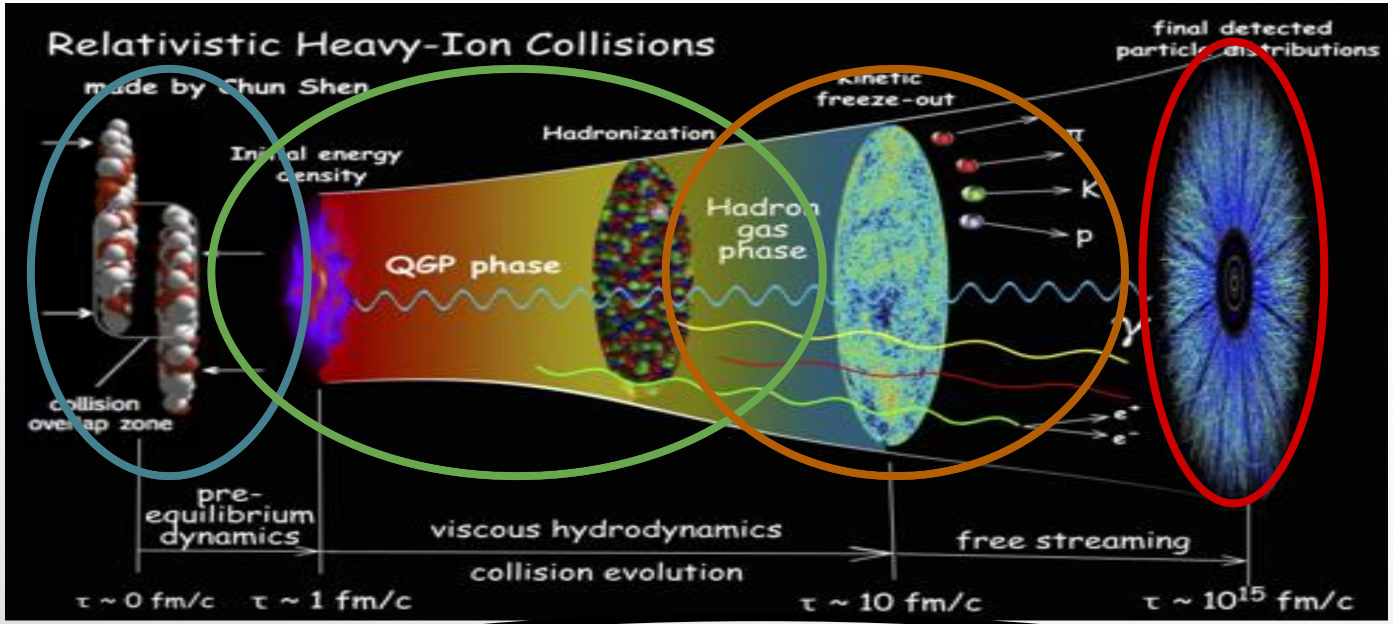
EQUILIBRIUM AND DYNAMIC LENSING EFFECTS NEAR A CRITICAL POINT IN THE QCD PHASE DIAGRAM

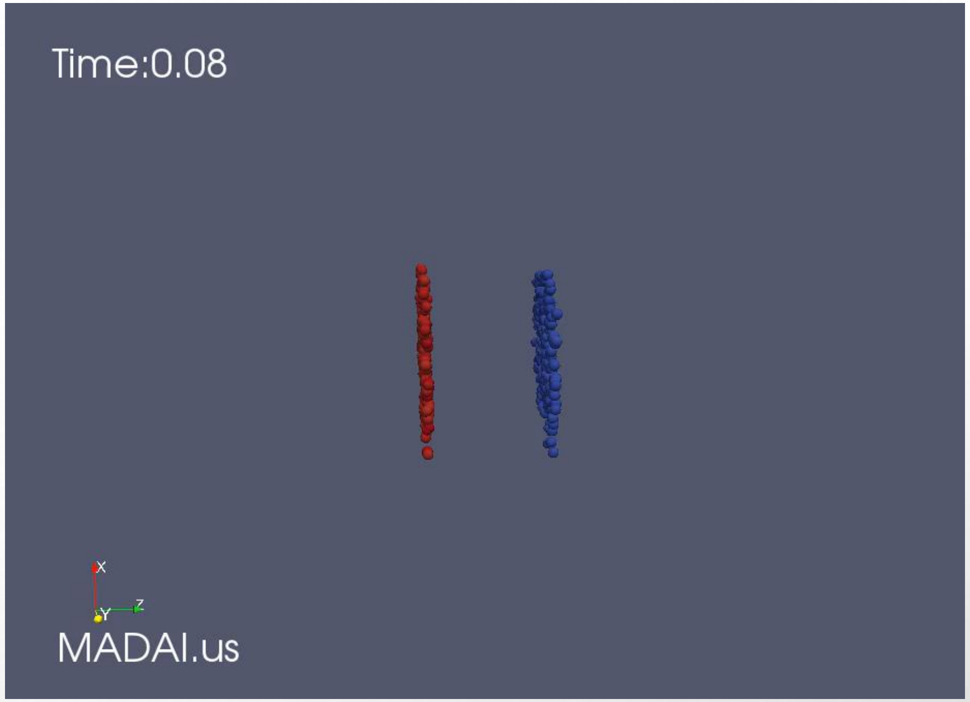
TRAVIS DORE

TD, ET AL., PHYS.REV.D 106 (2022) 9, 094024

In collaboration with: J. Karthein, I. Long, D. Mroczek, J. Noronha-Hostler, P. Parotto, C. Ratti, Y. Yamauchi

HEAVY-ION COLLISIONS IN A NUTSHELL





Initial conditions and pre-equilibrium

Viscous hydrodynamic evolution, Quark Gluon Plasma

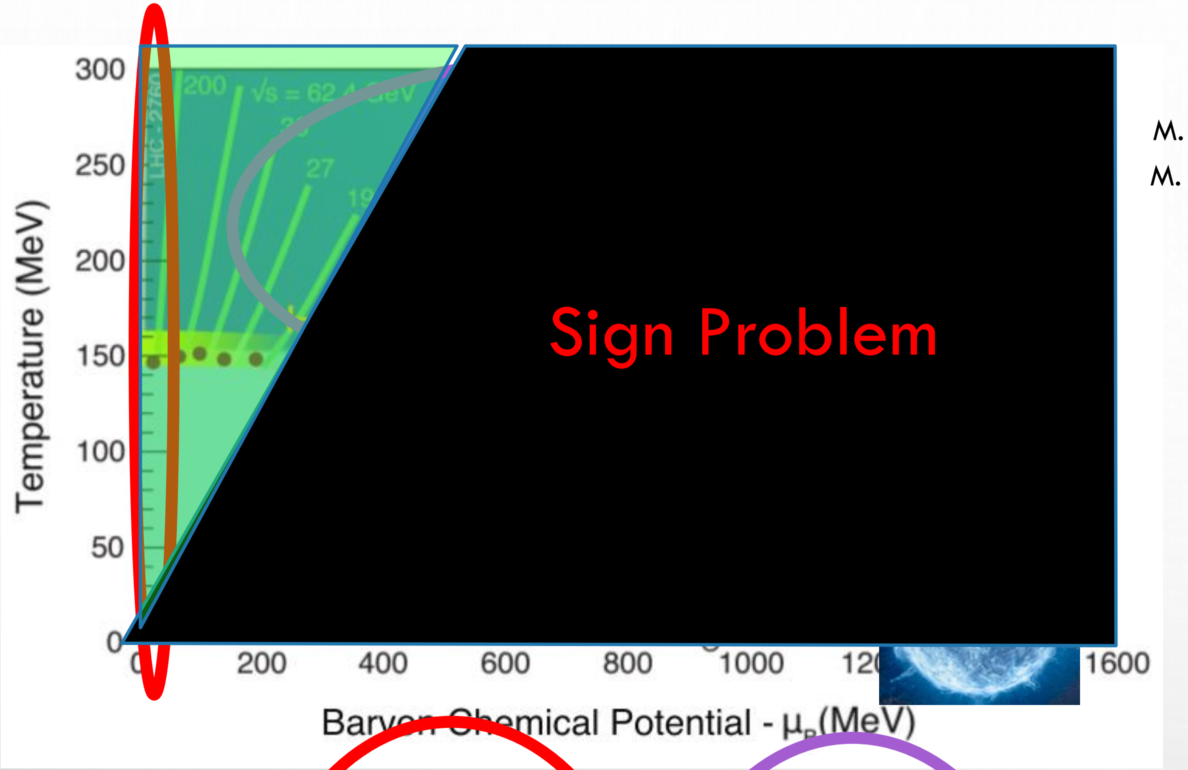
Hadronization, confinement transition, and freeze-out

Measurement of particle distributions

This talk will focus on the connection between the initial conditions and the following hydrodynamic evolution

QCD PHASE DIAGRAM: EQUILIBRIUM DYNAMICS

Lattice QCD



Crucial Experimental Observable:

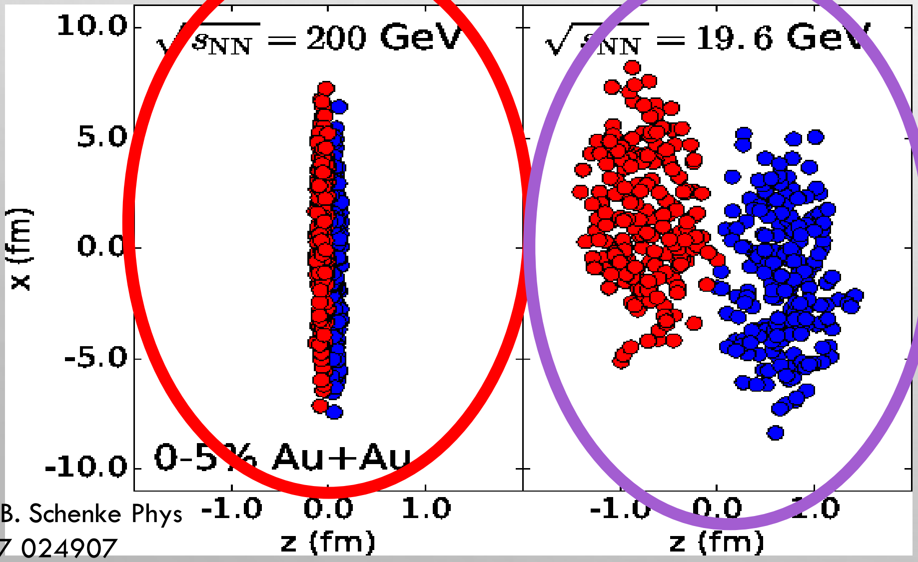
M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)

M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009)

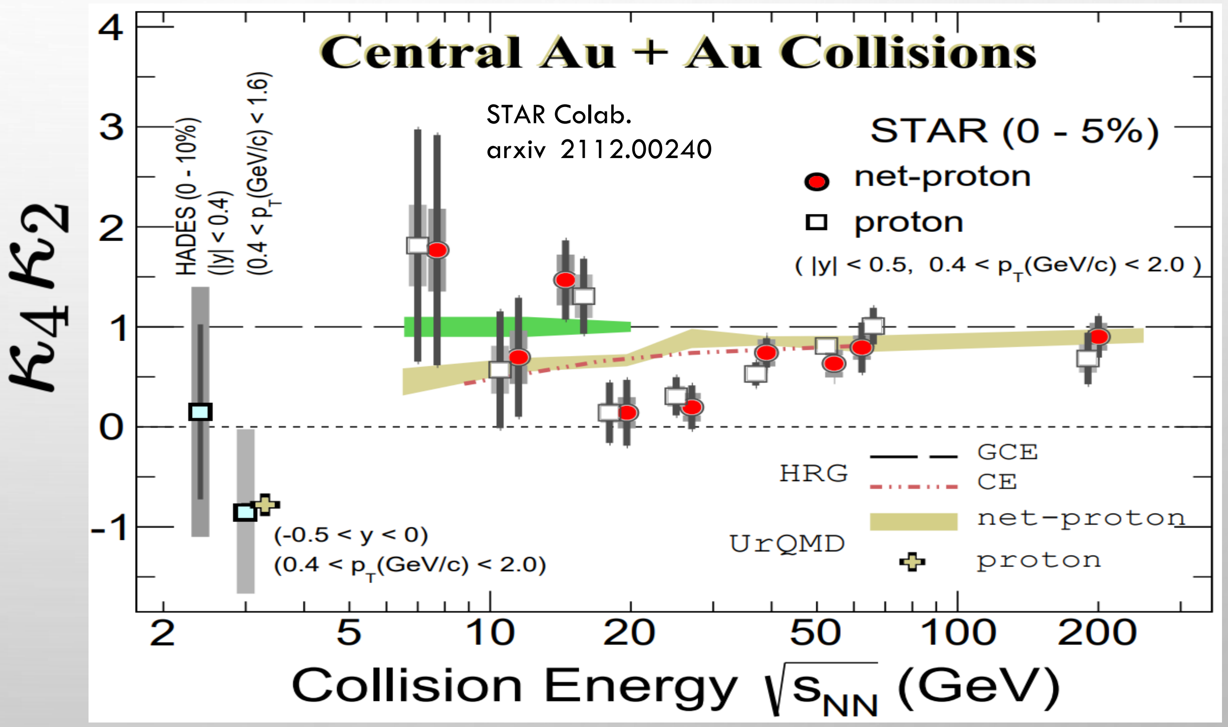
$$\kappa_4 \kappa_2 = \frac{\chi_4}{\chi_2} \sim \xi^9$$

$$\xi \rightarrow \infty$$

In the critical region



C. Shen, B. Schenke Phys Rev C. 97 024907



Final State Measurements of Fluctuations

HYDRODYNAMICS

$$T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}$$

$$J^{\mu} = J_{(0)} + n^{\mu}$$

Local Thermodynamic Equilibrium

$$T^{\mu\nu}_{(0)} = f\left(\epsilon, p, u^{\mu}, x^{\mu}\right)$$

$$\epsilon = \epsilon(T, \mu_B) \quad p = p(T, \mu_B)$$

Out-of-equilibrium Effects

$\Pi^{\mu\nu}$ Local shear and bulk viscous effects

$\partial_\mu T^{\mu\nu} = 0$	Even in equilibrium, evolution is dynamic

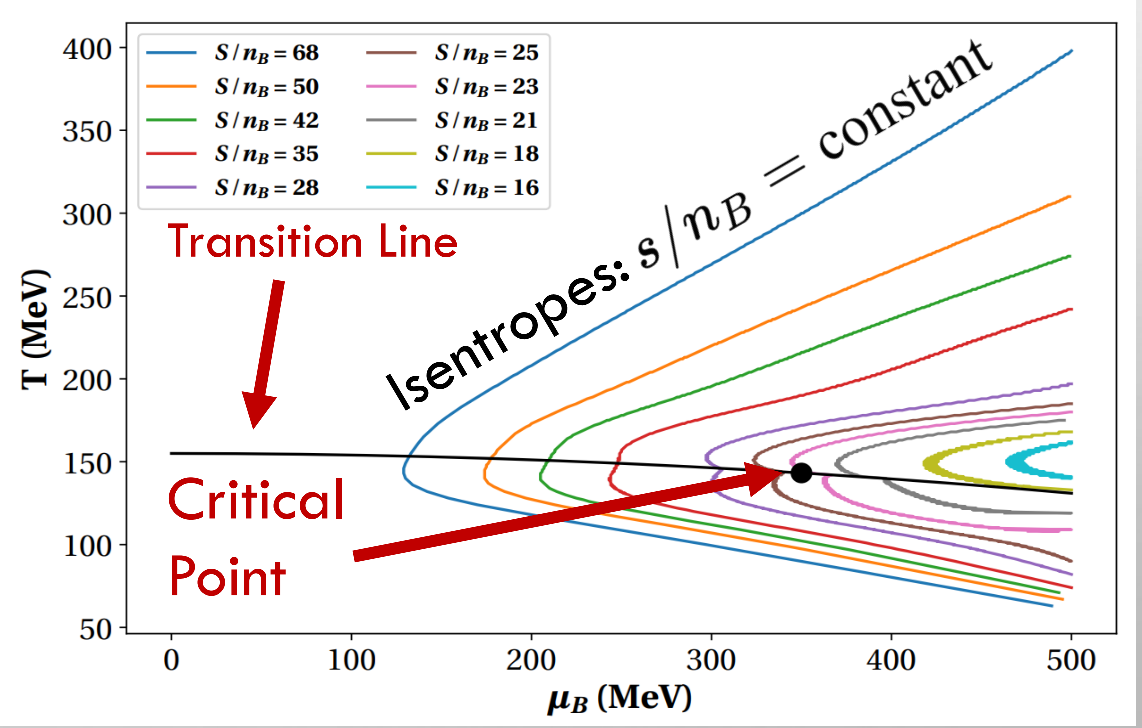
THERMODYNAMICS

P. Parotto Et al. *Phys.Rev.C* 101 (2020) 3, 034901

EoS = Non-critical + Parameterized Critical

$$p(T,\mu_B) = T^4 \sum_n \mathbf{c_n^{non-crit}}(T)\left(\frac{\mu_B}{T}\right)^n + \mathbf{p^{crit}(T,\mu_B)}$$

Ideal hydrodynamics evolves along isentropes: one initial (T, μ_B), unique evolutions

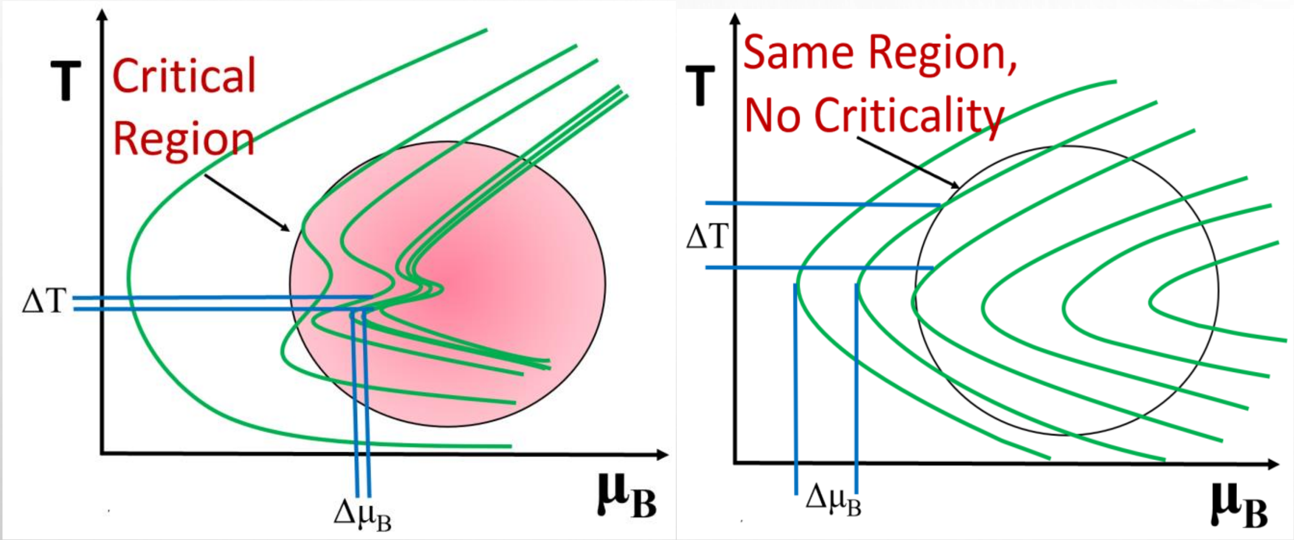


J. Karthein, Et al. Eur. Phys. J. Plus 136, 621 (2021),

THERMODYNAMICS AND EQUILIBRIUM LENSING

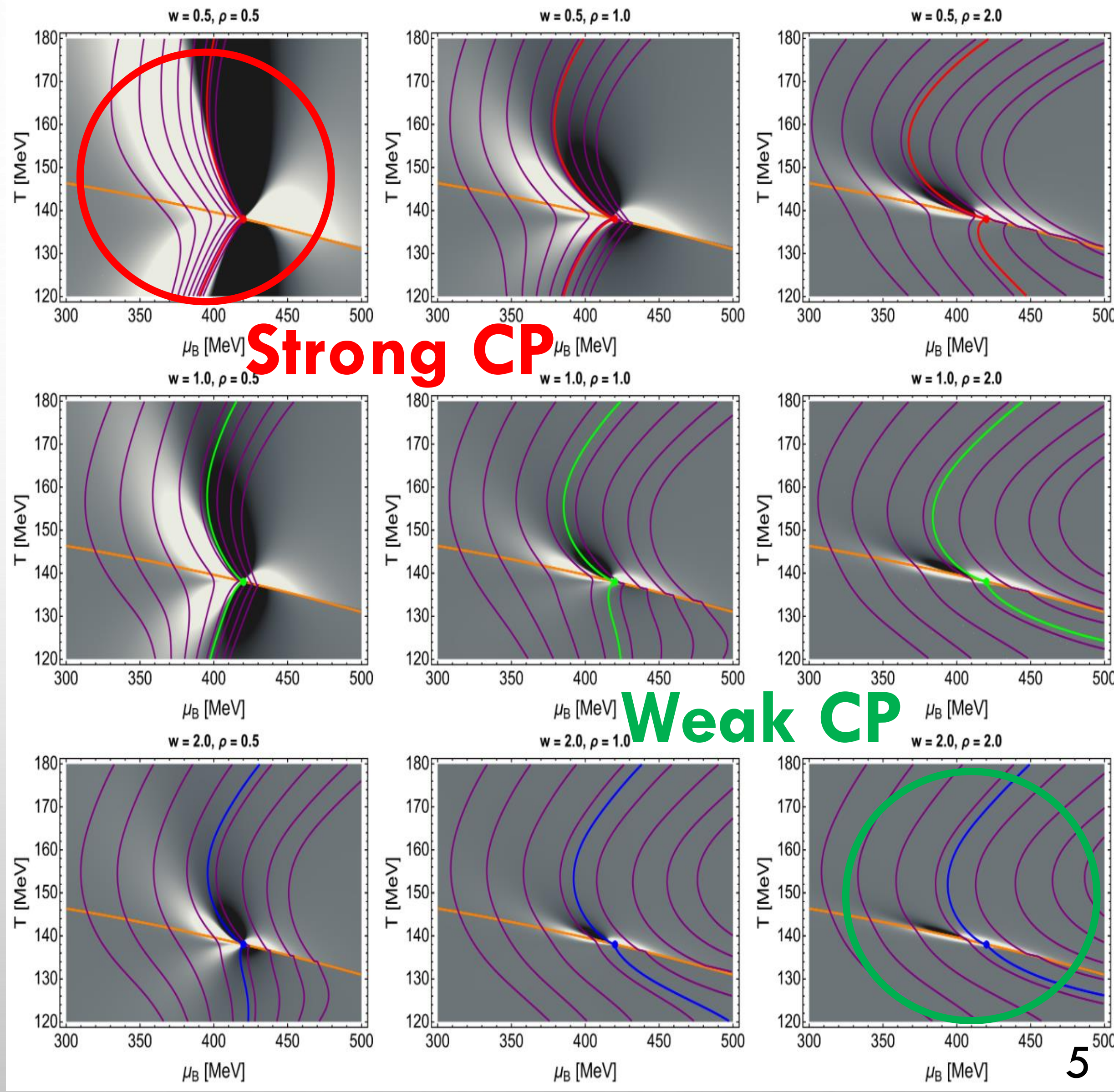
TD, et al., Phys.Rev.D 106 (2022) 9, 094024

$$\Pi^{\mu\nu} = 0$$



$$dT\big|_{\mu_B} = \frac{d\varepsilon\big|_{\mu_B} c^2_{\mu_B}}{s} \qquad d\mu_B\big|_T = \frac{dn\big|_T}{\chi_2}$$

$$c^2_{\mu_B}\big|_{crit} \to 0 \qquad \chi_2\big|_{crit} \to \infty$$

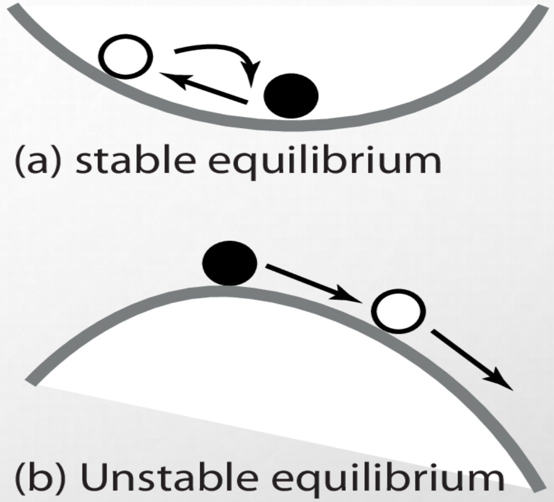


OUT-OF-EQUILIBRIUM HYDRODYNAMICS

Upgrading traditional Navier-Stokes equations to be relativistic…

$$\Pi = -\zeta \partial_\mu u^\mu$$

Leads to acausal (super-luminal) mode propagation and thermodynamic instabilities



One way to ensure linear stability and causality in your system:

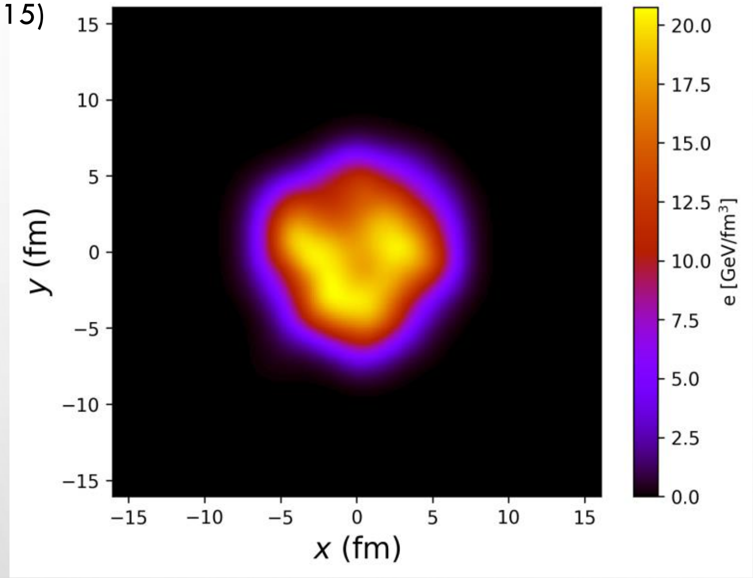
dynamic relaxation of viscous components

Must be initialized independently

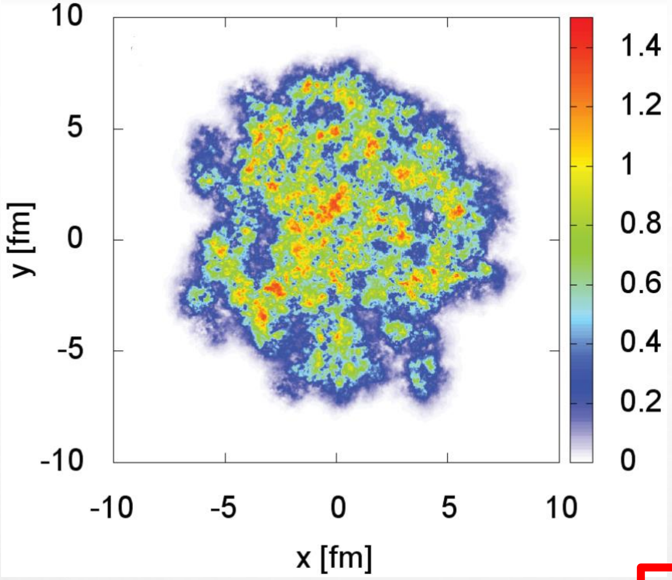
$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta \partial_\mu u^\mu + \ldots$$

WHAT DOES THE INITIAL STATE OF HIC LOOK LIKE?

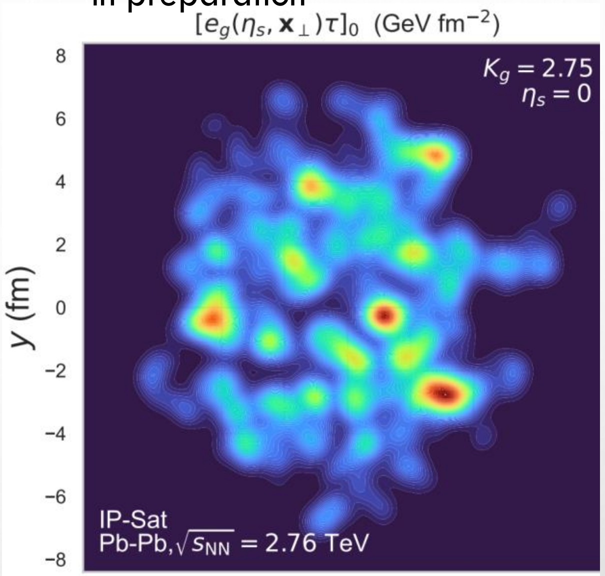
Courtesy Christopher Plumberg using **Trento** J. S. Moreland, J. E. Bernhard, and S. A. Bass, Phys. Rev. C92, 011901 (2015)



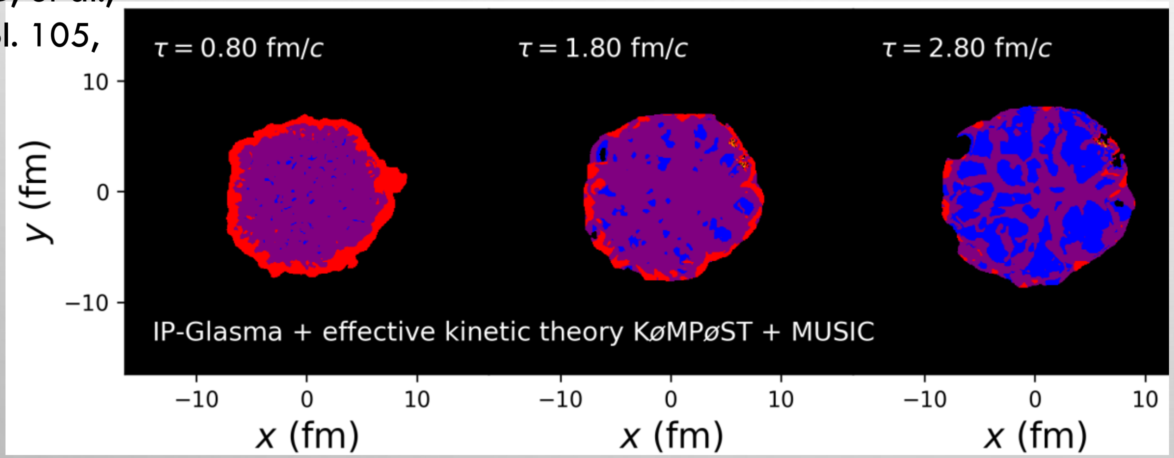
IP-Glasma S. Schliching, B. Schenke Phys.Rev.C 94 (2016) 4, 044907



IP-Sat courtesy Oscar Garcia-Montero, in preparation



C. Plumberg, TD, et al., Phys. Rev. C, vol. 105, p. L061901

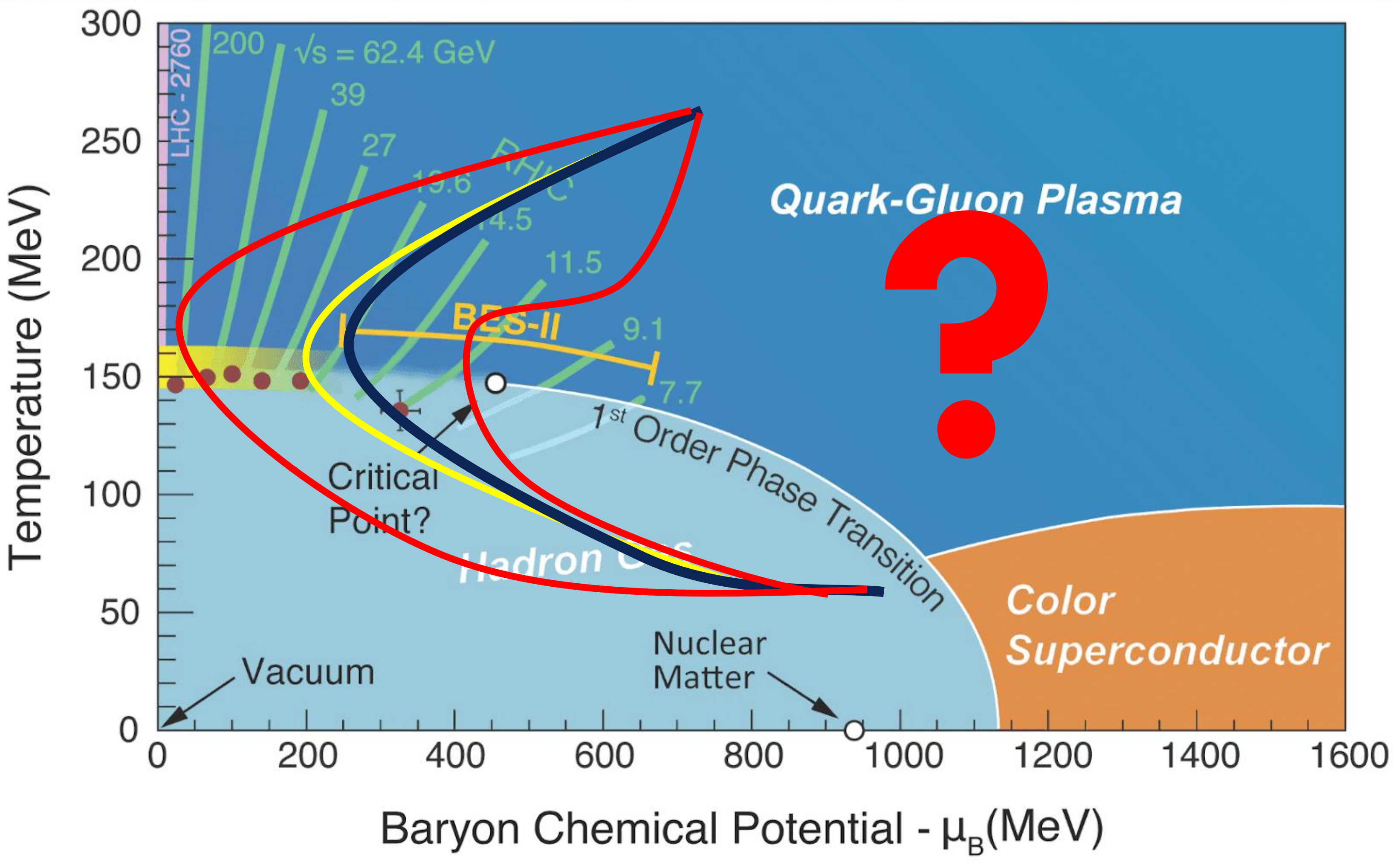


Red and Purple: Far from equilibrium

$$\Pi^{\mu\nu}\big|_{t_0} \neq 0$$

There is no reason to believe this fact changes at lower energies

THE BEHAVIOR OF T-μ_B TRAJECTORIES



SIMPLE MODEL, QUALITATIVE INVESTIGATION

Phys.Rev.D 102 (2020) 7, 074017

Toy model: Bjorken Symmetric Flow

Highly symmetric scenario, functions of space and time become *only* functions of time

$$\text{e.g. } \epsilon(\tau, \vec{x}) = \epsilon(\tau)$$

Coupled PDE'S become coupled ODE's

$$\dot{\epsilon} = -\frac{1}{\tau}\left[\epsilon + p + \Pi - \pi_\eta^\eta\right]$$

Energy Conservation

$$\rho(\tau) = \frac{\rho_0 \tau_0}{\tau}$$

Charge Conservation

Viscous Relaxation type equations

Denicol et al. *Phys. Rev. D* 85 (2012 11407

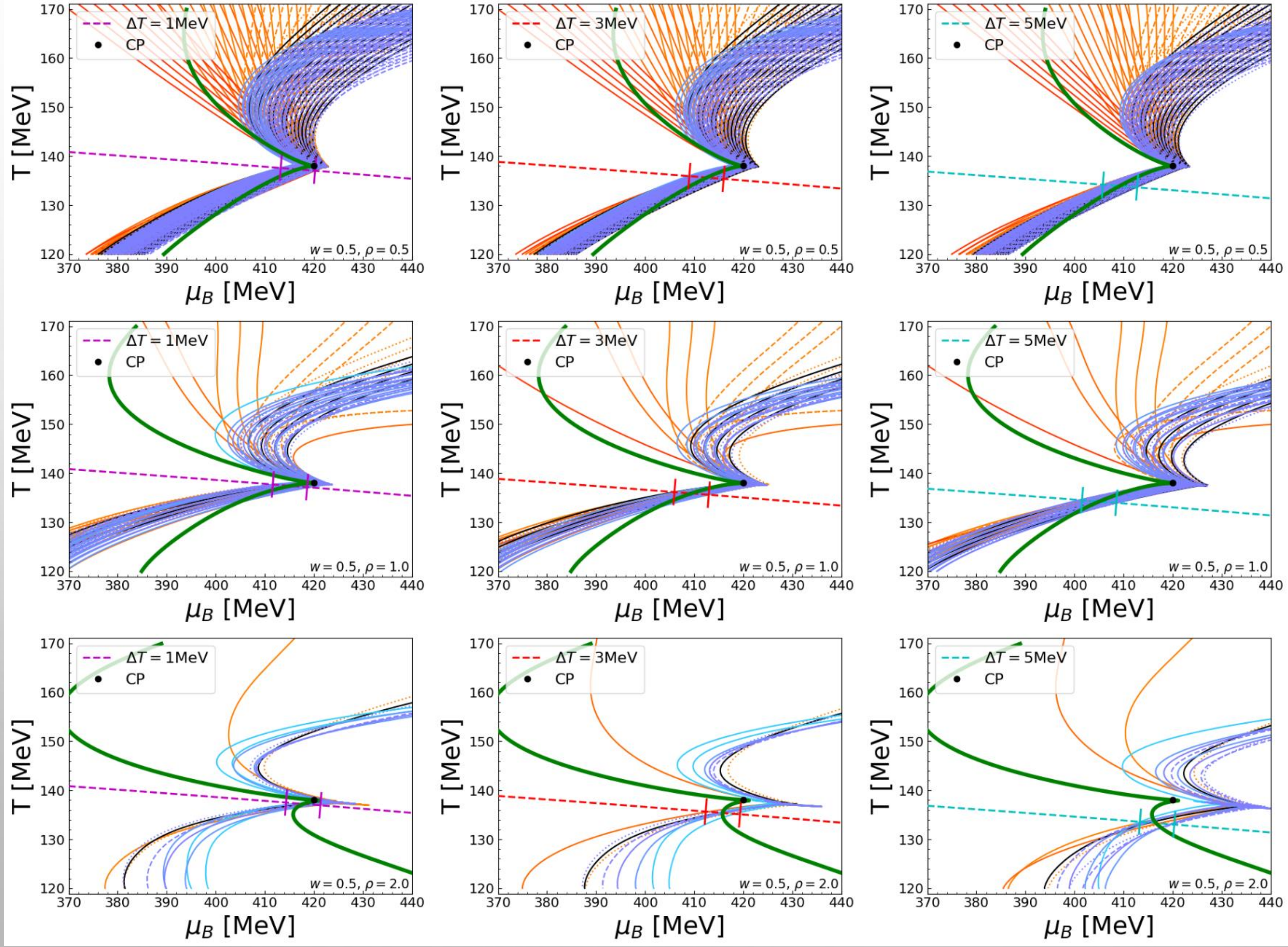
$$\tau_\pi \dot{\pi}_\eta^\eta + \pi_\eta^\eta = \frac{1}{\tau}\left[\frac{4\eta}{3} - \pi_\eta^\eta\left(\delta_{\pi\pi} + \tau_{\pi\pi}\right) + \lambda_{\pi\Pi}\Pi\right]$$

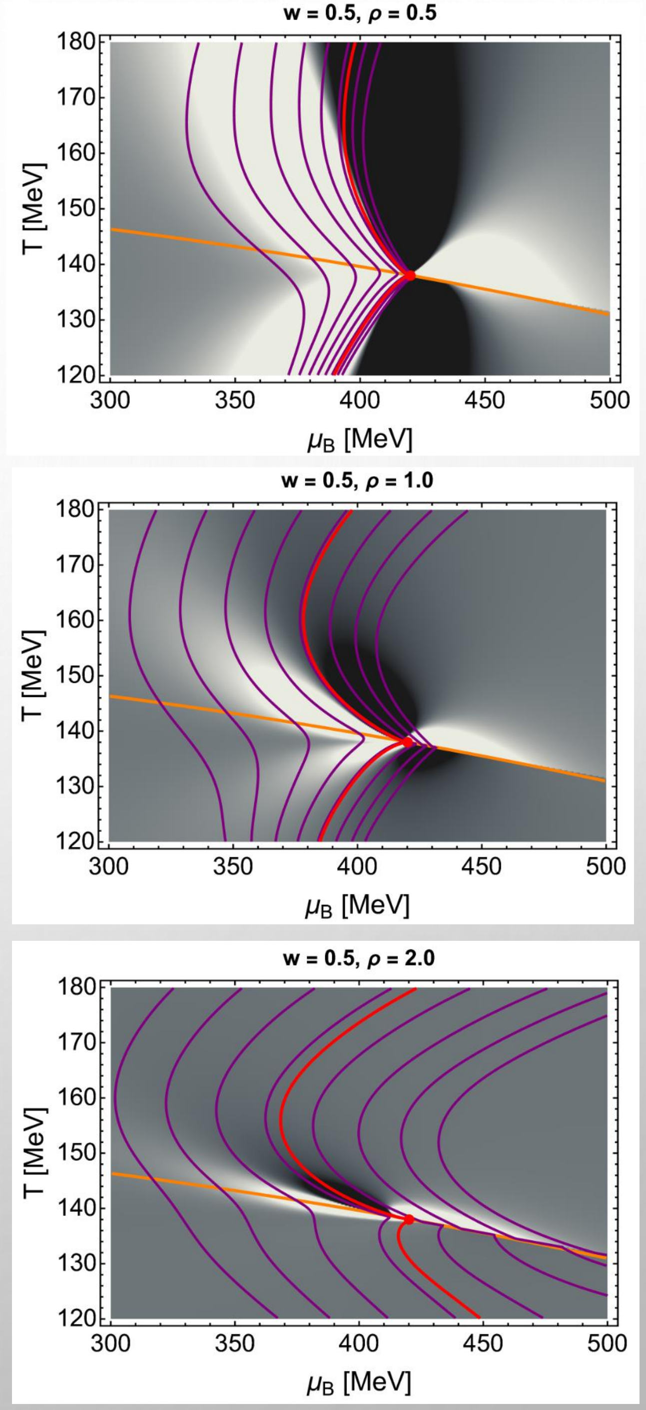
$$\tau_\Pi \dot{\Pi} + \Pi = -\frac{1}{\tau}\left(\zeta + \delta_{\Pi\Pi}\Pi + \frac{2}{3}\lambda_{\Pi\pi}\pi_\eta^\eta\right)$$

PROCEDURE

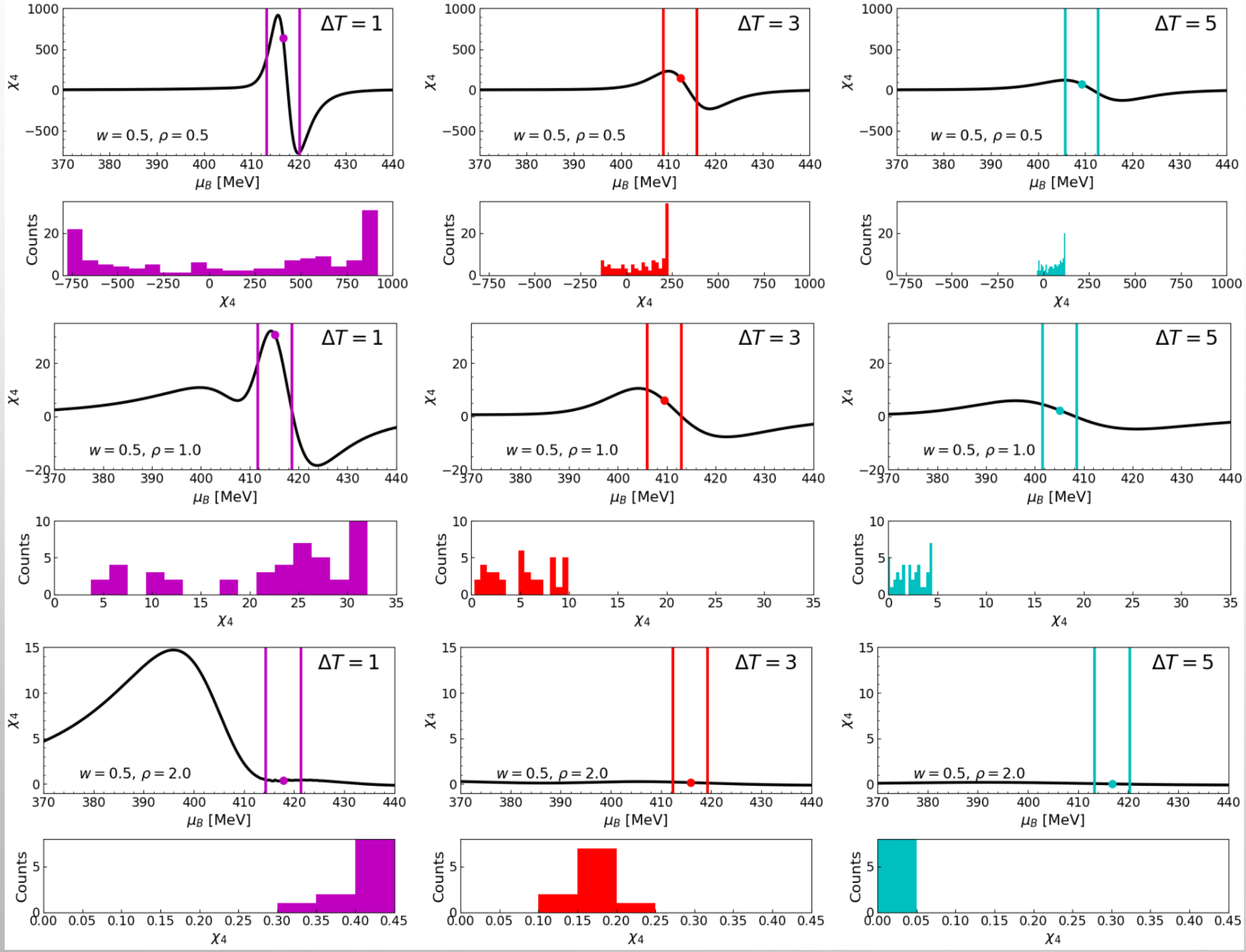
- INTIALIZE MANY DIFFERENT HYDRODYANMIC TRAJECTORIES SYSTEMATICALLY FROM A LIST OF n_{B_0} , Π_0, $\pi_0^{\mu\nu}$ (SAME ENERGY DENSITY)
- SELECT ON TRAJECTORIES THAT PASS THROUGH FREEZE-OUT WINDOW, CENTERED ON THE ISENTROPE THAT GOES THROUGH THE CRITICAL POINT, AND ALONG SHIFTED TRANSITION PARABOLAS
- REPEAT PROCEDURE FOR MANY DIFFERENT REALIZATIONS OF THE EQUATION OF STATE

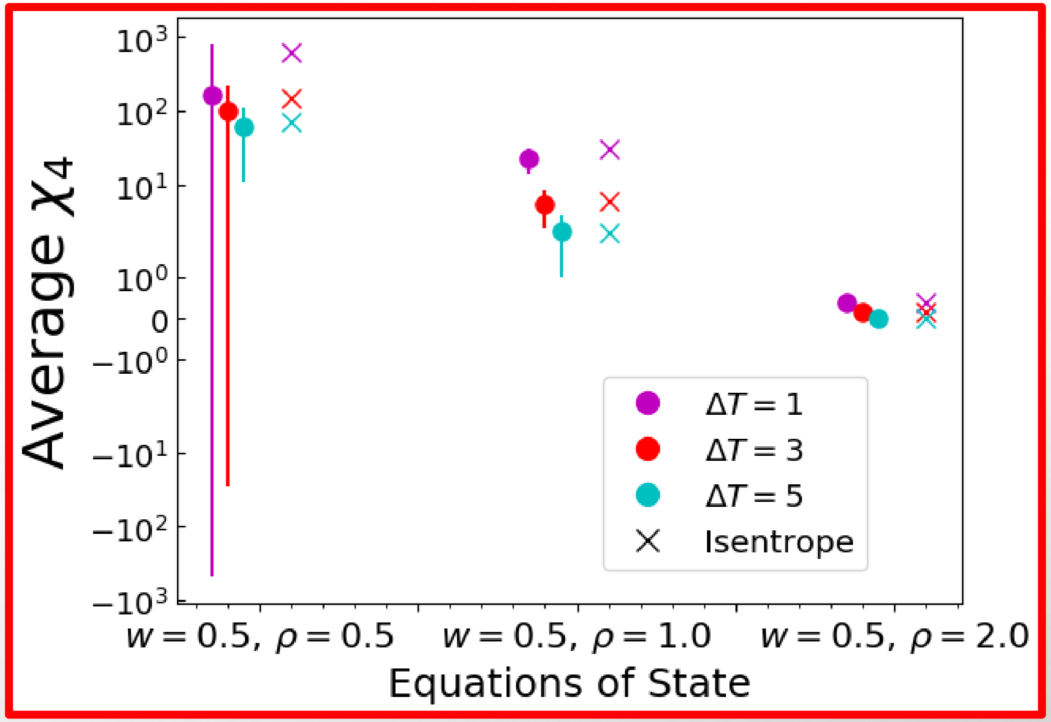
SIZE AND SHAPE OF REGION IS IMPORTANT





NON-TRIVIAL DISTRIBUTIONS OF FOURTH MOMENT

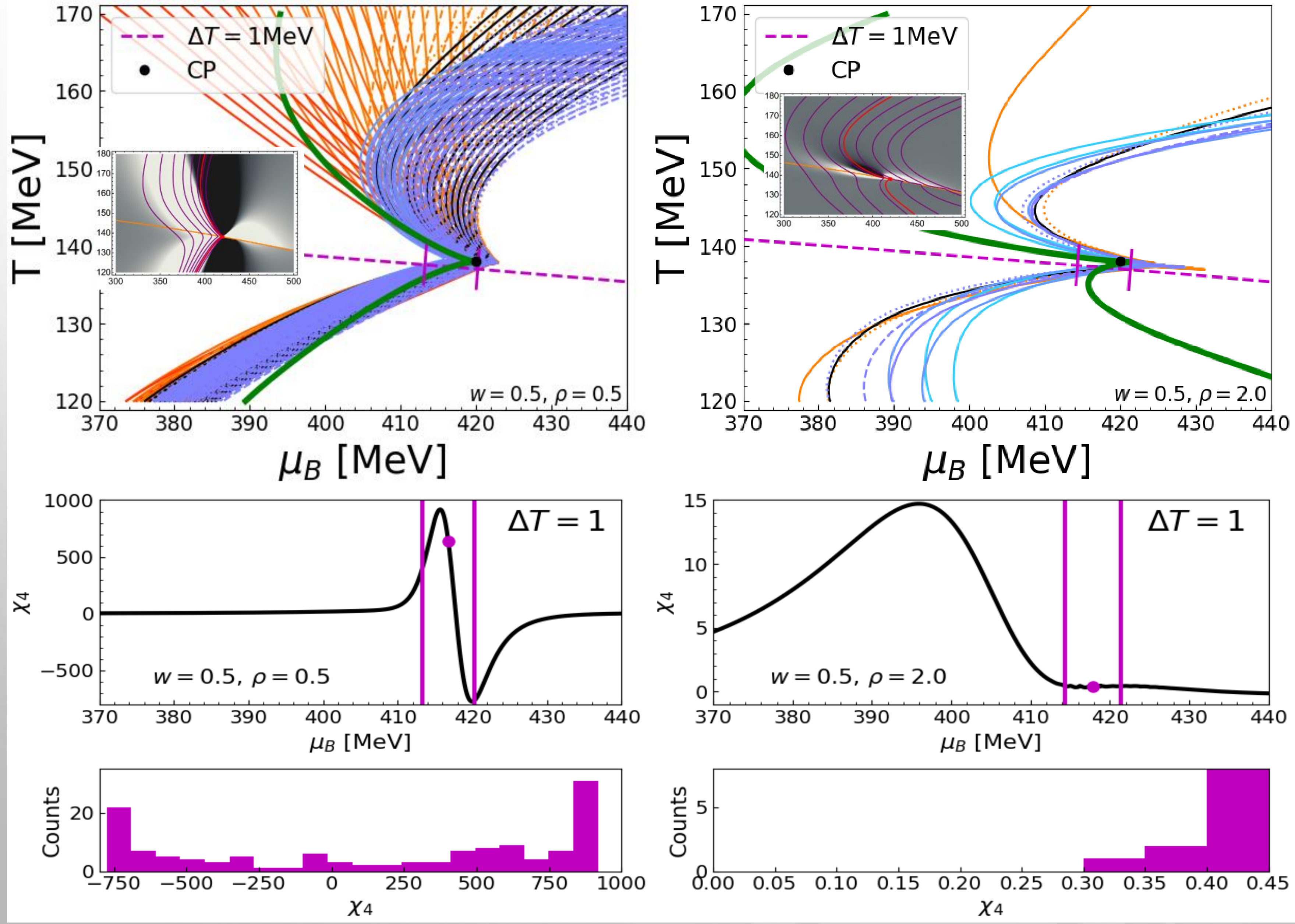




While the spread may be large when there are many trajectories, average values remain close to the central isentropic value.

A tight freeze-out window makes this possible

DYNAMIC LENSING AND KURTOSIS

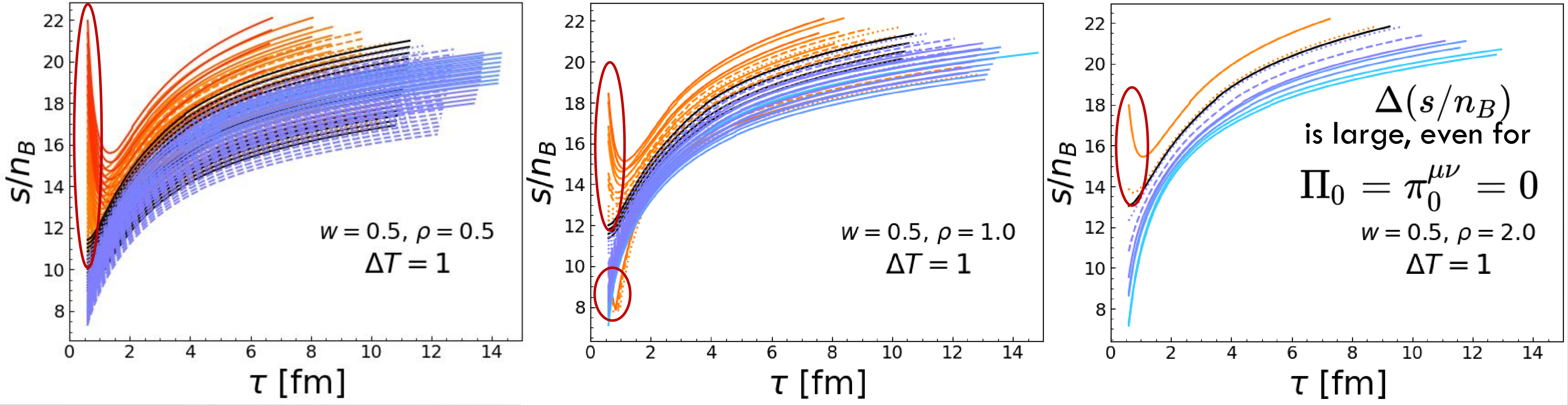


On the left, trajectories *pulled* to larger values of χ_4

Lensing effect may persist for strong viscous corrections

Very sensitive to EoS parameters

IS NEAR-IDEAL HYDRODYNAMICS A 'GOOD ENOUGH' APPROXIMATION?



"Thermal" entropy production, violation of second law?

$$\partial_\mu (s u^\mu) = \frac{1}{T}(\pi^{\mu\nu}\sigma_{\mu\nu} - \Pi\theta) < 0\ ?$$

Clearly negative for

$$\pi^{\mu\nu}\sigma_{\mu\nu} < 0,\ \Pi\theta > 0$$

Real entropy production:

$$\partial_\mu S^\mu \approx s u^\mu - \beta_\Pi \Pi \dot{\Pi} - \beta_\pi \pi^{\mu\nu} \dot{\pi}_{\mu\nu} > 0\ !$$

Recent work has confirmed this conjecture

C. Chattopadhyay, U. Heinz, T, Schaefer, arXiv: 2209.10483 [hep-ph].

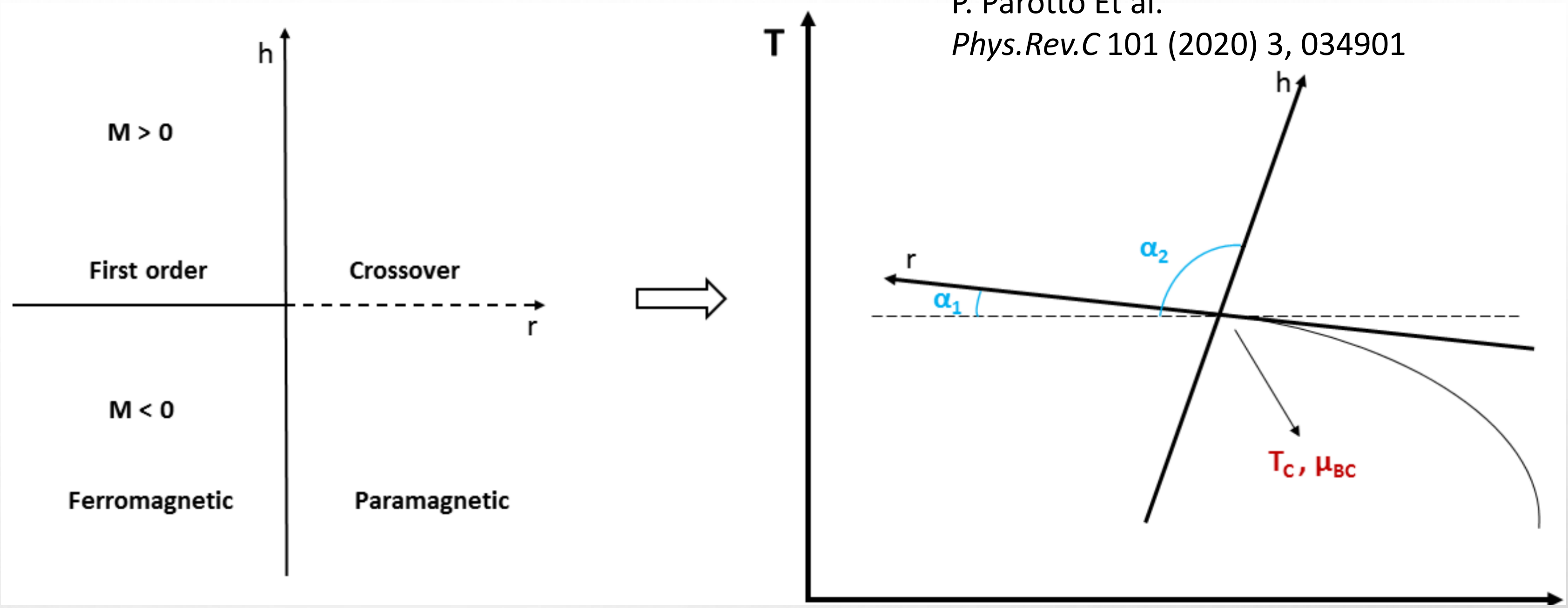
SUMMARY AND OUTLOOK

- Out of equilibrium effects will be very important to take into account in our search for the QCD critical point
- Work is ongoing to begin modelling charge dynamics in more realistic hydrodynamic models as well as in pre-hydrodynamic models
- Models which include the initialization of out-of-equilibrium components will be a crucial part of our ability to unambiguously find critical behavior if it is there
- Caveat: *Dynamic* criticality not included in this analysis

BACKUP

MAPPING THE 3D ISING MODEL TO QCD

Due to its symmetries, QCD is expected to be in the 3D Ising universality class



P. Parotto Et al.
Phys.Rev.C 101 (2020) 3, 034901

3D Ising

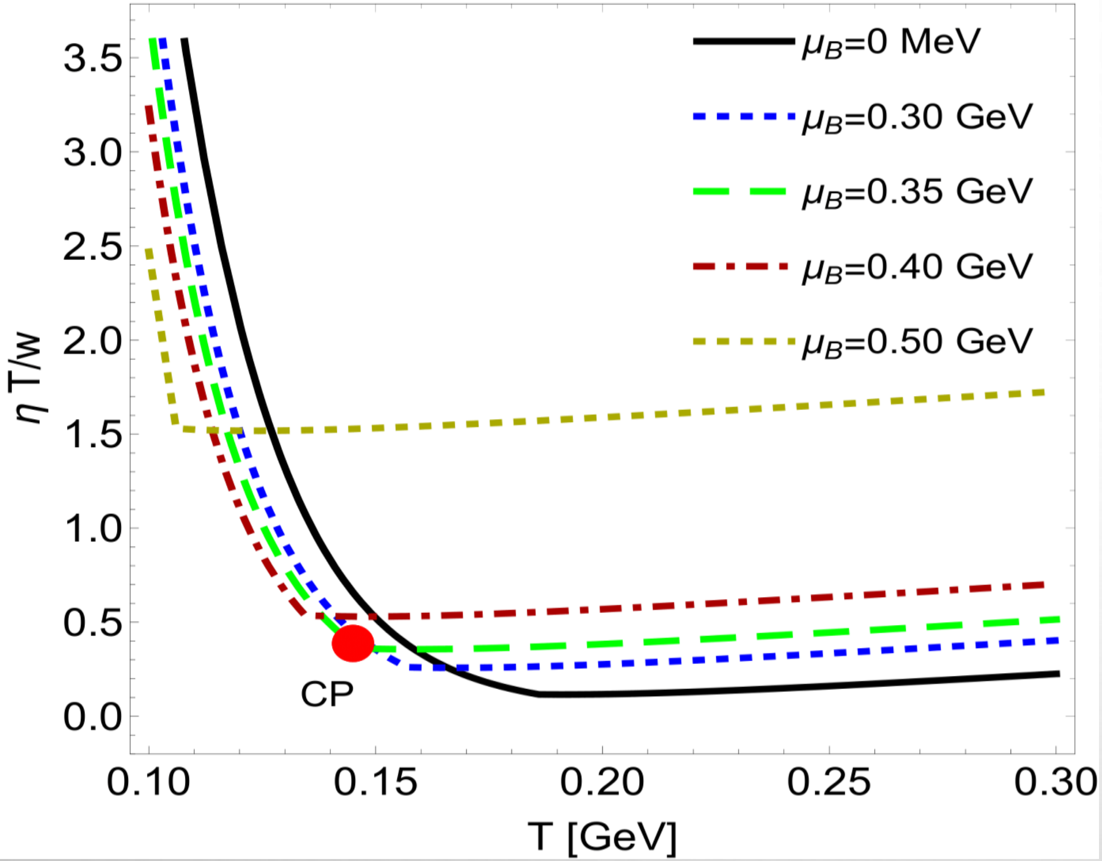
$$\xi \sim \left|\frac{T-T_c}{T_c}\right|^{-\nu} \quad \chi \sim \left|\frac{T-T_c}{T_c}\right|^{-\gamma}$$

QCD

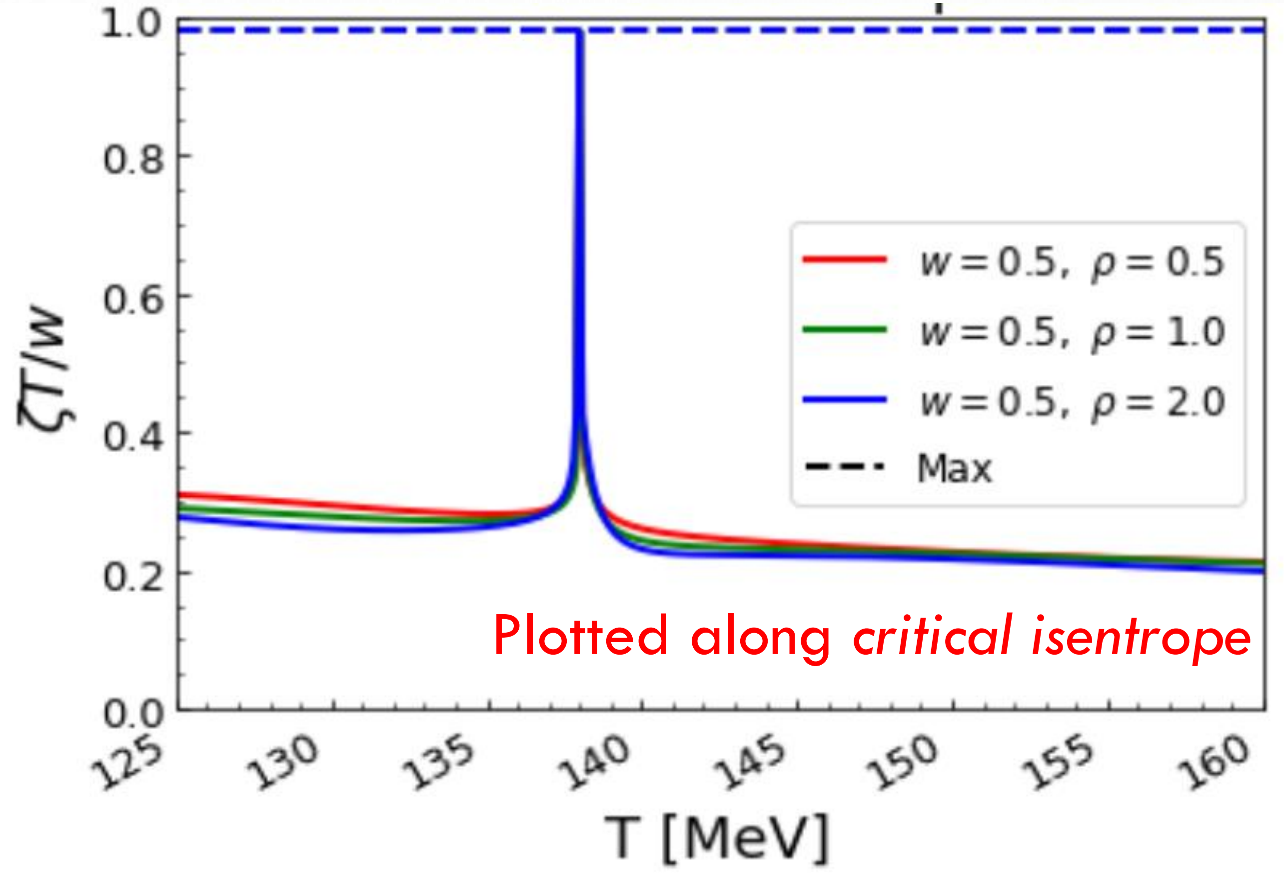
$$\chi_2^B \sim \xi^2 \quad \chi_4^B \sim \xi^{11}$$

M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)

Transport Coefficients



TD,E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7



Shear viscosity not sensitive to criticality explicitly

Critically Scaled Bulk:

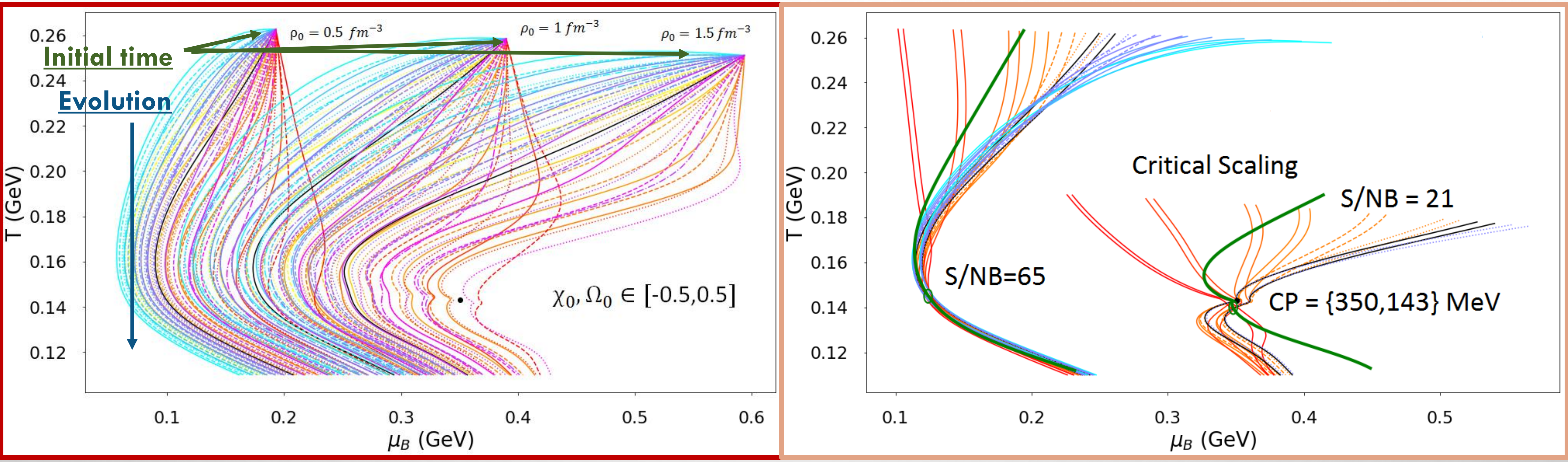
$$\left(\frac{\zeta T}{w}\right)_{CS} = \frac{\zeta T}{w}\left[1+\left(\frac{\xi}{\xi_0}\right)^3\right]$$

Monnai, Akihiko et al, Nucl. Phys. ,A967,2017

$$\chi_0 = \frac{\pi}{\epsilon+p}, \Omega_0 = \frac{\Pi}{\epsilon+p}$$

$$\{T(\tau), \mu_B(\tau)\}$$

Green Line: Equilibrium Hydro trajectory



TD, E. McLaughlin, J. Noronha-Hostler, *Phys.Rev.*D 102 (2020) 7

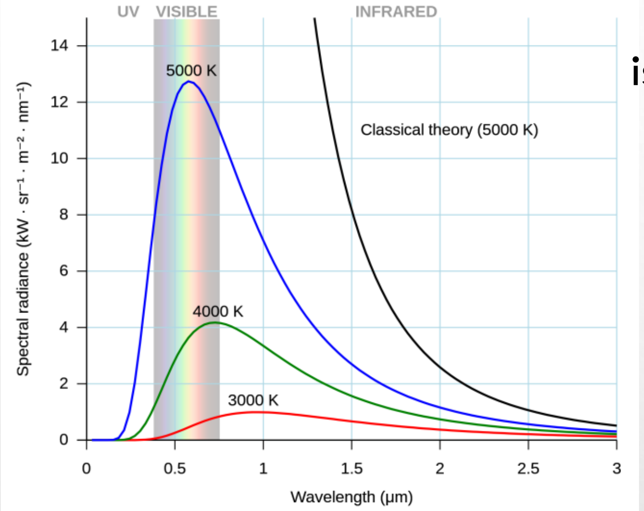
Takeaways:

1. Pushed to or away from CP on event-by-event basis
2. Degeneracy of final state mapping to initial state

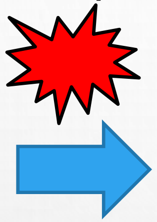
How does the kurtosis behave at freeze-out in the critical region?

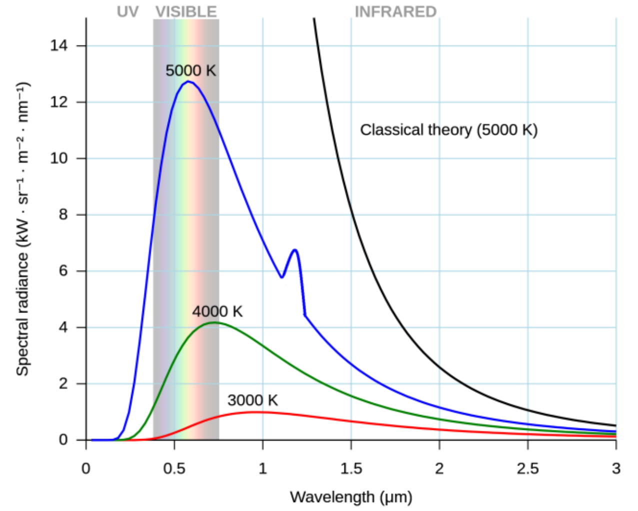
PHYSICALITY OF INDEPENDENT VISCOUS FIELDS

Consider the following thought experiment:



isotropic





Small deviation from eq

THERMALIZATION

What is expected to be in this black box?

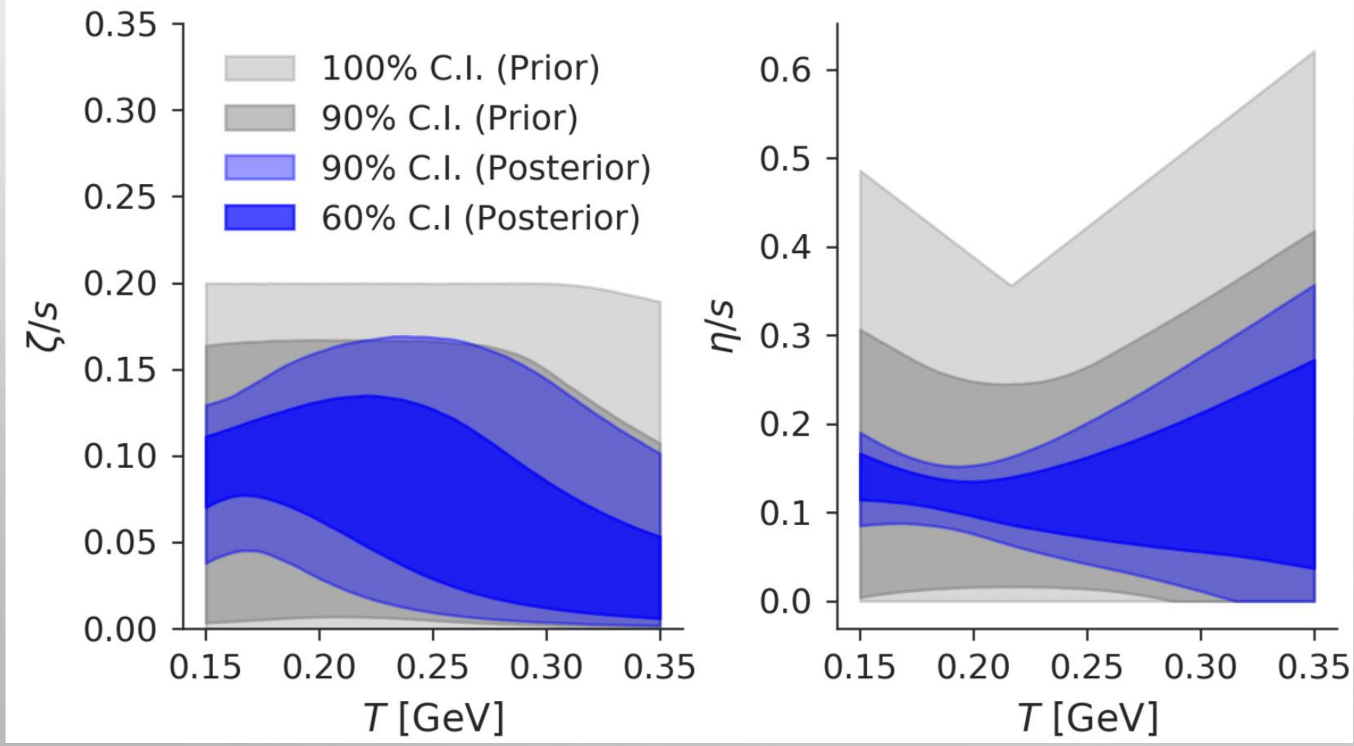
A kinetic theory perspective tells us that the viscous fields in relaxation hydro are given by moments of the distribution function

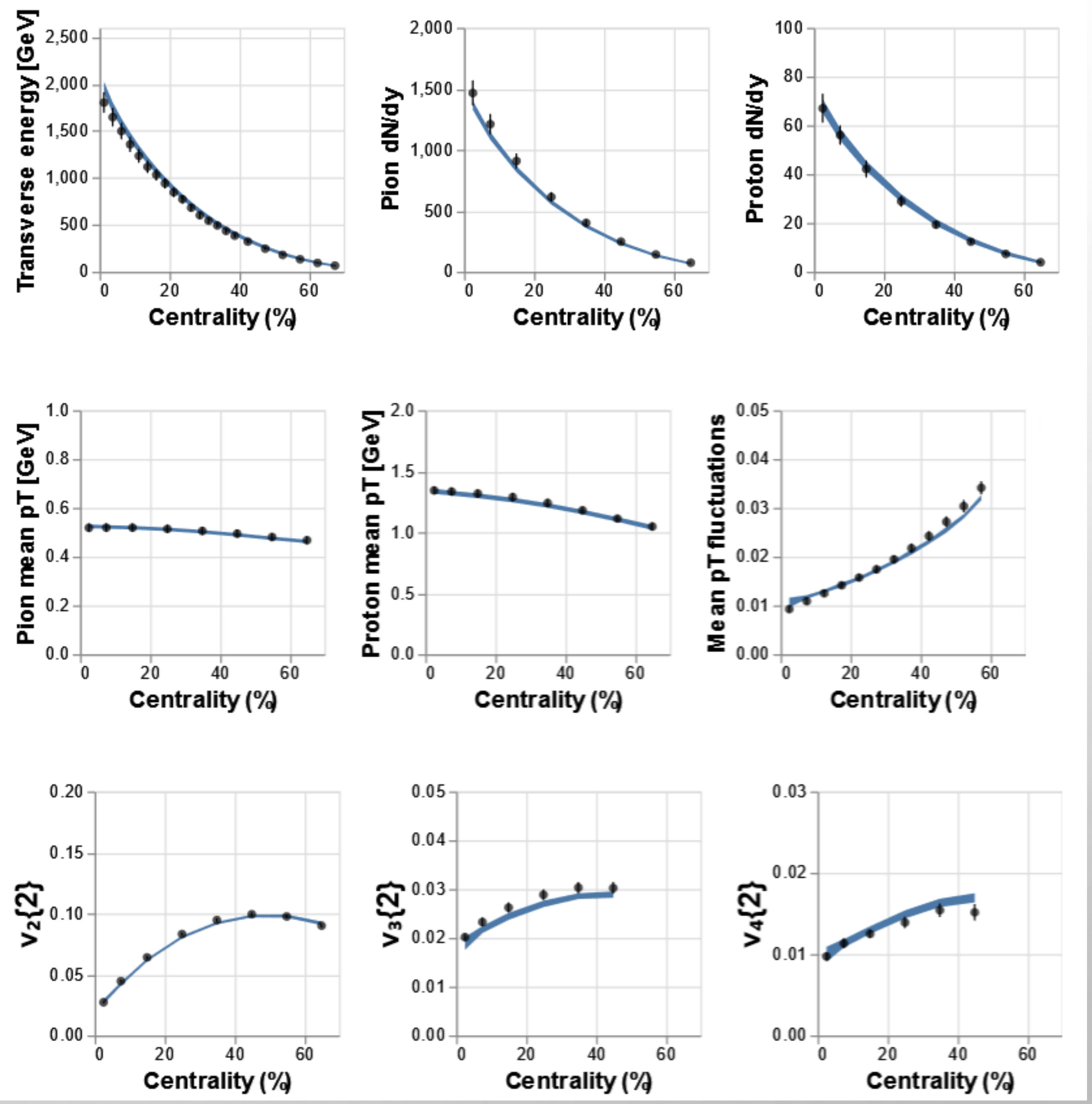
$$\Pi \sim \int dK \left(\Delta_{\mu\nu} k^\mu k^\nu \right) \delta f$$

This is more information than only spatial gradients

CAN WE REALLY EXTRACT NON-EQUILIBRIUM PROPERTIES?

Yes! Quite systematically using tools of Bayesian Analysis





http://eg1.jetscape.wayne.edu:443/