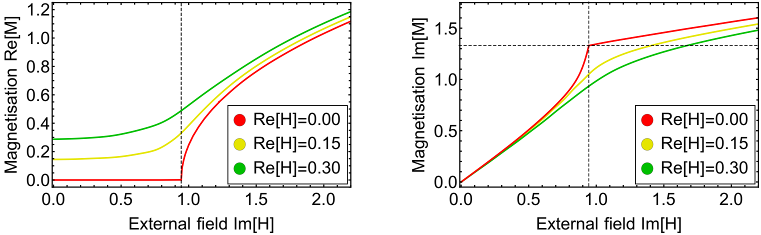
Towards locating the (real) critical endpoint

Friederike Ihssen

12/02 - CPOD 2022



Based on arXiv:2207.10057 in collaboration with J. M. Pawlowski

UNIVERSITÄT HEIDELBERG ZUKUNFT SEIT 1386

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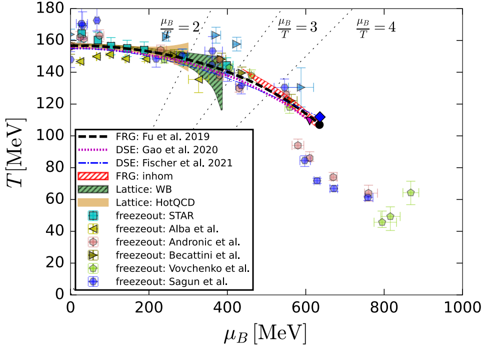
Why are we interested in Lee-Yang Zeroes?



PhysRevD.101.054032: Fu, Pawlowski, Rennecke
PhysLetB.2021.136584: Gao, Pawlowski
PhysRevD.104.054022: Gunkel, Fischer

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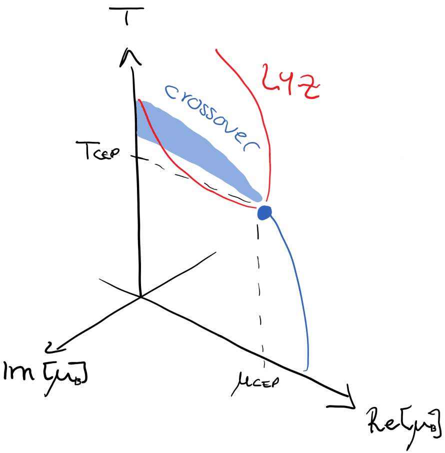

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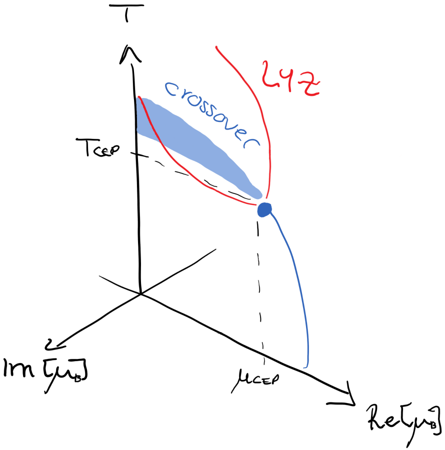

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- A zero-crossing of the generating functional/partition function $\mathcal{Z}$

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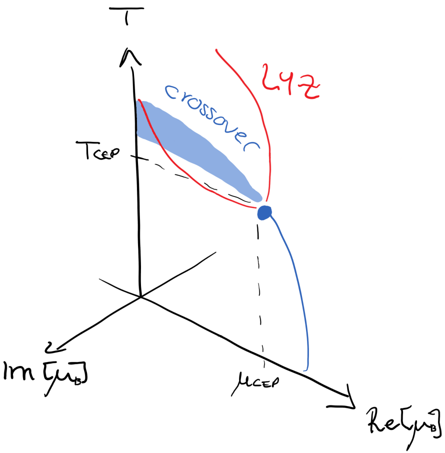

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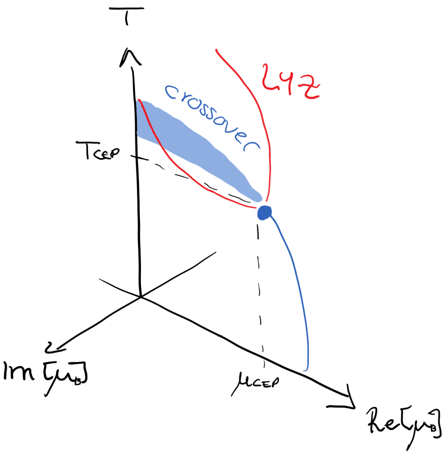

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$\Rightarrow$ How are complex LYZs connected to the (real) critical endpoint?

Complex phase transitions?

Partition function :

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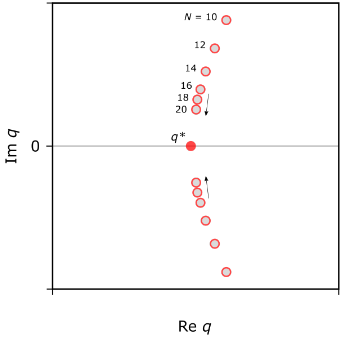
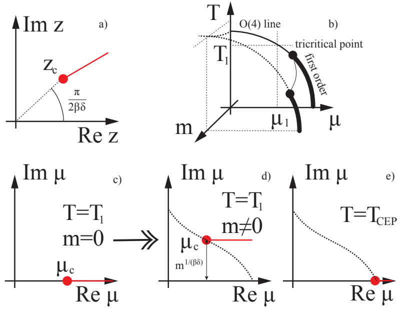
$\Rightarrow$ Predict real phase transition!



Lee Yang edge singularities in QCD



PhysRevLett.125.191602: Connelly, Johnson, Mukherjee, Skokov

- Simple LEFT for meson interactions: O(4) theory

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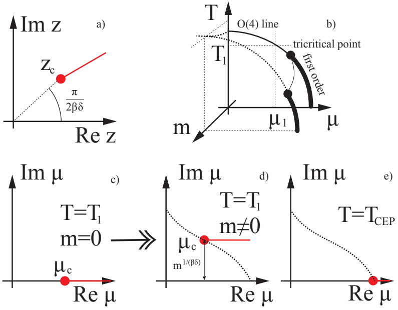

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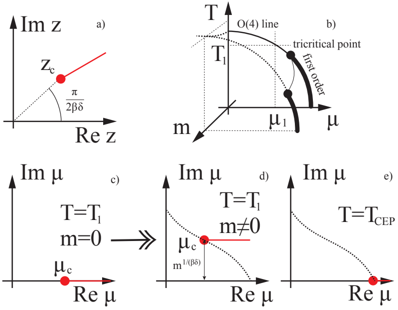

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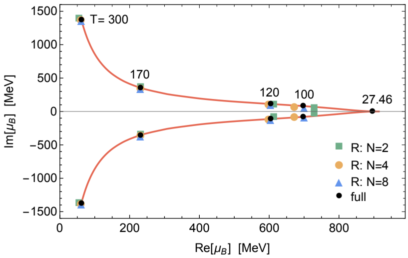
- Simple LEFT for meson interactions: O(4) theory
- At $m_\pi \to 0$ we find a second order phase transition at high T.
- As we go to physical pion masses the LY travels through the complex plane for increasing Re μ.



PhysRevD.105.014026: Mukherjee, Rennecke, Skokov

A Functional Approach

The path-integral / generating functional:

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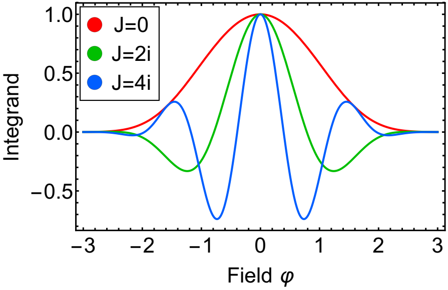
- $S[\varphi]$ classical action of the theory.
- Real function of a complex variable.
- Sample from oscillatory integrand (Lattice).

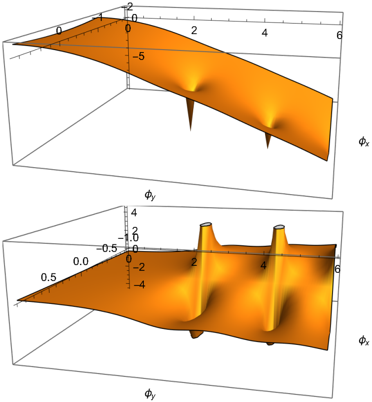


PoS LATTICE2021 (2022) 223: Attanasio, Bauer, Kades, Pawlowski

arXiv 2203.01243: Pawlowski, Urban

The $O(N)$ Model

$$S[\varphi] = \int_x \left\{ \frac{1}{2}\varphi(x)\left[-\partial_\mu^2 + m^2\right]\varphi(x) + \frac{\lambda}{4!}\varphi(x)^4 \right\}$$



Test case:

- $O(N)$ with $d = 0$, $N = 1$
- Classical input parameters $m = 1$, $\lambda = 1$
- Direct evaluation of $\mathcal{Z}$

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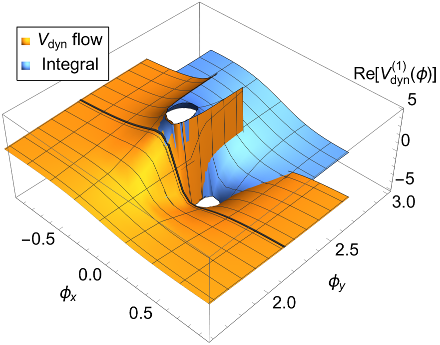
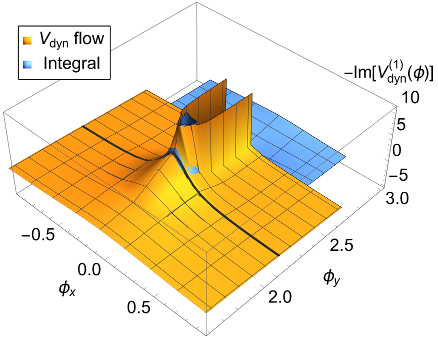
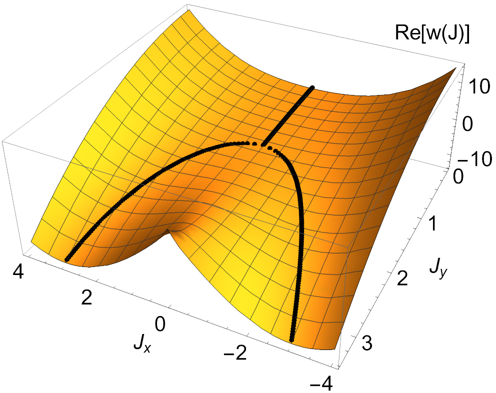




Magnetisation $M = \langle \phi \rangle$, at minimum of the effective potential $w[J]$.



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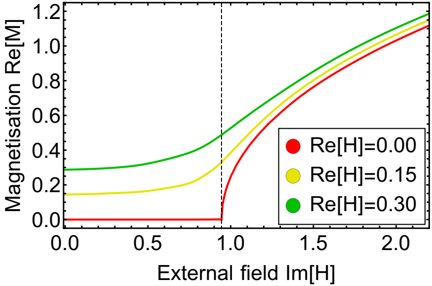

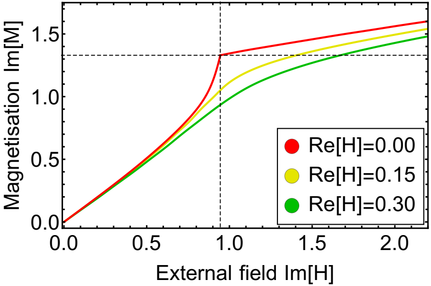

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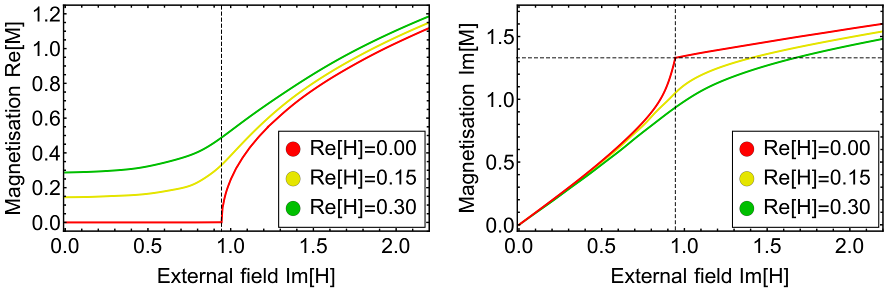

LY-location for $\mathrm{Im}[H] = 0.994$ and initial condition $m^2 = 1$ at $\mathrm{Im}[M] = 1.33$!

Recovering a Real Phase Transition

Scaling behaviour: $\dfrac{m^2 - m_c^2}{m_c^2} = const\, H^{1/\Delta}$

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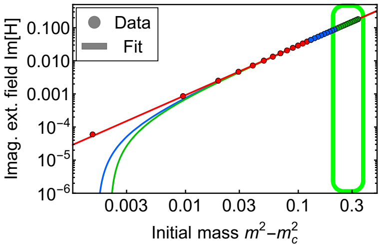

How close do we need to go to extrapolate m_c^2?

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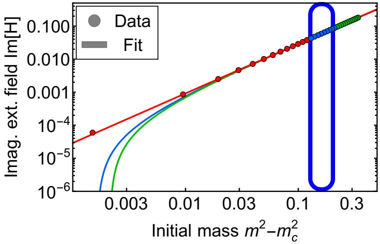

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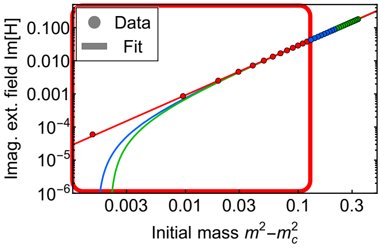

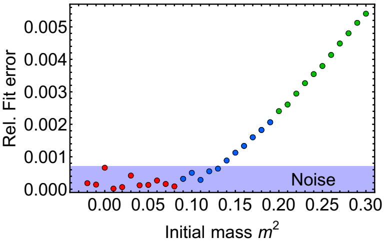

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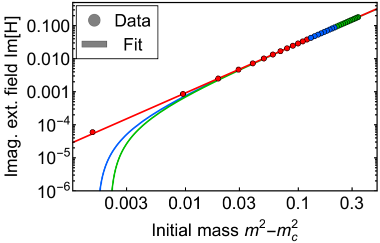

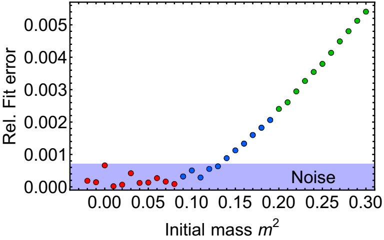

Δ	m_c^2
1.4969(66)	−0.03943(17)

$\Rightarrow$ Last computed physical mass:

$$m_0^2(m^2 = -0.039) = 4.2\ 10^{-4}$$

Summary & Outlook

- Investigated the convergence of functional flows in the complex plane using DG-methods.
- Retrieved location of the Lee-Yang singularity in $d = 0$ and $d = 4$.
- We extracted the critical mass m_c^2 of the real phase transition in $d = 4$!

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- Investigated the convergence of functional flows in the complex plane using DG-methods.
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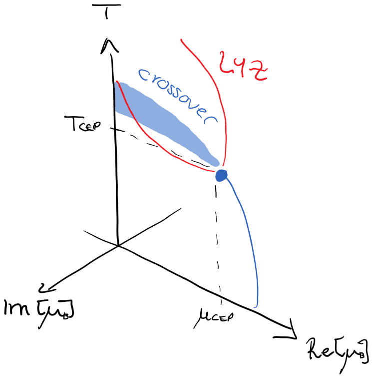

⇒ Go towards QCD phase structure and complex μ!

Thank you for your attention!

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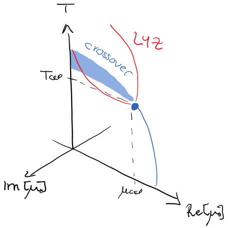
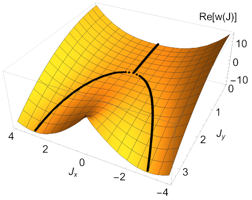
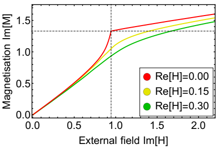
ITP Heidelberg







A Functional Approach

The infrared regularized path-integral / generating functional:

$$\mathcal{Z}_k[J] = \int [d\varphi]_{\mathrm{ren}, p^2 \geq k^2} \exp\Big\{ - S[\varphi] + J \cdot \varphi \Big\} \,,$$

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- Suppression of IR momentum modes by k-dependent mass term.

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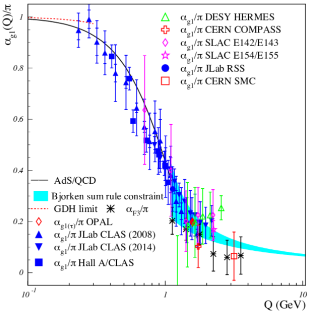
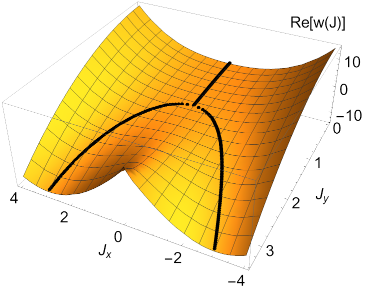
- Suppression of IR momentum modes by k-dependent mass term.
- Access to non-perturbative IR regime by stepwise integration of low energy momentum modes.



$$M(\phi_y, H) = \phi_{\rm EoM}\,, \quad \text{with} \quad \partial_{\phi_x} w[J(\phi_x,\phi_y)] - H|_{\phi_x=\phi_{x,\rm EoM}} = 0$$



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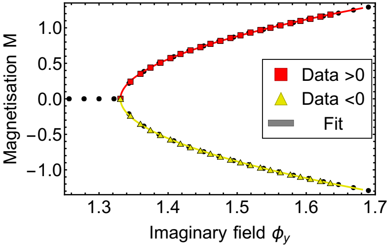

Mean Field: $\beta = 1/2$

Fit: $0.505(23)$

Scaling relations:

- Preserves symmetry:

$$\mathrm{Re}\left[M(\phi_y, H=0)\right] = B\left(\frac{\phi_y - \phi_c}{\phi_c}\right)^{\beta}$$

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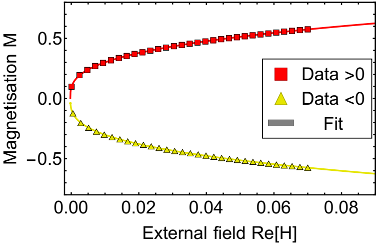

Mean Field: $\delta = 3$

Fit: $2.992(18)$

Scaling relations:

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$$\text{Re}\left[M(\phi_y, H = 0)\right] = B\left(\frac{\phi_y - \phi_c}{\phi_c}\right)^{\beta}$$

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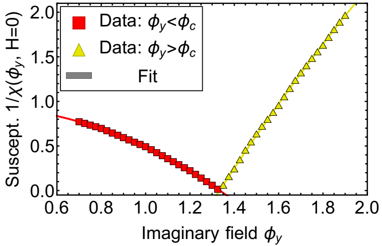

Mean Field: $|C_+| = 2|C_-|$

$$C_+ = -0.4369(11)\,,\ C_- = 0.2025(21)$$

$$R_\chi = \frac{C_+ B^{\delta-1}}{B_c^\delta} = 1.010(57)$$

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- Susceptibility:

$$\chi(\phi_y, H=0) = C_{+/-}\left(\frac{\phi_y - \phi_c}{\phi_c} + \text{subl.}\right)^{-\gamma}$$