Fluctuations in the mixed phase of the first order phase transition and nucleus-nucleus collisions

Roman Poberezhnyuk

CPOD 2022

In collaborations with: Volodymyr Vovchenko, Volodymyr Kuznietsov, Oleh Savchuk, Mark Gorenstein, Jan Steinheimer, Horst Stoecker, and others
Brief overview of the Subensemble method for fluctuations in the presence of conservation laws

Application to the fluctuations inside mixed phase of the FOPT

Results for mixed phase fluctuations in the context of HIC

HADES data on proton number fluctuations at $\sqrt{s_{NN}} = 2.4$ GeV
In HIC fluctuations are measured within Subensemble.
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**Subensemble**
- Reservoir (Thermostat) $V_2$
- Observed Subsystem $V_1$
- Correlation length $\xi$

**Grand Canonical Ensemble**
- Reservoir (Thermostat) $V_2$
- Observed Subsystem $V_1$
- Correlation length $\xi$

$\xi \sim V_1 \sim V_2$
Heavy Ion Collisions, Microscopic simulations

$\xi << V_1 << V_2$
Lattice QCD, Effective models of QCD
Ensembles are not equivalents w.r.t. fluctuations

\[ \xi \ll V_1 \ll V \]

1) \( \xi \ll V_1 \) (Thermodynamic limit)
ratios of extensive quantities become \( V \)-independent
2) \( V_1 \sim V; \quad \alpha \equiv V_1/V \quad (0 \ll \alpha \ll 1) \)

Solution:
There is a simple connection between GCE and Subensemble cumulants
in thermodynamic limit
Subensemble acceptance: Full result up to $\kappa_6$

$$\kappa_1[B_1] = \alpha VT^3 \chi_1^B$$

$$\kappa_2[B_1] = \alpha VT^3 \beta \chi_2^B$$

$$\kappa_3[B_1] = \alpha VT^3 \beta (1 - 2\alpha) \chi_3^B$$

$$\kappa_4[B_1] = \alpha VT^3 \beta \left[ \chi_4^B - 3\alpha\beta \frac{(\chi_3^B)^2 + \chi_2^B \chi_4^B}{\chi_2^B} \right]$$

$$\kappa_5[B_1] = \alpha VT^3 \beta (1 - 2\alpha) \left\{ [1 - 2\beta\alpha]\chi_5^B - 10\alpha\beta \frac{\chi_3^B \chi_4^B}{\chi_2^B} \right\}$$

$$\kappa_6[B_1] = \alpha VT^3 \beta [1 - 5\alpha\beta (1 - \alpha\beta)] \chi_6^B + 5 VT^3 \alpha^2 \beta^2 \left\{ 9\alpha\beta \frac{(\chi_3^B)^2 \chi_4^B}{(\chi_2^B)^2} - 3\alpha\beta \frac{(\chi_3^B)^4}{(\chi_2^B)^3} ight\} - 2(1 - 2\alpha)^2 \frac{(\chi_4^B)^2}{\chi_2^B} - 3[1 - 3\beta\alpha] \frac{\chi_3^B \chi_5^B}{\chi_2^B}$$

$$\chi_n^B = \frac{\partial^n(p/T^4)}{\partial(\mu_B/T)^n}$$  -- grand-canonical susceptibilities

$$P(B_1) \propto Z^{ce}(T, \alpha V, B_1) Z^{ce}(T, (1 - \alpha)V, B - B_1), \quad \alpha \equiv V_1/V$$

Model-independent

[Vovchenko., Savchuk, R.P., Gorenstein, Koch, PLB, 20]
Subensemble acceptance: Cumulant ratios

Some common cumulant ratios:

\[ \langle B \rangle = \kappa_1, \quad \omega = \frac{\kappa_2}{\kappa_1}, \quad S \sigma = \frac{\kappa_3}{\kappa_2}, \quad \kappa \sigma^2 = \frac{\kappa_4}{\kappa_2}. \]

scaled variance

\[ \frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1 - \alpha) \frac{\chi_2^B}{\chi_1^B}, \]

skewness

\[ \frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1 - 2\alpha) \frac{\chi_3^B}{\chi_2^B}, \]

kurtosis

\[ \frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1 - 3\alpha \beta) \frac{\chi_4^B}{\chi_2^B} - 3\alpha \beta \left( \frac{\chi_3^B}{\chi_2^B} \right)^2. \]

[Vovchenko., Savchuk, R.P., Gorenstein, Koch, PLB, 20]
Subensemble acceptance: van der Waals fluid

Calculate cumulants $\kappa_n[N]$ in a subvolume directly from the partition function

$$P(N) \propto Z_{vdW}^c(T, xV_0, N) Z_{vdW}^c(T, (1 - x)V_0, N_0 - N)$$

and compare with the subensemble acceptance results

Results agree with subsensemble acceptance in thermodynamic limit ($N_0 \to \infty$)

Finite size effects are strong near the critical point: a consequence of large correlation length $\xi$

[R.P., Savchuk, Gorenstein, Vovchenko, Taradiy, Begun, Satarov, Steinheimer, Stoecker, PRC, 20]
[Kuznetsov, Savchuk, Gorenstein, Koch, Vovchenko, PRC, 22]
Further developments of the Subensemble Acceptance Method

- Multiple conserved charges (e.g., B,Q,S) [Vovchenko, R.P., Koch, JHEP, 20]
- Factorial cumulants [Barej, Bzdak, PRC, 22]
- NLO corrections [Barej, Bzdak, 2210.15394, 22]
- Mixed phase of FOPT???
Mixed phase GCE fluctuations in homogeneous equilibrium

Model independent

\[ V_{\text{liquid}} = V_1, \quad V_{\text{gas}} = V_2, \quad x = V_1/V, \quad y = 1 - x \]

\[ \langle B^r \rangle = \langle (B_1 + B_2)^r \rangle = V^r \langle (x \rho_1 + y \rho_2)^r \rangle \]

\[ \langle \rho_1^l \rho_2^m x^n \rangle = \langle \rho_1^l \rangle \langle \rho_2^m \rangle \langle x^n \rangle \]

\[ \kappa_1 = \kappa_{1,1} + \kappa_{1,2} \]

\[ \kappa_j = \kappa_{j,1} + \kappa_{j,2} + [ (n_1 - n_2) V ]^j \kappa_{j,x} , \quad j \geq 2. \]

\[ \text{van der Waals model} \]

[R.P., Savchuk, Gorenstein, Vovchenko, Stoecker, PRC, 21]
Mixed phase at finite size and time

\[ \tilde{p} = p^*/p^*_c, \quad \tilde{n} = n^*/n^*_c, \quad \tilde{T} = T^*/T^*_c. \]

nucleation

spinodal decomposition

cavitation

[Evans, da Gamma, *Molecular Phys.*, 79]

[Kuznietsov, Savchuk, R.P., Vovchenko, Gorenstein, *in progress*]
Lennard-Jones fluid simulations

Model of non-interacting clusters in GCE:

\[ Z = \prod_{m \geq 1} \exp \left[ V(2\pi m_0 m T)^{3/2} g(m, T) \exp \left( \frac{\mu m}{T} \right) \right] \]

Distribution of clusters from LJ:

Potential: \[ V_{\text{LJ}} = 4\varepsilon \left[ \left( \frac{\sigma}{r} \right)^6 - \left( \frac{\sigma}{r} \right)^{12} \right] \]

\[ \alpha = 0.2, \; N = 400 \quad \tilde{\omega} = \omega/(1 - \alpha) \]

[Kuznietsov, Savchuk, R.P., Vovchenko, Gorenstein, in progress]
Application to HIC — UrQMD with Phase Transition

Using [Manjunath Omana Kuttan et al., *EPJ*, 22]

**UrQMD-3.5 with** Chiral SU(3)-flavor parity-doublet Polyakov-loop quark-hadron mean-field model (CMF) potential

[Savchuk, R.P., Motornenko, Steinheimer, Gorenstein, Vovchenko, 2211.13200, 22]
Application to HIC — Fluctuations in coordinate space

[ Savchuk, R.P., Motornenko, Steinheimer, Gorenstein, Vovchenko, 2211.13200, 22]
Application to HIC — Fluctuations in momentum space

At low collision energy space-momentum correlations are small, thus the signal of PT in fluctuations is washed out.

[Savchuk, R.P., Motornenko, Steinheimer, Gorenstein, Vovchenko, 2211.13200, 22]
HADES data on proton number fluctuations at $\sqrt{s_{NN}} = 2.4$ GeV

[HADES, PRC, 20]

\[
\frac{dN}{dy} = C \exp \left[ -\frac{y^2}{2a^2} \right]
\]

\[
\alpha = \frac{\Delta y/2}{\sqrt{\int_{-1/2}^{1/2} dy \frac{dN}{dy}}}
\]

[Savchuk, R.P., Gorenstein, PLB, 22]
HADES data on proton number fluctuations at $\sqrt{s_{NN}} = 2.4$ GeV

Binomial Acceptance formulas assume uncorrelated emission of protons:

$$p(n, \alpha) = \sum_{N=n}^{\infty} \frac{N!}{n!(N-n)!} \alpha^n(1-\alpha)^{N-n} P(N)$$

$$\omega_\alpha[n] = \frac{\kappa_2[n,\alpha]}{\kappa_1[n,\alpha]} = 1 - \alpha + \alpha \omega[N],$$

$$S\sigma_\alpha[n] = \frac{\kappa_3[n,\alpha]}{\kappa_2[n,\alpha]} = \frac{\omega[N]}{\omega_\alpha[n]} \left\{ \alpha^2 S\sigma[N] + 3\alpha(1-\alpha) \right\} + \frac{1-\alpha}{\omega_\alpha[n]} (1-2\alpha),$$

$$\kappa\sigma^2_{\alpha}[n] = \frac{\kappa_4[n,\alpha]}{\kappa_2[n,\alpha]} = \frac{\omega[N]}{\omega_\alpha[n]} \left\{ \alpha^3 \kappa\sigma^2[N] \right\} + \frac{\omega[N]}{\omega_\alpha[n]} (1-\alpha) \left\{ 6\alpha^2 S\sigma[N] + \alpha(7-11\alpha) \right\} + \frac{1-\alpha}{\omega_\alpha[n]} \left\{ 1 - 6\alpha(1-\alpha) \right\},$$

Consequence of picture with uncorrelated protons:

$$\rho(n_1, n_2) \equiv \frac{\langle N \rangle \langle n_1 n_2 \rangle - \langle n_1 \rangle \langle n_2 \rangle}{\langle n_1 \rangle \langle n_2 \rangle} = \omega[N] - 1$$

[Savchuk, R.P., Gorenstein, PLB, 22]
Subensemble acceptance formulas allow to compare fluctuations measured in different subensembles and grand canonical calculations. The method requires a sufficiently large system such as created in central HIC and sufficient space-momentum correlations. It is applicable in the metastable region of FOPT.

Directly in the vicinity of the CP or in the spinodal region the system is “never large enough”. Here different approximations should be used that account for finite size effects.

The expanding system created in HIC exhibits large fluctuations when crossing the spinodal region of FOPT. This signal survives till the later stages of a collision via memory effect.

However, at low energies the space-momentum correlation is small and this signal is not transferred to second and third order cumulants measured in momentum subspace.

This agrees with HADES data on proton number fluctuations at $\sqrt{s_{NN}} = 2.4\text{GeV}$ which are consistent with binomial baseline of non-interacting hadrons.

Thank you for attention!