Testing new proxies of 2nd order cumulants of B, Q and S in Au-Au collisions at BES energies with EPOS 4

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1. Phase diagram & the critical point
2. EPOS
3. Motivations & goals
4. Results & Discussion
5. Conclusion & Outlooks
Understanding thermodynamics of nuclear matter, or how matter in atomic nuclei behaves under extreme conditions like in the early Universe ⇔ studying the phase diagram of nuclear matter.

- **Theoretically**: crossover established thanks to lattice QCD simulations; 1\textsuperscript{st} order phase transition + critical endpoint (CEP) predicted by several models (PNJL, fRG...)

- **Experimentally**: exploration of QCD phase diagram thanks to the Beam Energy Scan (BES) program (checking collective flow, particle production...)

**Question(s) of interest**: is there a 1\textsuperscript{st} order phase transition and a critical endpoint (CEP) between QGP and hadronic gas phases? If yes, where?
Susceptibilities

**Background of this work:** the search for the CEP.

To probe its existence, one can use the **susceptibilities of conserved charges B,Q,S**.

In a grand-canonical ensemble (formalism often used to describe HICs), the partition function $\mathcal{Z}(T, V, \mu)$ carries information on possible microscopic system configurations.

$$\chi_{X,Y}^{i,j} = \frac{1}{VT^3} \cdot \left[ \frac{\partial^{i+j}(\mathcal{Z}(T, V, \mu))}{(\partial \mu_X)^i(\partial \mu_Y)^j} \right]_{\mu_X,\mu_Y}$$

Characterise how much $\mathcal{Z}$ is modified under the variation of the chemical potentials

⇒ sensitive to radical changes in the state of nuclear matter (i.e., phase transition)

$$\chi_\alpha \propto \xi^\beta$$ (correlation length)

which diverges at the vicinity of the CEP.

In particular: $\chi_2 \propto \xi^2$.

To make the link with experimental collisions, susceptibilities can also be written as a function of the cumulants of net-charges $N_X$ ($X = B, Q, S$), averaged over the events: $\langle \ldots \rangle$.

They represent event-by-event fluctuations of the considered net charges.

One uses ratios to compare with susceptibilities, to remove the trivial dependence on $V$ and $T$ (not directly measurable in experiments).

Also, conserved charges themselves are not accessible in experiments (unmeasured particles), so one generally uses net-number of hadrons as proxies ($\pi \leftrightarrow Q / p \leftrightarrow B, Q / K \leftrightarrow Q, S...$):

- $N_\alpha = n_\alpha - n_{\bar{\alpha}}$ (baryons)
- $N_\alpha = n_{\alpha^+} - n_{\alpha^-}$ (mesons)
There are **fundamental differences** between the **theoretical framework** used to calculate susceptibilities, and **HICs** where net-particles cumulants are measured:

<table>
<thead>
<tr>
<th>Theory</th>
<th>Heavy-ion collisions</th>
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</thead>
<tbody>
<tr>
<td>- GCE (global charge conservation, on average)</td>
<td>- global and local strict charge conservation</td>
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<tr>
<td>- in equilibrium with a particle heat bath</td>
<td>- inhomogeneous, evolve in the vacuum</td>
</tr>
<tr>
<td>- static &amp; in thermal equilibrium</td>
<td>- short-lived &amp; dynamical system</td>
</tr>
<tr>
<td>- spatially infinite</td>
<td>- small system (finite-size effects)</td>
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<tr>
<td>- integrated over the whole system</td>
<td>- restrained phase-space covered by detector</td>
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</table>

Furthermore, several stages of HICs can be source of **additional event-by-event fluctuations**, blurring the signal of interest: initial conditions, hadronic rescatterings...

**Important to disentangle non-critical effects from actual fluctuations due to CEP**
1 Phase diagram & the critical point

2 EPOS
   - Generalities
   - Simulation of an event with EPOS 4

3 Motivations & goals

4 Results & Discussion

5 Conclusion & Outlooks
The role of event generators

**Event generators**, codes computing models in order to simulate every step of a collision, are the most appropriate tools to bridge the gap between theory and experiments.

**Advantages**: - perfect detector, as final-state particles are all listed (no uncertainties)
- gives access to the whole history of the collision

Energy conserving quantum mechanical approach, based on
Partons, parton ladders, strings,
Off-shell remnants, and
Saturation of parton ladders

Multi-purpose event generator, based on a multiple scattering approach, and a hybrid evolution of matter including hydrodynamics to reproduce the fluid behaviour of the QGP.

Developed to simulate, above the GeV scale, any type of collision with the same formalism(s):

\[
e^{-} - e^{+}, \quad e^{-} - p, \quad p - p, \quad p - A, \quad A - A
\]
Simulate of an event with EPOS 4

Initial conditions

Details on the complete formalism to simulate events for hadronic collisions ($p$-$p$, $p$-$A$ or $A$-$A$).

Parton-based Gribov-Regge theory

Nuclei (A and B) interact by the simultaneous exchange of multiple Pomerons in parallel.

⇒ can be seen as parton ladders which are cut (particle production) or uncut ($\sigma$ calculation)

Core-corona separation

Those ladders are identified as strings, or color flux tubes ($q - g - \ldots - g - \bar{q}$ chains) with "kinks" due to tranverse gluons.

In HIC (but not only !), many strings may overlap, so we can separate:

- **core** = high string density region ($> \varepsilon_c$)
- **corona** = escaping segments (with high $p_T$) ($< \varepsilon_c$)
Medium evolution, hadronisation and hadronic cascades

**Core evolution**

- vHLLE: viscous 3+1D relat. hydrodynamics based on a cross-over equation of state
- Microcanonical hadronisation of the medium (local & global conservation of charges)

**Corona evolution**

- Strings evolution + fragmentation to produce hadrons

Re-scatterings between formed hadrons using the UrQMD model until reaching the chemical (no more inelastic scatterings) & kinetic freeze-outs (no more elastic scatterings).

K. Werner, 37th Joliot-Curie School (2018)
Phase diagram & the critical point

EPOS

Motivations & goals
- Motivations
- Objectives of my work

Results & Discussion

Conclusion & Outlooks
STAR collaboration has published measurements of 2nd order cumulants of $\pi$, $p$, and $K$ within $|\eta| < 0.5$ and for $0.4 < p_T < 1.6$ GeV/c:

- (co)variances $\sigma_{p,\pi,K}^{11,2}$ (proxies for $\chi_{11,2}^{B,Q,S}$)
  - as a function of $\langle N_{\text{part}} \rangle$ ($\equiv$ event centrality)

- ratios $C_{Qp,QK,pK}$ (proxies for $C_{QB,QS,BS}$)
  - as a function of $\langle N_{\text{part}} \rangle$ ($\equiv$ event centrality)
  - as a function of $\sqrt{s_{NN}}$ (0-5% and 70-80%)

arXiv:1903.05370
Theoretical context

Study of the breakdown of hadronic contributions to lattice QCD susceptibilities with Hadron Resonance Gas model calculations (statistical model of a gas of non-interacting hadrons).

**Purpose:** build proxy ratios matching corresponding susceptibility ratios

*(based on hadrons usually measured in experiments)*

\[
\tilde{C}_{BS} = \frac{\sigma_{11}^{pK}}{\sigma_{K}^2} \quad \text{or} \quad \frac{\sigma_{\Lambda}^2 + 2\sigma_{\Xi}^2 + 3\sigma_{\Omega}^2}{\sigma_{\Lambda}^2 + 4\sigma_{\Xi}^2 + 9\sigma_{\Omega}^2 + \sigma_{K}^2}
\]

and

\[
\tilde{C}_{QS} = \frac{\sigma_{11}^{QK}}{\sigma_{K}^2} \quad = \quad \frac{1}{2} \cdot \frac{\sigma_{K}^2}{\sigma_{\Lambda}^2 + \sigma_{K}^2}
\]

proposed, based on the fact that proxies for both numerator and denominator should correspond to the same proportion of their corresponding susceptibility

arXiv:1910.14592
No proxy ratio proposed for $C_{QB}$ though.

**Argument:** isospin randomisation blurs the fluctuations carried by the involved particles because of reactions like

$$p + \pi^0 \leftrightarrow \Delta^+ \leftrightarrow n + \pi^+$$

$$p + \pi^- \leftrightarrow \Delta^0 \leftrightarrow n + \pi^0$$

Nevertheless, I propose here a possible proxy ratio:

(based on results from the previous study)

$$\tilde{C}_{QB} \left( = \frac{\sigma_{11}^{11}}{\sigma_p^2} \right)_{\text{STAR}} = \frac{\sigma_{11}^{11} + \sigma_p^2}{2\sigma_p^2 + \sigma_N^2}$$

$\sigma_{11}^{11} + \sigma_p^2$ being supposedly unaffected by iso. rand.
and $2\sigma_p^2 \approx \sigma_N^2$ because of the same effect.
EPOS allows to access all particles without identification uncertainties (even neutral ones) + to control feed-down from weak decays (violating $S$ conservation).

1. Test the ”enhanced” proxy ratios against STAR proxy ratios and to compare them with ratios of conserved charges $B, Q, S$

Modularity of EPOS provides access to the distributions of particles just after hadronisation, before the hadronic cascades stage.

2. Assess quantitatively the impact of hadronic cascades on 2nd order cumulants

⇒ this work is NOT focused on the search for CEP itself but on the study of observables used to probe its existence
1 Phase diagram & the critical point

2 EPOS

3 Motivations & goals

4 Results & Discussion
   - Comparison of proxies with cumulants of $B, Q, S$
   - Effects of the hadronic stage

5 Conclusion & Outlooks
Simulations of Au-Au collisions (using $^{197}$Au ions) at RHIC energies, to have experimental data to compare with.

Limited number of events simulated for each energy, due to substantial computation time required:

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$ (GeV)</th>
<th>7.7</th>
<th>11.5</th>
<th>14.5</th>
<th>19.6</th>
<th>27</th>
<th>39</th>
<th>62.4</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{evts}$</td>
<td>240k</td>
<td>240k</td>
<td>240k</td>
<td>240k</td>
<td>300k</td>
<td>300k</td>
<td>270k</td>
<td>240k</td>
</tr>
</tbody>
</table>

All results shown in this section are coming from simulations obtained with EPOS 3.451 (advanced beta version of EPOS 4).

For all of the curves, results are obtained from particles within $|\eta| < 0.5$ and with $0.4 < p_T < 1.6$ GeV to enable comparison with STAR data.
EPOS is able to reproduce STAR data both qualitatively and quantitatively, except at 7.7 GeV/A.

All enhanced proxies $\tilde{C}_{XY}$ (blue) are in almost perfect agreement with respective $C_{XY}$ (green), but $\tilde{C}_{BS}$ (cyan) brings not much more.
Enhanced proxy ratios reproduce ratios of exact conserved charges (co)variances remarkably well & way better than STAR proxies.

Reminder:

\[
\tilde{C}_{BS} = \frac{\sigma^2_{\Lambda}}{\sigma^2_{K} + \sigma^2_{\Lambda}} \left(, \tilde{C}'_{BS} = \frac{\sigma^2_{\Lambda} + 2\sigma^2_{\Xi} + 3\sigma^2_{\Omega}}{\sigma^2_{\Lambda} + 4\sigma^2_{\Xi} + 9\sigma^2_{\Omega} + \sigma^2_{K}} \right), \tilde{C}_{QB} = \frac{\sigma_{\pi p}^1 + \sigma^2_{p}}{2\sigma^2_{p} + \sigma^2_{\Lambda}} , \tilde{C}_{QS} = \frac{1}{2} \cdot \frac{\sigma^2_{K}}{\sigma^2_{K} + \sigma^2_{\Lambda}}
\]

Using variances almost exclusively seems to be enough to get good proxy ratios
⇒ less statistical fluctuations on final observables!

Adding only \( \sigma^2_{\Lambda} \) to \( \pi, p, K \) (co)variances is enough to build excellent proxies.

Good news: measurements of \( \sigma^2_{\Lambda} \) published by STAR in *Phys.Rev.C* 102 (2020)
⇒ these proxies can be measured experimentally!
Effects of the hadronic stage

Impact of hadronic cascades

... on variances of particle net-multiplicities
The rescatterings during hadronic stage increase significantly the (co)variances amplitude.

Expected because of global multiplicity increase due to reactions like $2 \rightarrow 3, 4...$, and increase of correlations due to reactions like $X \rightarrow \alpha + \beta (+...)$.
Effects of the hadronic stage

Impact of hadronic cascades
... on ratios of proxies and B, Q, S (co)variances

Major impact of hadronic cascades on $\sigma_{2/11}^2$ though numerators and denominators seem modified by the same amplitude

Ratios (conserved charges & proxies) are little affected by the hadronic stage of the collision.

Exception made of $\tilde{C}_{BS} = \frac{\sigma_{\Lambda}^2}{\sigma_{K}^2 + \sigma_{\Lambda}^2}$, for which amplitude is significantly modified by the hadronic cascades.

\[
\frac{\sigma_{\Lambda}^2}{\sigma_{K}^2 + \sigma_{\Lambda}^2} \text{ bad proxy for BS correlation}
\]
Main results:

- enhanced proxies $\tilde{C}_{XY}$ found to reproduce $B, Q, S$ (co)variances ratios way better than STAR proxies
  
  (only $\sigma_\Lambda^2$ needed in addition + covariances mostly useless)

- despite strong impact on (co)variances from hadronic cascades, most of ratios are little modified by this stage (good ”memory” of these ratios)
  
  (except for $\tilde{C}_{BS} = \frac{\sigma_\Lambda^2}{\sigma_\Lambda^2 + \sigma_K^2}$)

Some ideas to go further:

- using simulations from last EPOS 4 version, with a better particle production at low collision energies

- finding a proxy little affected by the hadronic stage for $C_{BS}$

- extending this study to higher-order cumulants (more sensitive to criticality) and other fluctuation observables

Important news: EPOS 4 is now publicly available

https://klaus.pages.in2p3.fr/epos4/
Thank you for your attention!
Additional material
The motivations behind the PBGRT

**Parton model**
Mainly used for inclusive cross-section calculations

[Diagram: Deep Inelastic Scattering]

**Problems**:  
- can only calculate cross-section for hard processes → not suitable alone for HIC

**Gribov-Regge theory**
EFT for Multiple *Pomeron* Interaction  

(K. Werner et al., 2000)

**Inconsistencies**:
- energy conserved for particle production but NOT for cross-section calculations  
- although multiple scattering approach, all interactions are not treated equally

**Solution**: merge both into a formalism treating consistently hard and soft scattering  
⇒ Parton-based Gribov-Regge Theory!
Main principle of PBGRT

Based on S-matrix approach where, from:
\[ \langle f | \hat{S} | i \rangle = S_{fi} = \delta_{fi} + i(2\pi)^4 \delta(P_f - P_i) T_{fi} \]
we got the total inelastic cross-section \( \sigma_{tot} \) with the optical theorem

\[ \sigma_{tot} = \frac{1}{2s} (2\pi)^4 \delta(p_f - p_i) \sum_f \lvert T_{fi} \rvert^2 \]

In the PBGRT, one elementary interaction is modeled as a Pomeron, each of them giving a contribution to the total T-matrix.

- **Soft process** \((Q^2 < 1 \text{ GeV})\): mainly elastic scatterings, parametrised T-matrix (Regge poles)
- **Hard process** \((Q^2 > 1 \text{ GeV})\): pQCD applicable, computed T-matrix (DGLAP equation)
- **Semi-hard process** \((Q^2 > 1 \text{ GeV} q_{sea}/\bar{q}_{sea}/g)\): using both previous formalisms
In the simplest system which is \( e^+ e^- \), the \( q\bar{q} \) pair created is linked by a color field, forming what we call a relativistic string.

Such string can indeed have transverse kinks, caused by gluon emission, and evolves following the dynamics of a gauge invariant Lagrangian.

Eventually, it will fragment via production of \( q(q) - \bar{q}(\bar{q}) \) pairs, thus forming hadrons, following a so-called area law:

\[
\frac{dP_{\text{break}}}{dA} = \lambda \cdot dA
\]

\( (dA : \text{infinitesimal area}) \)

Meson and baryon production from string breaking
consider each fluid cell of mass $dm$ & volume $V$ in its rest mass ("removing" the flow)

decay the mass element $dm$ according to the distribution probability below

"give back" the flow to produced particles from the 4-velocity vector $u^\mu$ of initial fluid cell

$$V = \int \Sigma u^\mu d\Sigma_{\mu}, \quad M = \int \Sigma \sqrt{T^{\mu\nu}d\Sigma_{\mu}T_{\mu\nu}d\Sigma_{\nu}}$$

$$dP = \frac{V^n}{(2\pi\hbar)^3n} \times \prod_{i=1}^{n} g_i \times \prod_{\alpha} \frac{1}{n_\alpha!} \times \prod_{i=1}^{n} d^3 p_i \times \delta(E - \sum E_i) \times \delta(\sum ||\vec{p}_i||) \times \prod_A \delta(Q_A - \sum q_i^A)$$

with:
- $n$ = number of produced hadrons
- $\alpha$ = hadronic specie
- $Q_A$ = value of conserved charge $B, Q, S, ..$ for the whole volume $V$
- $q_i^A$ = value of conserved charge $B, Q, S, ..$ carried by particle $i$
Assumes a gas of interacting hadrons in ground states can be described by a gas of non-interacting hadrons and resonances.

Re-writing of \( \mathcal{Z}_R \) to consider kinematic cuts via phase space integration:

\[
\ln(\mathcal{Z}_R) = \eta_R \frac{V \cdot dR}{2\pi^2 T^3} \int_0^\infty p^2 \cdot dp \cdot \ln \left( 1 - \eta_R z_R e^{-\epsilon_R/T} \right)
\]

\[\Rightarrow \text{decomposition of susceptibilities as a function of hadronic species:}\]

\[
\chi^{BQS}_{ijk}(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S) = \sum_R \sum_{i \in \text{stable}} (P_{R \rightarrow p})^l \times B^i_p Q^j_p S^k_p \times I^R_l(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S)
\]

with:

- \( I = i + j + k \)
- \( P_{R \rightarrow p} = \sum_\alpha N^\alpha_{R \rightarrow p} \times n^R_{p, \alpha} : \langle n_p \rangle \text{ produced in process } \alpha \text{ by each resonance } R \)
- \( B^i_p, Q^j_p, S^k_p : \) quantum numbers of particle specie \( p \)
- \( I^R_l(T, \hat{\mu}_B, \hat{\mu}_Q, \hat{\mu}_S) = \frac{\partial^l}{\partial \mu^l_R} \left[ \frac{1}{VT^3} \sum_R \ln(\mathcal{Z}_R) \right] \) \( (\hat{\mu}_R = \hat{\mu}_B B_R + \hat{\mu}_Q Q_R + \hat{\mu}_S S_R) \)
Breakdown of hadronic contribution to susceptibilities

\[ \chi_{1B}^{(1)} \]

\[ \chi_{1B}^{(2)} \]

\[ \chi_{2Q} \]

\[ \chi_{2S} \]
**Goal:** ensuring particle production is correctly reproduced by EPOS in the phase space of interest for the cumulants study ($|\eta| < 0.5$ and $p_T < 2$ GeV).

Checking all particles of interest: $\pi^{\pm}$, $K^{\pm}$, $p$, $\bar{p}$, $\Lambda$, $\bar{\Lambda}$, $\Xi^-$, $\Xi^+$, $\Omega^-$ and $\bar{\Omega}^+$.  

For reasons of data availability and time, only sparse overview of the results. I will spot the important points of the results, or span widely in energy range to see the evolution with $\sqrt{s_{NN}}$.

All analyses are performed as close as possible to the experimental methods.

**Only differences:**
- centrality definition (using impact parameter $b$ in EPOS)
- strict weak decays feed-down corrections in EPOS (no DCA selection)
Basic spectra for BES energies

Pseudorapidity densities of charged particles

Charged particles distributions, corrected from weak feed-down.

Distributions well reproduced for $|\eta| < 0.5$

(except for 0-3% at 19.6 GeV/A)

Bumpy structure at low energy due to excited remnants
(requires additional work to be corrected)
Basic spectra for BES energies

Pseudorapidity density for charged particles @ 62.4 GeV
Basic spectra for BES energies

Transverse momentum spectra of light hadrons

Results very well reproduced quantitatively for $\sqrt{s_{NN}} > 20$ GeV/A despite slight overestimation towards more peripheral collisions (centrality definitions might play a role)
Basic spectra for BES energies

Transverse momentum spectra of light hadrons

Results very well reproduced quantitatively for $\sqrt{s_{NN}} > 20$ GeV/A despite slight overestimation towards more peripheral collisions (centrality definitions might play a role)

Softening of the $p_T$ spectra $\Leftrightarrow$ steeper slopes for lowest energies ($\sqrt{s_{NN}} < 20$ GeV) $\Rightarrow$ lack of flow from the core (fixable by parametrisation)
Basic spectra for BES energies

Transverse momentum spectra of light hadrons

Results very well reproduced quantitatively for $\sqrt{s_{NN}} > 20$ GeV/A despite slight overestimation towards more peripheral collisions (centrality definitions might play a role)

Softening of the $p_T$ spectra $\iff$ steeper slopes for lowest energies ($\sqrt{s_{NN}} < 20$ GeV)

$\Rightarrow$ lack of flow from the core (fixable by parametrisation)

Overproduction of $p$ compared to $\bar{p}$ for $< 20$ GeV/A

$\Rightarrow$ instantaneous interactions of PBGRT (all protons from targ./proj. did not have time to interact)
Basic spectra for BES energies

Transverse momentum spectra of strange baryons

For strange baryons, correction for weak feed-down, but $\Sigma^0$ decays contamination.

Here also:

- very good data agreement for $\sqrt{s_{NN}} > 20$ GeV
- softening of spectra at lower energies
For strange baryons, correction for weak feed-down, but $\Sigma^0$ decays contamination.

Here also:

- very good data agreement for $\sqrt{s_{NN}} > 20$ GeV
- softening of spectra at lower energies

Particle production in Au-Au collisions at RHIC energies for EPOS 3.451 is OK for low-$p_T$ hadrons at mid-rapidity.

(great improvement since previous versions)
$\sigma^2$ dependence for collisions above 20 GeV/A; starts to deviate below.

Good quantitative agreement overall, deviations due to hadronic specie over/underproduction or particle/antiparticle imbalance (e.g., protons at 7.7 GeV/A).
Large fluctuations because of too low number of events analysed.

Good quantitative agreement, within fluctuations, for most of the displayed results
+ sign change of $\sigma_{pK}^{11}$ with $\sqrt{s_{NN}}$ reproduced (unlike UrQMD alone).
Ratios vs. centrality

Ratios seem to show almost no dependence on the centrality, like the experimental data.
Impact of hadronic cascades on $B$, $Q$, $S$ (co)variances - 200 GeV/A

$\sigma^2_{B_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |
|-----------------|
| line: EPOS (full event) |
| dots: EPOS (no hacas) |

$\sigma^2_{Q_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |

$\sigma^2_{S_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |

$\sigma^2_{B_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |

$\sigma^2_{Q_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |

$\sigma^2_{S_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |

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| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |

$\sigma^2_{Q_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |

$\sigma^2_{S_0}$

| $|\eta| < 0.5$ , $0.4 < p_T < 1.6$ GeV |
Impact of hadronic cascades on $B, Q, S$ (co)variances - 7.7 GeV/A