Critical Point and Onset of Deconfinement 2022 28 November 2022

Chiral spin symmetry and the QCD phase diagram

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Based on: Glozman, O.P., Pisarski, arXiv:2204.05083
Lowdon, O.P., arXiv:2207.14718

Chiral spin symmetry

Trafo:

Dirac: $\psi \to \psi' = \exp\left(i \frac{\varepsilon^n \Sigma^n}{2} \right) \psi$

Generators:

$\Sigma^n = \{\gamma_k, -i\gamma_5\gamma_k, \gamma_5\} \qquad k = 1, 2, 3, 4$

$[\Sigma^a, \Sigma^b] = 2\mathrm{i}\epsilon^{abc}\Sigma^c \qquad su(2)$

Obviously: $SU(2)_{CS} \supset U(1)_A$

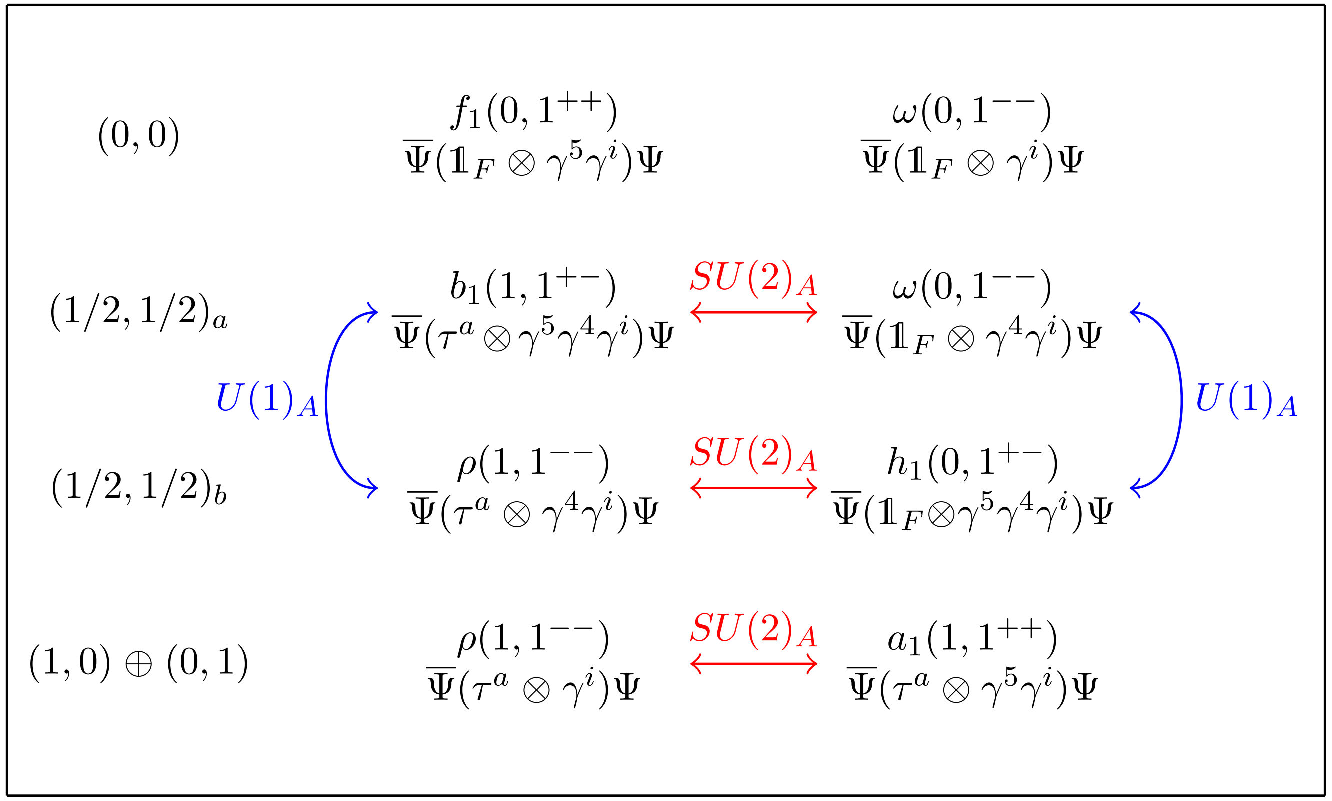
Not so obvious $SU(2)_{CS} \otimes SU(2)_F : \quad \{(\vec{\tau} \otimes \mathbb{1}_D), (\mathbb{1}_F \otimes \vec{\Sigma}_k), (\vec{\tau} \otimes \vec{\Sigma}_k)\}$ 15 generators

$$\Big\downarrow$$

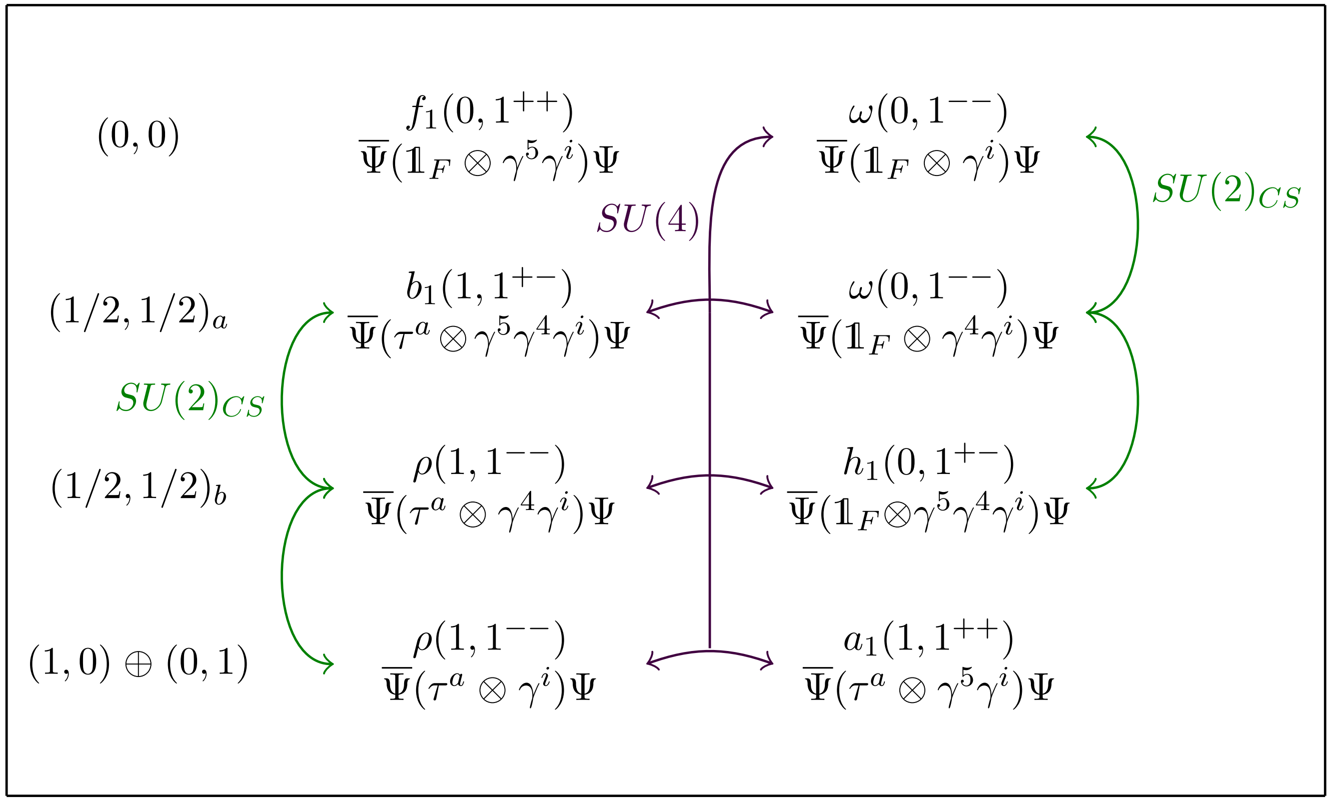
$SU(4) \supset SU(2)_L \times SU(2)_R \times U(1)_A$ [Glozman, EPJA 15]

Relations in meson multiplets

chiral rep. (I, J^{PC}) mesons



$(0,0)$	$f_1(0,1^{++})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^i)\Psi$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^i)\Psi$
$(1/2,1/2)_a$	$b_1(1,1^{+-})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^4\gamma^i)\Psi$	$\omega(0,1^{--})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^4\gamma^i)\Psi$
$(1/2,1/2)_b$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^4\gamma^i)\Psi$	$h_1(0,1^{+-})$ $\overline{\Psi}(\mathbb{1}_F \otimes \gamma^5\gamma^4\gamma^i)\Psi$
$(1,0) \oplus (0,1)$	$\rho(1,1^{--})$ $\overline{\Psi}(\tau^a \otimes \gamma^i)\Psi$	$a_1(1,1^{++})$ $\overline{\Psi}(\tau^a \otimes \gamma^5\gamma^i)\Psi$



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chiral symmetry

CS symmetry

[Rohrhofer et al., PLB 20]

[Glozman, Pak, PRD 15]

Emergent CS symmetry: where does it come from?

QCD quark action, chiral limit: $\bar{\psi}\gamma^{\mu}D_{\mu}\psi = \underset{\substack{\uparrow \\ \text{CS invariant}}}{\bar{\psi}\gamma^{0}D_{0}\psi} + \underset{\substack{\uparrow \\ \text{breaks CS}}}{\bar{\psi}\gamma^{i}D_{i}\psi}$

$[\Sigma^{a}, \gamma^{0}\gamma^{0}] = 0, \; [\Sigma^{a}, \gamma^{0}\gamma^{i}] \neq 0$

- The classical QCD action in the chiral limit is not CS symmetric!
- The free quark action in the chiral limit is not CS symmetric!

Quark gluon interactions:

colour-electric

$\bar{\psi}\gamma_{0}T^{a}\psi \; A_{0}^{a}$

CS invariant

colour-magnetic

$\bar{\psi}\gamma_{i}T^{a}\psi \; A_{i}^{a}$

breaks CS

Necessary condition for approximate CS symmetry:

Quantum effective action Γ_{k} dominated by colour-electric interactions!

Spatial and temporal correlators at finite T

Chiral symmetry restoration at finite T

$$C_\Gamma(\tau, \mathbf{x}) = \langle O_\Gamma^\dagger(\tau, \mathbf{x}) O_\Gamma(0, \mathbf{0})\rangle$$

$$C_\Gamma(\tau, \boldsymbol{p}) = \int_0^\infty \frac{d\omega}{2\pi}\, K(\tau,\omega)\rho_\Gamma(\omega, \boldsymbol{p}) \ ,$$

$$K(\tau,\omega) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \ .$$

$$C_\Gamma^s(z) = \sum_{x,y,\tau} C_\Gamma(\tau, \boldsymbol{x})$$

$$C_\Gamma^\tau(\tau) = \sum_{x,y,z} C_\Gamma(\tau, \boldsymbol{x})$$

Spectral function contains all information about degrees of freedom

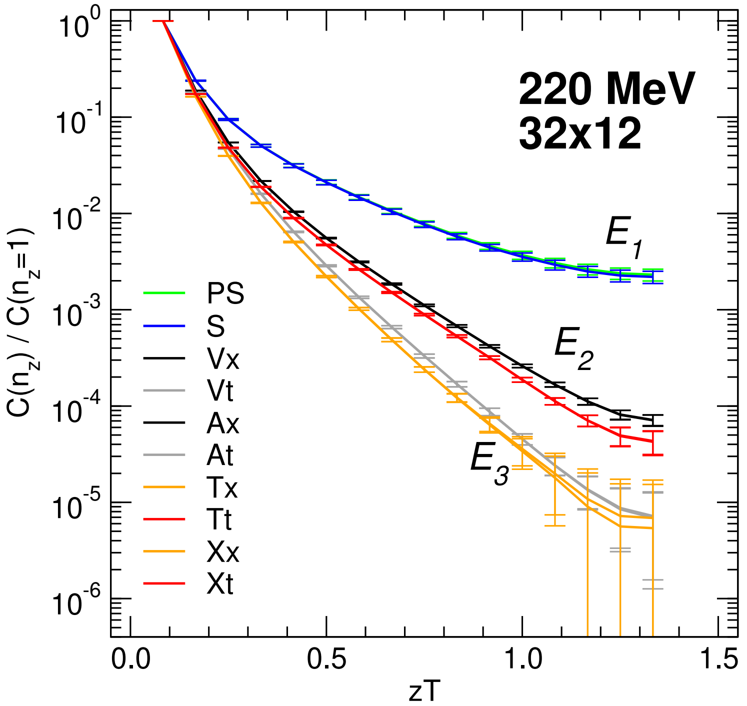
Inversion from discrete data ill-posed problem

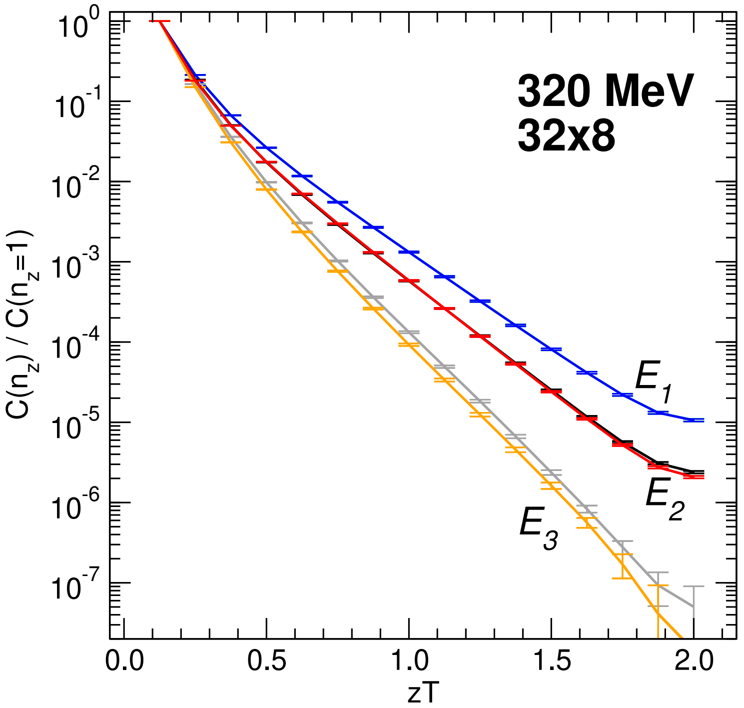
Finite T has preferred reference frame: colour-electric and colour magnetic distinguishable!

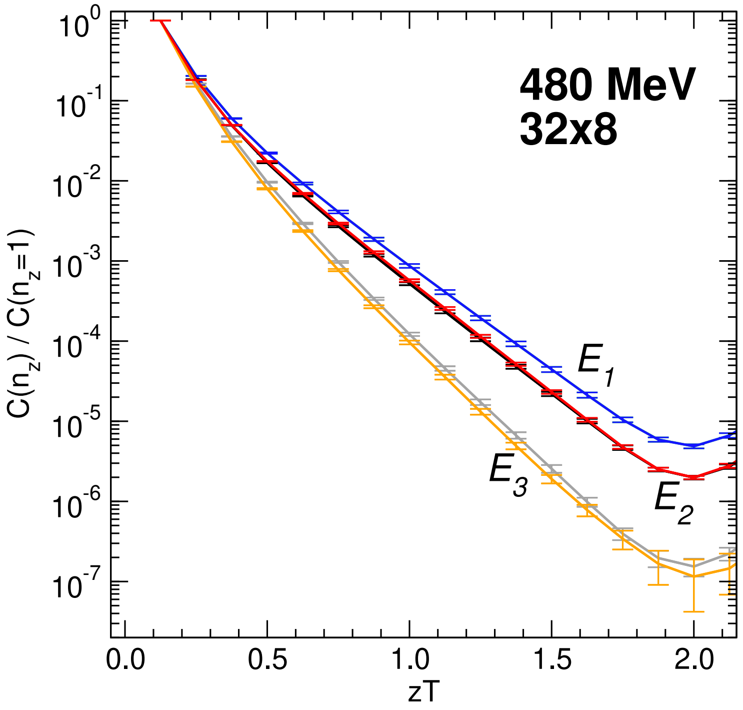
Spatial correlators at finite T

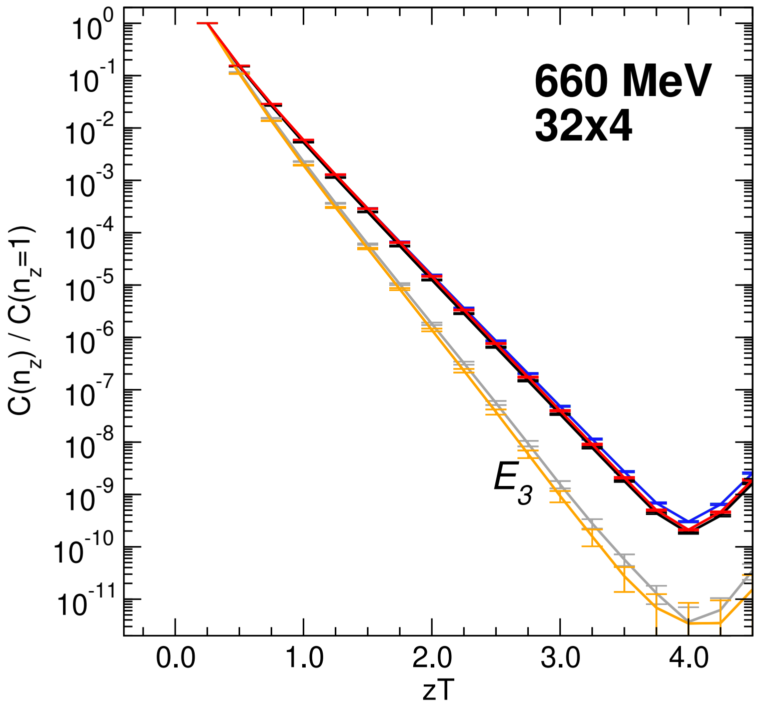
Multiplet structure

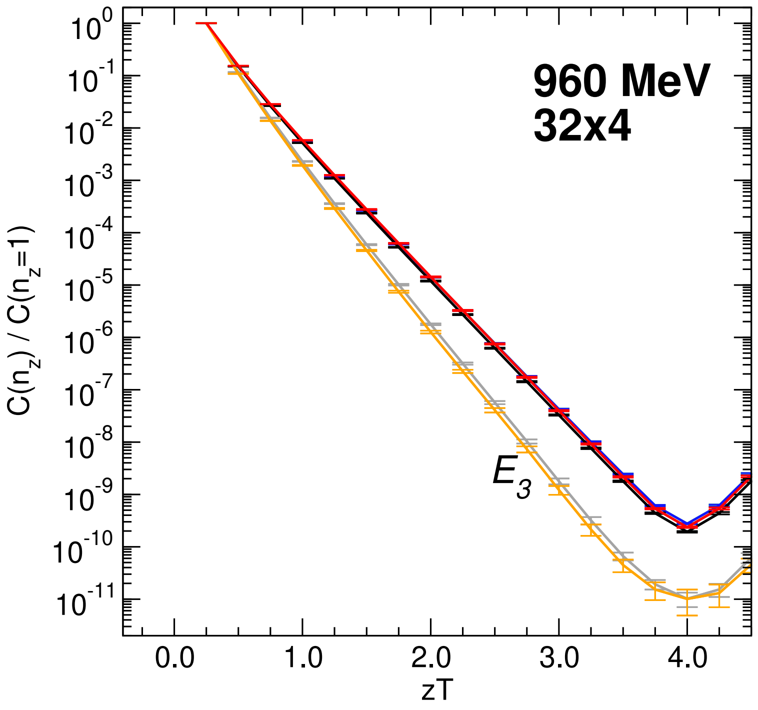
$$\begin{array}{lll} E_1: & PS \leftrightarrow S\ , & U(1)_A \\ E_2: & V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x\ , & SU(4) \\ E_3: & V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t\ . & SU(2)_L \times SU(2)_R \times U(1)_A \end{array}$$











JLQCD domain wall fermions

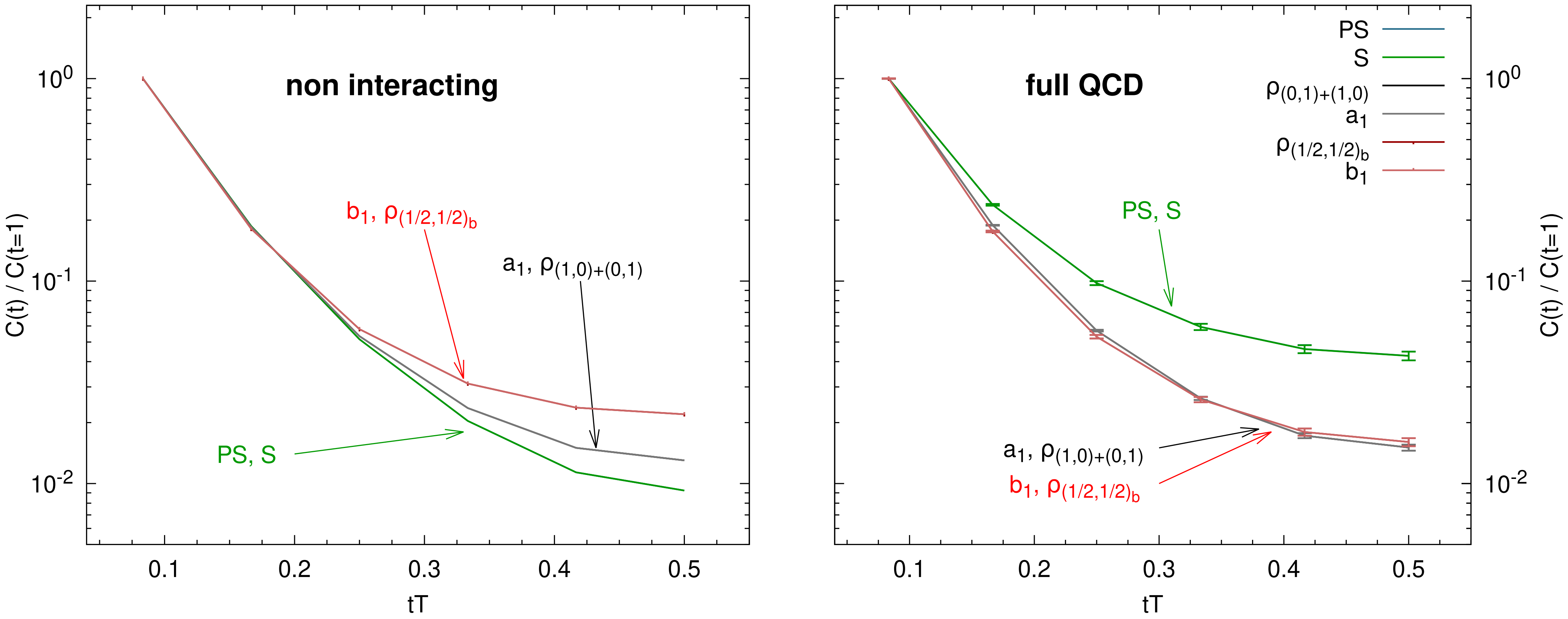
$$N_f = 2, a \leq 0.1 \text{ fm}$$

physical masses

[Rohrhofer et al., PRD19]

Temporal correlators at finite T

JLQCD domain wall fermion configurations [Rohrhofer et al., PLB 20]



$$48^3 \times 12 \qquad T = 220\text{MeV}\ (1.2T_c) \qquad (a = 0.075\ \text{fm})$$

Three temperature regimes of QCD

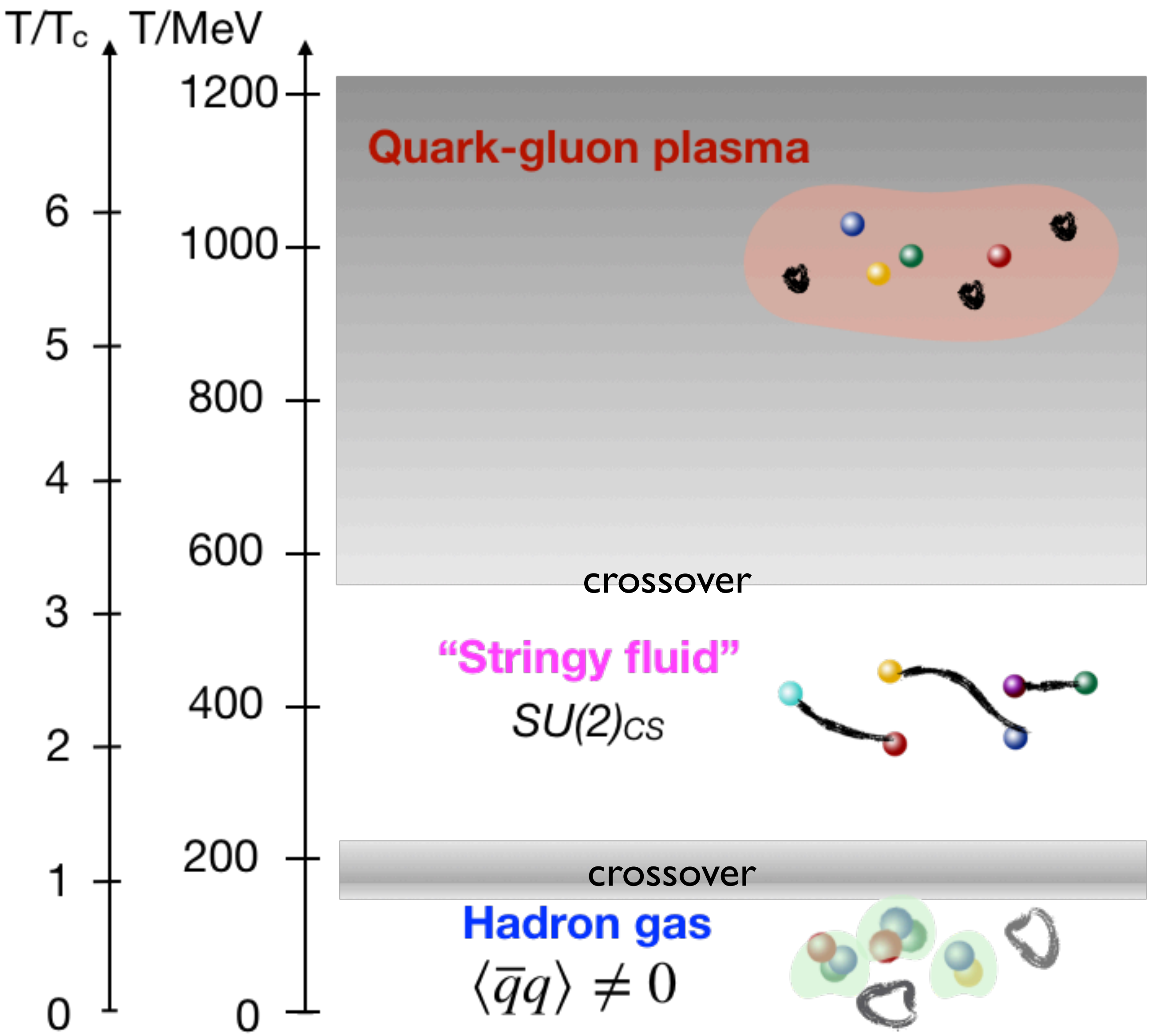
Symmetries (verified):

Degrees of freedom (to be verified):

Chiral symmetry (approximate)

Chiral spin symmetry (approximate)

Chiral symmetry broken



Rohrhofer et al., Phys. Rev. D100 (2019)

Check well-studied observables: screening masses

$$C^s_\Gamma(z) = \sum_{x,y,\tau} C_\Gamma(\tau, \boldsymbol{x}) \quad \xrightarrow{z\to\infty} \text{const. } e^{-m_{scr} z}$$

Directly related to the partition function and equation of state

by transfer matrices: $T = e^{-aH}, T_z = e^{-aH_z}$

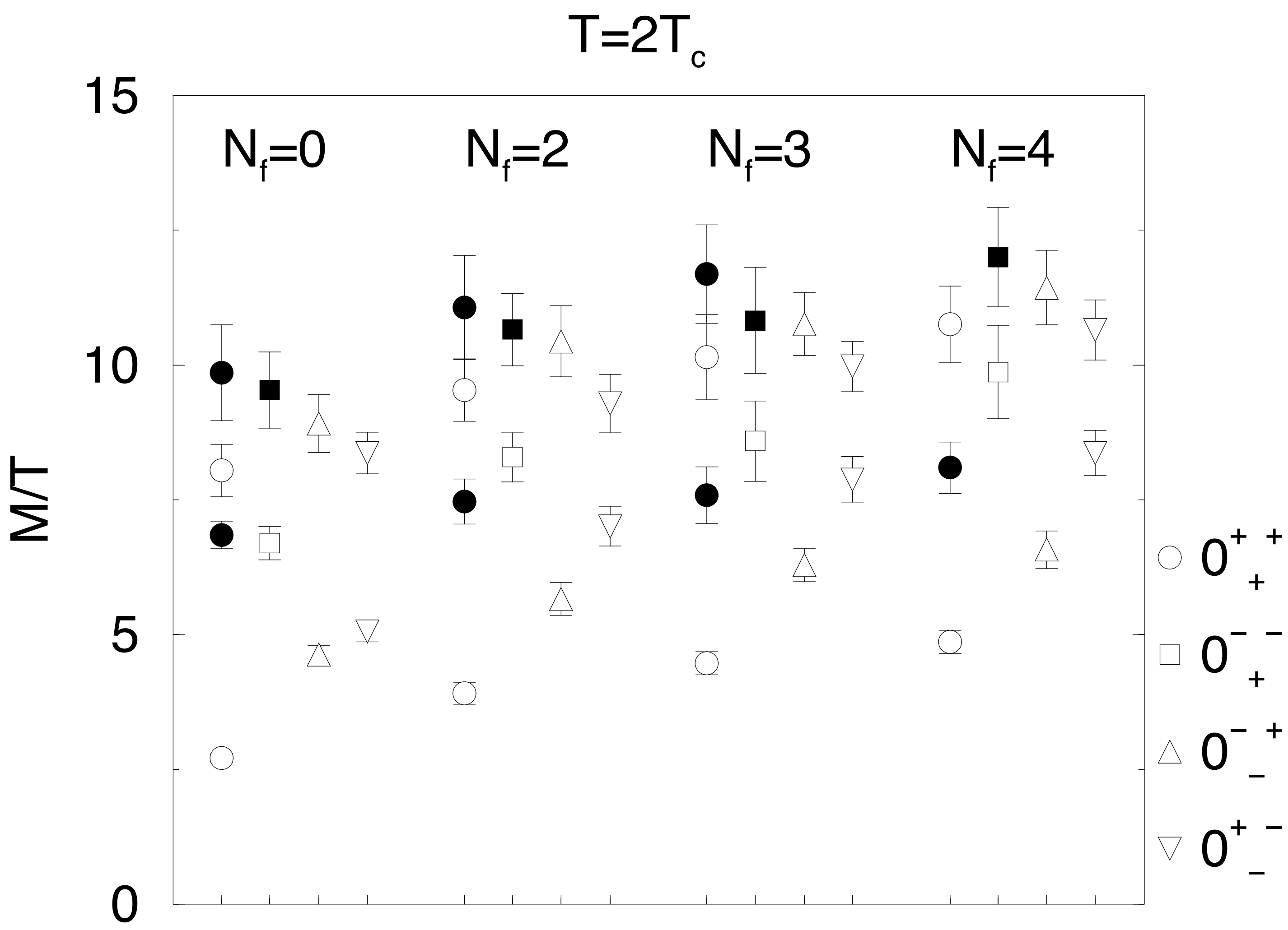
$$e^{pV/T} = Z = \mathrm{Tr}(e^{-aHN_\tau}) = \sum_n e^{-aE_n N_\tau}$$
$$= \mathrm{Tr}(e^{-aH_z N_z}) = \sum_{n_z} e^{-aE_{n_z} N_z}$$

Screening masses: eigenvalues of H_z

For T=0 equivalent to eigenvalues of H, for $T \neq 0$ "finite size effect"

Colour-electric vs. colour magnetic fields

Scales at finite T:	$\text{Matsubara} \sim \pi T,$	$\text{hard modes, fermions}$	QCD
	$\text{Debye/electric} \sim gT,$	A_0	EQCD
	$\text{magnetic} \sim g^2 T,$	A_i	MQCD



[Hart, Laine, O.P., NPB 00]

Screening masses in EQCD

● ops. constructed only from A_i

○ ops. containing also A_0

Colour-electric fields dynamically dominant, perturbative ordering reversed!

Meson screening masses at high temperatures

[Dalla Brida et al., JHEP 22]

Nf=3, T=1 GeV -160 GeV

Highly non-trivial technically:
shifted b.c. + step-scaling techniques
(Alpha-Collaboration)

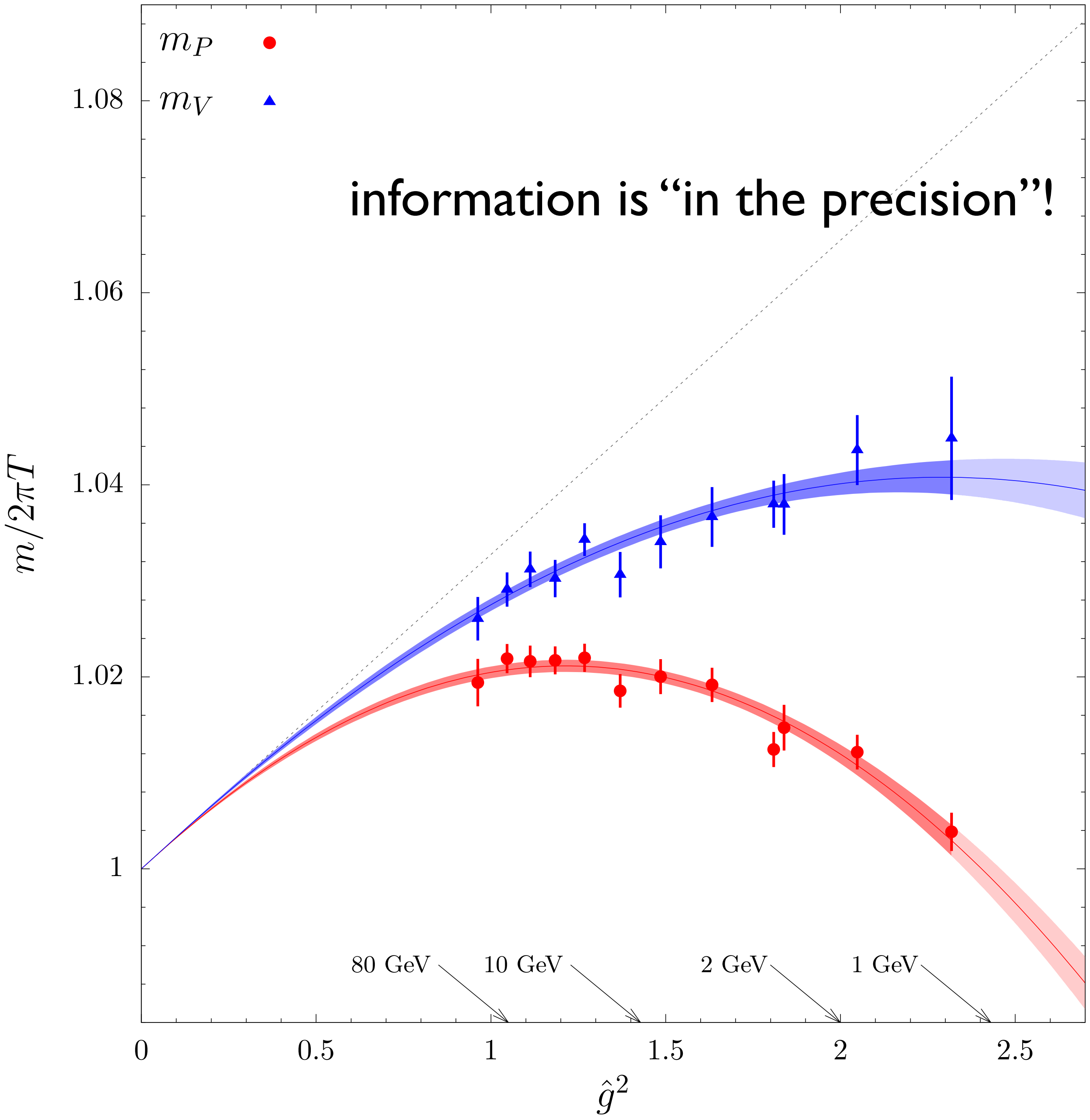
$$\frac{m_{PS}}{2\pi T} = 1 + p_2\, \hat{g}^2(T) + p_3\, \hat{g}^3(T) + p_4\, \hat{g}^4(T)$$

$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\, \hat{g}^4(T)$$

$$p_2 = 0.032739961$$

[Laine, Vepsäläinen., JHEP 04]

p_3, p_4, s_4 fitted, excellent $\chi^2_{\rm dof}$



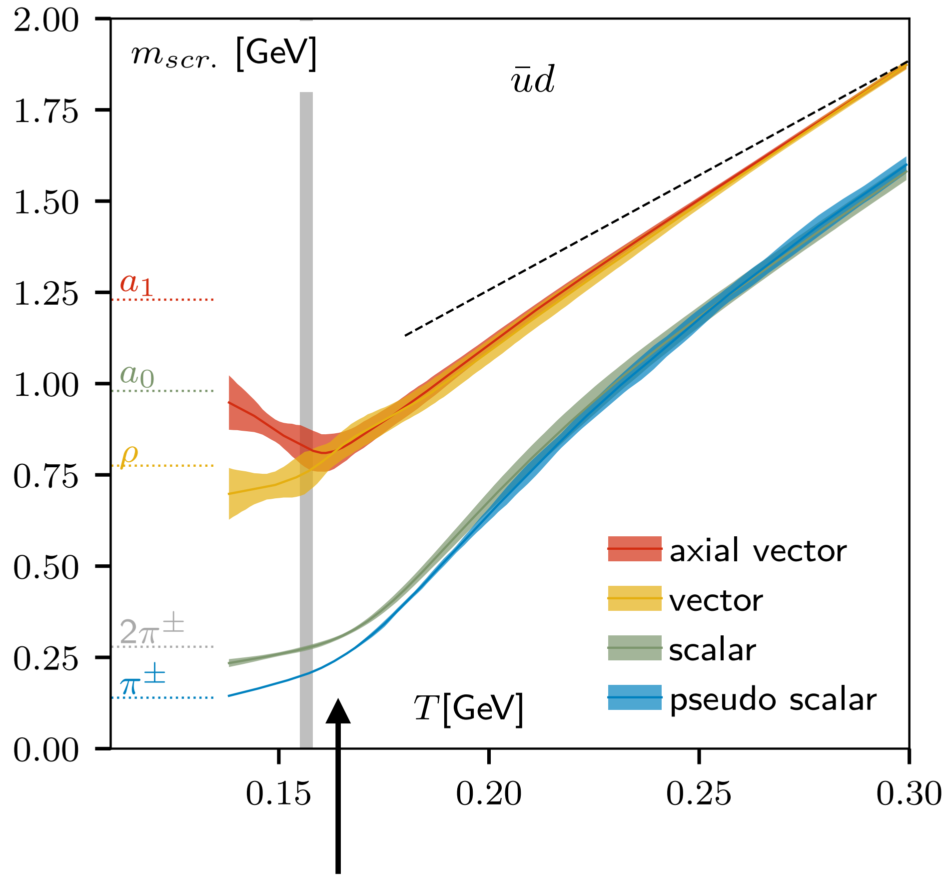
information is “in the precision”!

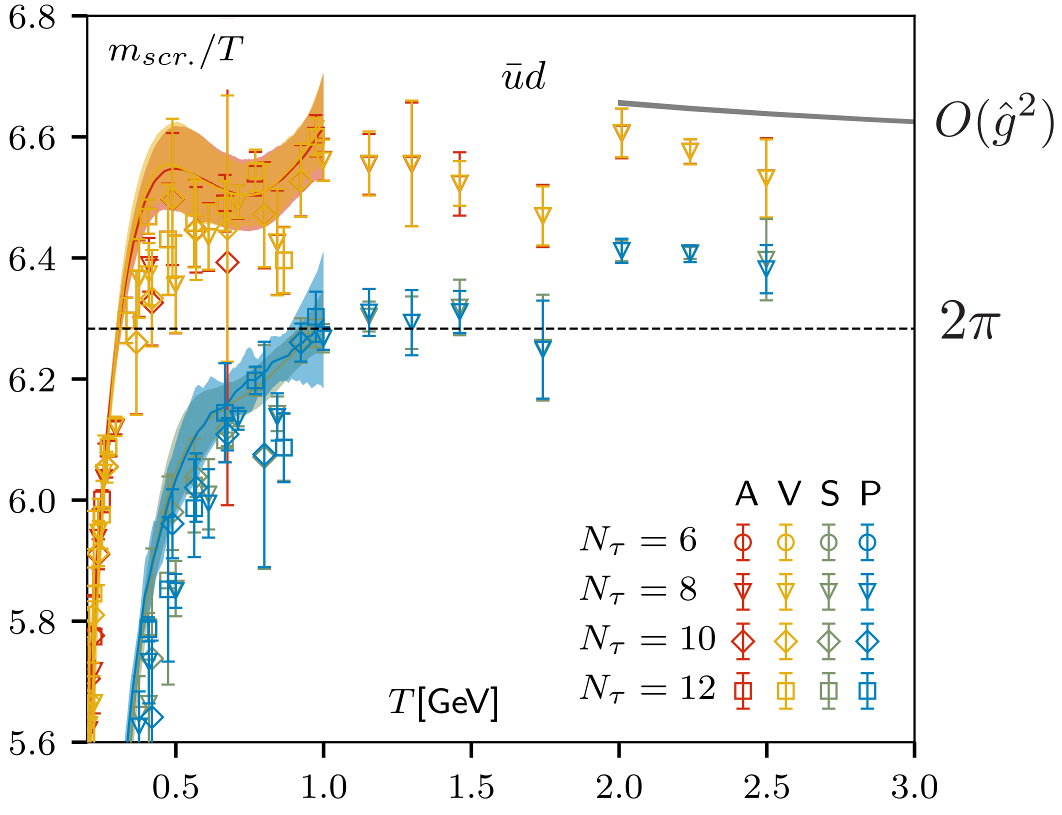
$$\frac{1}{\hat{g}^2(T)} \equiv \frac{9}{8\pi^2} \ln \frac{2\pi T}{\Lambda_{\overline{\rm MS}}} + \frac{4}{9\pi^2} \ln \left(2 \ln \frac{2\pi T}{\Lambda_{\overline{\rm MS}}} \right)$$

Meson screening masses

HotQCD, Phys. Rev. D100 (2019)

HISQ, physical point, continuum extrapolated





Chiral symmetry restoration

Drastic change: "vertical" - "horizontal"

Heavy chiral partners "come down" in all flavour combinations

➡ pressure increases

Resummed pert. theory:

$$\frac{m_{PS}}{2\pi T} = 1 + p_2\,\hat{g}^2(T) + p_3\,\hat{g}^3(T) + p_4\,\hat{g}^4(T)\;,$$

$$\frac{m_V}{2\pi T} = \frac{m_{PS}}{2\pi T} + s_4\,\hat{g}^4(T)\;,$$

[Laine, Vepsäläinen., JHEP 04]

[Dalla Brida et al., JHEP 22]

Cannot describe the "bend"

Change of dynamics at $T \approx 0.5$ GeV in 12 lightest meson channels! CS symmetry!

Finite density

- Finite density: $\mu\bar{\psi}\gamma_0\psi$ is CS invariant; regime must continue to finite density

- Upper "boundary" of CS band: screening radius at "bend" (one possible def.)

$$r_V^{-1} \equiv m_V(\mu_B = 0, T_\mathrm{s}) = C_0 T_\mathrm{s} \quad \Rightarrow \quad \begin{array}{ll} T < T_\mathrm{s} & \text{unscreened} \\ T > T_\mathrm{s} & \text{screened} \end{array}$$

- For small μ_B, line of constant r_V^{-1}

$$\frac{m_V(\mu_B)}{T} = C_0 + C_2 \left(\frac{\mu_B}{T}\right)^2 + \ldots \quad \Rightarrow \quad \frac{dT_\mathrm{s}}{d\mu_B} = -\frac{2C_2}{C_0}\frac{\mu_B}{T} - \frac{2C_2^2}{C_0^2}\left(\frac{\mu_B}{T}\right)^3 + \ldots$$

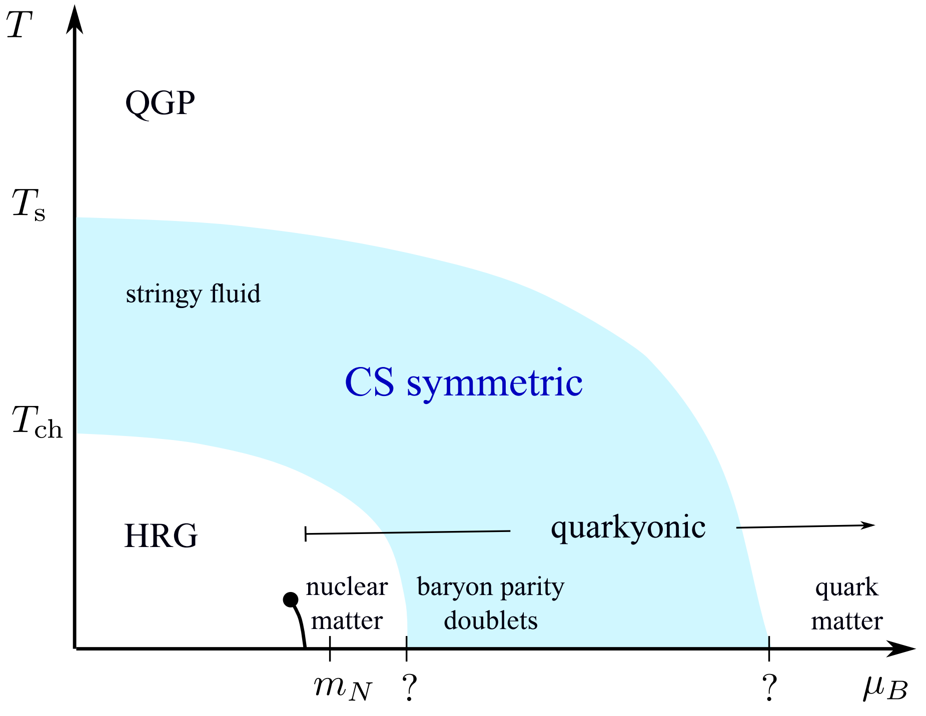
$$C_2 > 0$$

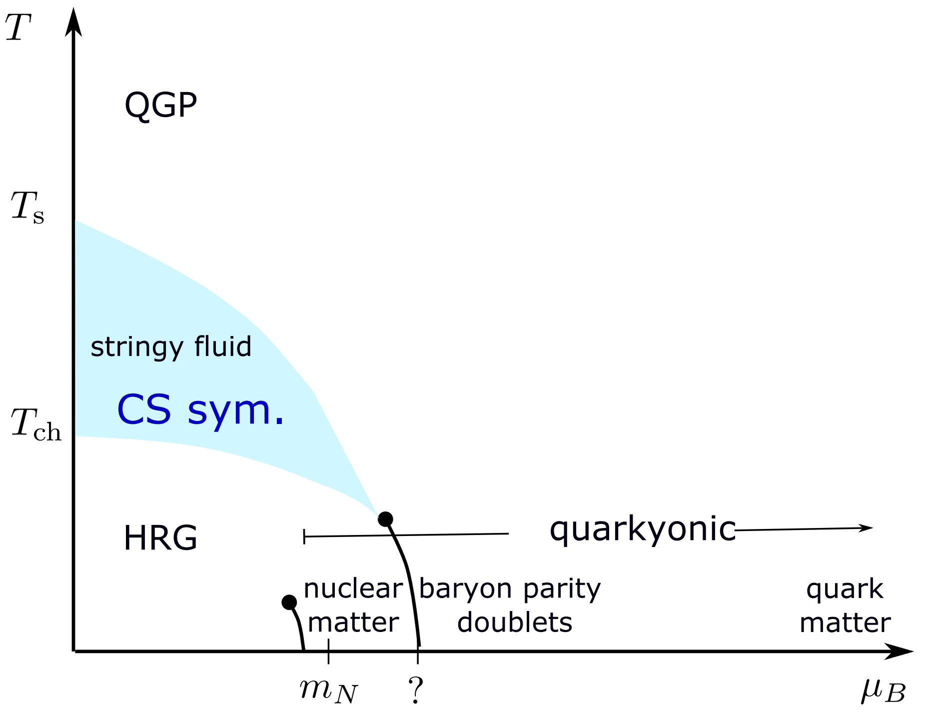
- Lower "boundary" of CS band: (this is a lower bound only)

$$\frac{T_\mathrm{pc}(\mu_B)}{T_\mathrm{pc}(0)} = 1 - 0.016(5)\left(\frac{\mu_B}{T_\mathrm{pc}(0)}\right)^2 + \ldots \qquad \approx \frac{T_\mathrm{ch}(\mu_B)}{T_\mathrm{ch}(0)}$$

Separate order parameters for $SU(2)_A, U(1)_A, SU(4)$?

Possibilities for the QCD phase diagram



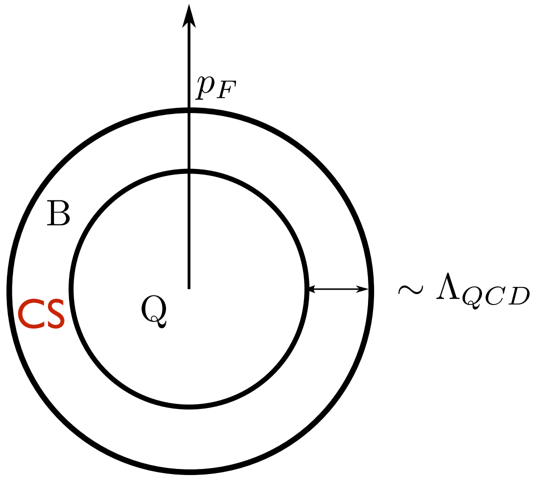


etc …

- Cold and dense candidate: baryon parity doublet models CS symmetric
  Total symmetry depends on couplings to mesons
  [Glozman, Catillo PRD 18]

- Quarkyonic matter [McLerran, Pisarski, NPA 07; O.P., Scheunert JHEP 19]
  Contains regime with chirally symmetric baryon matter
  Consistent with intermediate CS regime!

- Can be realized with or without non-analytic chiral phase transition!



Effective degrees of freedom…? → Spectral functions

Based on micro-causality of scalar, local quantum fields at finite T:

[Bros, Buchholz., NPB 94, Ann. Inst. Poincare Phys. Theor. 96]

$$\rho_{\mathrm{PS}}(p_0,\vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\ \epsilon(p_0)\,\delta\big(p_0^2-(\vec{p}-\vec{u})^2-s\big)\ \underset{\substack{\uparrow\\ \text{thermal spectral density}}}{\widetilde{D}_\beta(\vec{u},s)}$$

Exact, goes to Källen-Lehmann representation for $T \to 0$

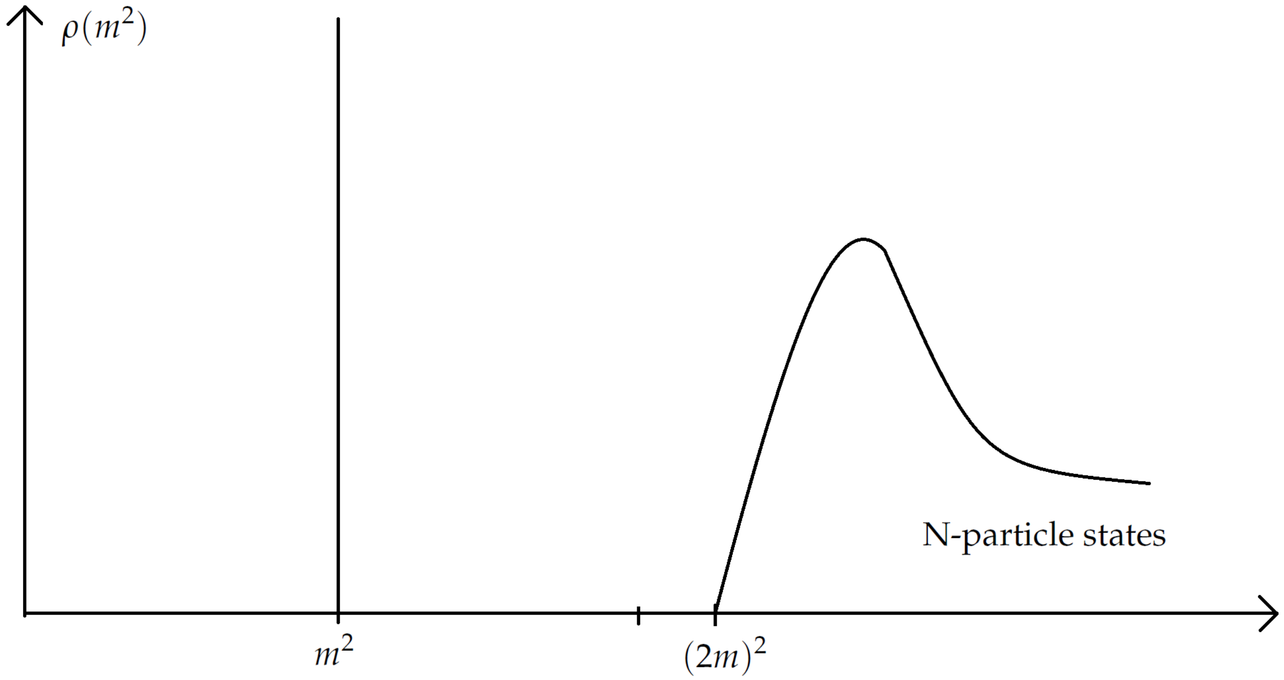
→ Relation between spatial correlators and thermal spectral density

$$C^s_{PS}(z) = \frac{1}{2}\int_0^\infty ds \int_{|z|}^\infty dR\; e^{-R\sqrt{s}} D_\beta(R,s)$$

[Lowdon, O.P., JHEP 22]

For stable massive particle with gap to continuum states (QCD pions!):

Vacuum spectral function:



Ansatz $\tilde{D}_\beta(\vec{u}, s) = \tilde{D}_{m,\beta}(\vec{u})\,\delta(s - m^2) + \tilde{D}_{c,\beta}(\vec{u}, s)$ [Bros, Buchholz., NPB 02]

Analytic structure inherited from vacuum in absence of phase transition

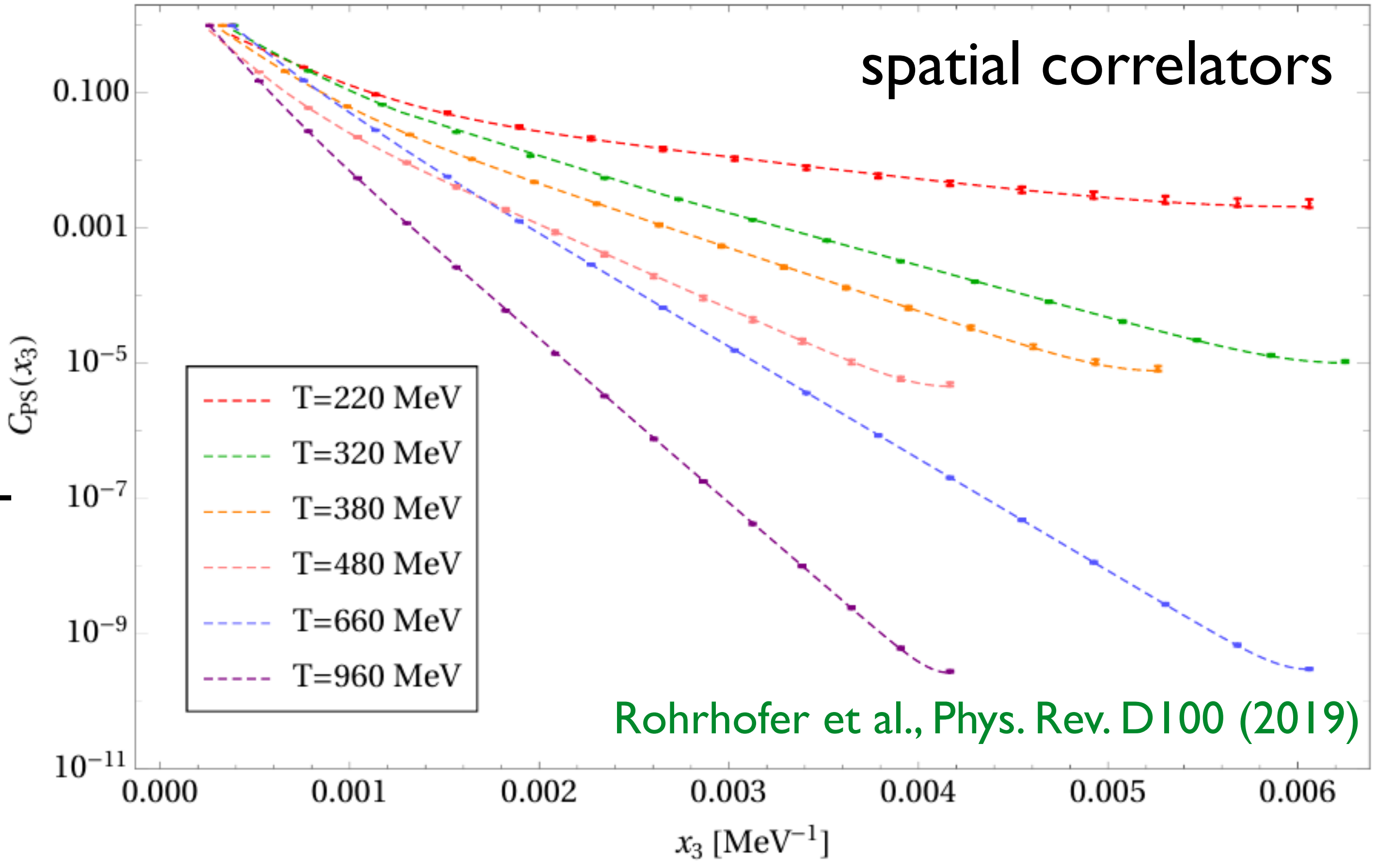
➡ low T behaviour influenced (dominated) by vacuum particle states

The pion spectral function

[Lowdon, O.P., JHEP 22]

2-state fits π, π^*

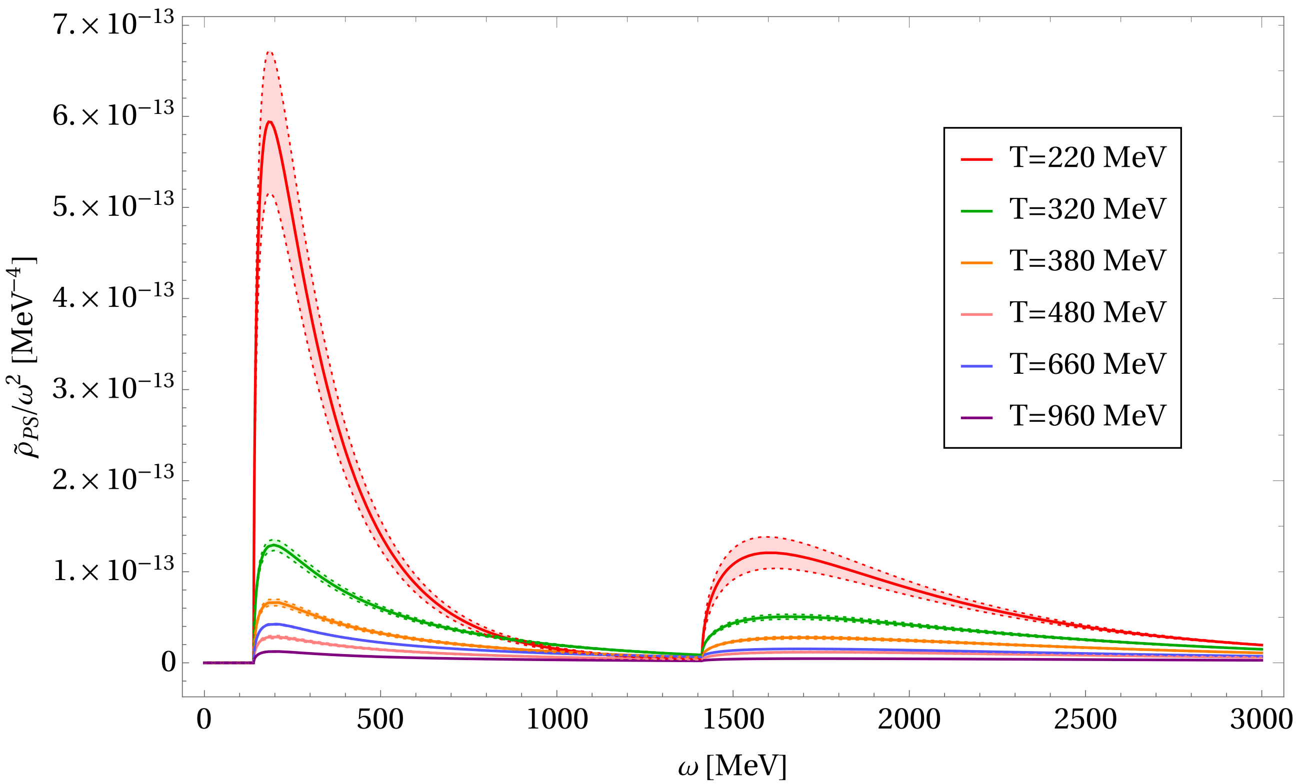
$$D_{m_{\pi^{(*)}},\beta} = \alpha_{\pi^{(*)}} e^{-\gamma_{\pi^{(*)}} x_3}$$

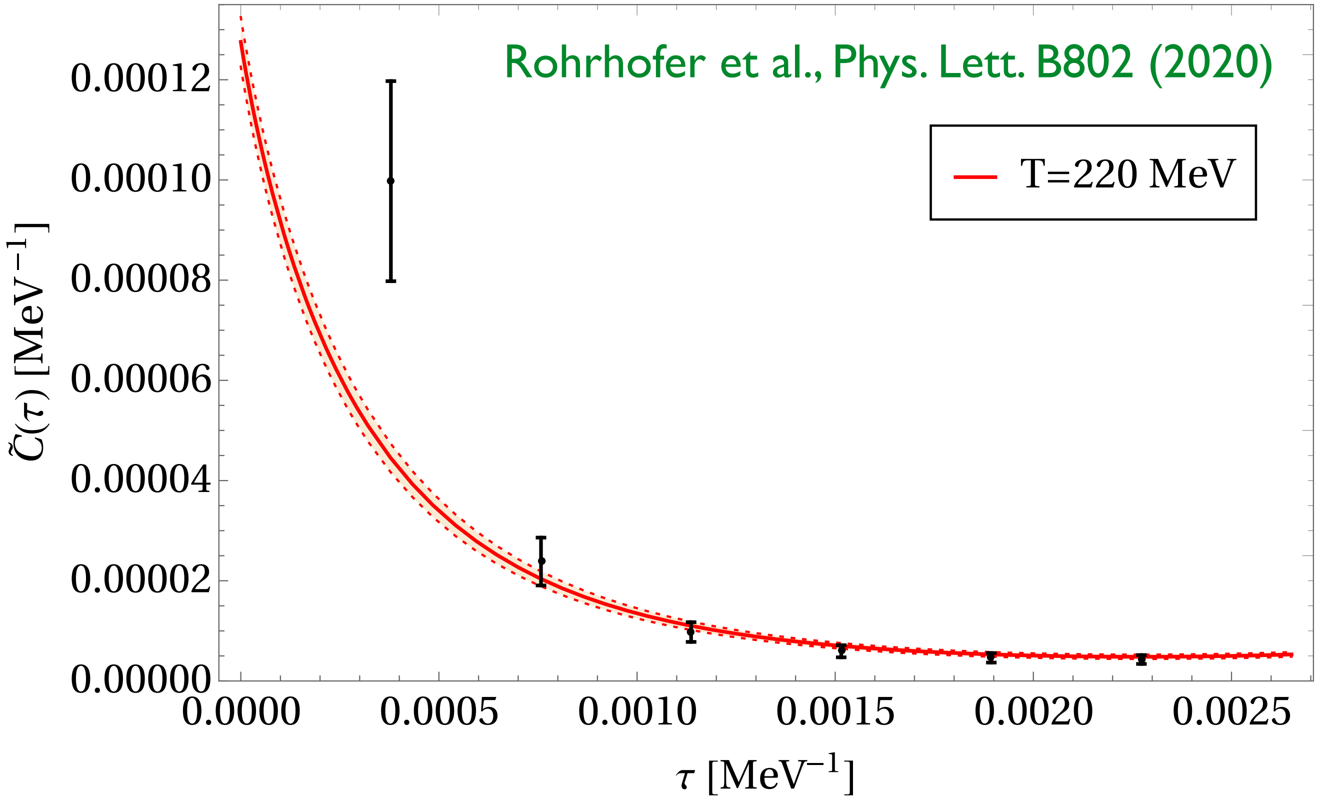

spatial correlators

Rohrhofer et al., Phys. Rev. D100 (2019)

spectral functions

predict temporal correlators, compare with data




Rohrhofer et al., Phys. Lett. B802 (2020)

Comparison with plasmon ansatz

Bros+Buchholz Ansatz

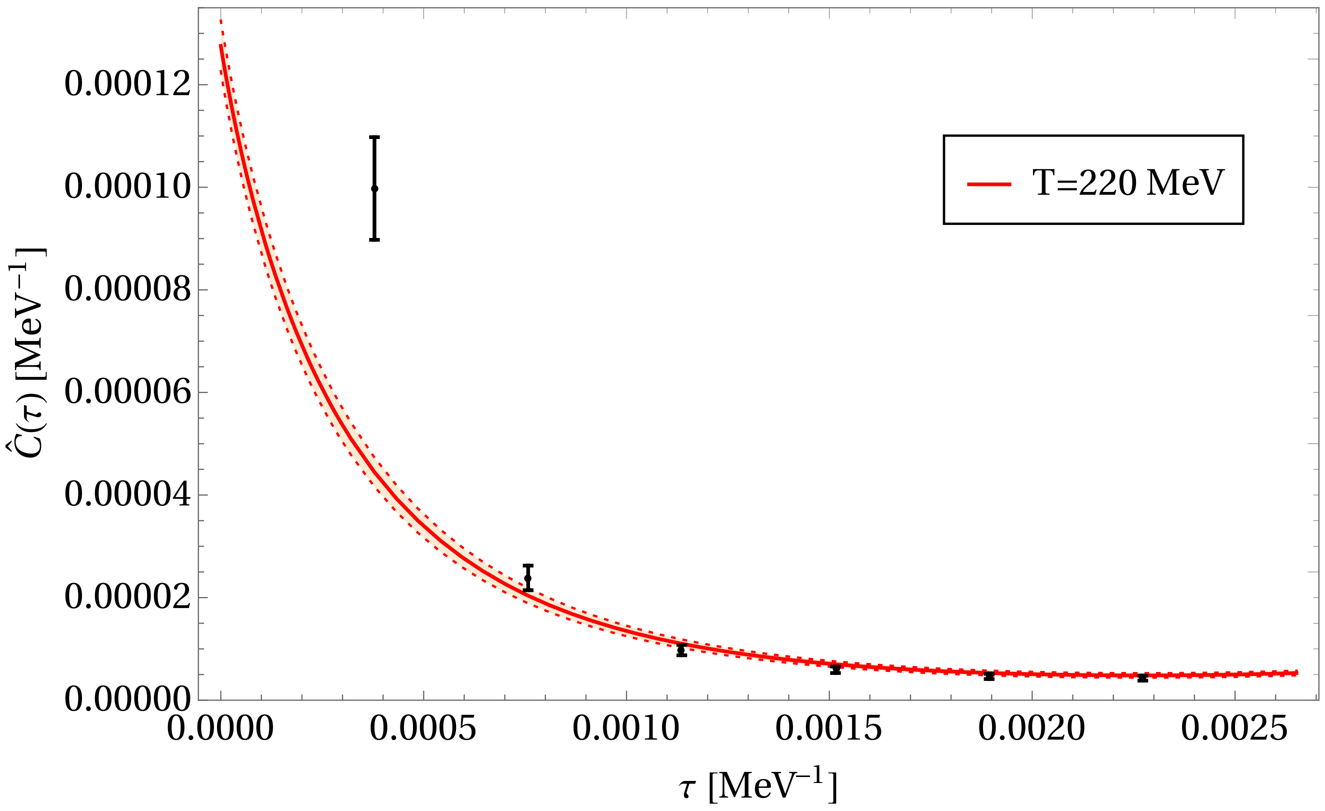
Perturbative plasmon: Breit-Wigner shape

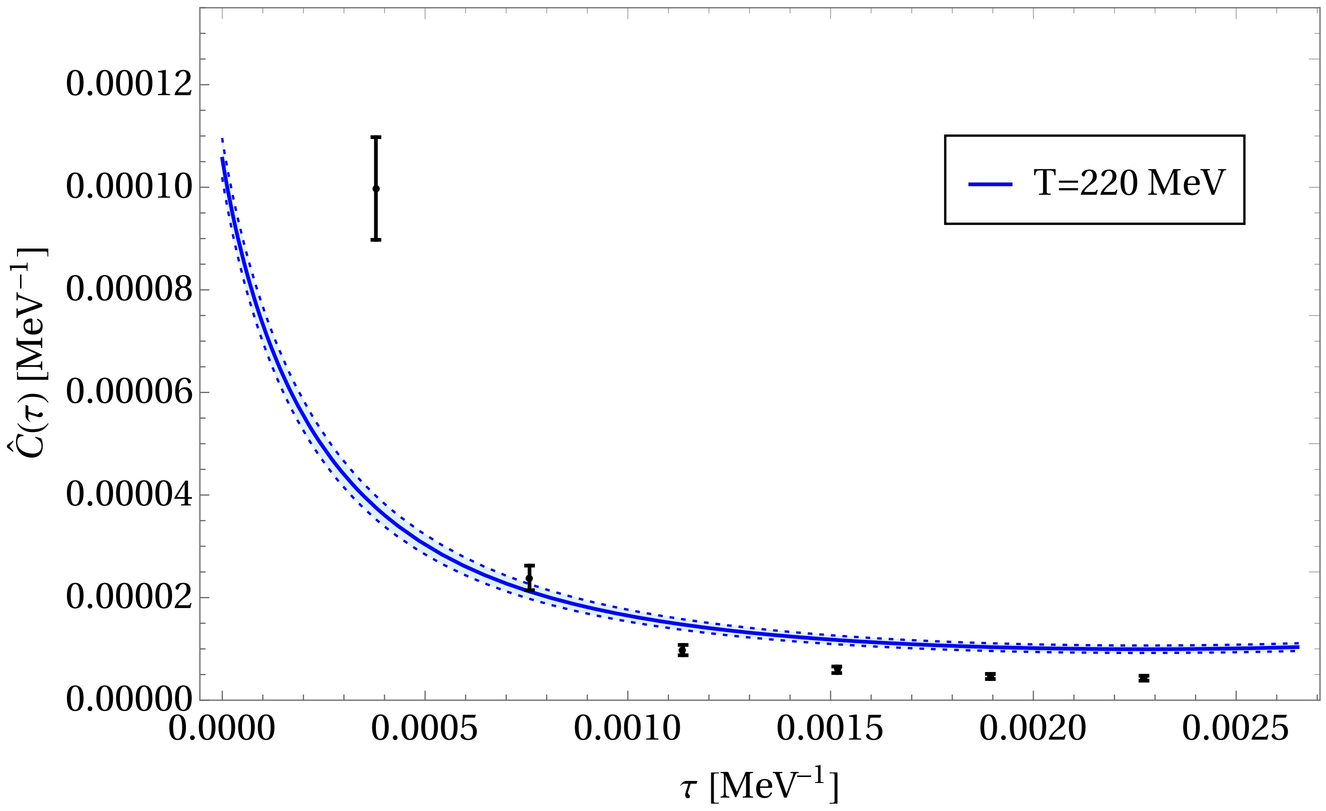
Both fit spatial correlator

$$\rho_{\mathrm{PS}}(\omega, \mathbf{p} = 0) = \epsilon(\omega) \left[\theta(\omega^2 - m_\pi^2) \frac{4\,\alpha_\pi\,\gamma_\pi \sqrt{\omega^2 - m_\pi^2}}{(\omega^2 - m_\pi^2 + \gamma_\pi^2)^2} + \theta(\omega^2 - m_{\pi^*}^2) \frac{4\,\alpha_{\pi^*}\,\gamma_{\pi^*} \sqrt{\omega^2 - m_{\pi^*}^2}}{(\omega^2 - m_{\pi^*}^2 + \gamma_{\pi^*}^2)^2} \right]$$

$$\rho_{PS}^{BW}(\omega, \mathbf{p} = 0) = \frac{4\alpha_\pi \omega \Gamma_\pi}{(\omega^2 - m_\pi^2 - \Gamma_\pi^2)^2 + 4\omega^2\Gamma_\pi^2} + \frac{4\alpha_\pi^* \omega \Gamma_{\pi^*}}{(\omega^2 - m_{\pi^*}^2 - \Gamma_{\pi^*}^2)^2 + 4\omega^2\Gamma_{\pi^*}^2}$$

Predicted temporal correlators:



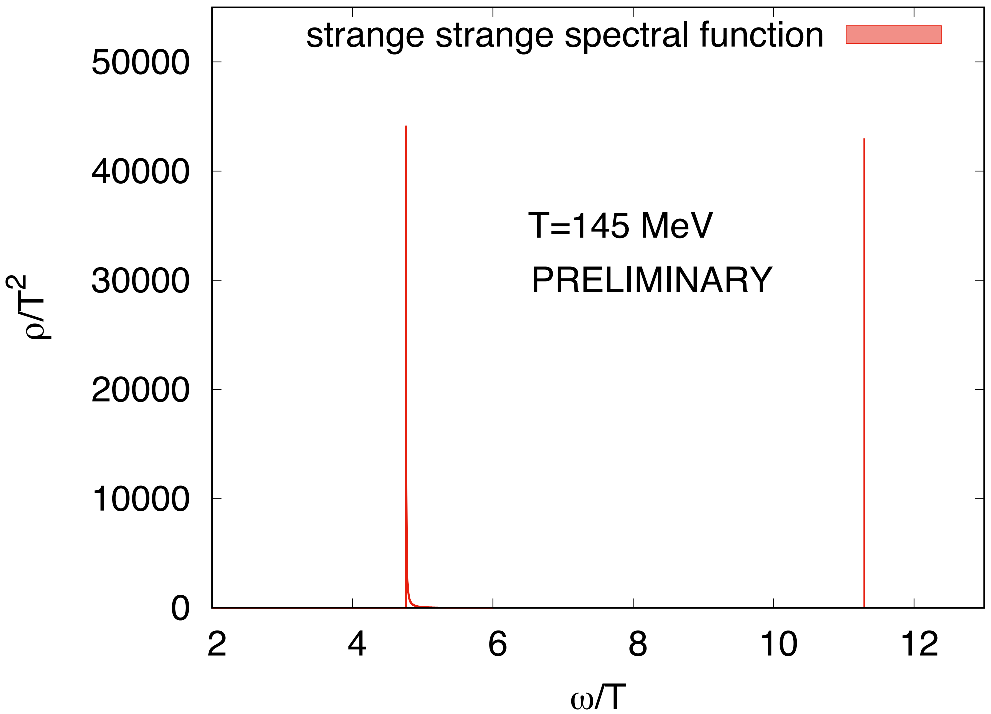


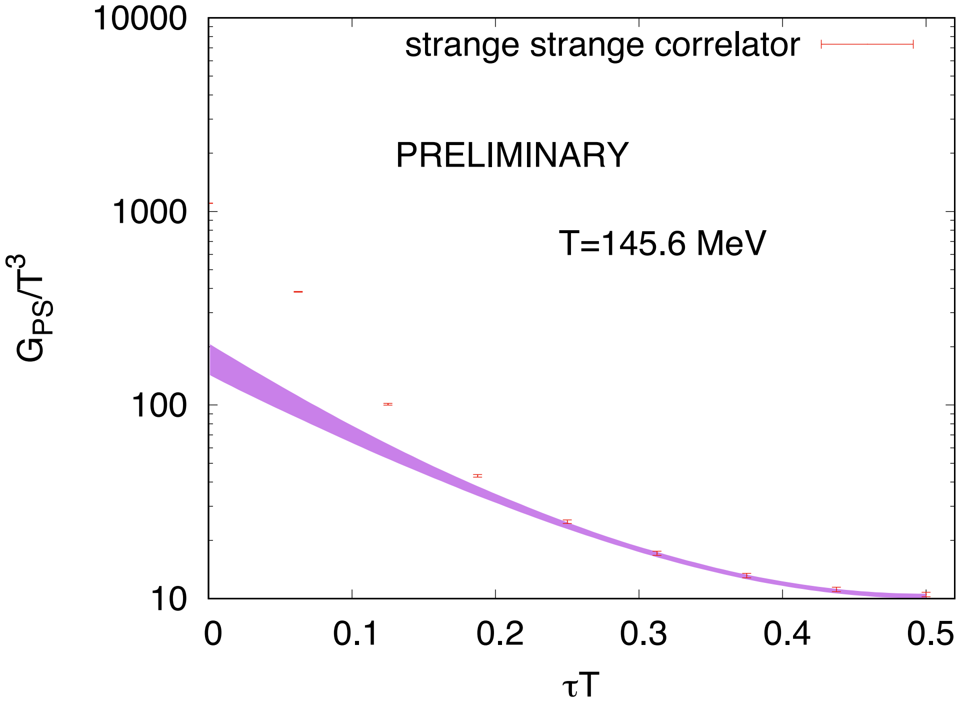
In progress: same analysis for additional states

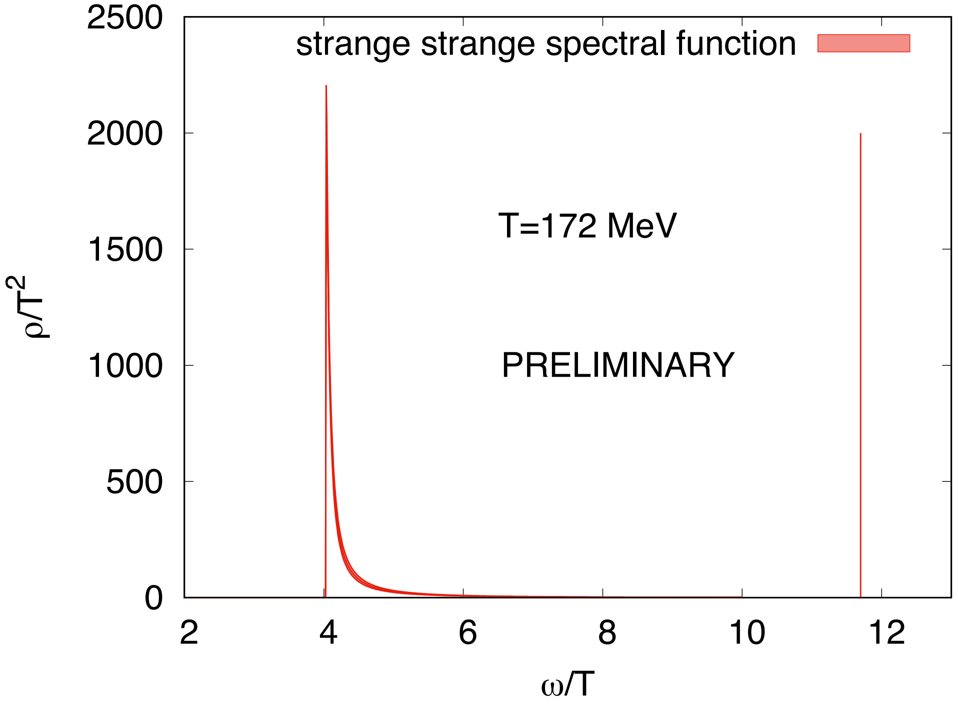
[Bala, Kaczmarek, Lowdon, O.P., Ueding]

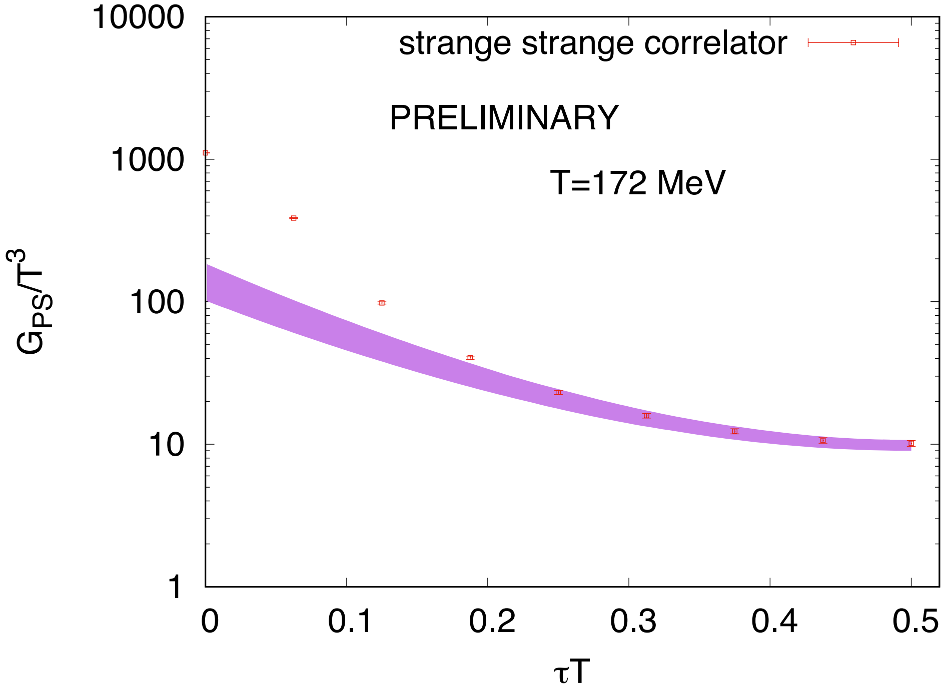
$N_f = 2+1$ HISQ sea + domain wall valence quarks, physical masses on $64^3 \times 16$

Goal: analyse all scalar and pseudo-scalar correlators, here: $\bar{s}s$ - channel, PS









Conclusions

- QCD has an emergent approximate Chiral Spin symmetry in an intermediate temperature and density range
- Screening masses entirely non-perturbative in that window
- Spectral functions from spatial lattice correlators, based on locality
- Effective degrees of freedom in CS-regime hadron-like
- CS-regime extends as a band into QCD phase diagram; natural connection to quarkyonic matter