Spinodal Enhancement of Light Nuclei Yield Ratio in Relativistic Heavy-Ion Collisions

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Outline

1. Motivation: Why light nuclei? Why $N_t N_p / N_d^2 (t p / d^2)$?

2. Spinodal enhancement of $t p / d^2$ from the first-order QCD phase transition

3. Summary and Outlook
1.1 QCD phase transition & light nuclei production (1)

Critical Point: long-range correlation
First-order Phase Transition: Spinodal instability

1.2 1\textsuperscript{st} order QCD phase transition & light nuclei production (2)

Phase separation, spinodal decomposition (SD)

Density matrix formulation (coalescence)

\[ N_d \propto \text{Tr} [\hat{\rho}_s \hat{\rho}_d] \quad \text{and} \quad N_t \propto \text{Tr} [\hat{\rho}_s \hat{\rho}_t] \]

\[ \frac{N_t N_p}{N_d^2} \propto \Delta \rho_n \]

\[ \Delta \rho_n = \frac{\int dx (\delta \rho(x))^2}{\int dx \sigma_n^2} \]

characterizes density inhomogeneity

\[ B_\Lambda = 0.41 \text{ MeV} \]

\[ N_t N_p / N_d^2 \]

See [PLB 816, 136258 (2021)] for critical effects on \( N_t N_p / N_d^2 \)
1.3 QCD phase transition & light nuclei production

**Our works**

- Probing QCD phase transition with light nuclei production
  - PLB 774, 103 (2017)
  - K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu
  - \[ \frac{N_t N_p}{N_A^2} \approx \frac{1}{2\sqrt{3}} [1 + \Delta \rho_n] \]
  - PLB 781, 499 (2018)
  - PLB 816, 136258 (2021) (criticality)

- 1st-order QCD phase transition
  - PRD 103, 014006 (2021)
  - EPJA 57, 313 (2021) (Transport)
  - arxiv:2205.11010 (Transport) (First-order PT in BES)

**Other works**

- First-order phase transition & composite particle production
  - J. Steinheimer et al., PRC 87, 054903 (2013)
  - PRL 109, 212301 (2012) (Hydrodynamics)
  - JHEP 12, 122 (2019) (Machine learning)

- Baryon clustering near the critical point
  - E. Shuryak, J.M. Torres-Rincon et al., PRC 100, 024903 (2019)
  - PRC 101, 034914 (2020)
  - EPJA 56, 241 (2020)
  - PRC 104, 024908 (2021)

- Background effects
  - S. Wu et al., PRC 106, 034905 (2022)

**Timeline**

- 2012
- 2014
- 2015
- 2016
- 2017
- 2018
- 2019
- 2020
- 2021
- 2022
2. Spinodal enhancement of $tp/d^2$ from the first-order QCD phase transition
2.1 Equation of State (extended NJL model) (4)

The eNJL provides a flexible equation of state (EoS). The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for eNJL

\[
\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m})\psi + G_S \sum_{a=0}[(\bar{\psi}\lambda^a \psi)^2 + (\bar{\psi}i\gamma_5 \lambda^a \psi)^2] - K \{\text{det}[\bar{\psi}(1 + \gamma_5)\psi] + \text{det}[\bar{\psi}(1 - \gamma_5)\psi]\}
+ G_{SV} \left\{\sum_{a=1}^3[(\bar{\psi}\lambda^a \psi)^2 + (\bar{\psi}i\gamma_5 \lambda^a \psi)^2]\right\}
\times \left\{\sum_{a=1}^3[(\bar{\psi}\gamma_5 \lambda^a \psi)^2 + (\bar{\psi}i\gamma_5 \gamma^\mu \lambda^a \psi)^2]\right\},
\]

\[\Lambda [\text{MeV}] = 602.3 \quad M_{u,d} [\text{MeV}] = 367.7 \quad G_A^2 = 1.835 \quad M_s [\text{MeV}] = 549.5 \quad K\Lambda^6 = 12.36 \quad (\langle \bar{u}u \rangle)^{1/3} [\text{MeV}] = -241.9 \quad m_{u,d} [\text{MeV}] = 5.5 \quad (\langle s\bar{s} \rangle)^{1/3} [\text{MeV}] = -257.7 \quad m_s [\text{MeV}] = 140.7\]

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)
2.2 Box Simulation

Effective mass:

\[ M_u = m_u - 4G_S \phi_u + 2K \phi_d \phi_s - 2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d), \]
\[ M_d = m_d - 4G_S \phi_d + 2K \phi_u \phi_s - 2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d), \]
\[ M_s = m_s - 4G_S \phi_s + 2K \phi_u \phi_d \]

\[ \phi_i = \frac{1}{2N_c} \int_0^\Lambda \frac{d^3p}{(2\pi \hbar)^3} \frac{M_i}{E_i} (1 - f_i - \bar{f}_i) \]

\[ \rho_i = \frac{1}{2N_c} \int_0^\Lambda \frac{d^3p}{(2\pi \hbar)^3} (f_i - \bar{f}_i) \]


\[ \frac{dr}{dt} = \mathbf{v}, \]
\[ \frac{dp}{dt} = -\frac{M}{E^*} \nabla_r L_m \pm \mathbf{E} \pm \mathbf{v} \times \mathbf{B} \]

Strong EM fields

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)
### 2.3 Relativistic Heavy-Ion Collisions

- **Initialization**
- **Parton evolution**
  - Mean field (eNJL) + scattering
  - \( \text{Exhibits dynamical chiral phase transition} \)
  - \( \text{Hadronization: Quark coalescence} \)
  - \( \text{ART (A Relativistic Transport model for hadrons)} \)
  - \( \text{Nucleon coalescence} \)
- **Light nuclei**
2.4 Trajectories in the phase diagram

Phase trajectories of central cells in the phase diagram

\[ \rho^{N} = \frac{\int dx \rho^{(N+1)}(x)}{\int dx \rho(x)} \]

\[ y_{2} = \frac{\left[ \int dx \rho^{2}(x) \right]^{2}}{\int dx \rho^{3}(x)} \]
2.5 Survival of density fluctuation in an expanding fireball (8)

Off-equilibrium effects

Density moment:
\[
\overline{\rho^N} = \frac{\int dx \rho^{(N+1)}(x)}{\int dx \rho(x)}
\]
\[
y_2 = \frac{\int dx \rho(x)[\int dx \rho^3(x)]}{[\int dx \rho^2(x)]^2}
\]

If the expansion is self-similar or scale invariant

\[
\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)
\]

then \(y_2(t) = y_2(t_h)\), i.e., remains a constant

‘Memory effects’: Large density inhomogeneity survives to kinetic freezeout
2.7 Collision energy dependence

1. Without a critical point:
The energy dependence of \( tp/d^2 \) is almost flat.

2. With a first-order phase transition:
The spinodal instability induced enhancement of \( tp/d^2 \) during the first-order phase transition increases as increasing the critical temperature.

STAR, arXiv:2209.08058(2022)
Hui Liu (STAR), QM2022
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The spinodal enhancement of $tp/d^2$ subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.
2.8 Centrality dependence

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The slope with EoS-I is 5 times smaller.
2.9 Possible critical effects
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With long-range correlation:

\[ \frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[ 1 + \Delta \rho_n + \frac{\lambda}{\sigma} G \left( \frac{\xi}{\sigma} \right) \right] \]

\[ G(z) = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{z} e^{\frac{1}{2z^2}} \text{erfc} \left( \frac{1}{\sqrt{2z}} \right) \]

Further investigations are needed
3. Summary and Outlook

Main findings:

1. With scans of the collision energy and centrality as well as the equation of state using a novel transport model, we find that large density inhomogeneities generated by the spinodal instability during the first-order QCD phase transition can survive the fast expansion of the subsequent hadronic matter and lead to an enhanced $t_p/d^2$ in central Au+Au collisions at $\sqrt{s_{NN}}=3-5$ GeV for $T_c \geq 80$ MeV, which is in accordance with the STAR measurements.

2. We also find that the spinodal enhancement of $t_p/d^2$ subsides with increasing collision centrality because of the shortening of fireball lifetime, and this effect results in an almost flat centrality dependence of $t_p/d^2$ at $\sqrt{s_{NN}}=3$ GeV, which can also be used as a signal for the occurrence of a first-order phase transition.

Future developments:

1. Incorporation of Polyakov loop
2. Inclusion of long-range correlation