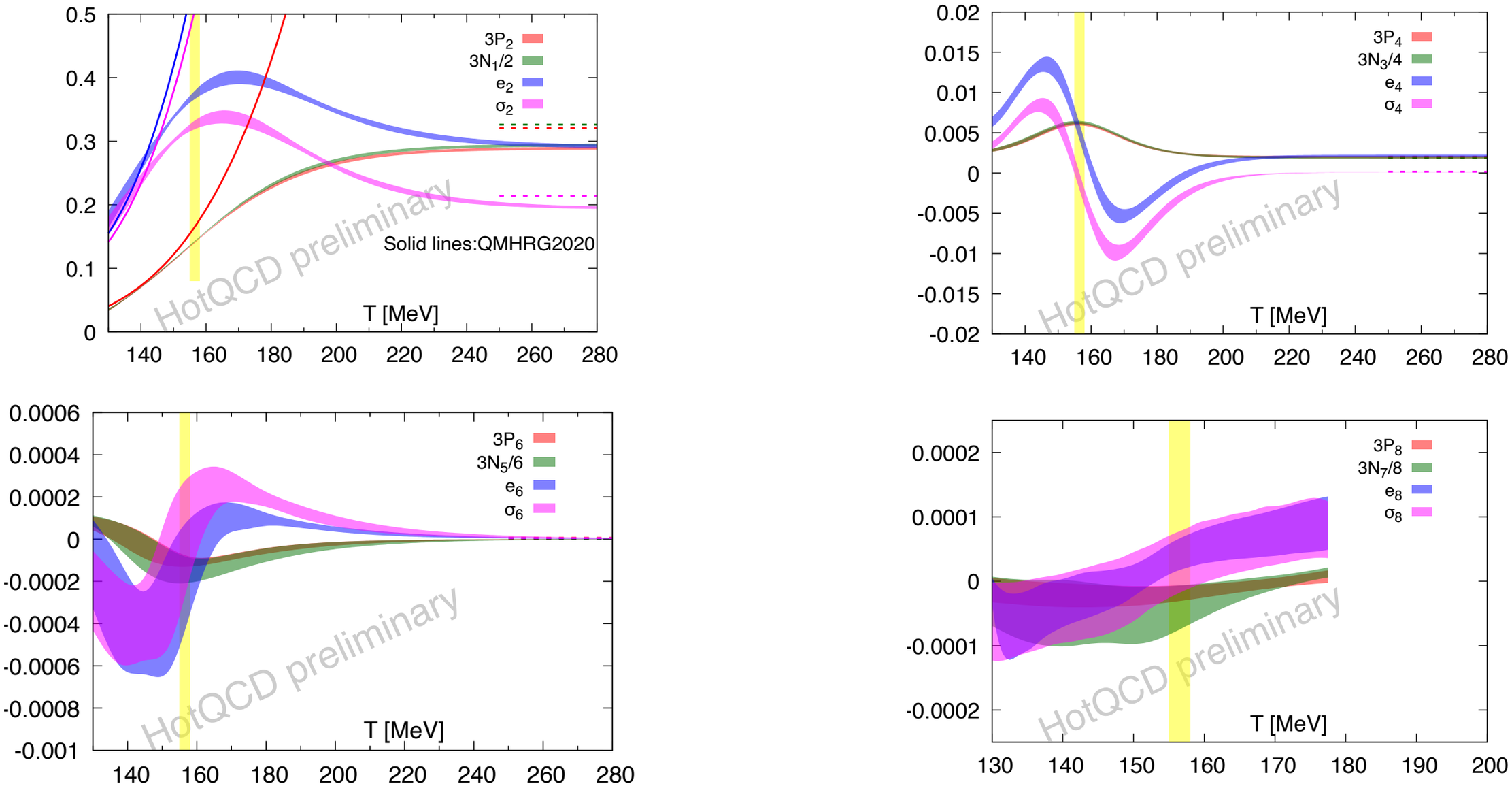
Parametrization of μ_Q, μ_S $[n_Q/n_B = r, n_S = 0]$ **:**

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + ...$$
$$\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + ...$$

Condition in HIC, $n_S = 0, \ \ n_Q/n_B = 0.4, \ \ \Rightarrow \mu_B, \mu_S \neq 0, \ \mu_Q \ll \mu_B$

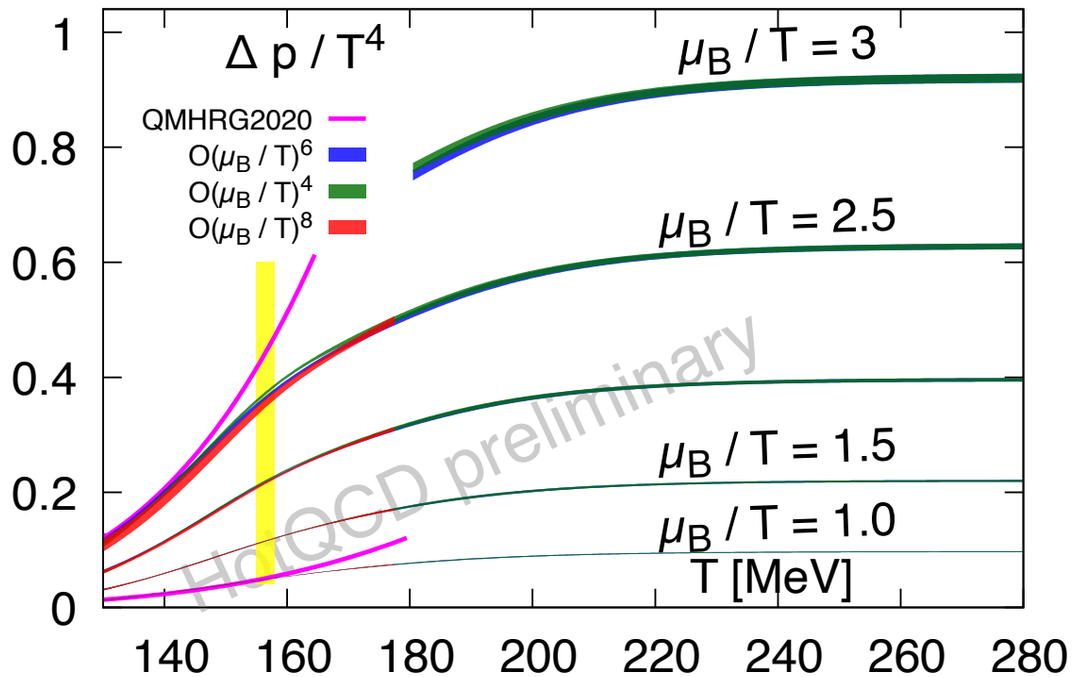
Isospin symmetric case, $n_S = 0, \ \ n_Q/n_B = 0.5, \ \ \Rightarrow \mu_B, \mu_S \neq 0, \ \mu_Q = 0,$

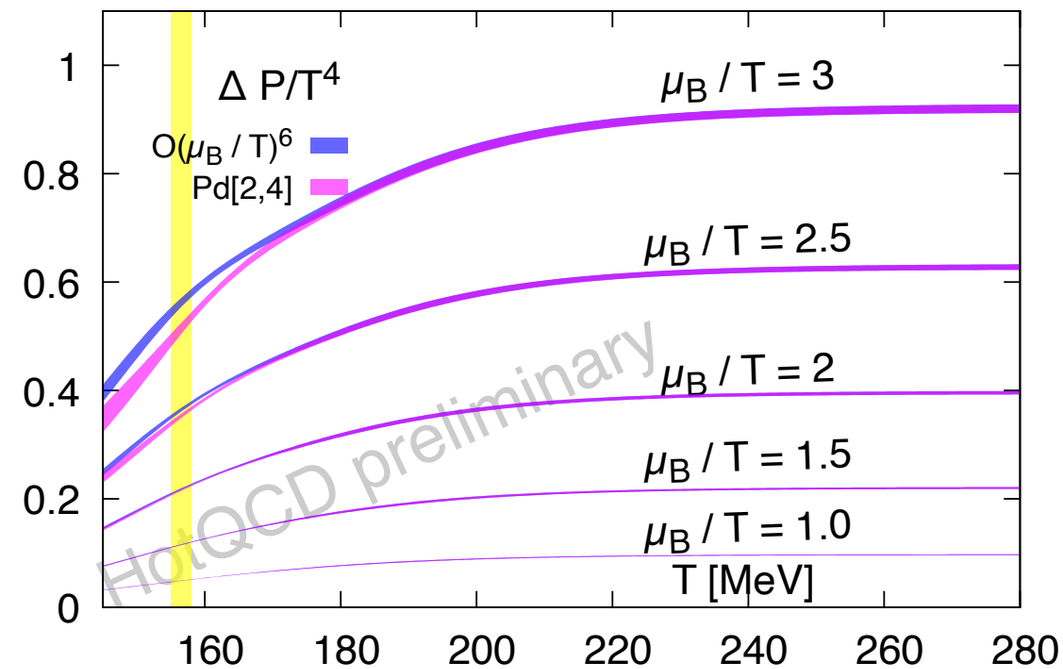
Expansion coefficients of the Thermodynamic observables



- Expansion coefficients show pronounced structure only at $T < 200$ MeV.
- Hence, higher order expansion coefficients will only be important for Eos at $T < 200$ MeV.

Taylor expansion and Padé approximants of pressure



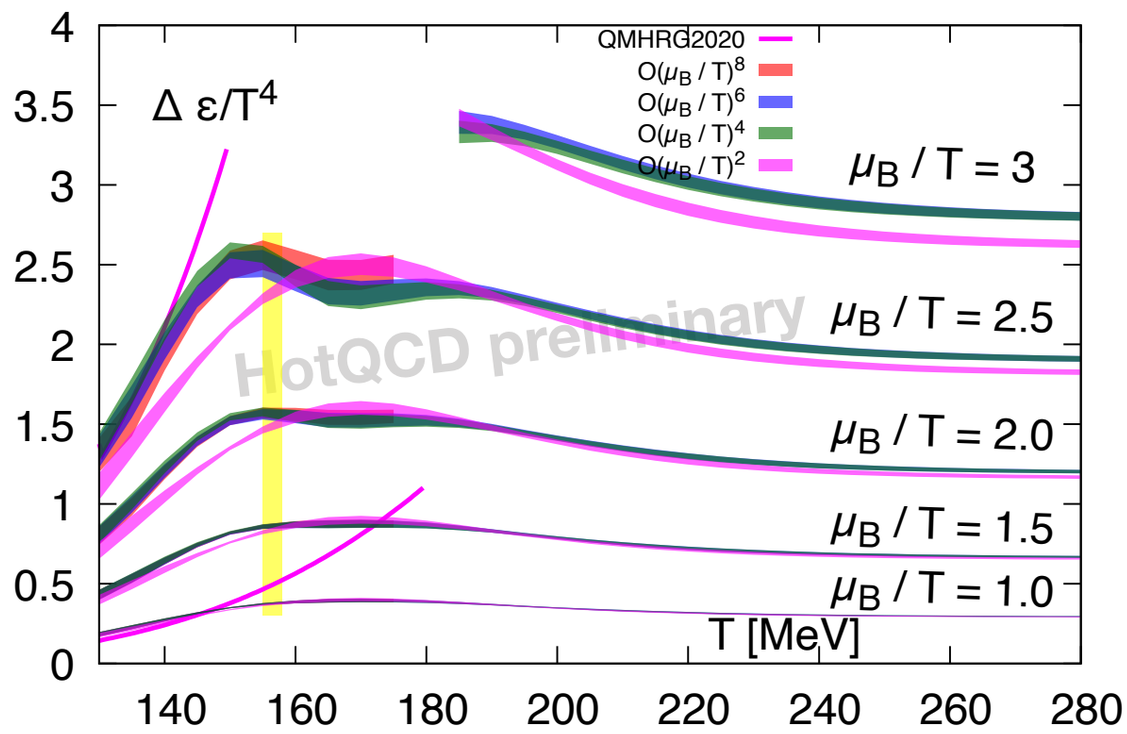


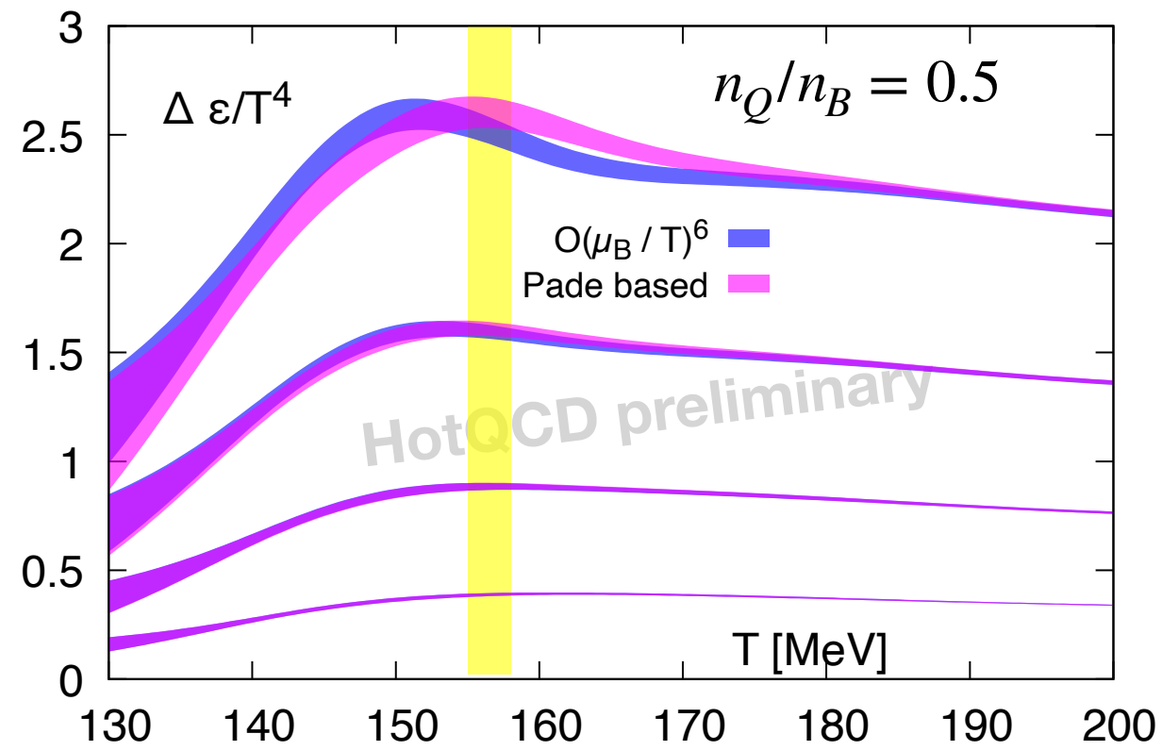
- There is good agreement between the eighth-order, sixth-order and fourth-order Taylor series up to $\mu_B/T = 2.5$ for $T \leq 175$ and $\mu_B/T \simeq 3$ for $T \geq 175$.

- Sixth order Taylor series and $[2,4]-$Padé approximant agree with each other up to $\mu_B/T \leq 2.5$ for $T \leq 175$ and $\mu_B/T \simeq 3$ for $T \geq 175$.

Reliability of pressure : $\mu_B/T \leq 2.5$ for $T \leq 175$ and $\mu_B/T \leq 3$ for $T \geq 175$.

Taylor expansion of ϵ and s



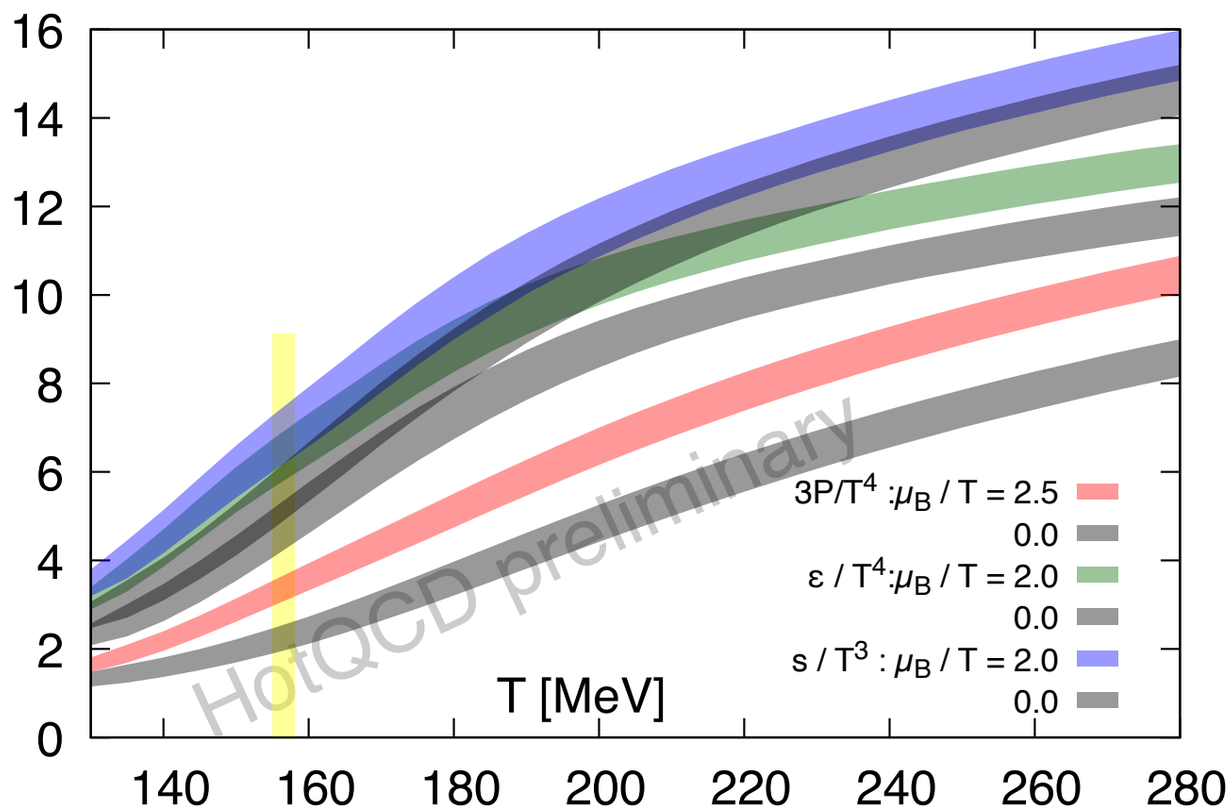


Reliability of energy and entropy density : $2 \leq \mu_B/T \leq 2.5$ for $T \leq 175$ and $\mu_B/T \leq 3$ for $T \geq 175$.

At, large μ_B, increasing order make the “wiggles” less pronounced.

Padé based resummation is able to remove the “wiggles” of ϵ and s at $\mu_B/T > 2.0$: Work in progress [HotQCD]

EoS of (2+1)-flavor QCD



Fixed μ_B/T EoS

This is an update on the EoS [Taylor exp.]:
***Phys.Rev.D* 95 (2017) 5, 054504**

Consistent with [Img. mu] :
Phys. Rev. D 105, 114504

- P_0, ϵ_0 and s_0 taken from Phys.Rev.D 90 (2014) 094503.

The isentropic equation of state of 2-flavor QCD

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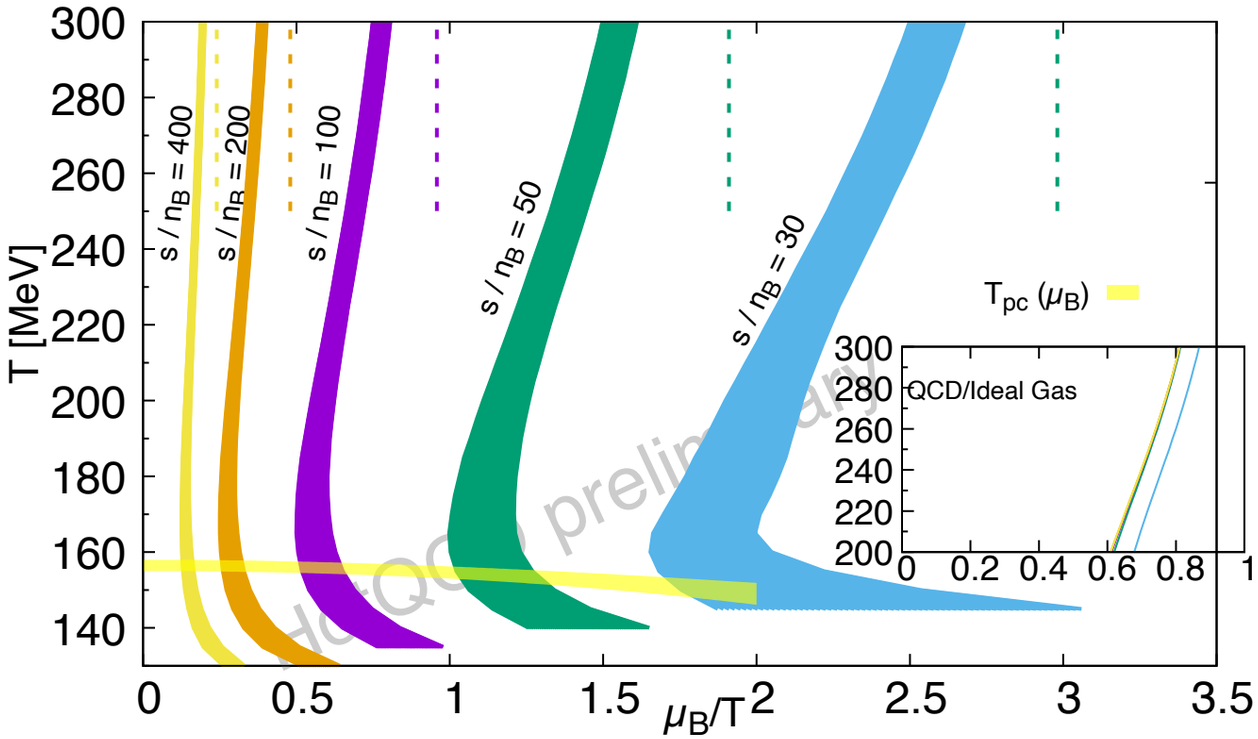
(Dated: November 6, 2018)

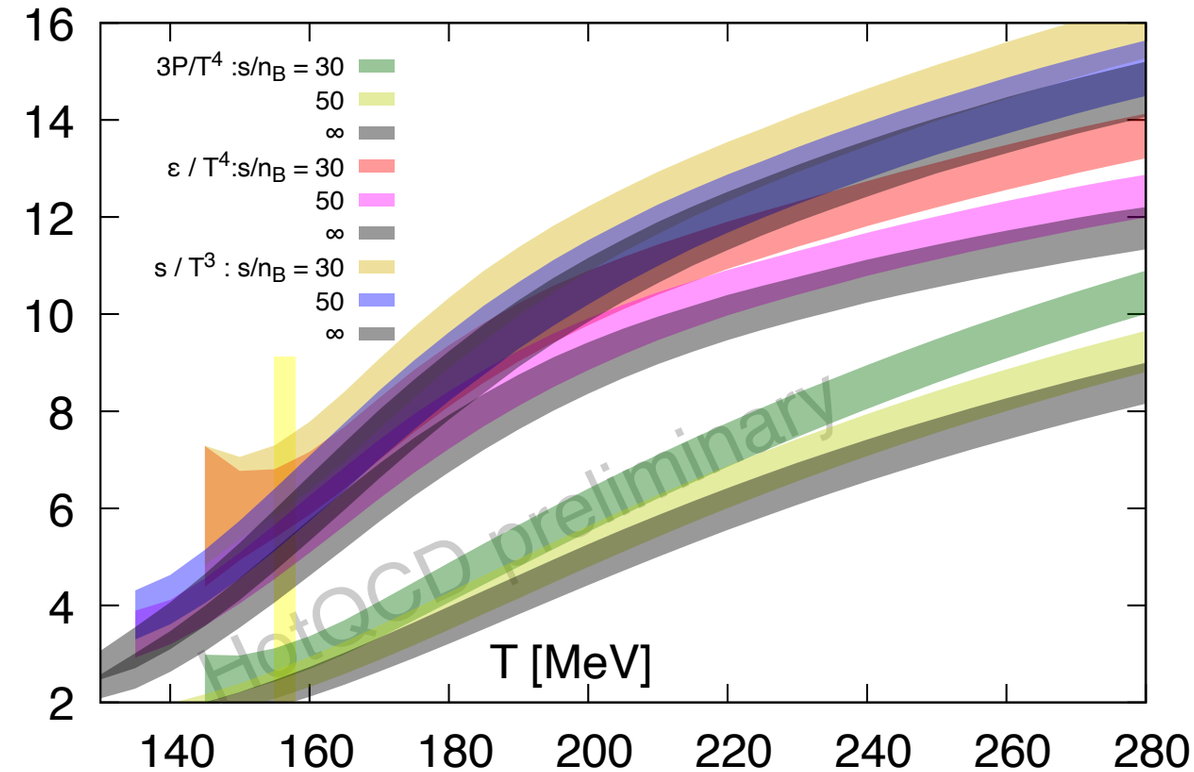
Need EoS for eg fixed $s/n_B, n_Q/n_B$, and $n_S = 0$

Phys. Rev. D 73, 054506

In a heavy ion collision a dense medium is created which after thermalization is expected to expand without further generation of entropy (S) and with fixed baryon number (N_B) or, equivalently, with fixed quark number $N_q = 3N_B$. During the isentropic expansion the ratio S/N_B thus remains constant[1]. The cooling of the expanding system then is controlled by the equation of state on lines of constant S/N_B. From a knowledge of the energy density and pressure at non-vanishing quark chemical potential we can calculate trajectories in the μ_q-T phase diagram of QCD that correspond to constant S/N_B and can determine the *isentropic equation of state* on these trajectories.

Baryon chemical potentials in the constant s/n_B trajectories

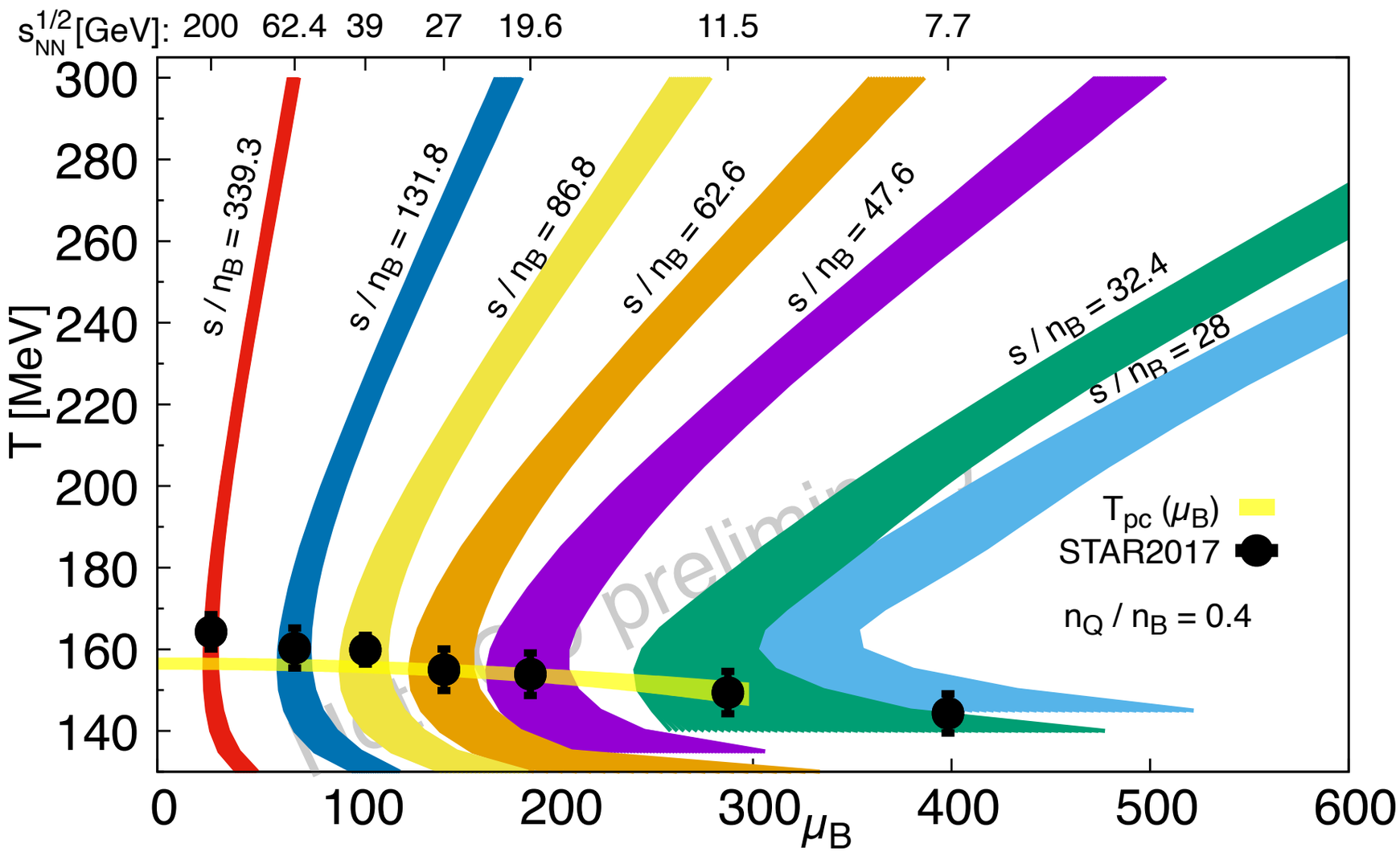




$$\hat{s}/\hat{n}_B = (\hat{P} + \hat{\epsilon})/\hat{n}_B - \hat{\mu}_B - \hat{\mu}_Q \hat{n}_Q/\hat{n}_B$$

- μ_B/T is almost constant at high temperature for $\hat{s}/\hat{n}_B \geq 100$.
- μ_B/T slightly increases with decreasing temperature at $T \leq 160$ MeV, that is in accordance with the HRG models.
- At, smaller $\hat{s}/\hat{n}_B$, μ_B/T increases both at high and low temperatures. The deviation from the Ideal gas in all cases is about $20\,\%$ at the highest temperature.

Comparison of s/n_B QCD trajectories with RHIC Freezeout



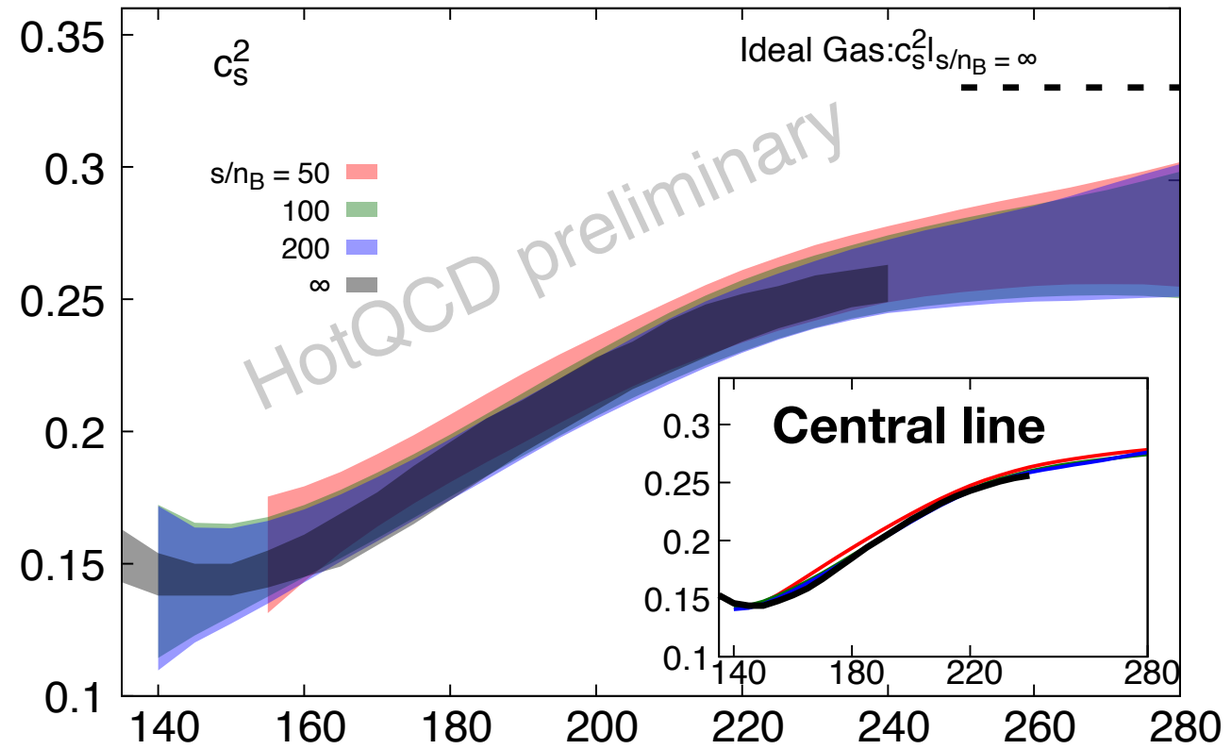
The result is consistent with : The QCD equation of state at finite density from analytical continuation, J.N. Guenther et al,
***Nucl.Phys.A* 967 (2017) 720-723**

Isentropic Eos in (2+1)-flavor is well described down to the smallest beam energy of BESII in collider mode.

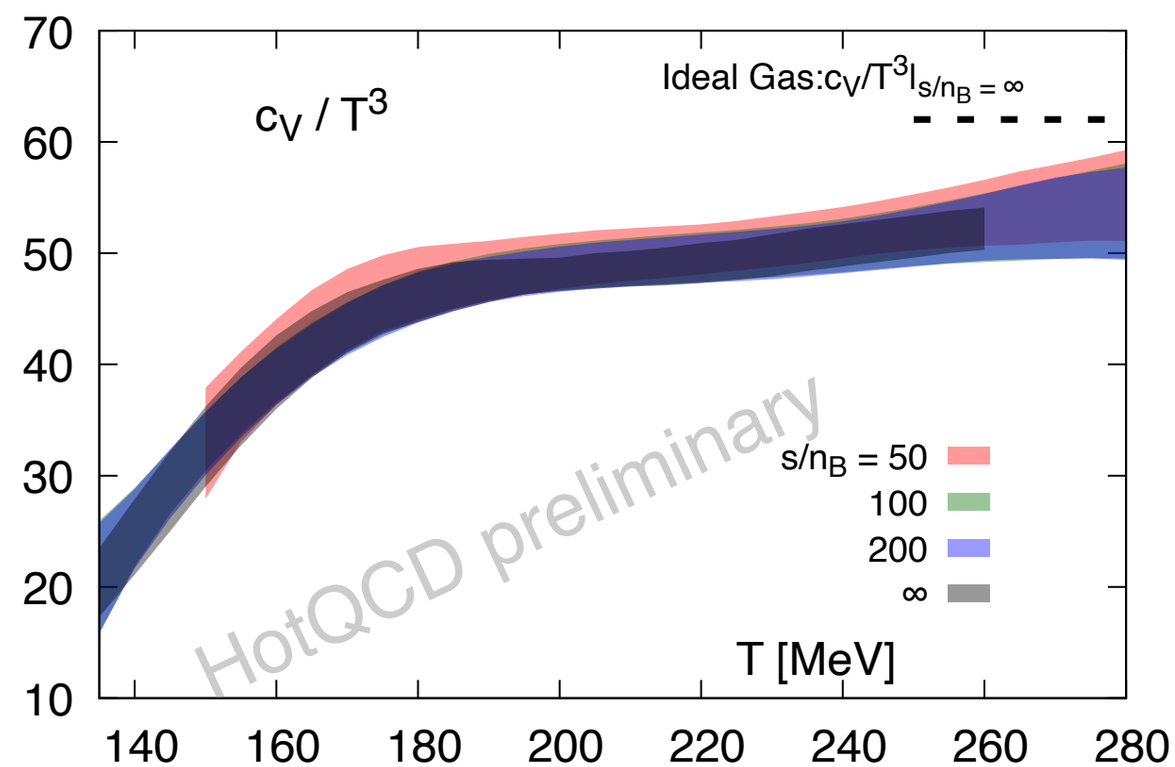
Determination of μ_B at freeze out from HRG model motivated relations maybe problematic for smaller beam energies.

Specific heat and speed of sound in $(2+1)-$flavor QCD

$$c_s^2 = \left.\frac{4\hat{p} + T\frac{\partial \hat{p}}{\partial T}}{4\hat{\epsilon} + T\frac{\partial \hat{\epsilon}}{\partial T}}\right|_{s/n_B, n_Q/n_B, n_S=0}$$



$$c_V/T^3 = 3\hat{s} + T\left.\frac{\partial \hat{s}}{\partial T}\right|_{s/n_B, n_Q/n_B, n_S=0}$$

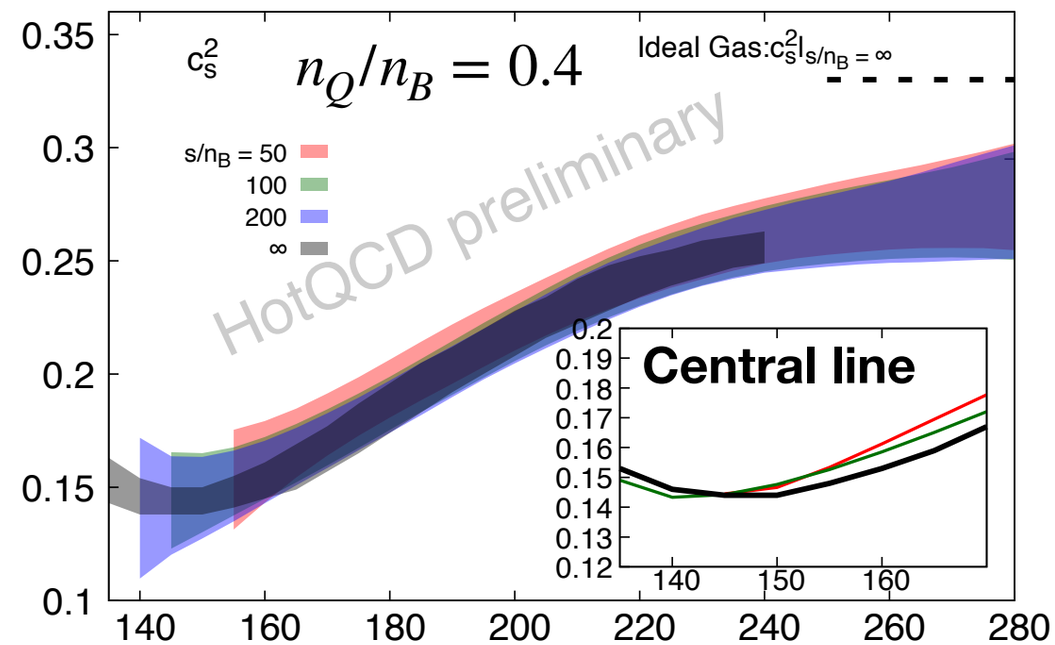


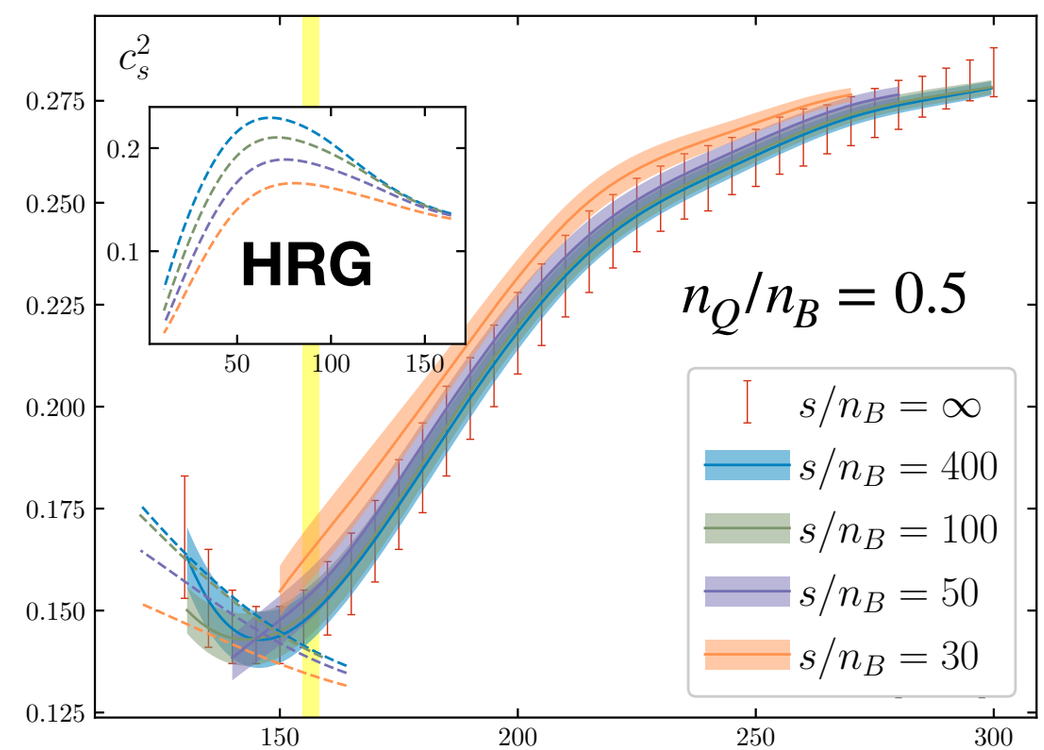
HotQCD in preparation,
D. Clarke Lattice 2022.

Specific heat and speed of sound in $(2+1)-$flavor QCD

- c_s^2 and c_V/T^3 increases rapidly close to the T_{pc} and then slowly increase.
- c_s^2 develop a dip at lower temperature and we see a hint of an s/n_B ordering change at low temperatures.
- This is consistent with model calculations.

Wei-bo He et. al, Phys. Rev. D 105, 094024



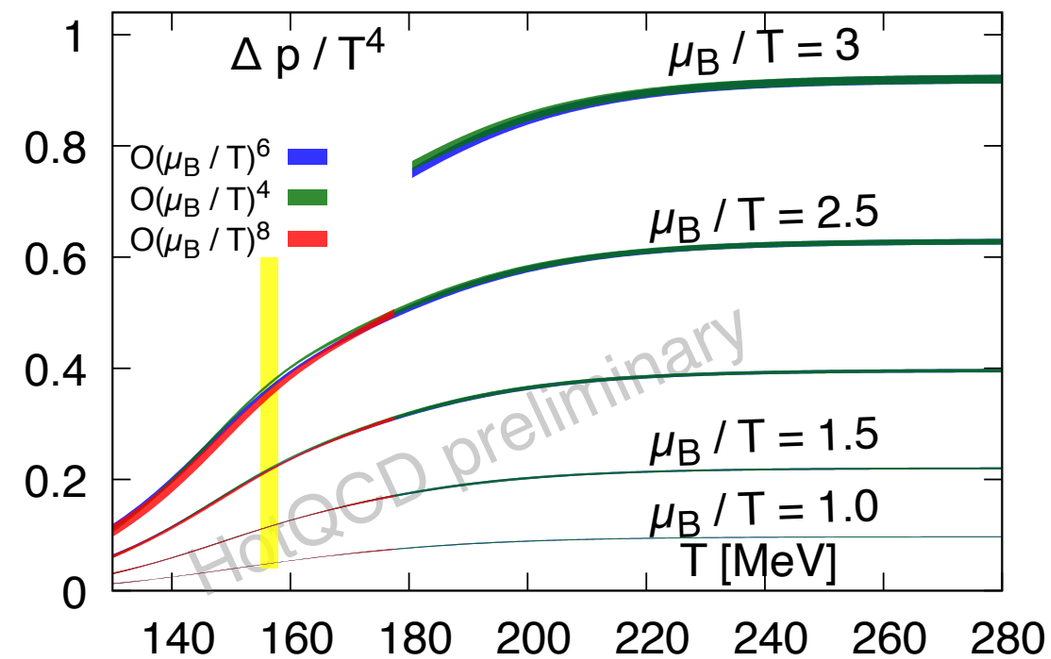


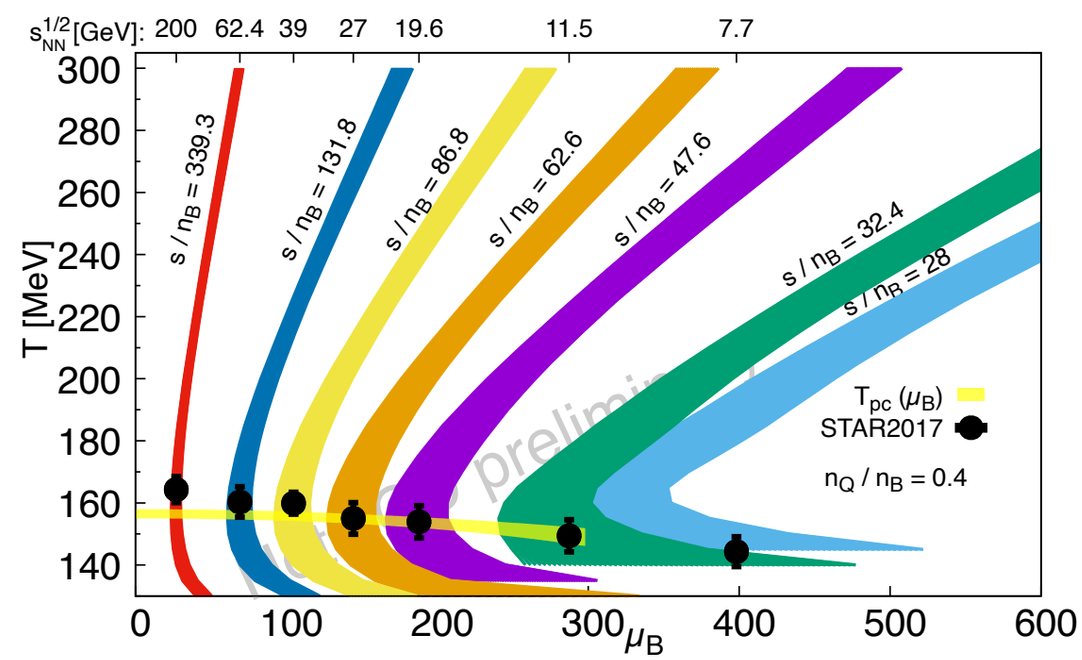
HotQCD in preparation, D. Clarke Lattice 2022.

Conclusions

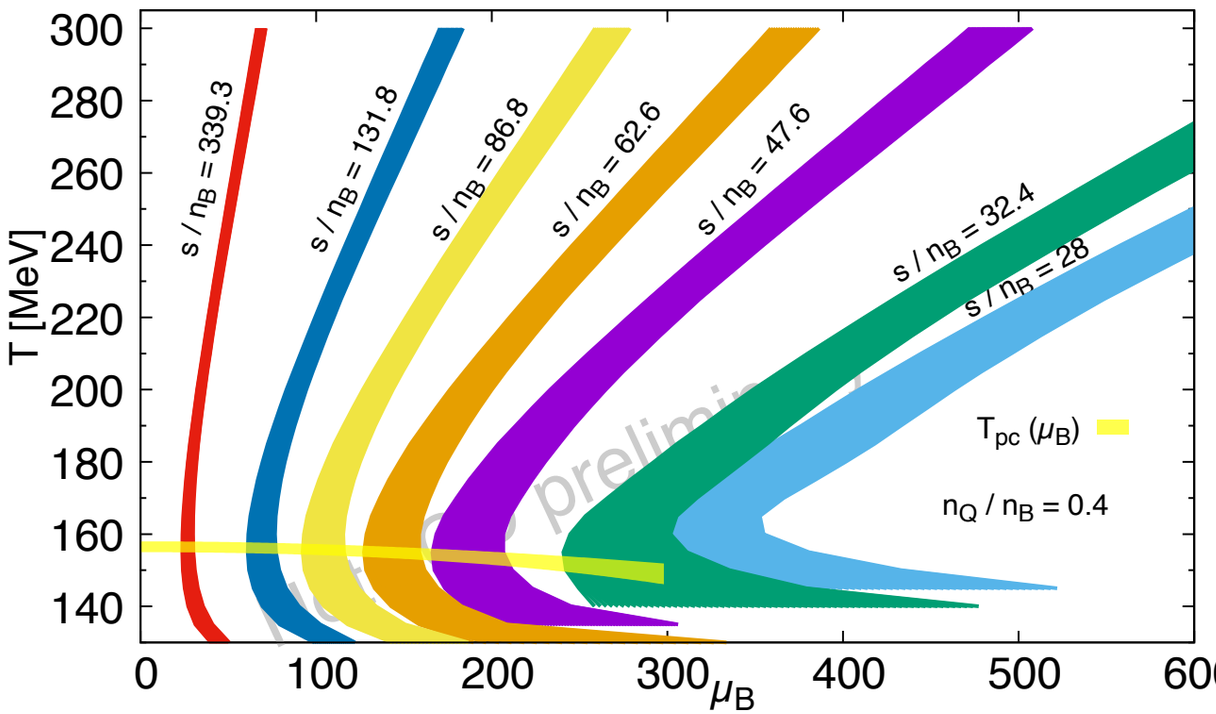
★ Reliable Eos of $(2+1)-$flavor QCD for $2 \leq \mu_B/T \leq 2.5$ for $T < 175$ and $\mu_B/T \simeq 3$ for $T > 175$.

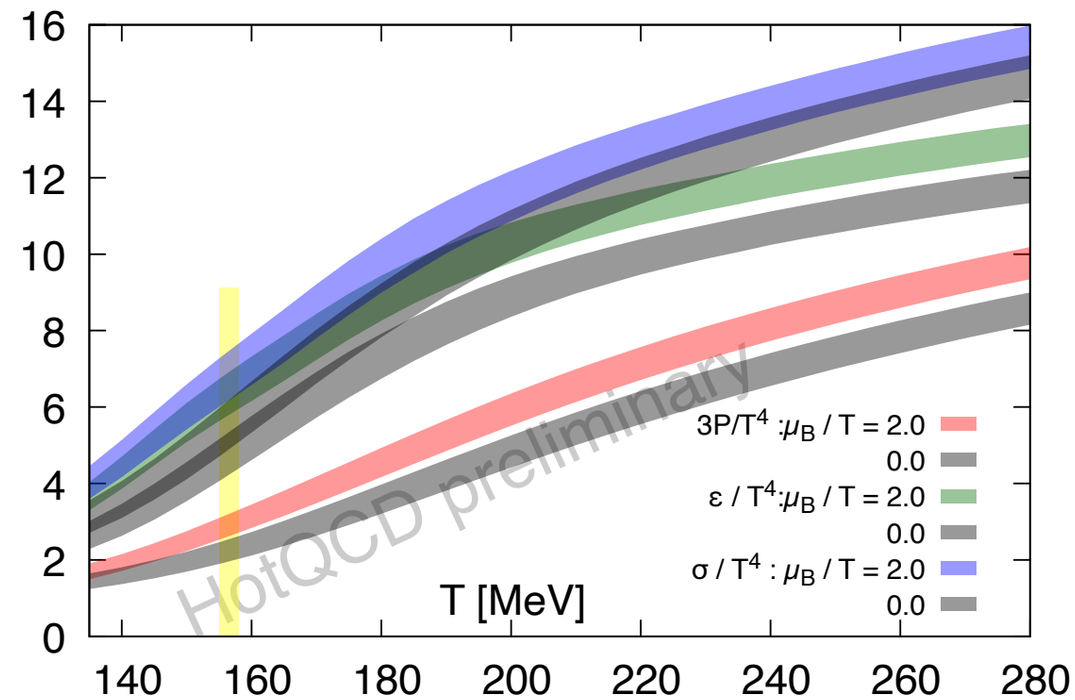
★ This range covers the beam energy, $\sqrt{s} \geq 7.7$ GeV.





Isentropic EoS of (2+1)-flavor QCD : relevant for HIC





$$\hat{s}/\hat{n}_B = (\hat{P} + \hat{\epsilon})/\hat{n}_B - \hat{\mu}_B - \hat{\mu}_Q \hat{n}_Q/\hat{n}_B$$

$$\hat{\mu}_Q = q_1 \hat{\mu}_B + q_3 \hat{\mu}_B^3 + \ldots, \; \hat{n}_Q/\hat{n}_B = 0.4$$

$$\hat{\mu}_B = \sum_{n=1}^{\infty} h_{2n-1} \left(\frac{\hat{s}}{\hat{n}_B} \right)^{1-2n}$$