Fluctuations in the mixed phase of the first order phase transition and nucleus-nucleus collisions

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CPOD 2022

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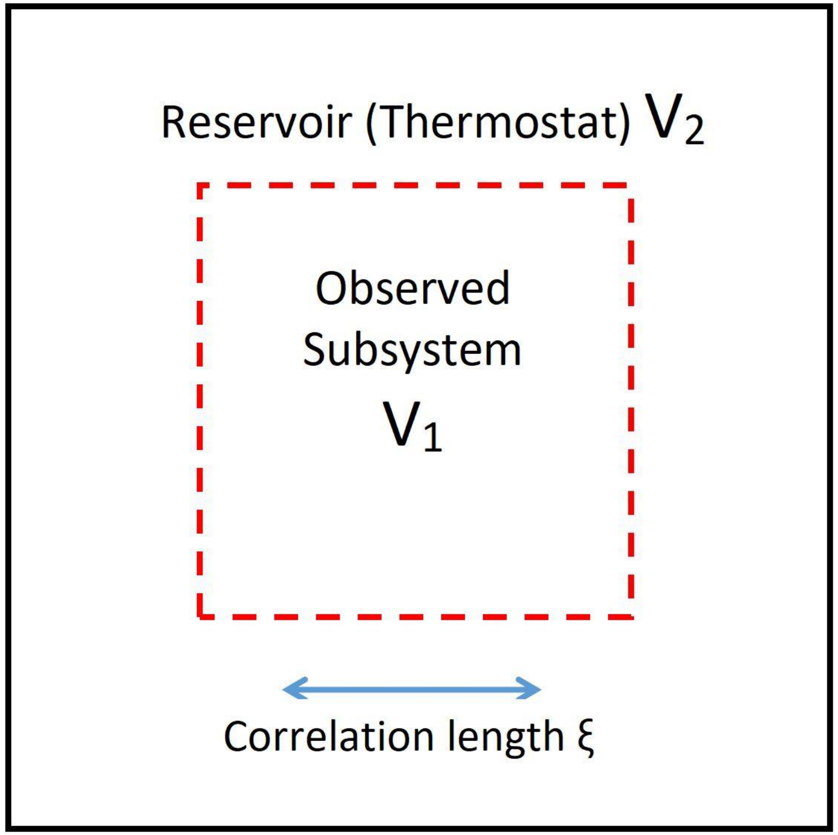
Outline

- Brief overview of the Subensemble method for fluctuations in the presence of conservation laws
- Application to the fluctuations inside mixed phase of the FOPT
- Results for mixed phase fluctuations in the context of HIC
- HADES data on proton number fluctuations at $\sqrt{s_{NN}}$ = 2.4 GeV

In HIC fluctuations are measured within Subensemble

Subensemble



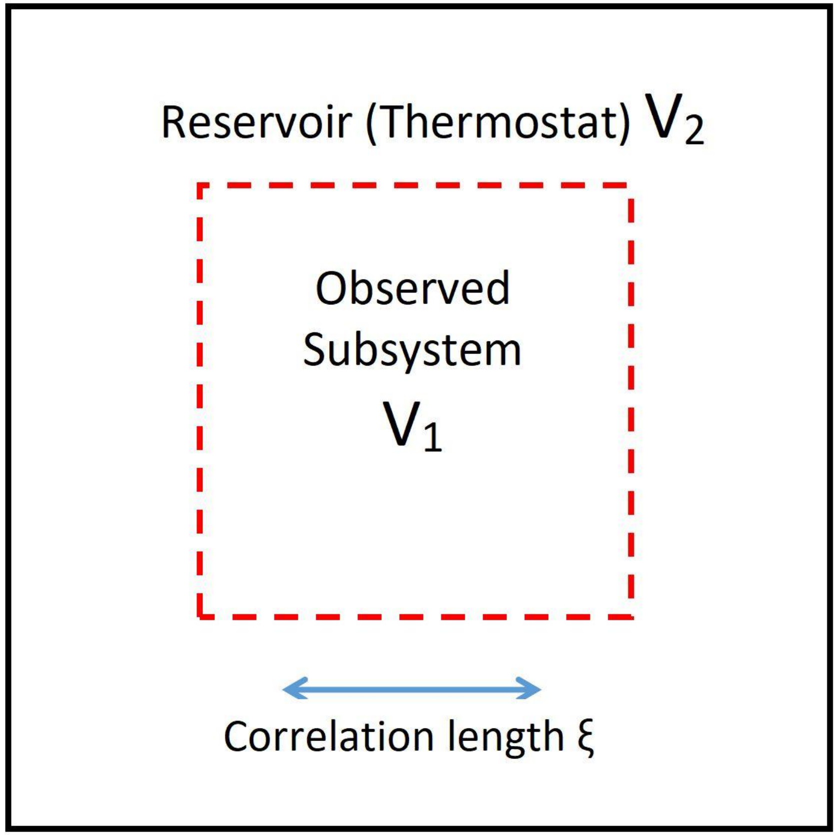
Total system V

$\xi \sim V_1 \sim V_2$

Heavy Ion Collisions, Microscopic simulations

In HIC fluctuations are measured within Subensemble

Subensemble



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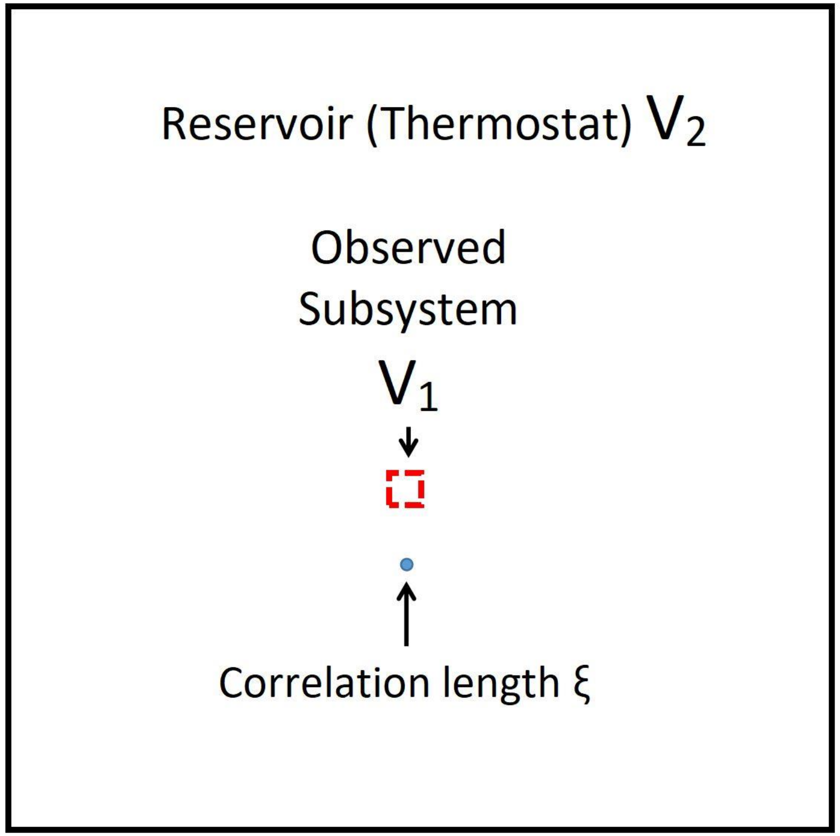
Grand Canonical Ensemble



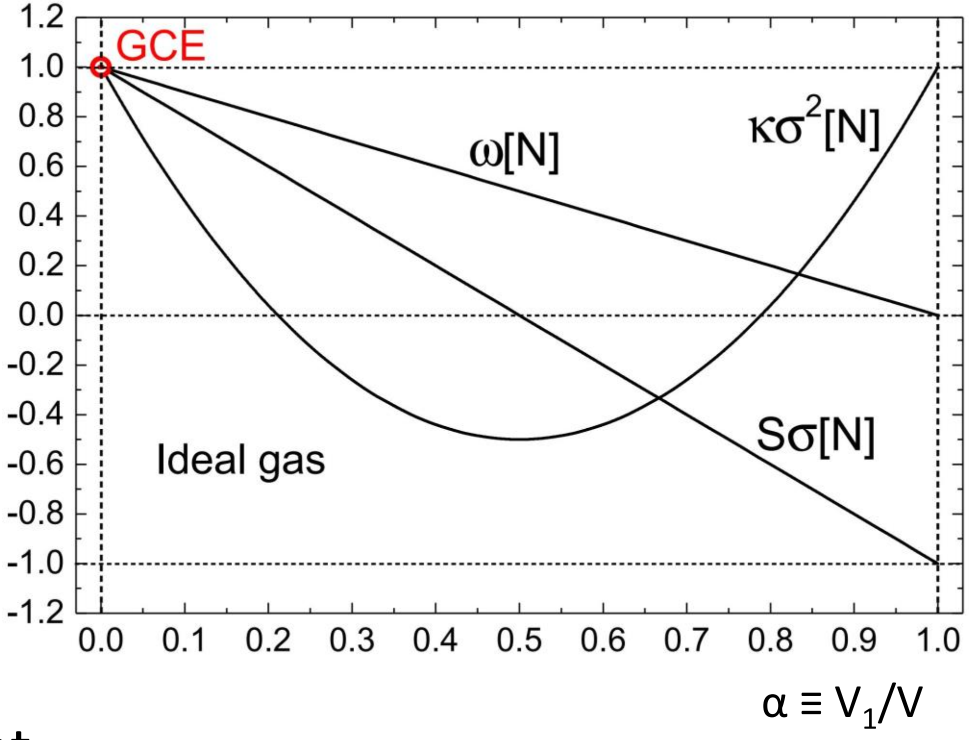
$\xi << V_1 << V_2$

Lattice QCD, Effective models of QCD

Ensembles are not equivalents w.r.t. fluctuations

In HIC the system is too small to satisfy both GCE requirements

$$\xi \ll V_1 \ll V$$



1) $\xi \ll V_1$ (Thermodynamic limit)
ratios of extensive quantities become V-independent
2) $V_1 \sim V$; $\alpha \equiv V_1/V$ $(0 \ll \alpha \ll 1)$

<u>Solution:</u>
There is a simple connection between GCE and Subensemble cumulants in thermodynamic limit

Subensemble acceptance: Full result up to $\boldsymbol{\kappa_6}$

$$\kappa_1[B_1] = \alpha\, VT^3\, \chi_1^B$$

$$P(B_1) \propto Z^{\rm ce}(T, \alpha V, B_1)\, Z^{\rm ce}(T, (1-\alpha)V, B - B_1), \qquad \alpha \equiv V_1/V$$

$$\kappa_2[B_1] = \alpha\, VT^3\, \beta\, \chi_2^B$$

Model-independent

$$\kappa_3[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha)\, \chi_3^B$$

$$\kappa_4[B_1] = \alpha\, VT^3\, \beta\, \left[\chi_4^B - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B\, \chi_4^B}{\chi_2^B}\right]$$

$$\kappa_5[B_1] = \alpha\, VT^3\, \beta\, (1-2\alpha) \left\{[1-2\beta\alpha]\chi_5^B - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B}\right\}$$

$$\kappa_6[B_1] = \alpha\, VT^3\, \beta\, [1 - 5\alpha\beta(1-\alpha\beta)]\, \chi_6^B + 5\, VT^3\, \alpha^2\, \beta^2 \left\{9\alpha\beta \frac{(\chi_3^B)^2\, \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} - 2(1-2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1-3\beta\alpha] \frac{\chi_3^B\, \chi_5^B}{\chi_2^B}\right\}$$

$$\chi_n^B = \frac{\partial^n (p/T^4)}{\partial(\mu_B/T)^n}$$ – grand-canonical susceptibilities

$$\beta = 1 - \alpha$$

[Vovchenko., Savchuk, **R.P.**, Gorenstein, Koch, *PLB*, 20]

Subensemble acceptance: Cumulant ratios

Some common cumulant ratios:

$$\langle B \rangle = \kappa_1, \quad \omega = \frac{\kappa_2}{\kappa_1}, \quad S\sigma = \frac{\kappa_3}{\kappa_2}, \quad \kappa\sigma^2 = \frac{\kappa_4}{\kappa_2}.$$

scaled variance
$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

skewness
$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

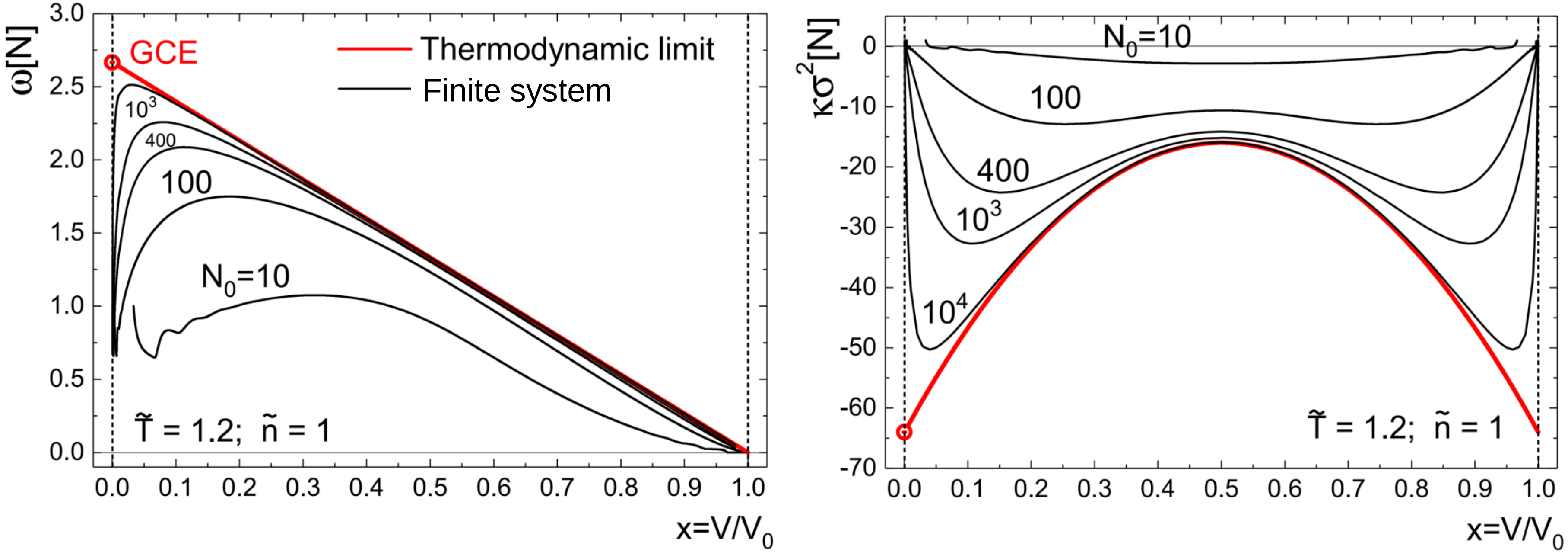
[Vovchenko., Savchuk, **R.P.**, Gorenstein, Koch, *PLB*, 20]

Subensemble acceptance: van der Waals fluid

Calculate cumulants $\kappa_n[N]$ in a subvolume directly from the partition function

$$P(N) \propto Z_{\rm vdW}^{\rm ce}(T, xV_0, N)\, Z_{\rm vdW}^{\rm ce}(T, (1-x)V_0, N_0 - N)$$

and compare with the subensemble acceptance results



Results agree with subsensemble acceptance in thermodynamic limit ($N_0 \to \infty$)

Finite size effects are strong near the critical point: a consequence of large correlation length ξ

[**R.P.**, Savchuk, Gorenstein, Vovchenko, Taradiy, Begun, Satarov, Steinheimer, Stoecker, *PRC*, 20]

[Kuznietsov, Savchuk, Gorenstein, Koch, Vovchenko, *PRC*, 22]

Further developments of the Subensemble Acceptance Method

- Multiple conserved charges (e.g., B,Q,S) [Vovchenko, **R.P.**, Koch, *JHEP*, 20]
- Factorial cumulants [Barej, Bzdak, *PRC*, 22]
- NLO corrections [Barej, Bzdak, *2210.15394*, 22]
- Mixed phase of FOPT???

Mixed phase GCE fluctuations in homogeneous equilibium

Model independent

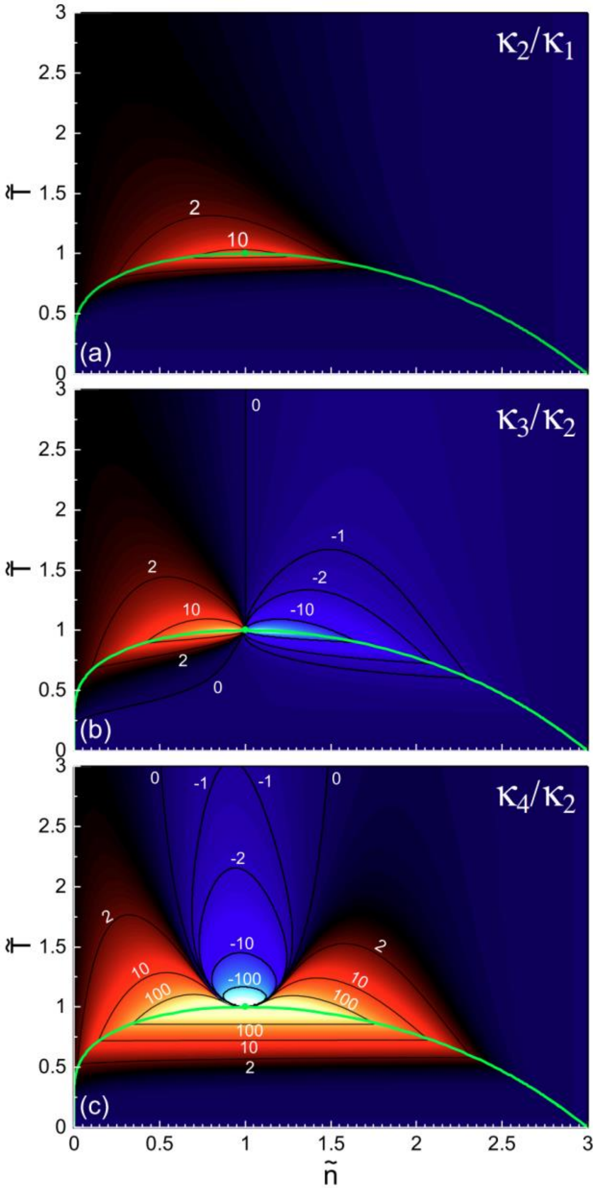
$$V_{liquid} \equiv V_1, \;\; V_{gas} \equiv V_2, \;\; x \equiv V_1/V, \;\; y \equiv 1 - x$$

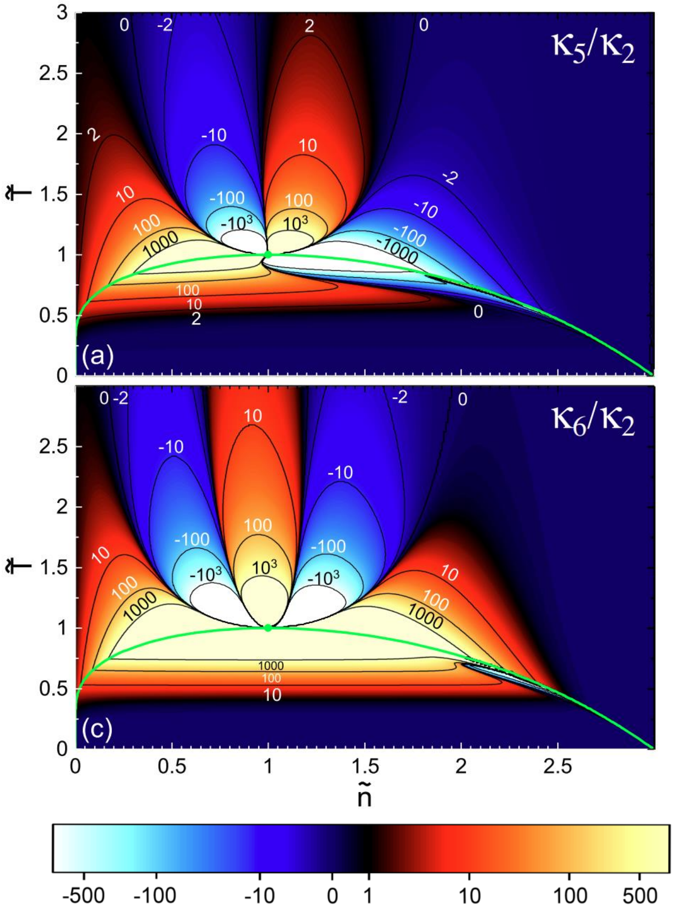
$$\langle B^r \rangle = \langle (B_1 + B_2)^r \rangle = V^r \, \langle (x\rho_1 + y\rho_2)^r \rangle$$

$$\langle \rho_1^l \, \rho_2^m \, x^n \rangle = \langle \rho_1^l \rangle \, \langle \rho_2^m \rangle \, \langle x^n \rangle$$

$$\kappa_1 = \kappa_{1,1} + \kappa_{1,2} \; ,$$

$$\kappa_j = \kappa_{j,1} + \kappa_{j,2} + [(n_1 - n_2)V]^j \; \kappa_{j,x} \; , \quad j \geq 2.$$

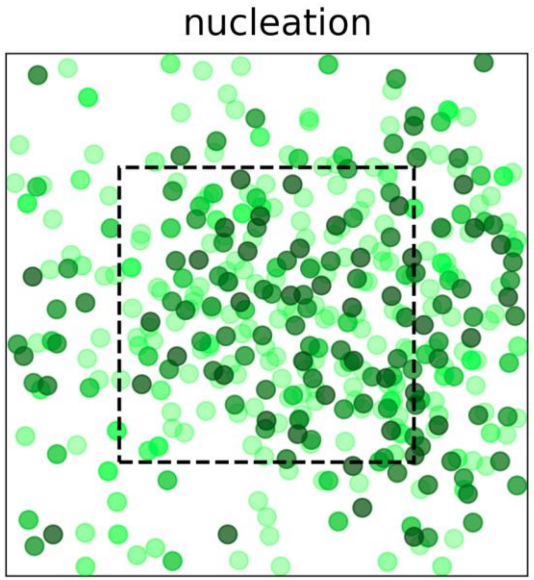




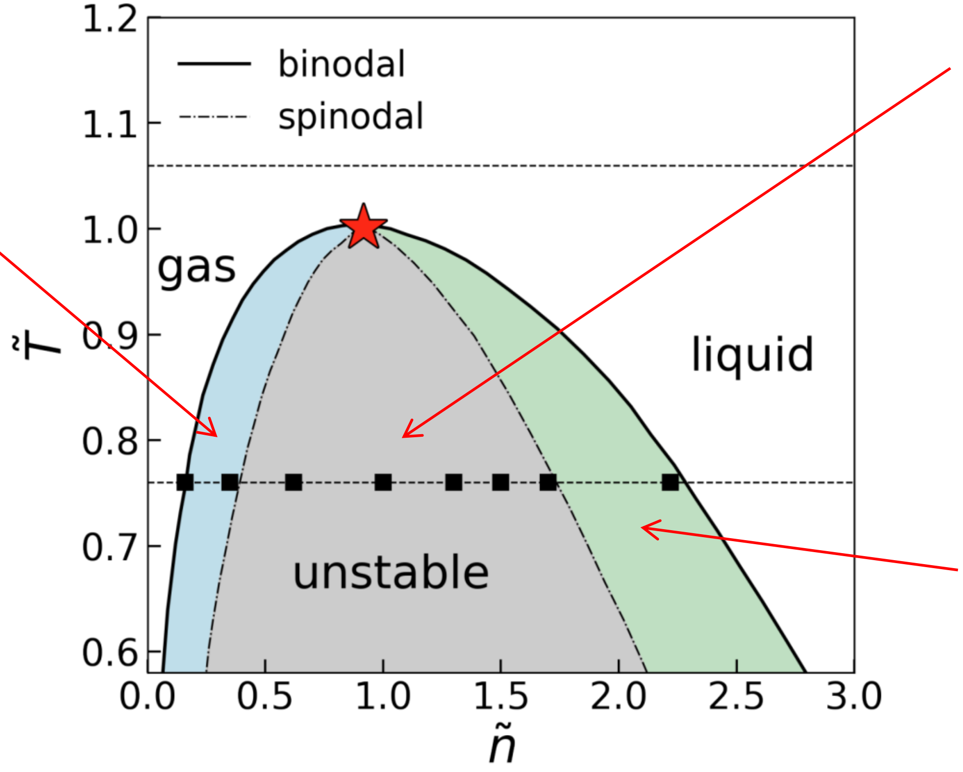
van der Waals model

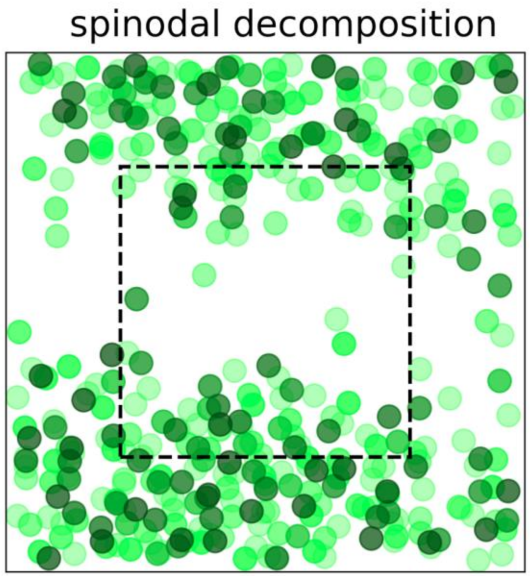
[**R.P.**, Savchuk, Gorenstein, Vovchenko, Stoecker, *PRC*, 21]

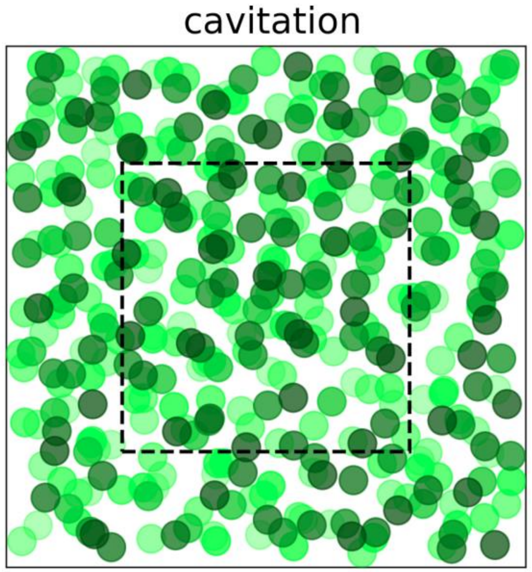
Mixed phase at finite size and time



$$\tilde{p} = p^*/p_c^*, \quad \tilde{n} = n^*/n_c^*, \quad \tilde{T} = T^*/T_c^*.$$







[Evans, da Gamma, *Molecular Phys.*, 79]

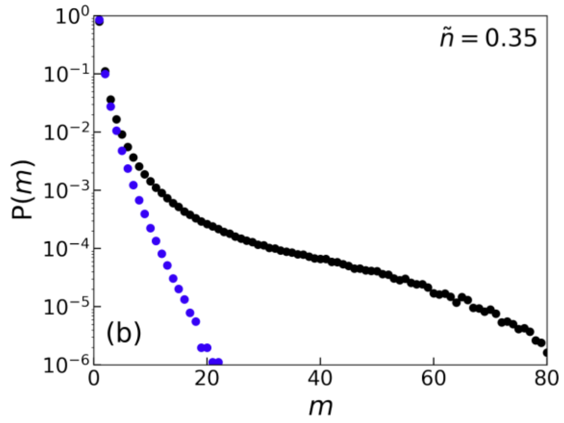
[Kuznietsov, Savchuk, **R.P.**, Vovchenko, Gorenstein, *in progress*]

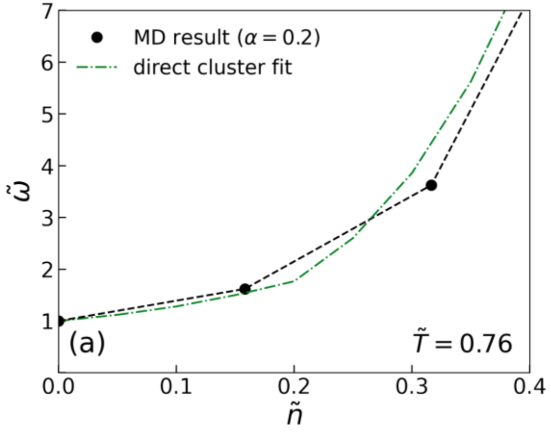
Lennard-Jones fluid simulations

Model of non-interacting clusters in GCE:

$$Z = \prod_{m \geq 1} \exp\left[V (2\pi m_0 m T)^{3/2} g(m,T) \exp\left(\frac{\mu m}{T}\right)\right]$$

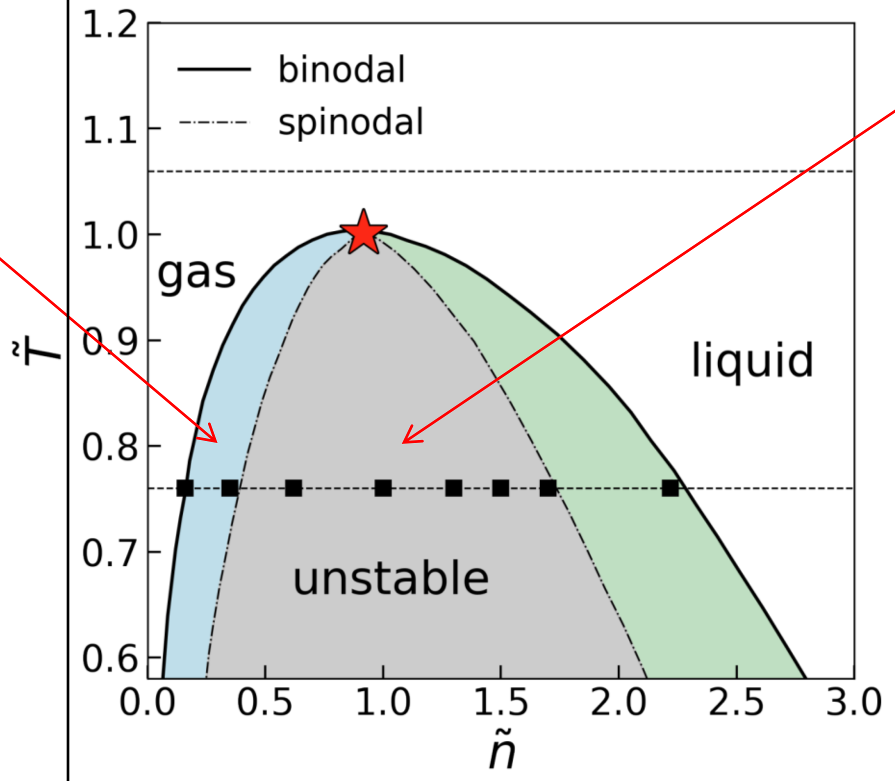
Distribution of clusters from LJ:

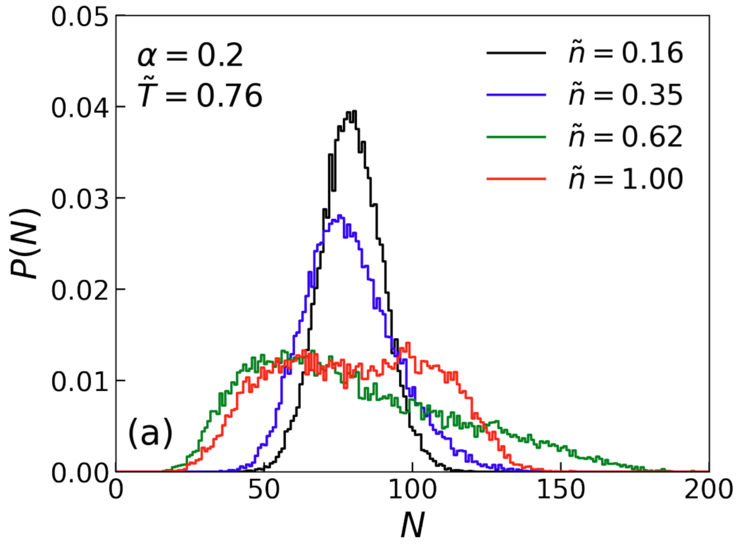


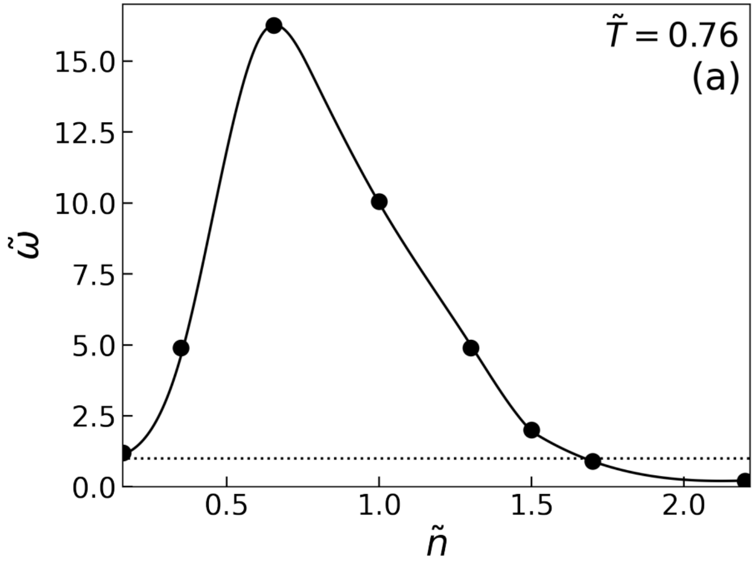


Potential: $V_{\mathrm{LJ}} = 4\varepsilon\left[\left(\frac{\sigma}{r}\right)^6 - \left(\frac{\sigma}{r}\right)^{12}\right]$

$\alpha = 0.2,\ N = 400 \quad \tilde{\omega} = \omega/(1-\alpha)$





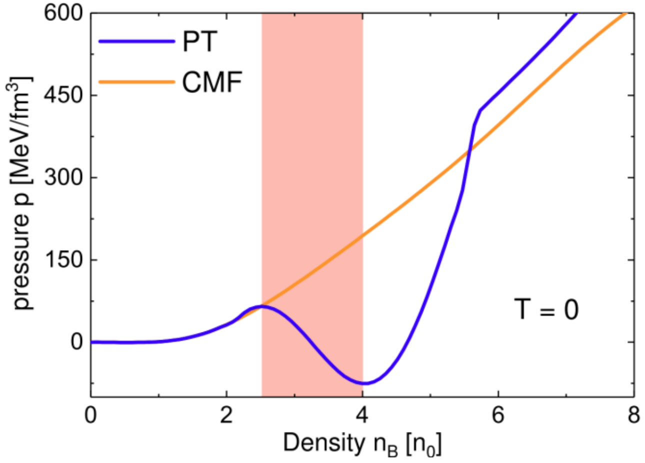


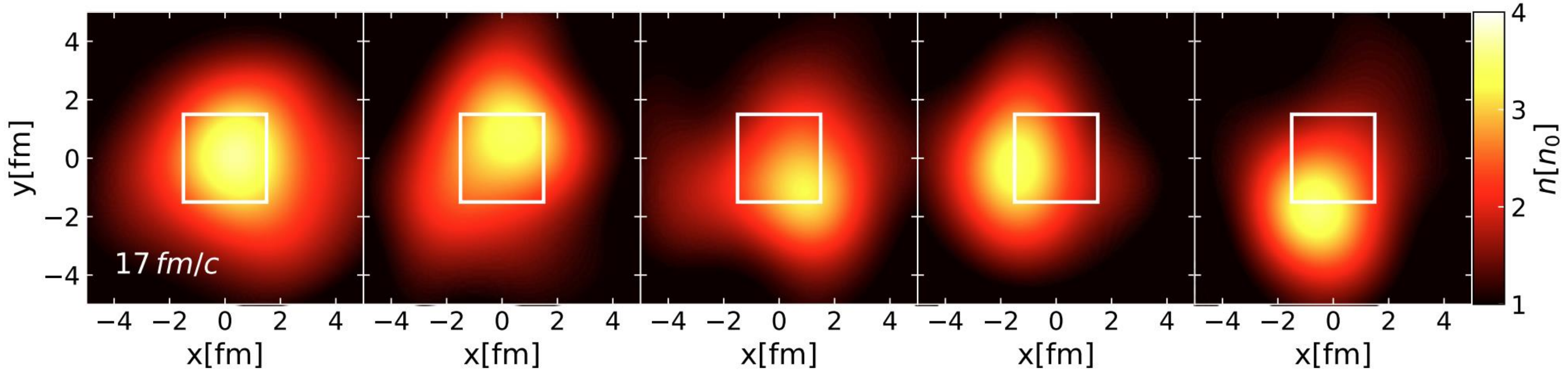
[Kuznietsov, Savchuk, **R.P.**, Vovchenko, Gorenstein, *in progress*]

Application to HIC — UrQMD with Phase Transition

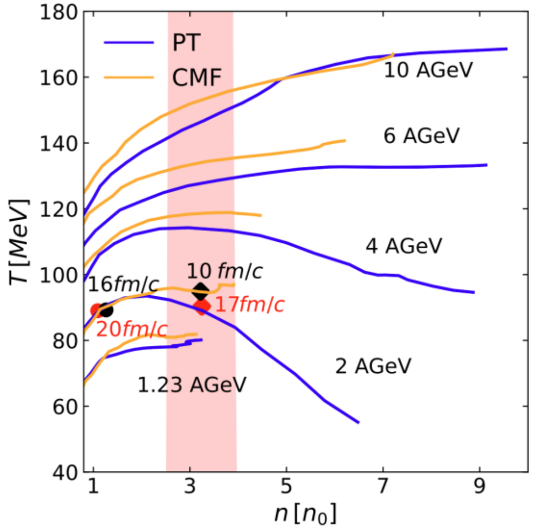
Using [Manjunath Omana Kuttan et al., *EPJ*, 22]

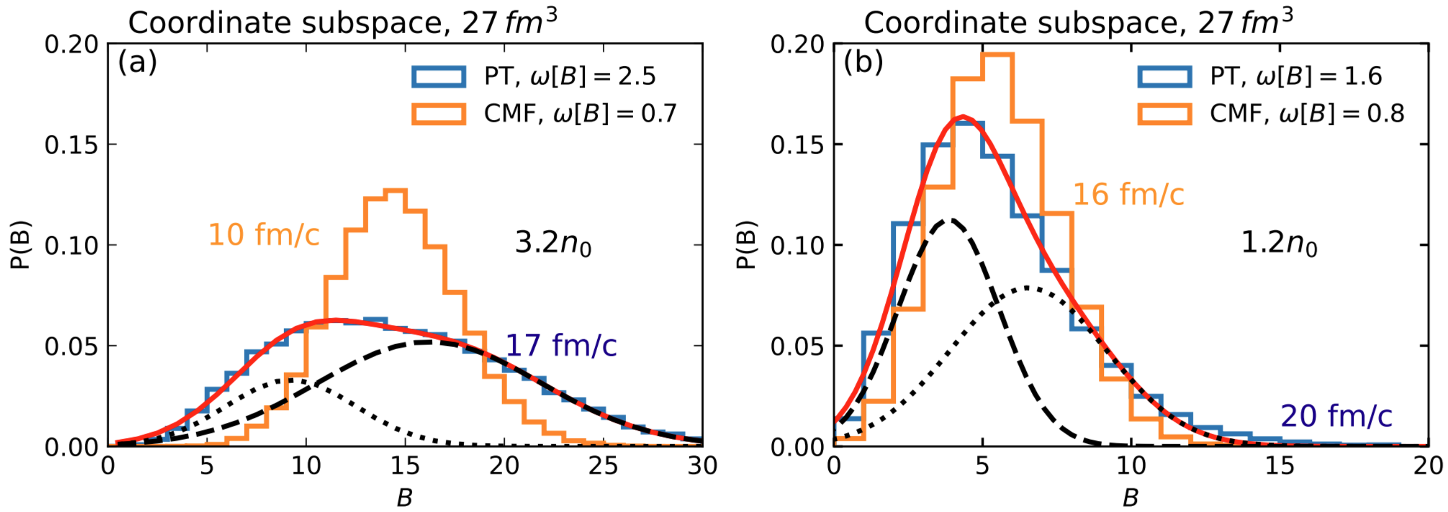
UrQMD-3.5 with Chiral SU(3)-flavor parity-doublet Polyakov-loop quark-hadron mean-field model **(CMF)** potential





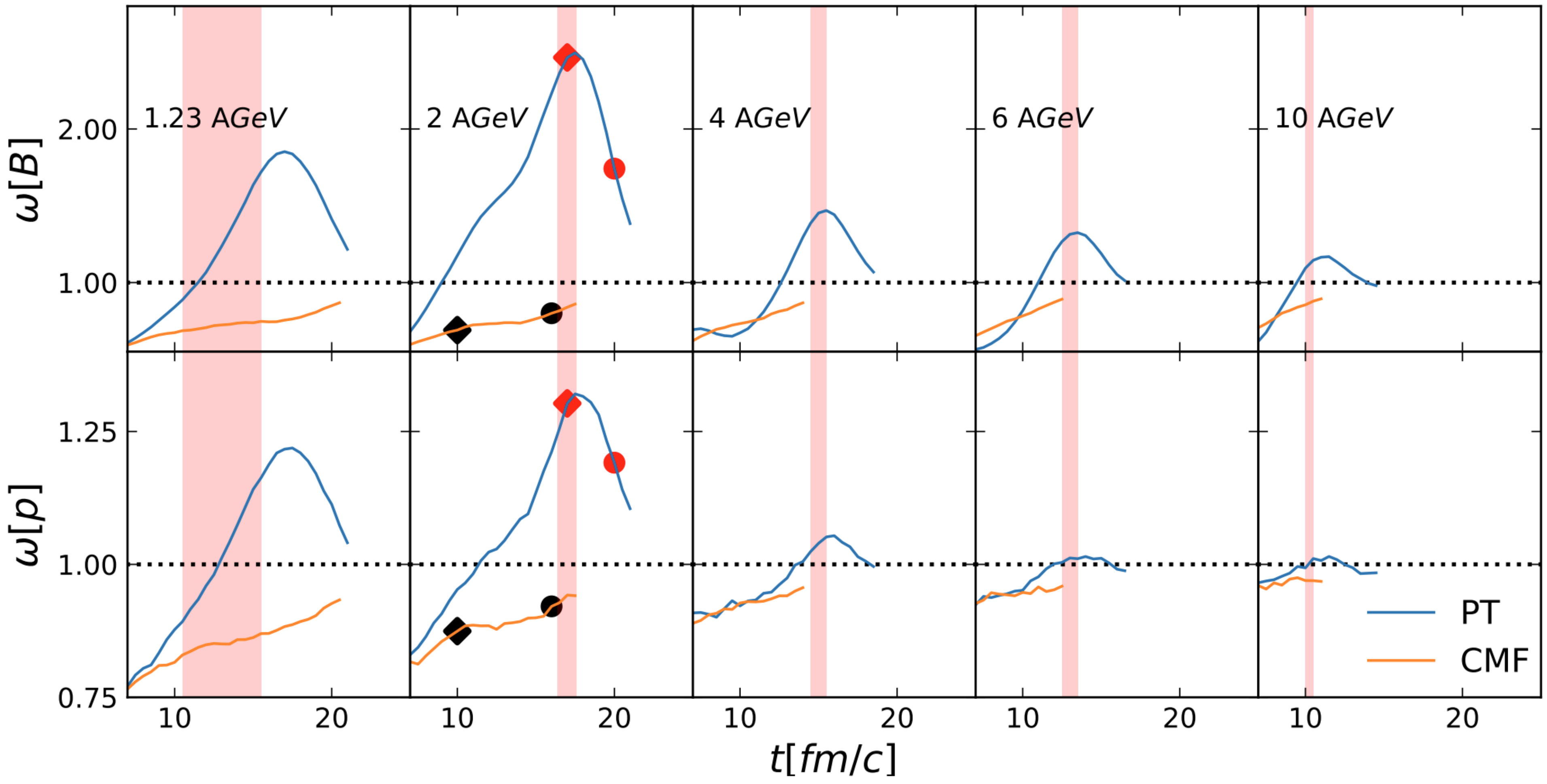
System Trajectories





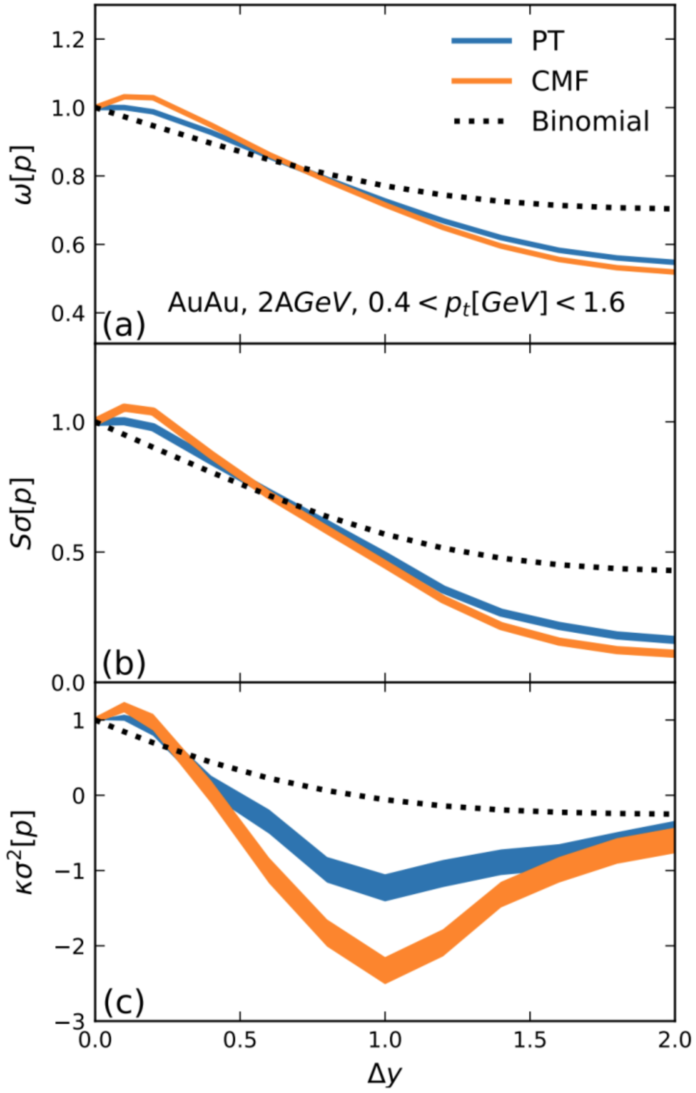
[Savchuk, **R.P.**, Motornenko, Steinheimer, Gorenstein, Vovchenko, *2211.13200*, 22]

Application to HIC — Fluctuations in coordinate space



[Savchuk, **R.P.**, Motornenko, Steinheimer, Gorenstein, Vovchenko, *2211.13200*, 22]

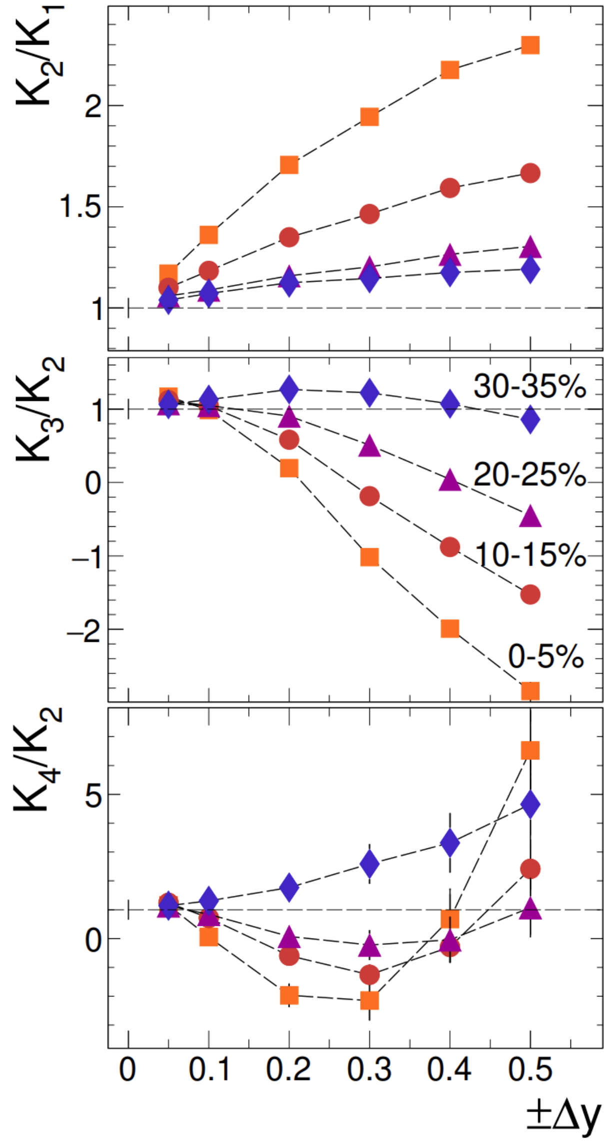
Application to HIC — Fluctuations in momentum space



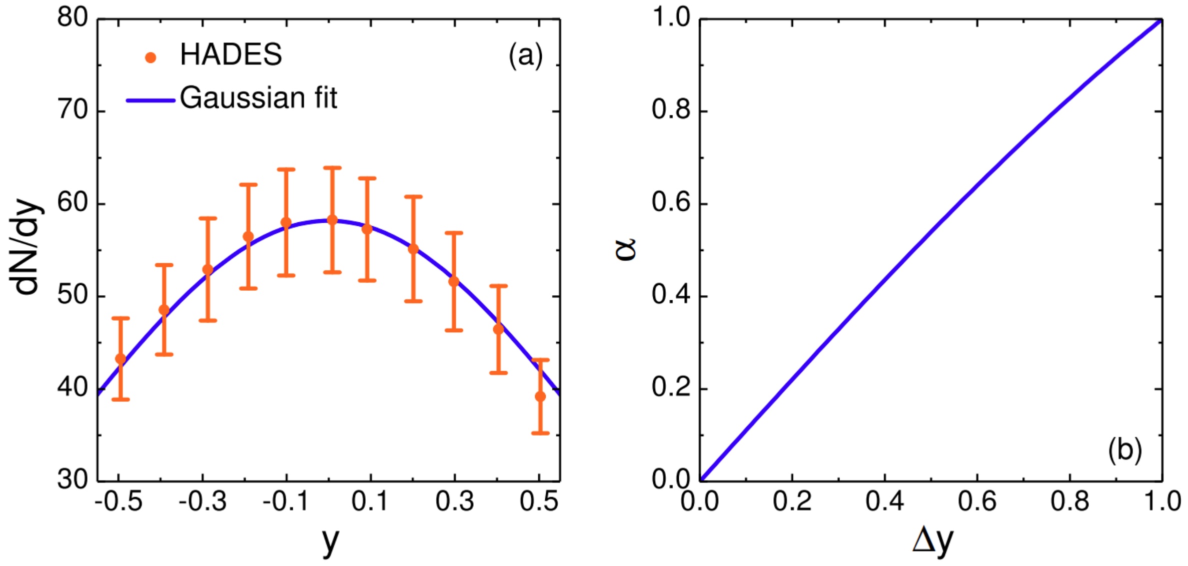
At low collision energy
space-momentum correlations are small,
thus the signal of PT in fluctuations is washed out

[Savchuk, **R.P.**, Motornenko, Steinheimer, Gorenstein, Vovchenko, *2211.13200*, 22]

HADES data on proton number fluctuations at at $\sqrt{s_{NN}}$ = 2.4 GeV



[HADES, *PRC*, 20]



$$\frac{dN}{dy} = C \exp\left[-\frac{y^2}{2a^2}\right]$$

$$\alpha = \frac{\int_{-\Delta y/2}^{\Delta y/2} dy\, dN/dy}{\int_{-1/2}^{1/2} dy\, dN/dy}$$

[Savchuk, **R.P.**, Gorenstein, *PLB*, 22]

HADES data on proton number fluctuations at at $\sqrt{s_{NN}}$ = 2.4 GeV

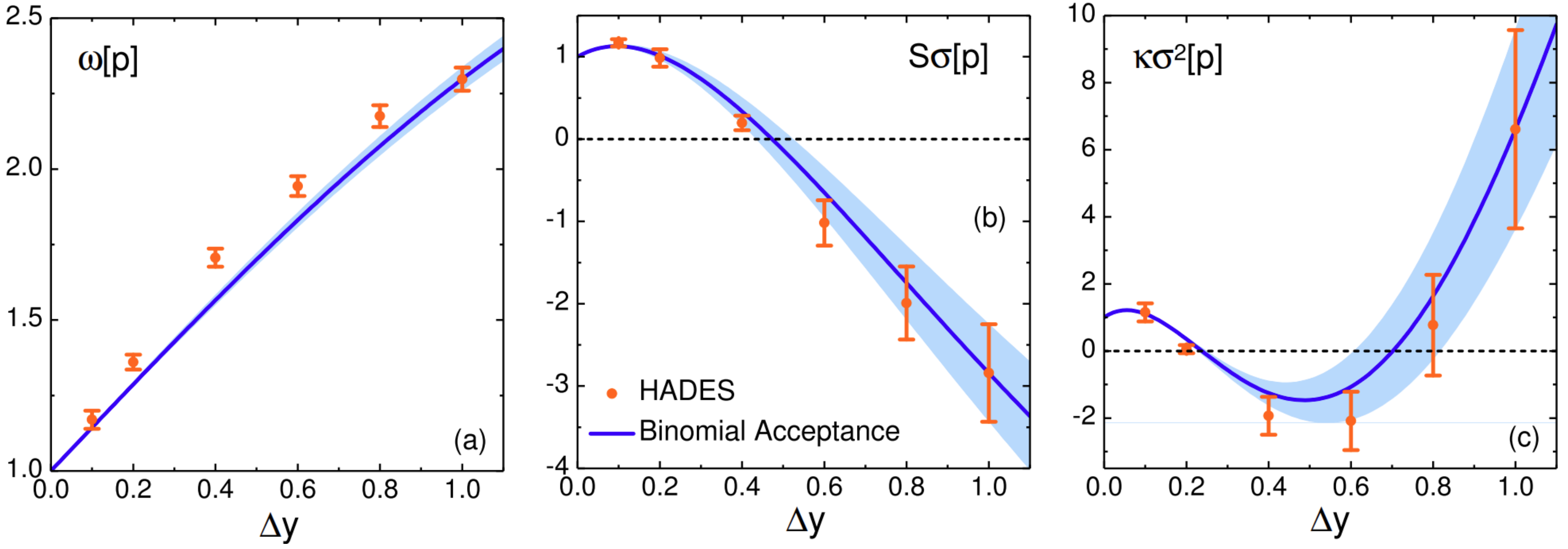
Binomial Acceptance formulas assume uncorrelated emission of protons:

$$p(n,\alpha) = \sum_{N=n}^{\infty} \frac{N!}{n!(N-n)!} \alpha^n (1-\alpha)^{N-n} P(N)$$

$$\omega_\alpha[n] \equiv \frac{\kappa_2[n|\alpha]}{\kappa_1[n|\alpha]} = 1 - \alpha + \alpha\omega[N] ,$$

$$S\sigma_\alpha[n] = \frac{\kappa_3[n|\alpha]}{\kappa_2[n|\alpha]} = \frac{\omega[N]}{\omega_\alpha[n]} \{\alpha^2 S\sigma[N] + 3\alpha(1-\alpha)\} + \frac{1-\alpha}{\omega_\alpha[n]}(1-2\alpha) ,$$

$$\kappa\sigma^2_\alpha[n] = \frac{\kappa_4[n|\alpha]}{\kappa_2[n|\alpha]} = \frac{\omega[N]}{\omega_\alpha[n]} \{\alpha^3 \kappa\sigma^2[N]\} + \frac{\omega[N]}{\omega_\alpha[n]}(1-\alpha)\{6\alpha^2 S\sigma[N] + \alpha(7-11\alpha)\} + \frac{1-\alpha}{\omega_\alpha[n]}\{1-6\alpha(1-\alpha)\} ,$$



Consiquence of picture with uncorrelated protons:

$$\rho(n_1,n_2) \equiv \langle N \rangle \frac{\langle n_1 n_2\rangle - \langle n_1\rangle\langle n_2\rangle}{\langle n_1\rangle\langle n_2\rangle} = \omega[N] - 1$$

[Savchuk, **R.P.**, Gorenstein, *PLB*, 22]

Summary

- Subensemble acceptance formulas allow to compare fluctuations measured in different subensembles and grand canonical calculations. The method requires a sufficiently large system such as created in central HIC and sufficient space-momentum correlations. It is applicable in the metastable region of FOPT.
- Directly in the vicinity of the CP or in the spinodal region the system is “never large enough”. Here different approximations should be used that account for finite size effects.
- The expanding system created in HIC exhibits large fluctuations when crossing the spinodal region of FOPT. This signal survives till the later stages of a collision via memory effect.
- However, at low energies the space-momentum correlation is small and this signal is not transferred to second and third order cumulants measured in momentum subspace.
- This agrees with HADES data on proton number fluctuations at $\sqrt{s_{NN}}$=2.4GeV which are consistent with binomial baseline of non-interacting hadrons.

Thank you for attention!