Nuclear Science Computing Center at CCNU

QCD phase transition from lattice QCD

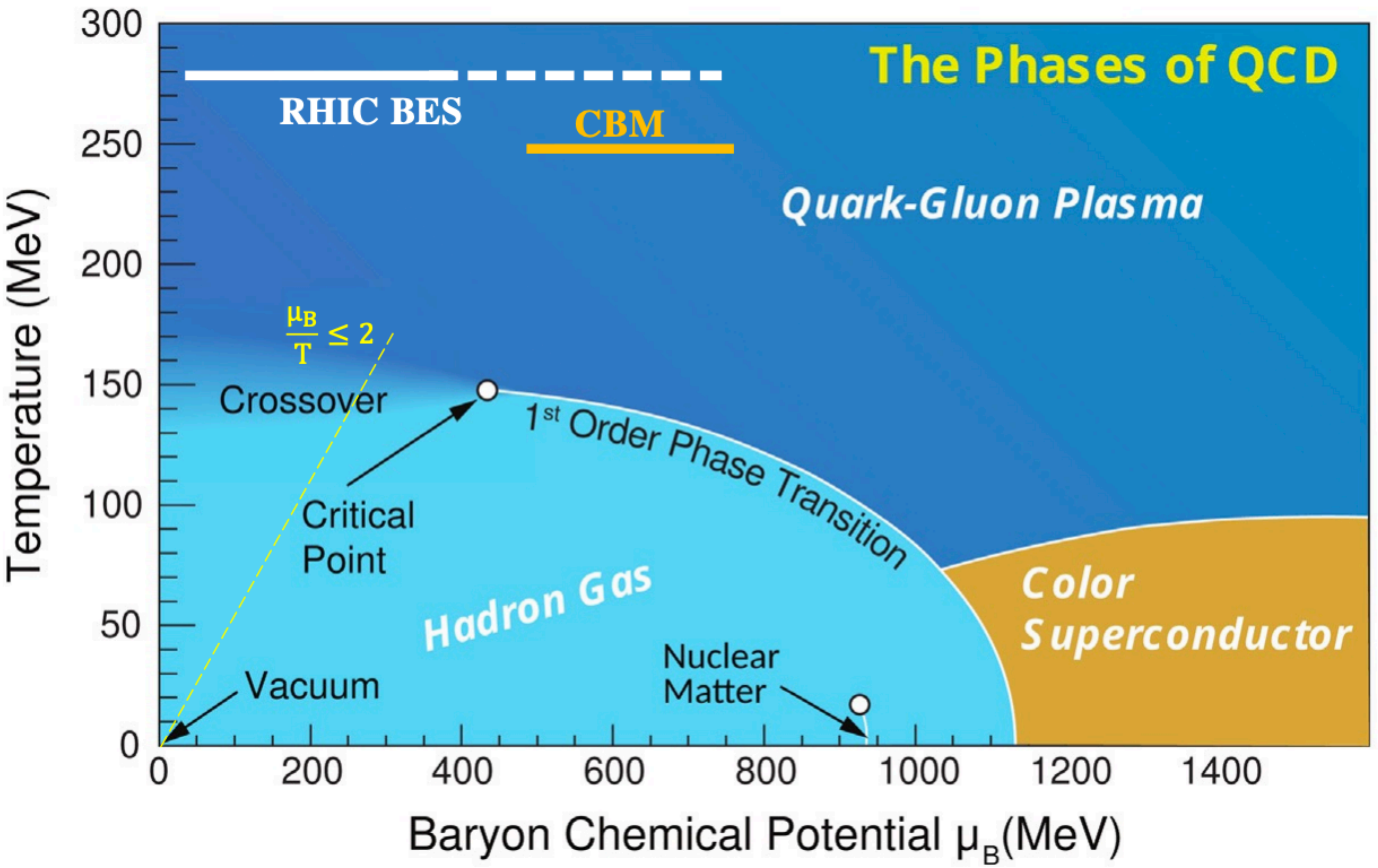
small quark mass & strong magnetic field

Heng-Tong Ding
Central China Normal University

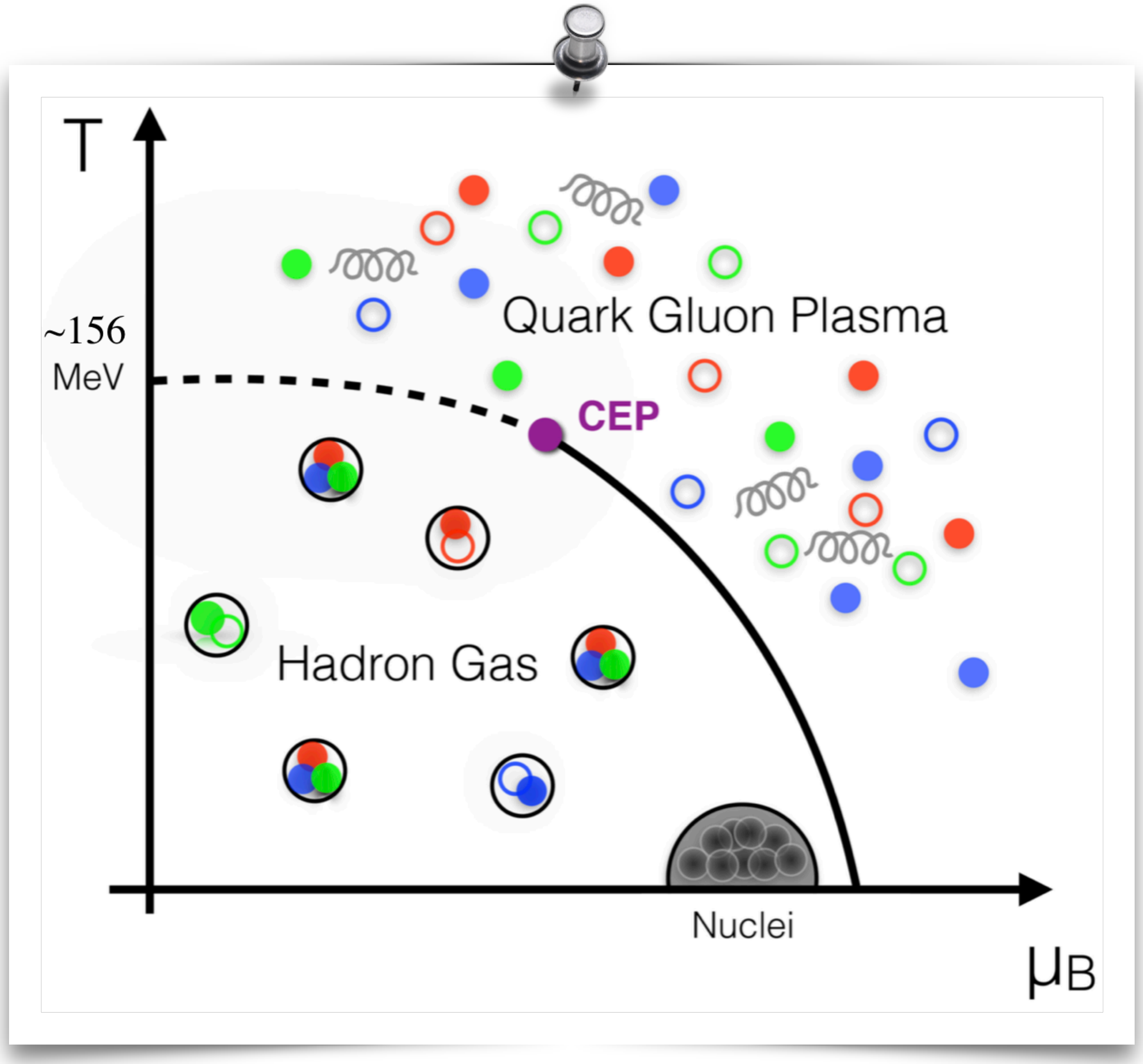
Online workshop on Critical Point and Onset of Deconfinement (CPOD2022)

28 Nov. - 2 Dec., 2022

Search for criticality



Almaalol et al., arXiv:2209.05009



HTD, F. Karsch, S. Mukherjee, arXiv:1504.05274

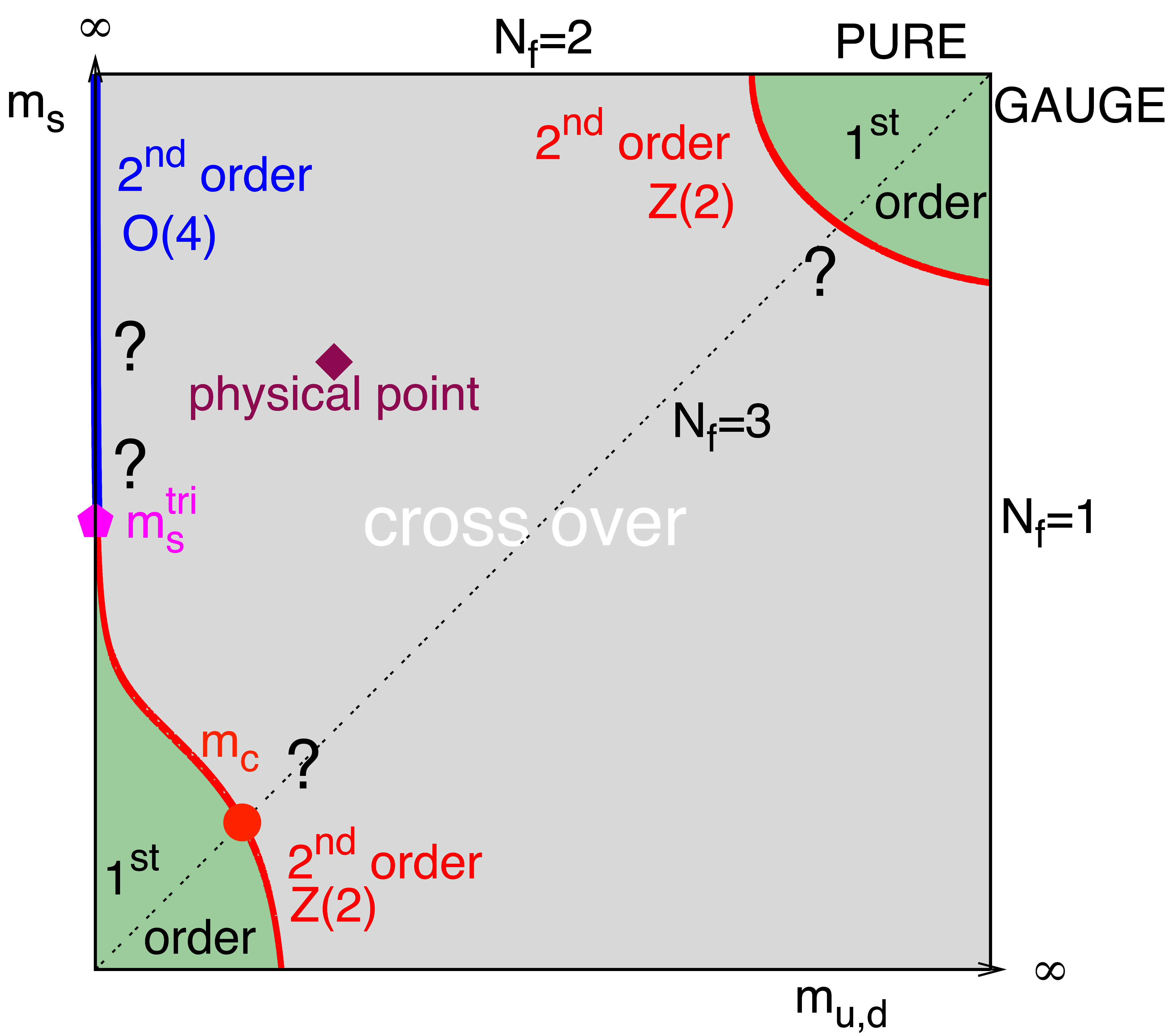
Sign Problem at μ_B=/=0

Taylor Expansion: Jishinu Goswami [Tue]

Imaginary μ_B: David Clark [Tue]

QCD criticality at μ_B=0: relevance to CEP?

Columbia plot:
QCD phase diagram in quark mass plane



RG arguments: Pisarski & Wilczek, PRD29 (1984) 338

- m_q=0 or ∞ with N_f=3: a first order phase transition
- Critical lines of 2nd order transition
  N_f=2: O(4) universality class
  N_f=3: Z(2) universality class

K. Rajagopal & F. Wilczek, NPB 399 (1993) 395
Gavin, Gocksch & Pisarski, PRD 49 (1994) 3079
F. Wilczek IJMPA 7(1992) 3911

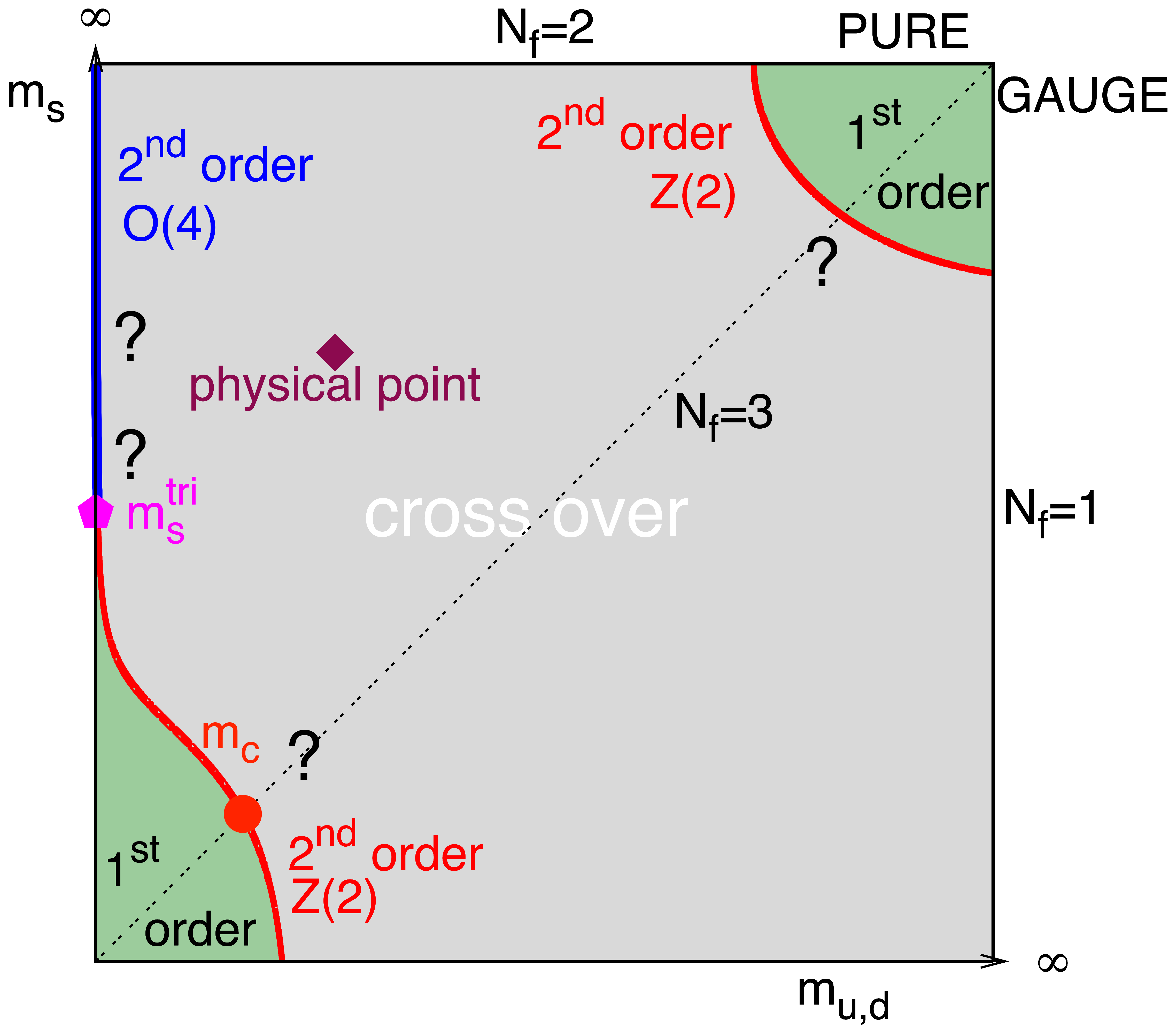
- Axial U(1) anomaly in Nf=2 QCD
  If manifested at T_c: 2nd order O(4)
  If not: 1st order or 2nd order $(U(2)_L \otimes U(2)_R/U(2)_V)$

Butti, Pelissetto and Vicari, JHEP 08 (2003) 029
Pelissetto & Vicari, PRD 88 (2013) 105018
Grahl, PRD 90 (2014) 117904

1st order deconfinement phase transition region

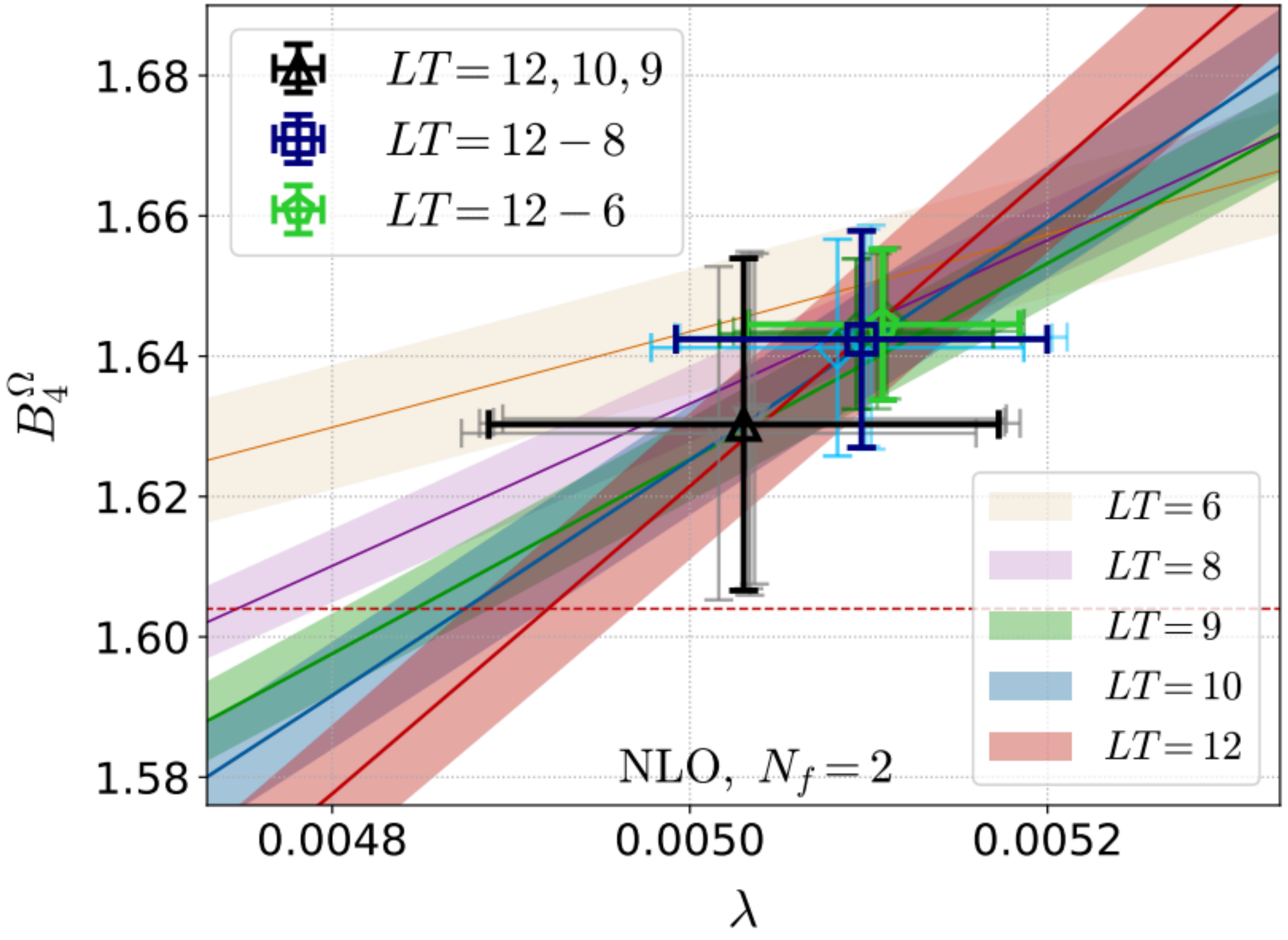
See Masakiyo Kitazawa's talk on Thu

Columbia plot:
QCD phase diagram in quark mass plane



Upper right corner of the Columbia plot:

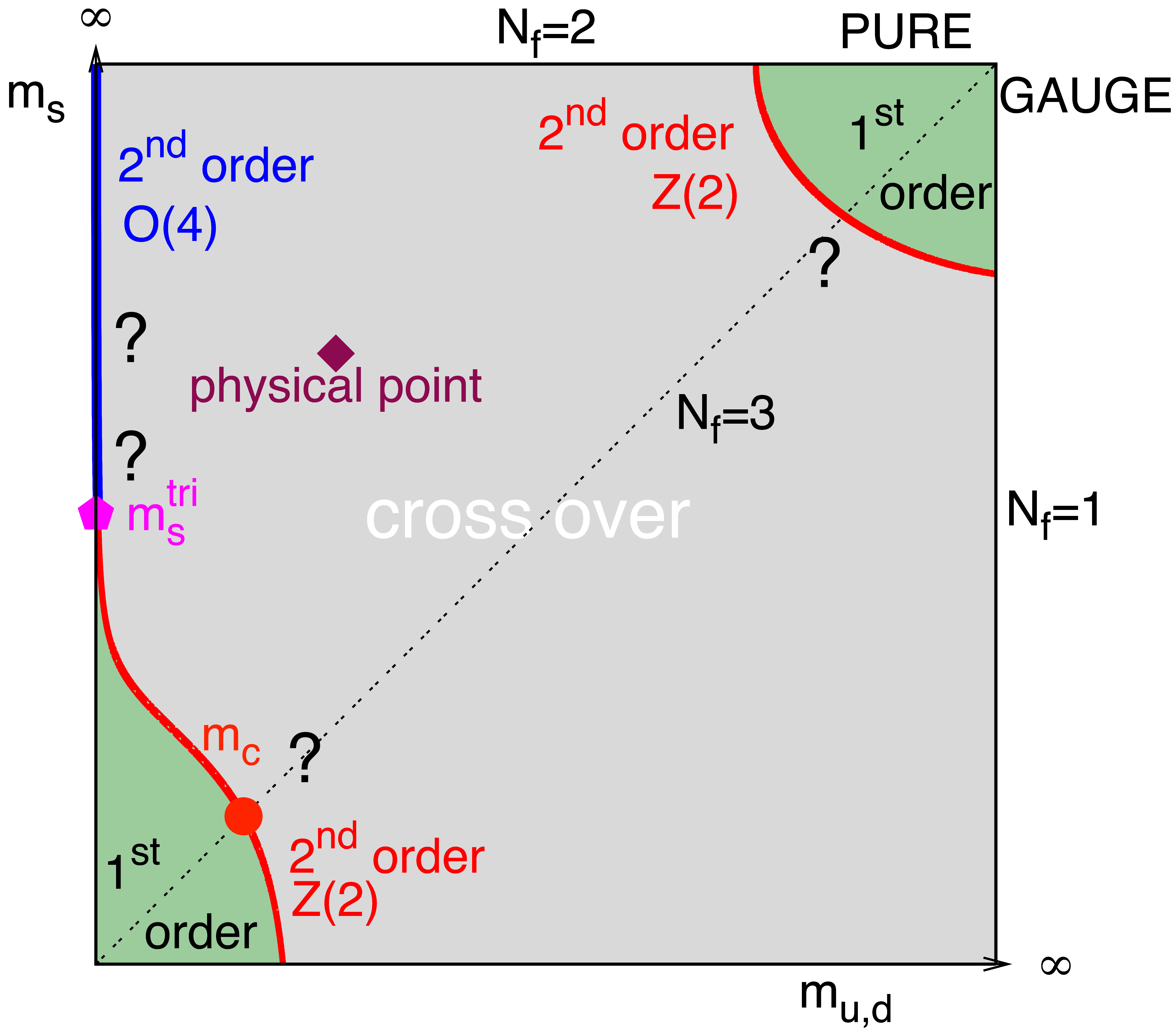
Observed a 2nd order Z(2) transition in N_f=2 QCD with sufficiently heavy quarks



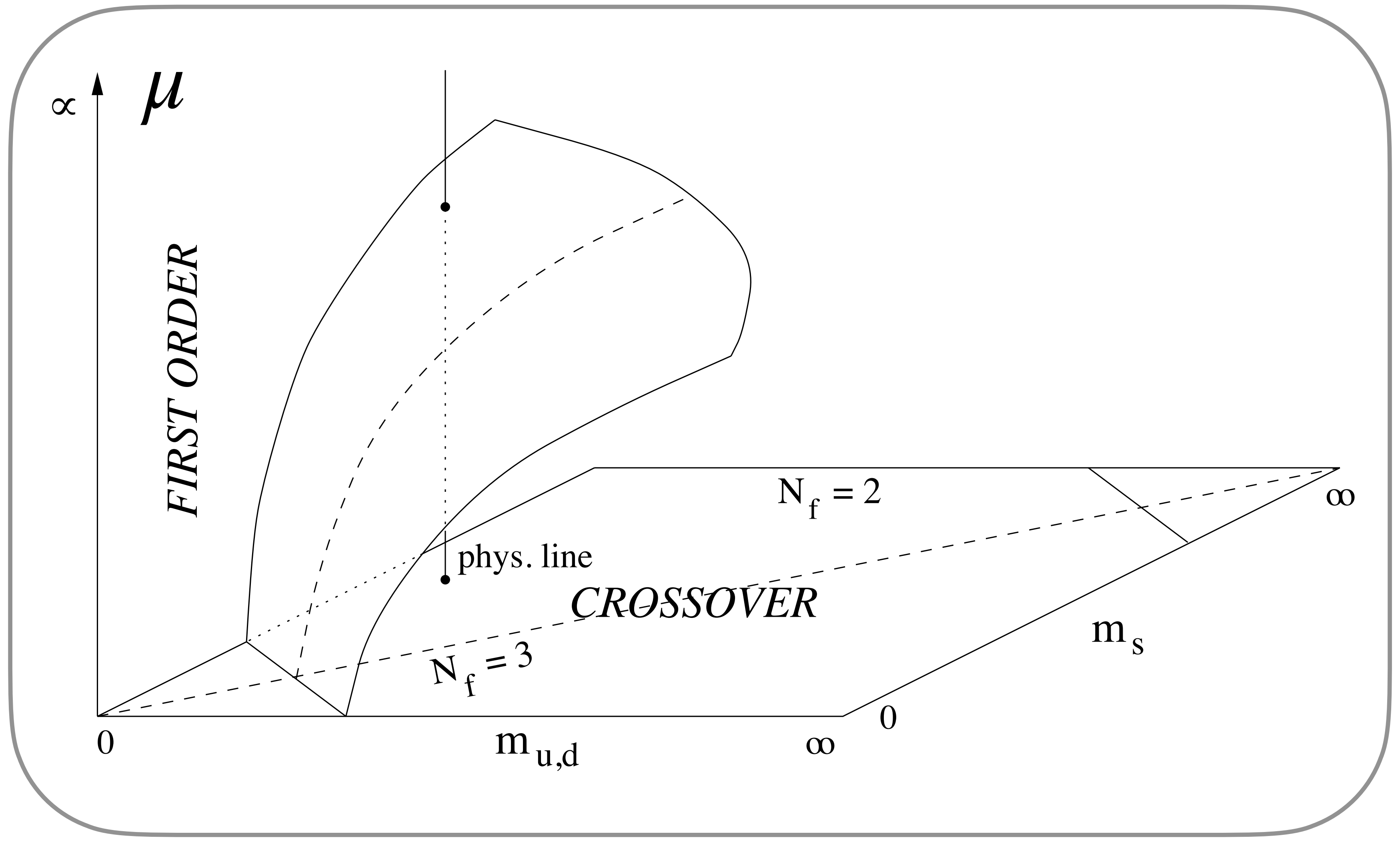
Kiyohara et al., PRD 104 (2021) 114509

1st order chiral phase transition region

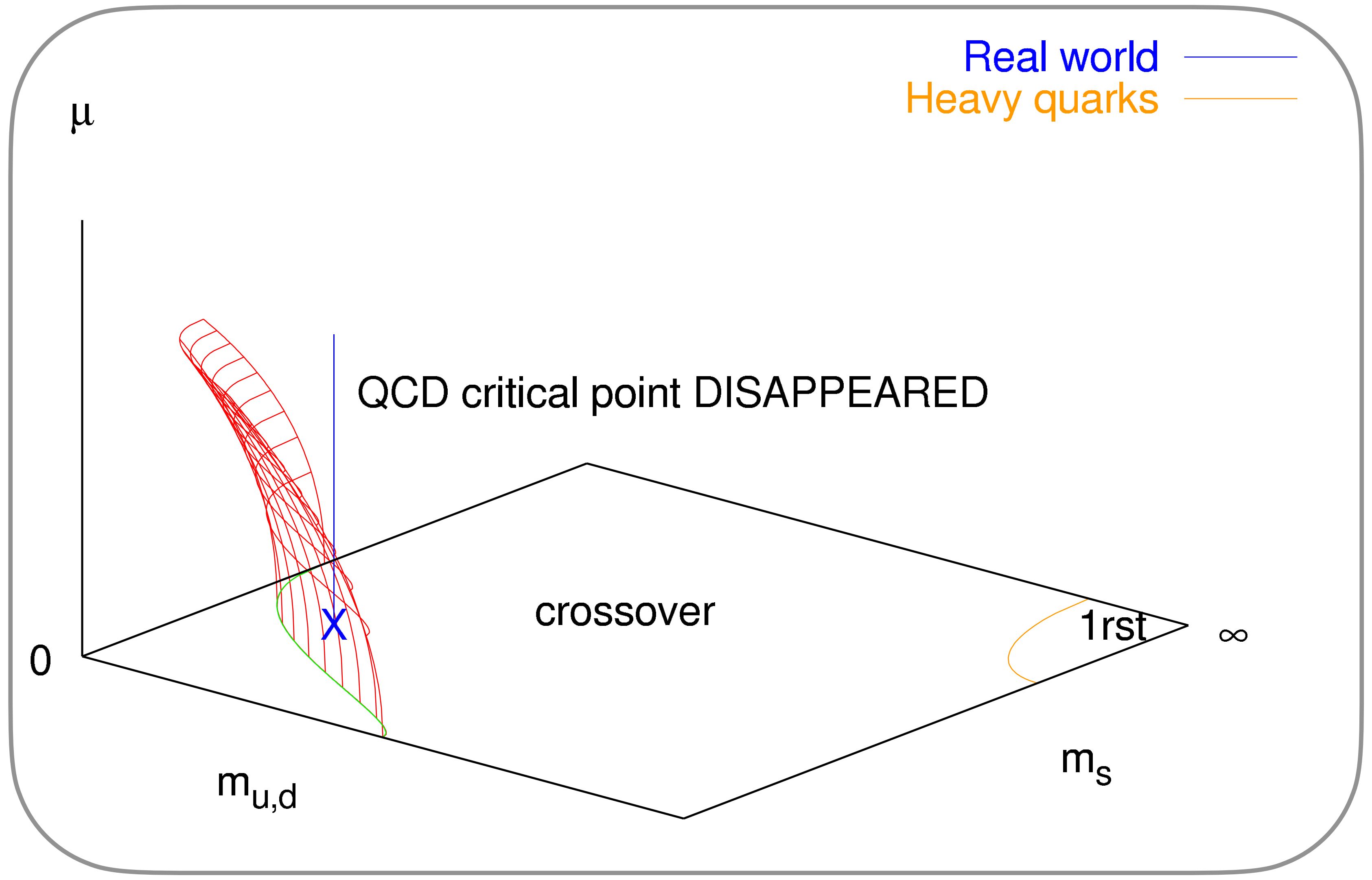
Columbia plot:
QCD phase diagram in quark mass plane



Bottom left corner of the Columbia plot:



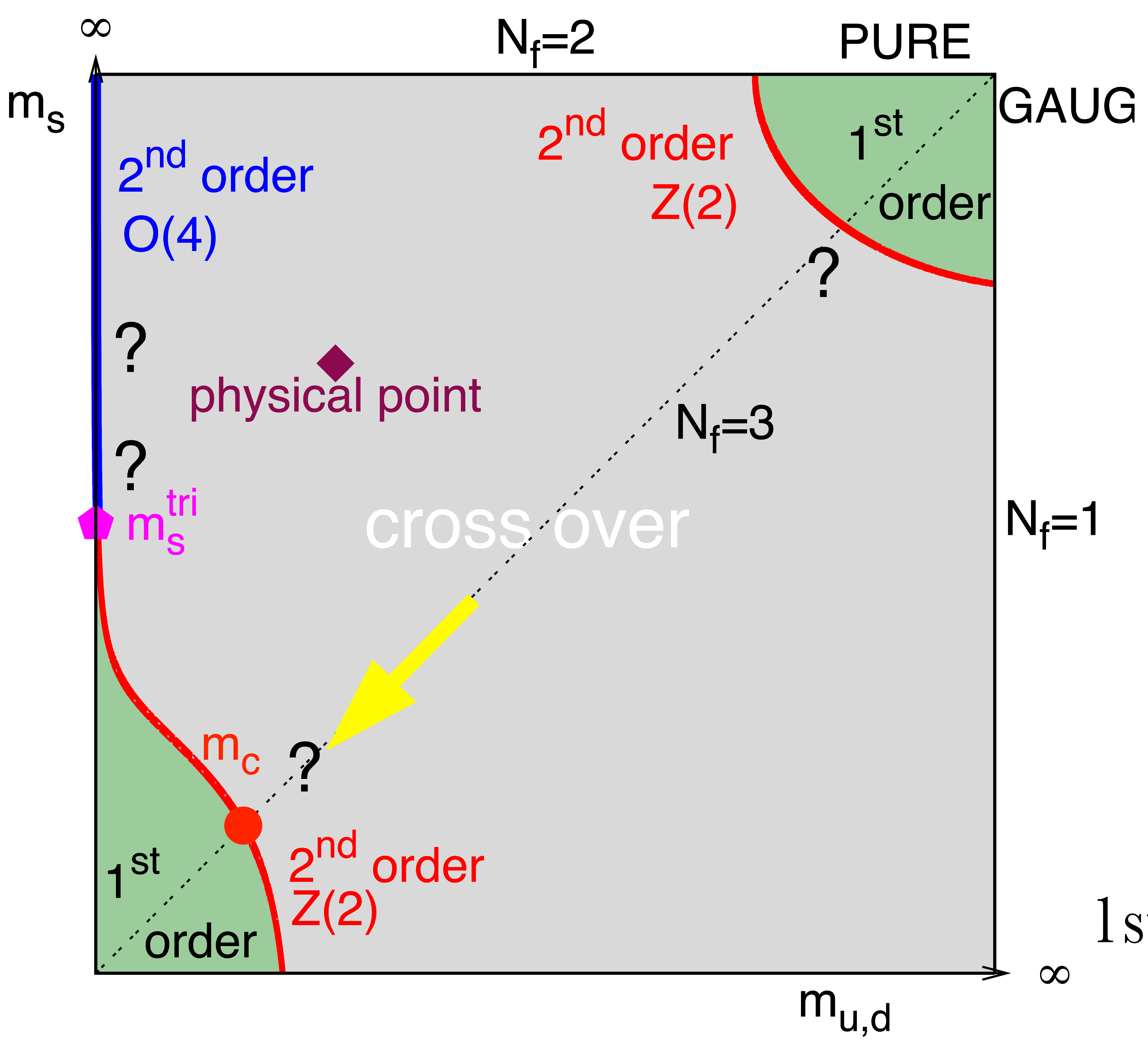
Karsch et al., '03,
Nakamura et al., 15'



de Forcrand & Philipsen,'07

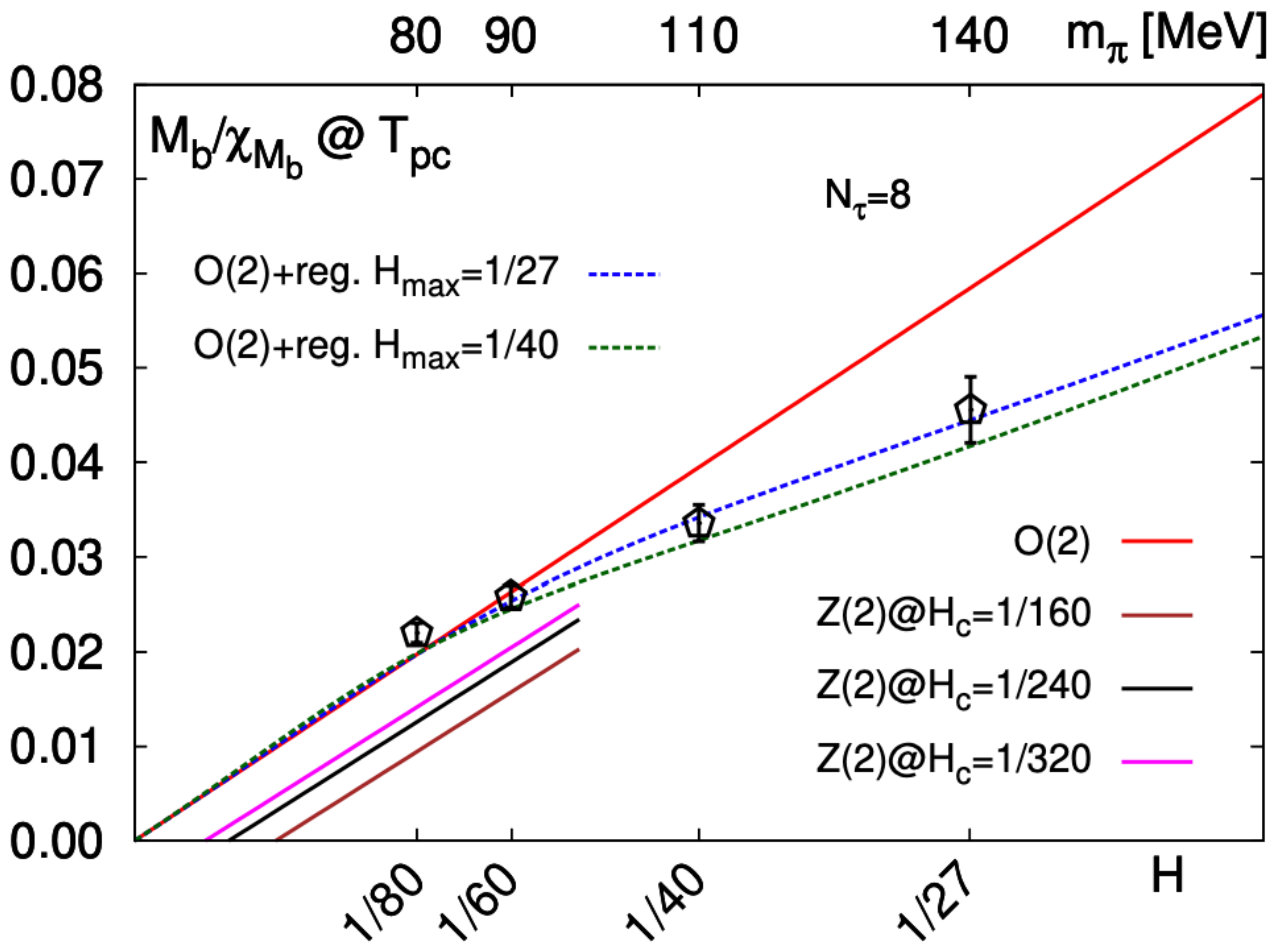
1st order chiral phase transition region

Columbia plot:
QCD phase diagram in quark mass plane



Bottom left corner of the Columbia plot:

HISQ fermions in Nf=3 QCD on Nt=8 lattices



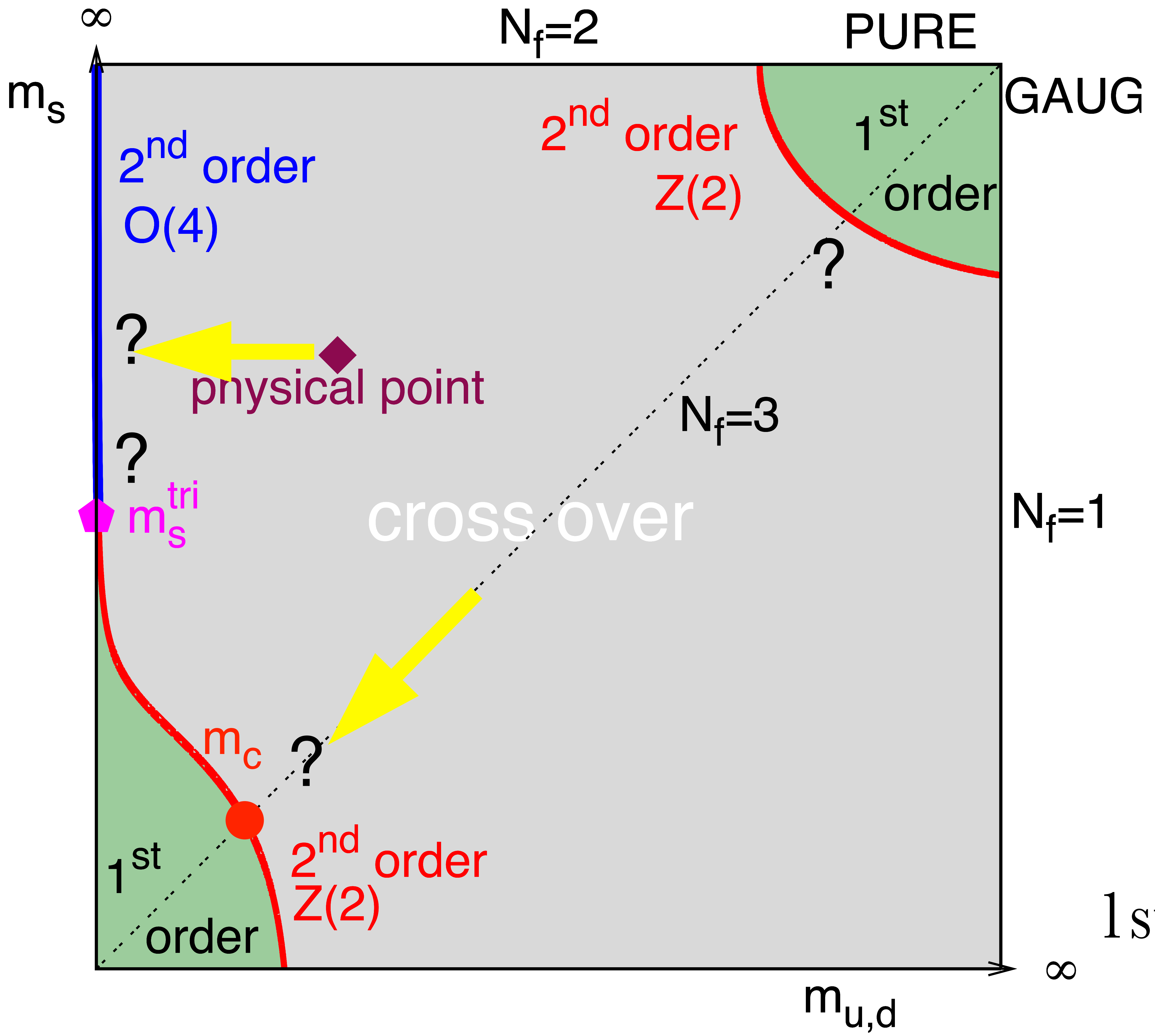
$$\frac{M}{\chi_M} \propto (H - H_c)\,\frac{f_G}{f_\chi}$$

1st order chiral phase transition region seems to be vanishing

Dini et al., Bielefeld-Bangalore, PRD 105 (2022) 3, 034510

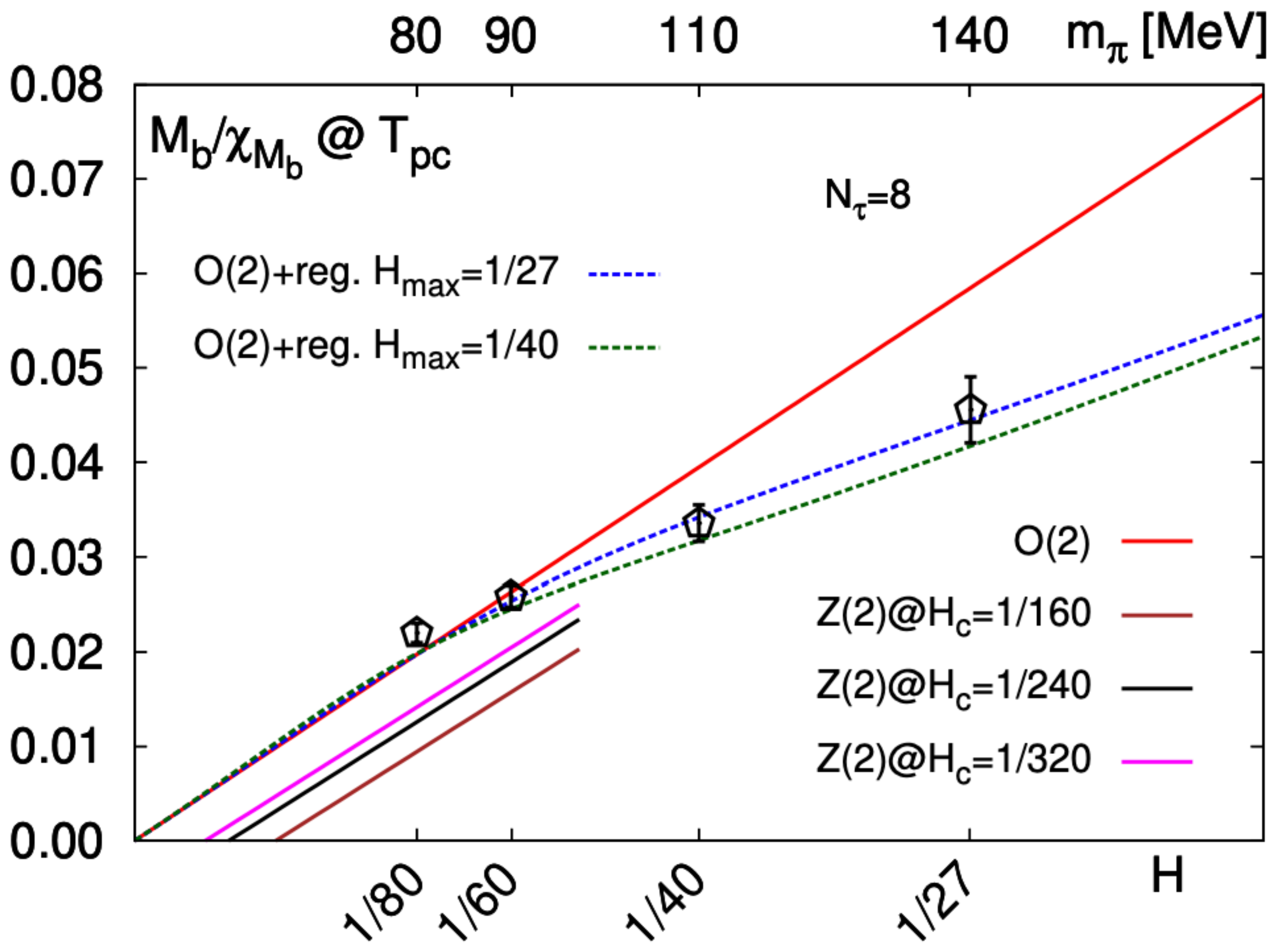
1st order chiral phase transition region

Columbia plot:
QCD phase diagram in quark mass plane



Bottom left corner of the Columbia plot:

HISQ fermions in Nf=3 QCD on Nt=8 lattices



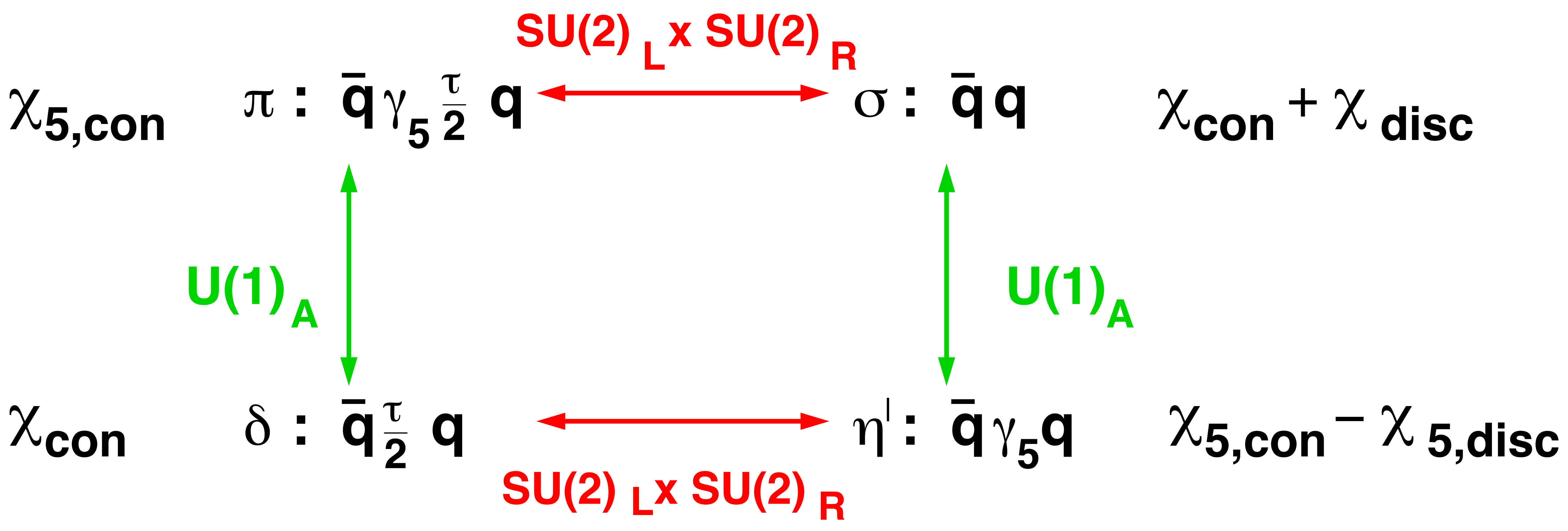
$$\frac{M}{\chi_M} \propto (H - H_c) \frac{f_G}{f_\chi}$$

1st order chiral phase transition region seems to be vanishing

Dini et al., Bielefeld-Bangalore, PRD 105 (2022) 3, 034510

Signatures of symmetry restorations

- Susceptibilities defined as integrated two point correlation functions of the local operators, e.g. $\chi_\pi = \int d^4x \langle \pi^i(x)\pi^i(0)\rangle$ with $\pi^i(x) = i\bar{\psi}_l(x)\gamma_5\tau^i\psi_l(x)$



Restoration of SU(2)$_L$xSU(2)$_R$:

$\chi_\pi - \chi_\sigma = 0$

$\chi_\delta - \chi_\eta = 0$

$\Rightarrow \chi_\pi - \chi_\delta = \chi_{disc} = \chi_{5,disc}$

$$\chi_{\rm disc} = \frac{T}{V}\int d^4x \left\langle \left[\bar{\psi}(x)\psi(x) - \langle\bar{\psi}(x)\psi(x)\rangle\right]^2 \right\rangle$$

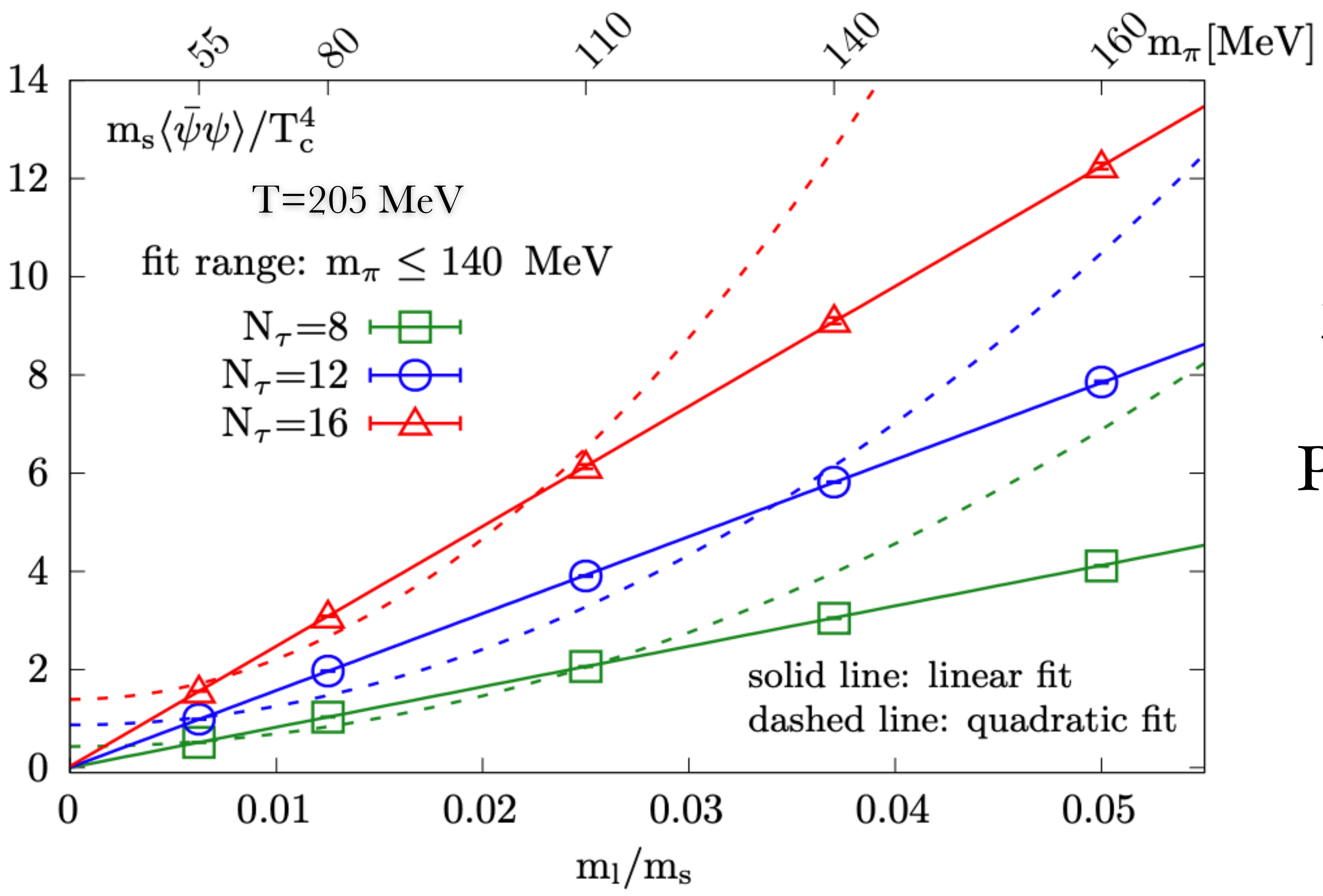
Effective restoration of U(1)$_A$:

$\chi_\pi - \chi_\delta = 0$

$\chi_\sigma - \chi_\eta = 0$

$\Rightarrow \chi_\pi - \chi_\delta = \chi_{disc} = \chi_{5,disc} = 0$

SU(2)xSU(2) symmetry restoration at T=205 MeV



HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang
PoS LATTICE2021(2022)619

In the chiral symmetric phase

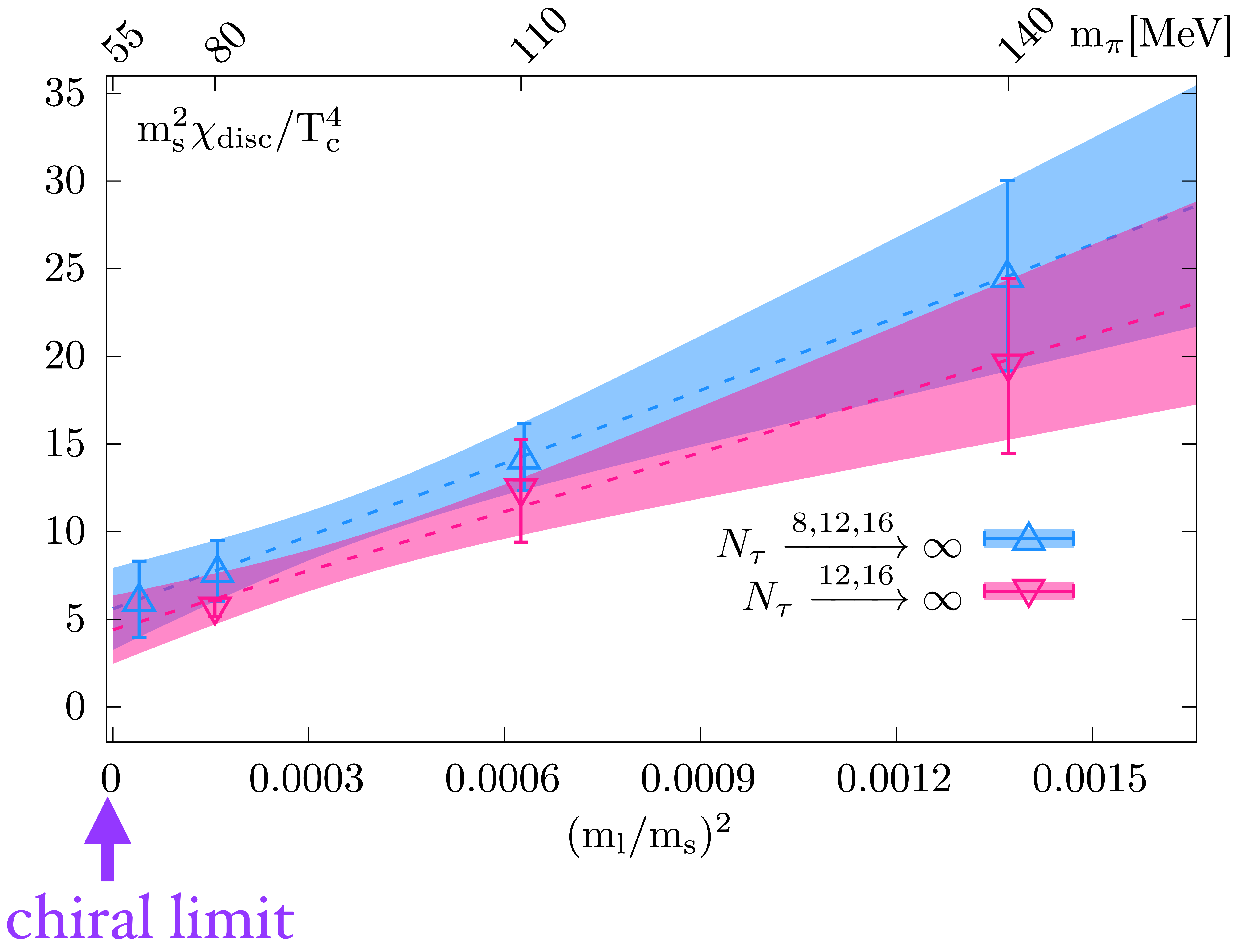
Z(2) subgroup of SU(2)xSU(2) sym.

Partition function: even function of m

$$\langle\bar{\psi}\psi\rangle \propto m \text{ as } m \to 0$$

$$\chi_{disc} \propto m^2 \text{ as } m \to 0$$

Continuum and chiral extrapolations with $m_\pi \leq 140$ MeV data at T≈ 205 MeV in N_f=2+1 QCD



HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang
PRL126(2021)082001

- **Joint fit**: simultaneous fits

  Continuum: $c_0 + c_1/N_\tau^2 + c_2/N_\tau^4$
  Chiral: quadratic in quark mass

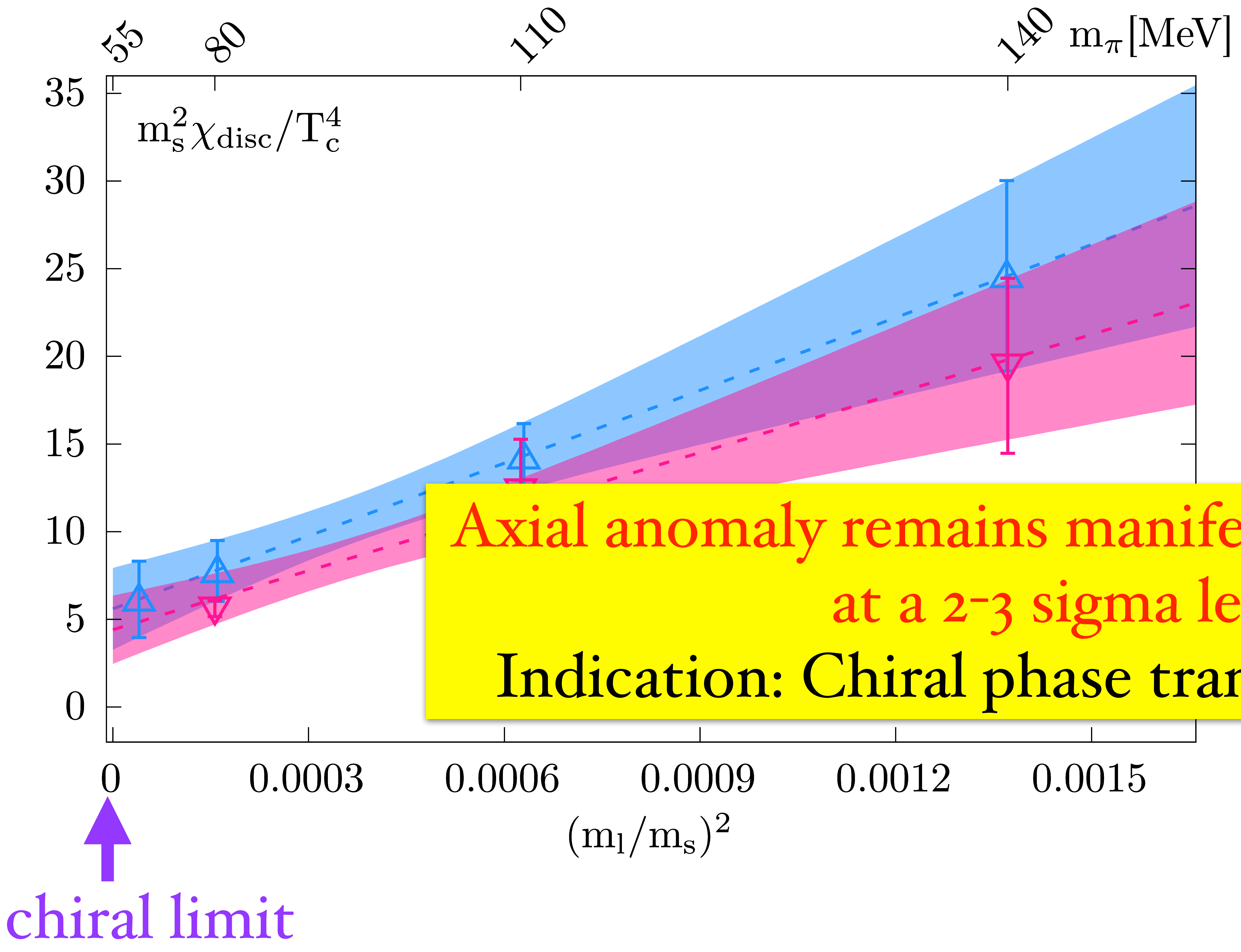
  Value at $N_\tau \rightarrow \infty$ and $m \rightarrow 0$:
  5.6 ± 2.3

- **Sequential fit:** first continuum and then chiral extrapo.

  Continuum: quadratic in $1/N_\tau$ with N_τ=12 & 16 data
  Chiral: quadratic in quark mass

  Value at $N_\tau \rightarrow \infty$ and $m \rightarrow 0$:
  4.4 ± 1.9

Continuum and chiral extrapolations with $m_\pi \leq 140$ MeV data at $T \approx 205$ MeV in $N_f=2+1$ QCD



Joint fit: simultaneous fits

Continuum: $c_0 + c_1/N_\tau^2 + c_2/N_\tau^4$

Chiral: quadratic in quark mass

Value at $N_\tau \to \infty$ and $m \to 0$:

5.6 ± 2.3

Axial anomaly remains manifested in the $U(1)_A$ measure at a 2-3 sigma level at $1.6T_c$

Indication: Chiral phase transition is 2nd order O(4)

...ontinuum and
...hiral extrapo.
...tic in $1/N_\tau$ with
N_τ=12 & 16 data

Chiral: quadratic in quark mass

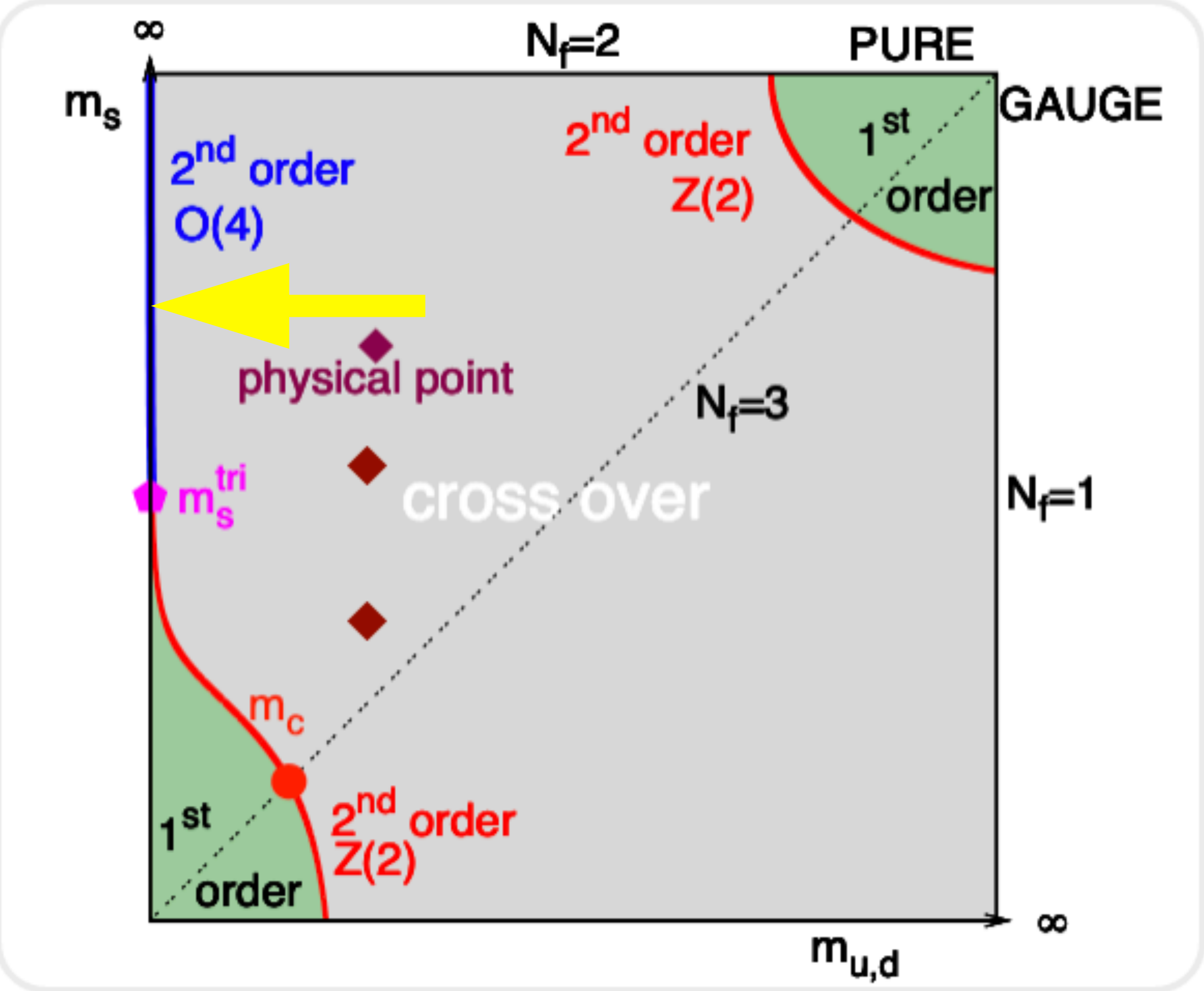
Value at $N_\tau \to \infty$ and $m \to 0$:

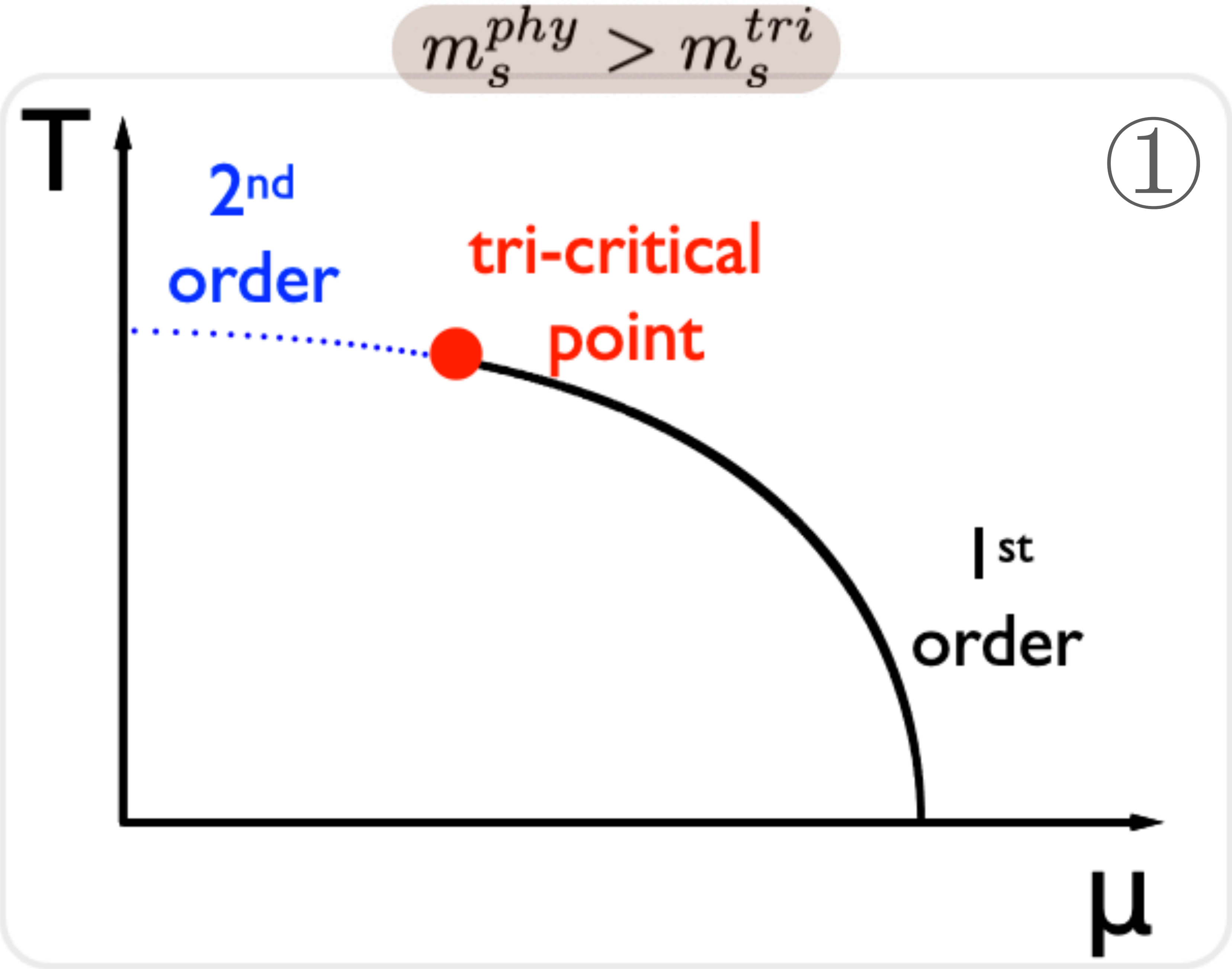
4.4 ± 1.9

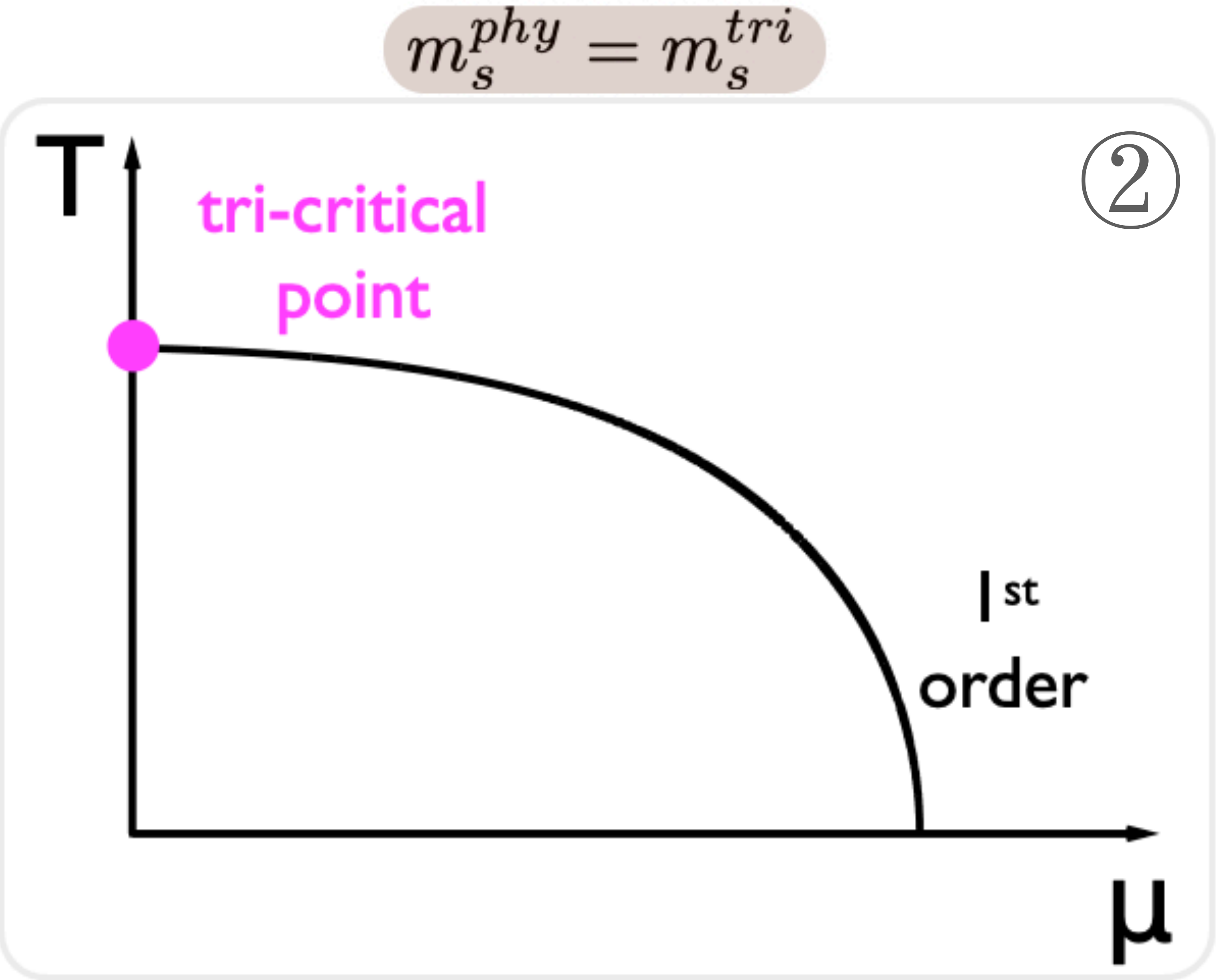
HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang
PRL126(2021)082001

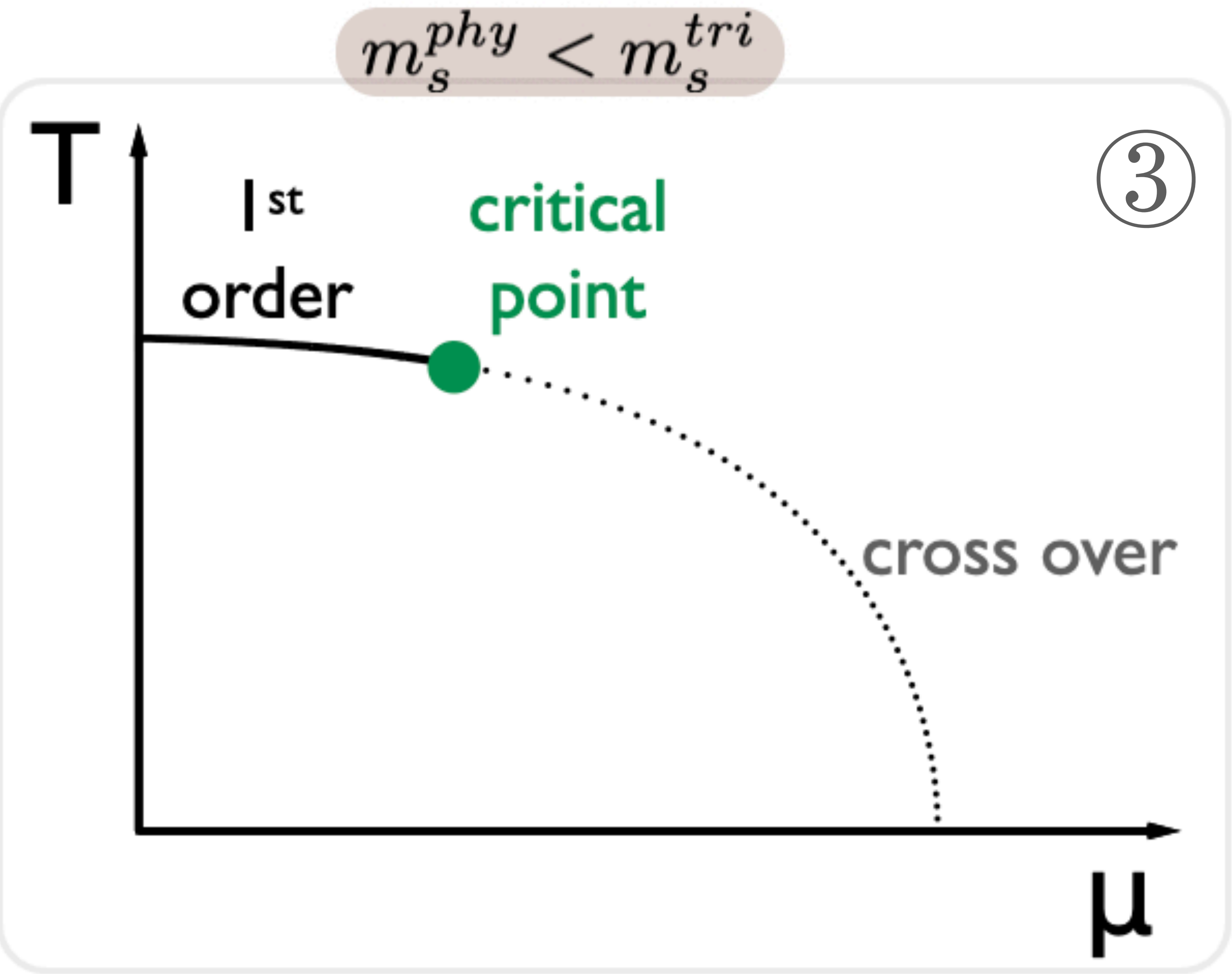
Scenarios of QCD chiral phase transition

at m_l=0

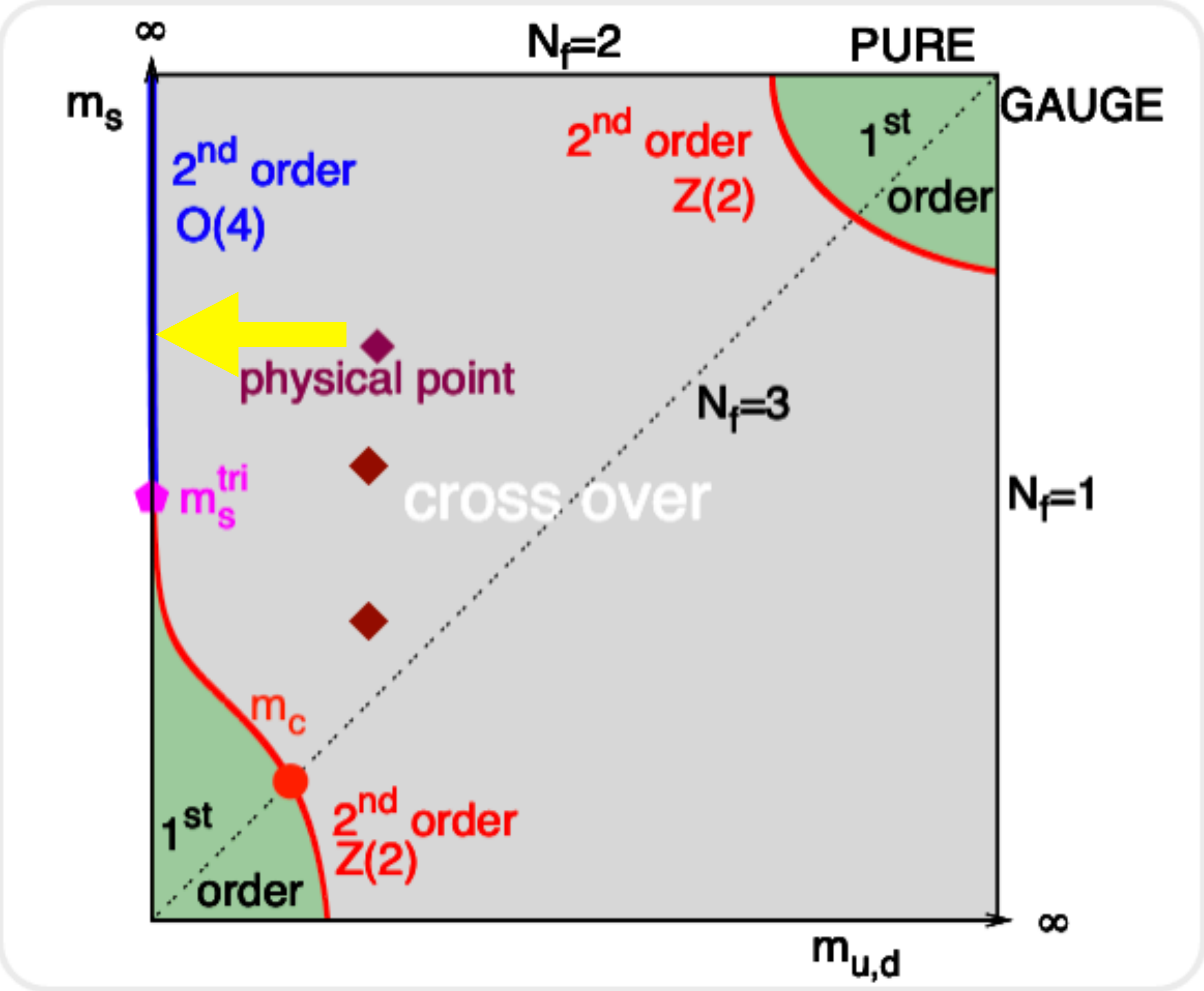




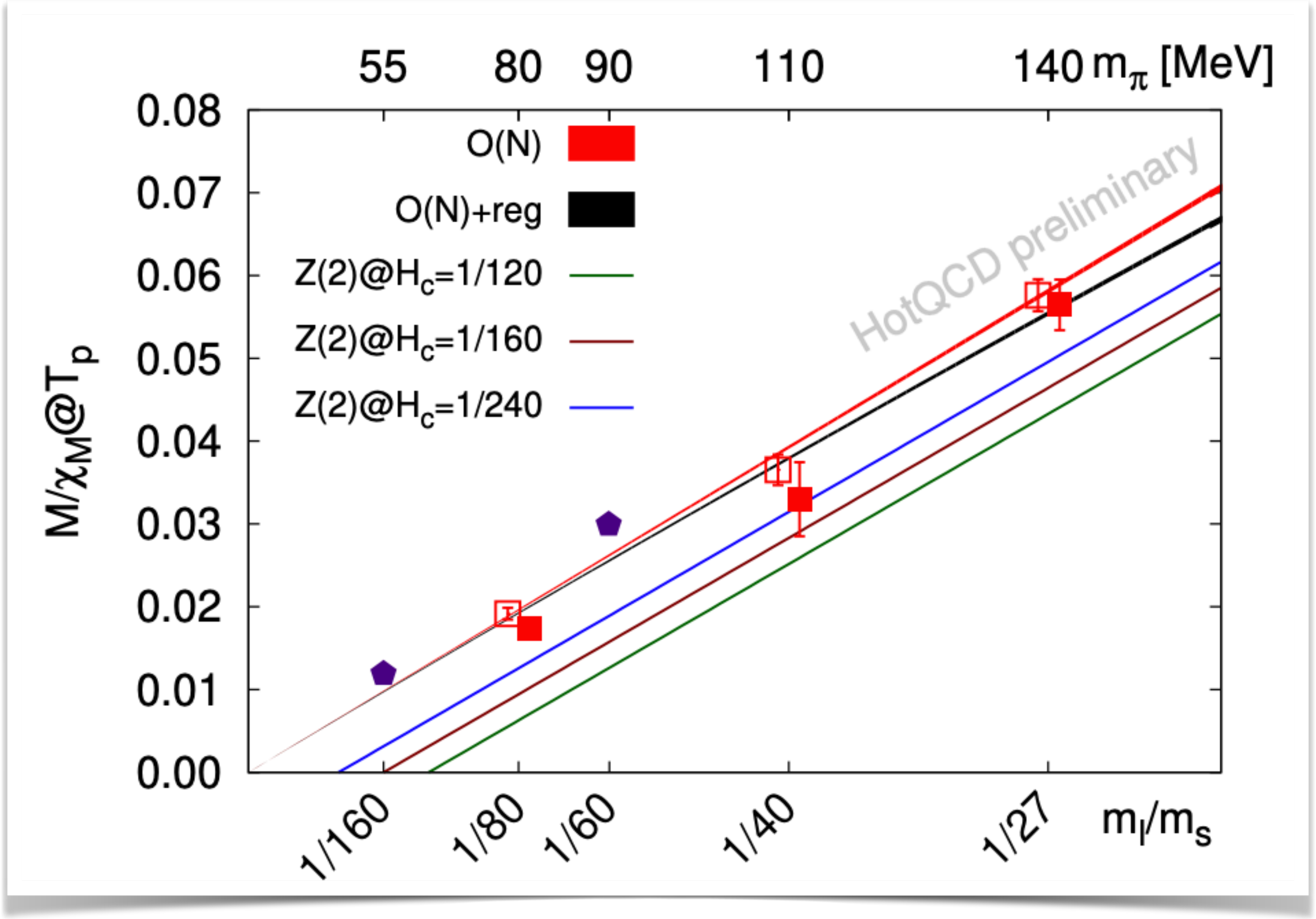




chiral phase transition in N_f=2+1 QCD



Ratio of order parameter and its sus.



O. Kaczmarek et al., Acta Phys. Pol. B Proc. Suppl. 14 (2021) 291

$$\frac{M}{\chi_M} \propto (\frac{m_l}{m_s} - H_c)\, \frac{f_G}{f_\chi}$$

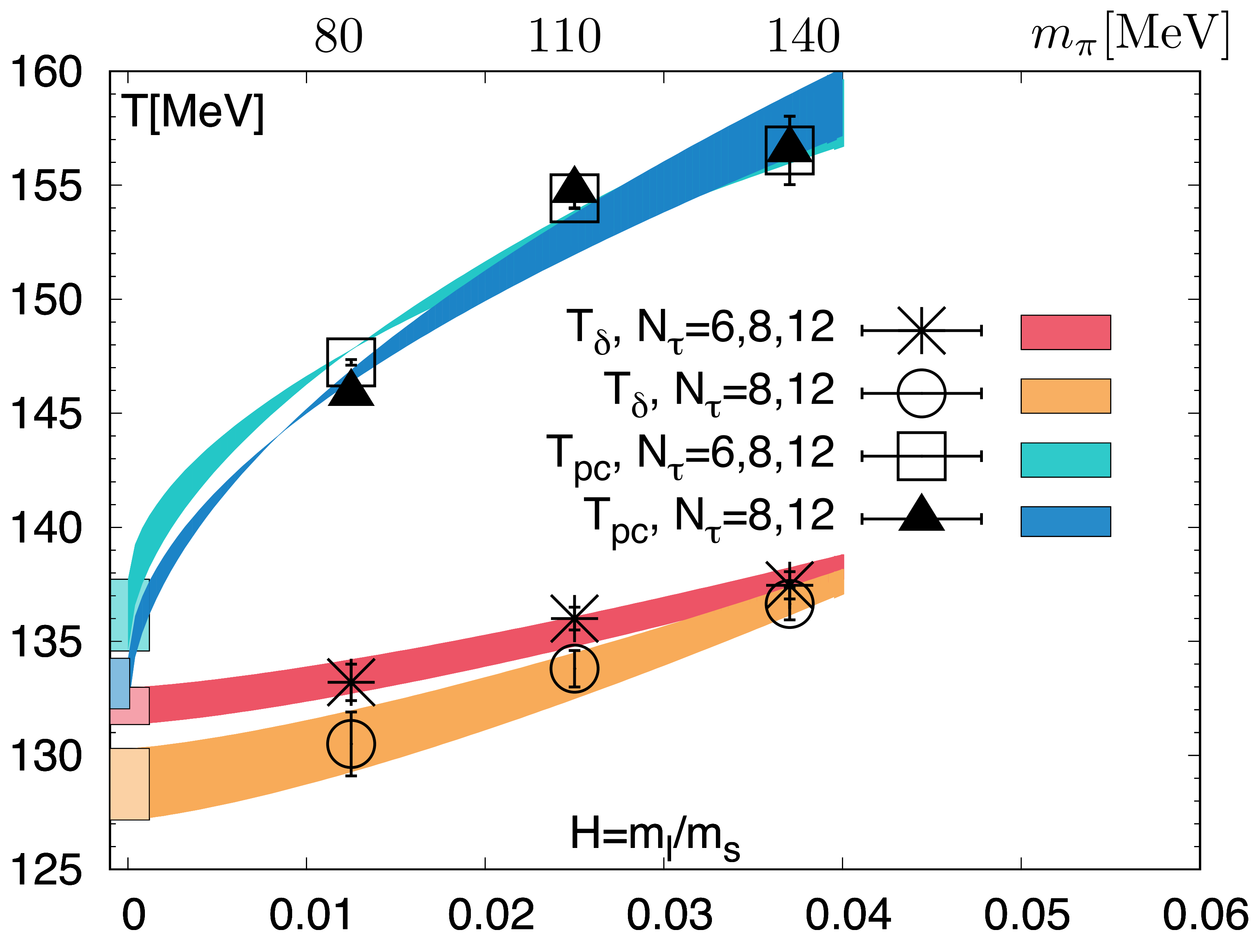
$$H_c \approx 0$$

- $m_s^{phy} > m_s^{tri}$: favors a 2nd order O(4) but not Z(2) chiral phase transition

chiral phase transition temperature in N$_f$=2+1 QCD

O(4) scaling fit: $T_X(H,L) = T_c^0 \left(1 + \left(\frac{z_X(z_L)}{z_0}\right) H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$

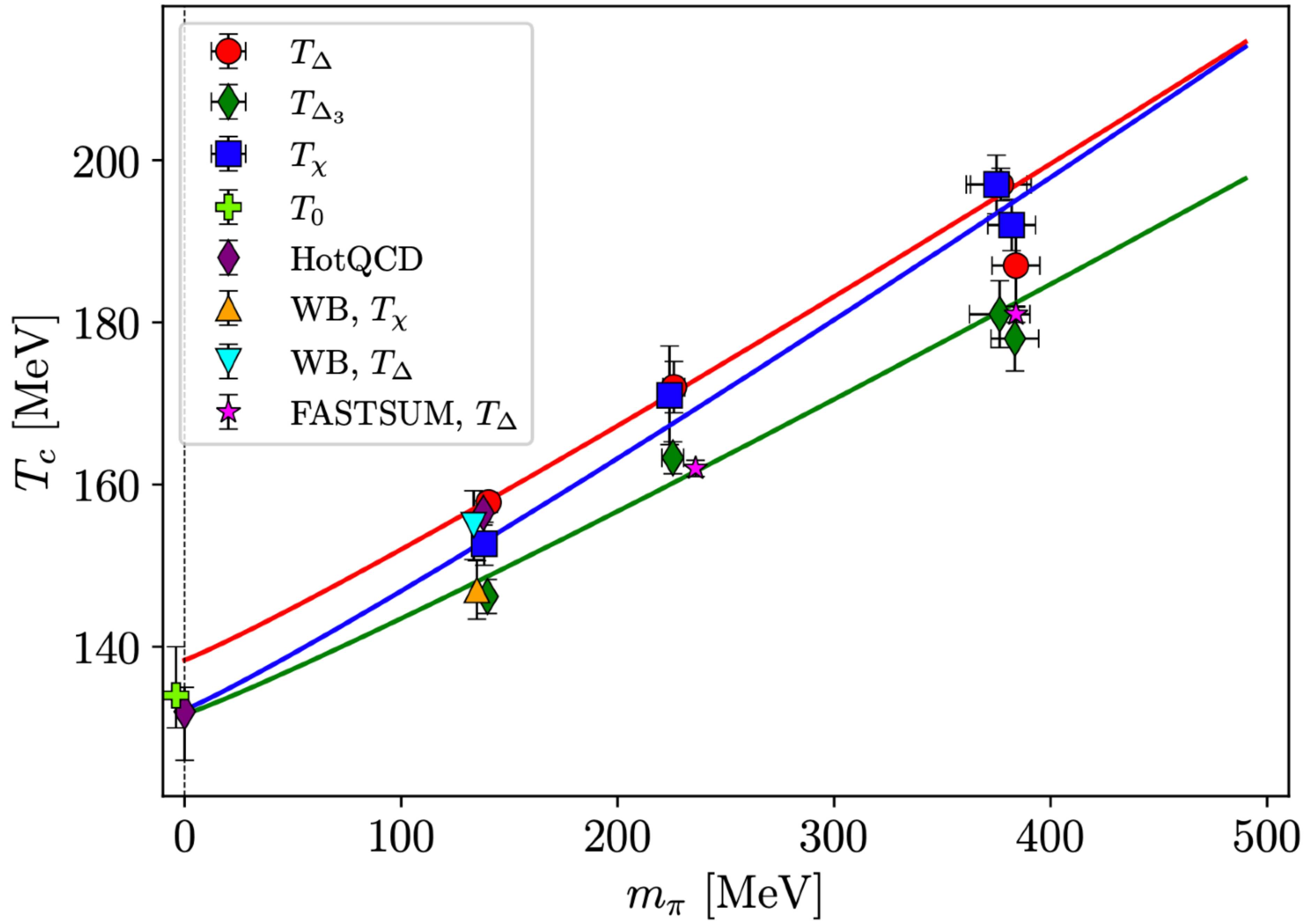
HISQ fermions



HTD et al.[HotQCD], Phys.Rev. Lett. 123 (2019) 062002,
S.-T. Li et al., Nucl.Phys.Rev. 37 (2020) 3, 674,
O. Kaczmarek et al., Acta Phys. Pol. B Proc. Suppl. 14 (2021) 291

$$T_c^0 = 132^{+3}_{-6}\ \text{MeV}$$

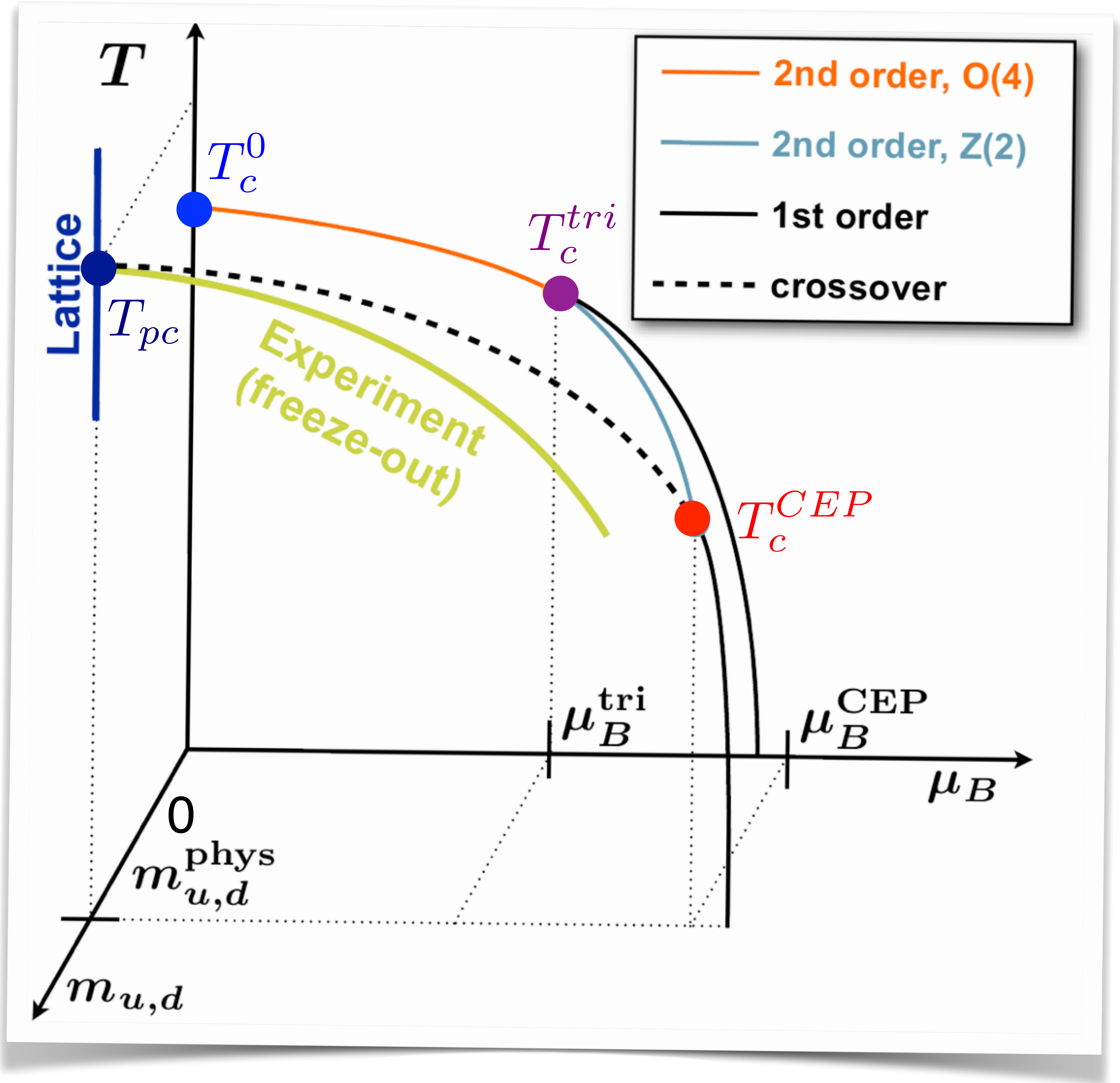
Twisted Wilson fermions



A.Y.Kotov, M.P.Lombardo and A.Trunin, Phys. Lett. B 823 (2021)136749

$$T_c^0 = 134^{+6}_{-4}\ \text{MeV}$$

Recap: relevance of criticality at μ_B=0 to CEP



T_{pc} : ≈ 156 MeV, chiral crossover T at μ_B=0

Bazavov et al., [HotQCD] Phys. Lett. B795 (2019) 15
Borsanyi et al.,Phys.Rev.Lett. 125 (2020) 052001

T_c^{CEP}: transition T at the critical end point

T_c^0 : ≈ 132 MeV chiral phase transition T at m_q=0 and μ_B=0

HTD et al.[HotQCD], PRL123 (2019) 062002
A.Y.Kotov et al., PLB 823 (2021)136749

T_c^{tri} : transition T at the tri-critical point

- Random Matrix Model & NJL suggests:

$$T_c^{tri} - T_c^{CEP}(m_q) \propto m_q^{2/5}$$

Y. Hatta & T. Ikeda, PRD67 (2003) 014028
M. A. Halasz et al, PRD 58 (1998) 096007
M. Buballa, S. Carignano, PLB791(2019)361

- $T_c^0(\mu_B)$ decreases as μ_B up to NLO from LQCD

O. Kaczmarek et al., PRD83 (2011) 014504
P. Hegde & HTD, PoS LATTICE2015 (2016) 141
O. Kaczmarek et al., *PoS* LATTICE2021 (2022) 429

Indication

$$T_c^0 > T_c^{tri} > T_c^{CEP}$$

Microscopic manifestation of the symmetry restorations: Dirac Eigenvalues and their correlations C_n

$$\langle\bar{\psi}\psi\rangle = \int_0^\infty \frac{4m_l\,\rho}{\lambda^2+m_l^2}\,\mathrm{d}\lambda \quad , \lim_{m_l\to 0}\langle\bar{\psi}\psi\rangle = \pi\rho(0)$$

$$\chi_1 \equiv \chi_{disc} = \int_0^\infty \frac{4m_l\,\partial\rho/\partial m_l}{\lambda^2+m_l^2}\mathrm{d}\lambda$$

$$\propto \left\langle \left(\bar{\psi}\psi - \langle\bar{\psi}\psi\rangle\right)^2 \right\rangle = \kappa_2(\bar{\psi}\psi)$$

$$\chi_2 = \int_0^\infty \frac{4m_l\,\partial^2\rho/\partial m_l^2}{\lambda^2+m_l^2}\mathrm{d}\lambda \propto \kappa_3(\bar{\psi}\psi) + \cdots$$

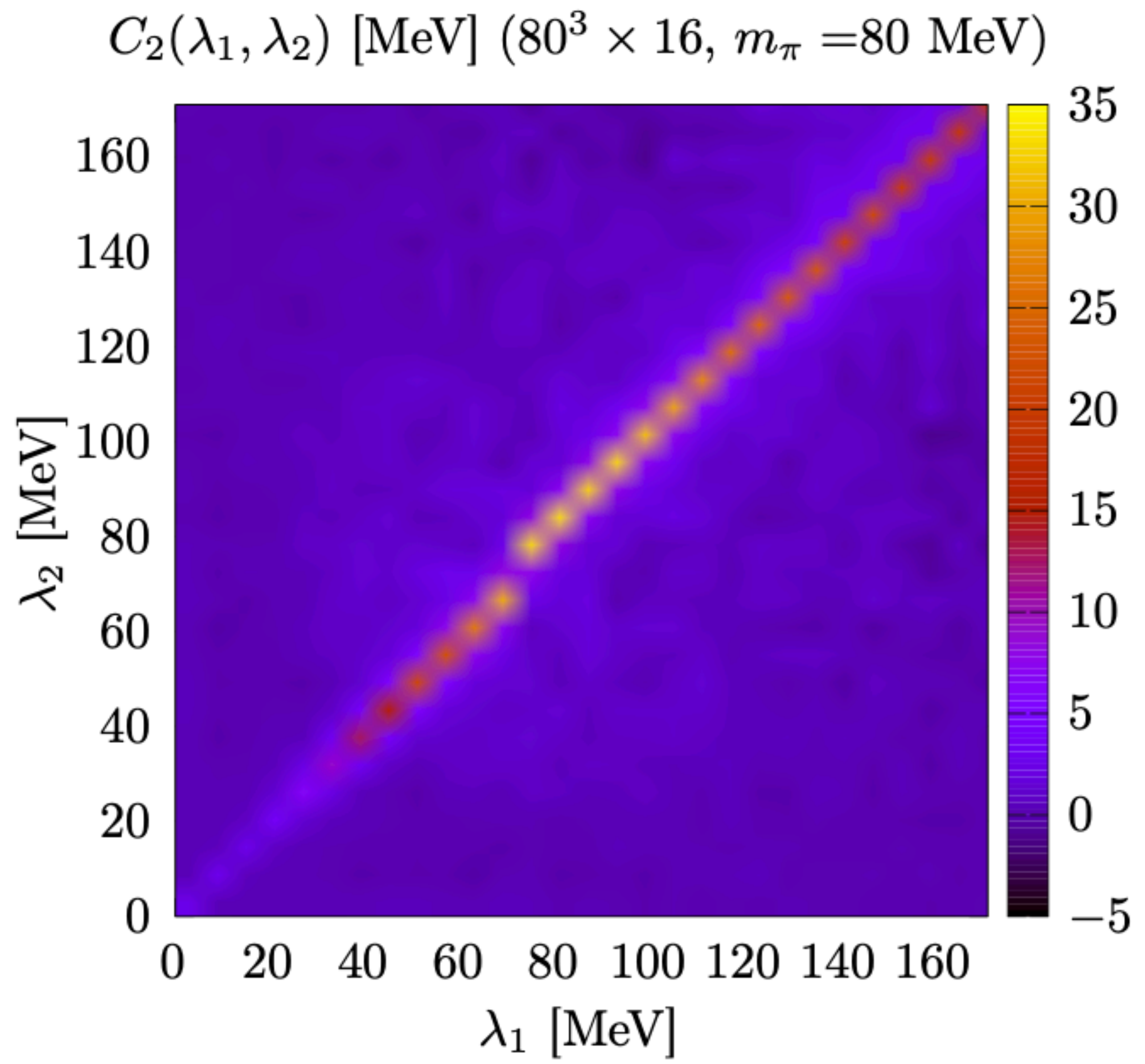
… …

$$\chi_n = \int_0^\infty \frac{4m_l\,\partial^n\rho/\partial m_l^n}{\lambda^2+m_l^2}\mathrm{d}\lambda \propto \kappa_{n+1}(\bar{\psi}\psi) + \cdots$$

$$\partial^n\rho(\lambda)/\partial m_l^n = f(C_{n+1}, C_n \cdots, C_2)$$

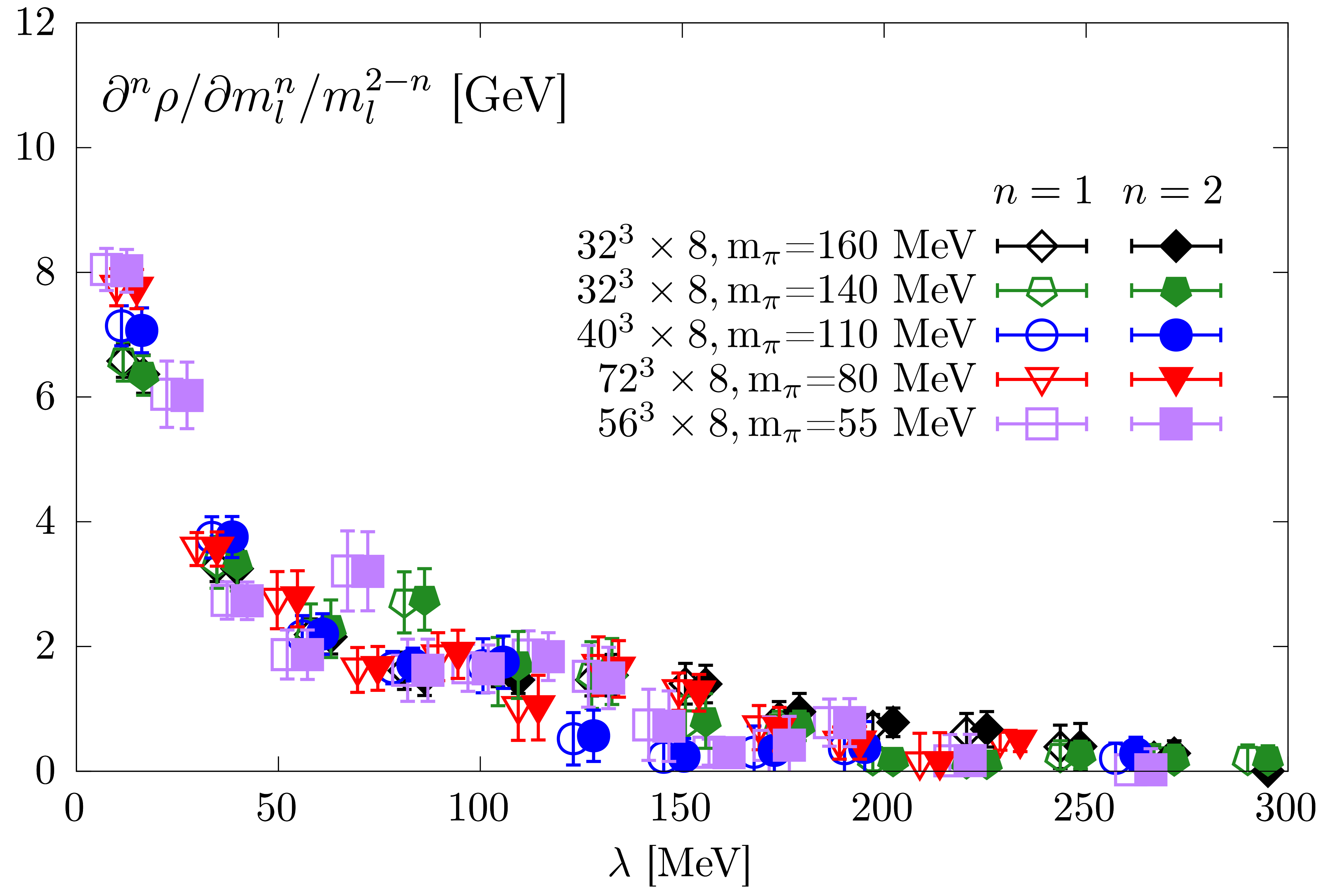
$$C_n = \kappa_n(\rho_U(\lambda))$$

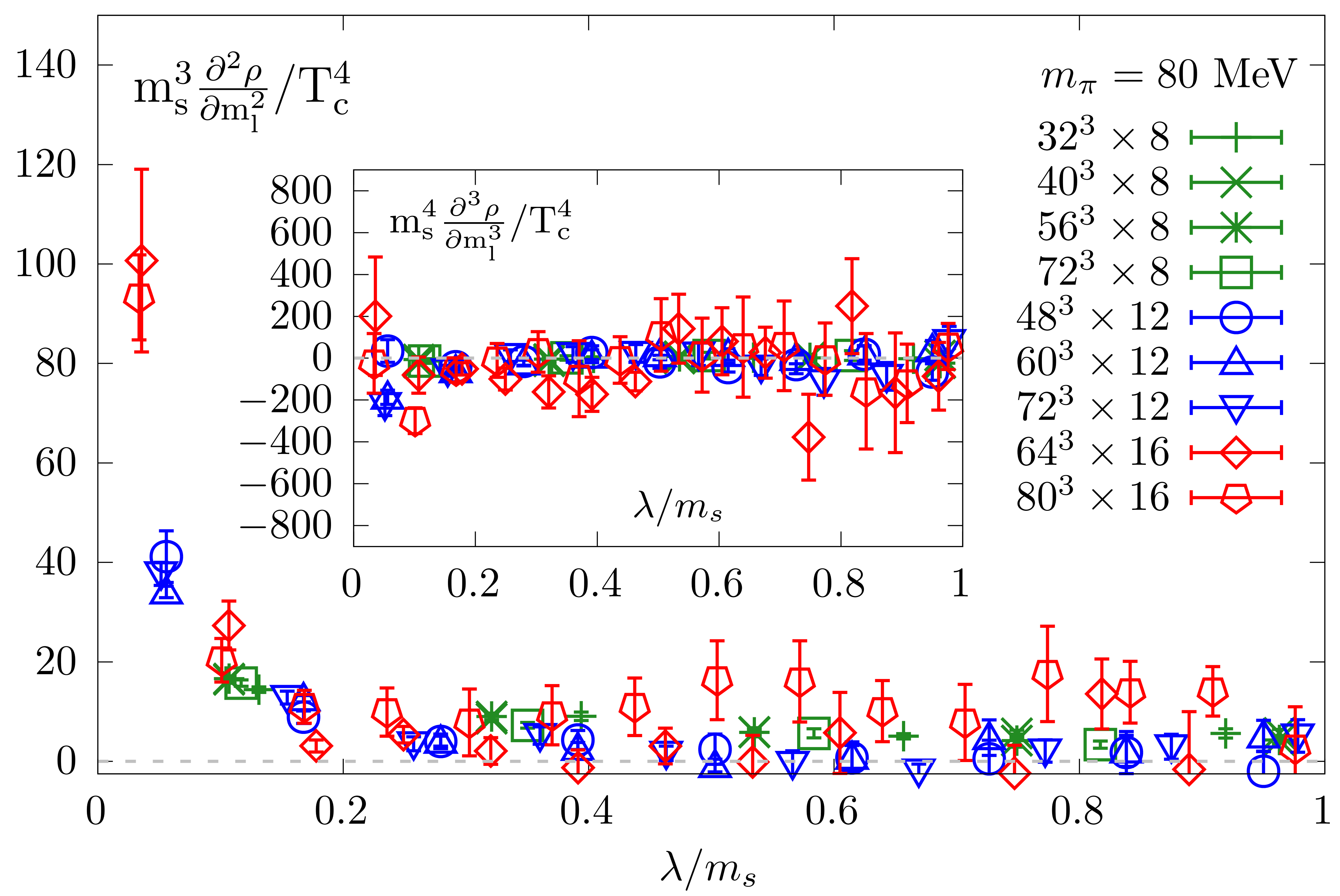
E.g.: $\frac{V}{T}\frac{\partial\rho}{\partial m_l} = \int_0^\infty \mathrm{d}\lambda_2\,\frac{4m_l\,C_2(\lambda,\lambda_2;m_l)}{\lambda_2^2+m_l^2}$



HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang, PRL126(2021)082001

1st, 2nd & 3rd quark mass derivative of ρ on N_τ=8 lattices at $T \approx 205$ MeV $\simeq 1.6T_c$





$$m_l^{-1}\partial\rho/\partial m_l \approx \partial^2\rho/\partial m_l^2$$

$$\partial^3\rho/\partial m_l^3 \approx 0$$

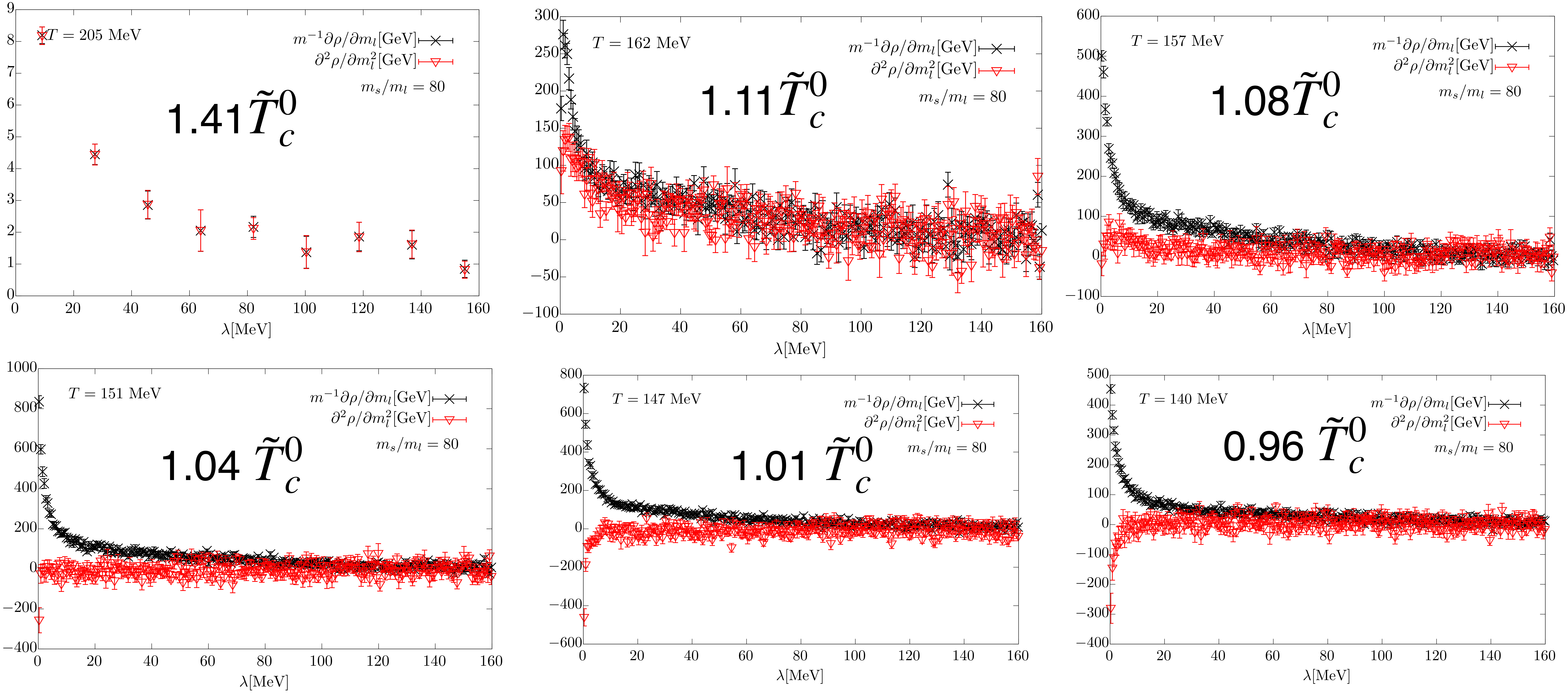
Dilute instanton gas approximation at 1.6 Tc: $\rho(\lambda\to 0, m_l\to 0)\propto m_l^2\delta(\lambda)$

HTD, S.-T. Li, A. Tomiya, S. Mukherjee, X.-D. Wang, Y. Zhang, PRL126(2021)082001

Dilute instanton gas approximation does not hold towards T_c

$$\partial^2\rho/\partial m_l \neq m_l^{-1}\partial\rho/\partial m$$

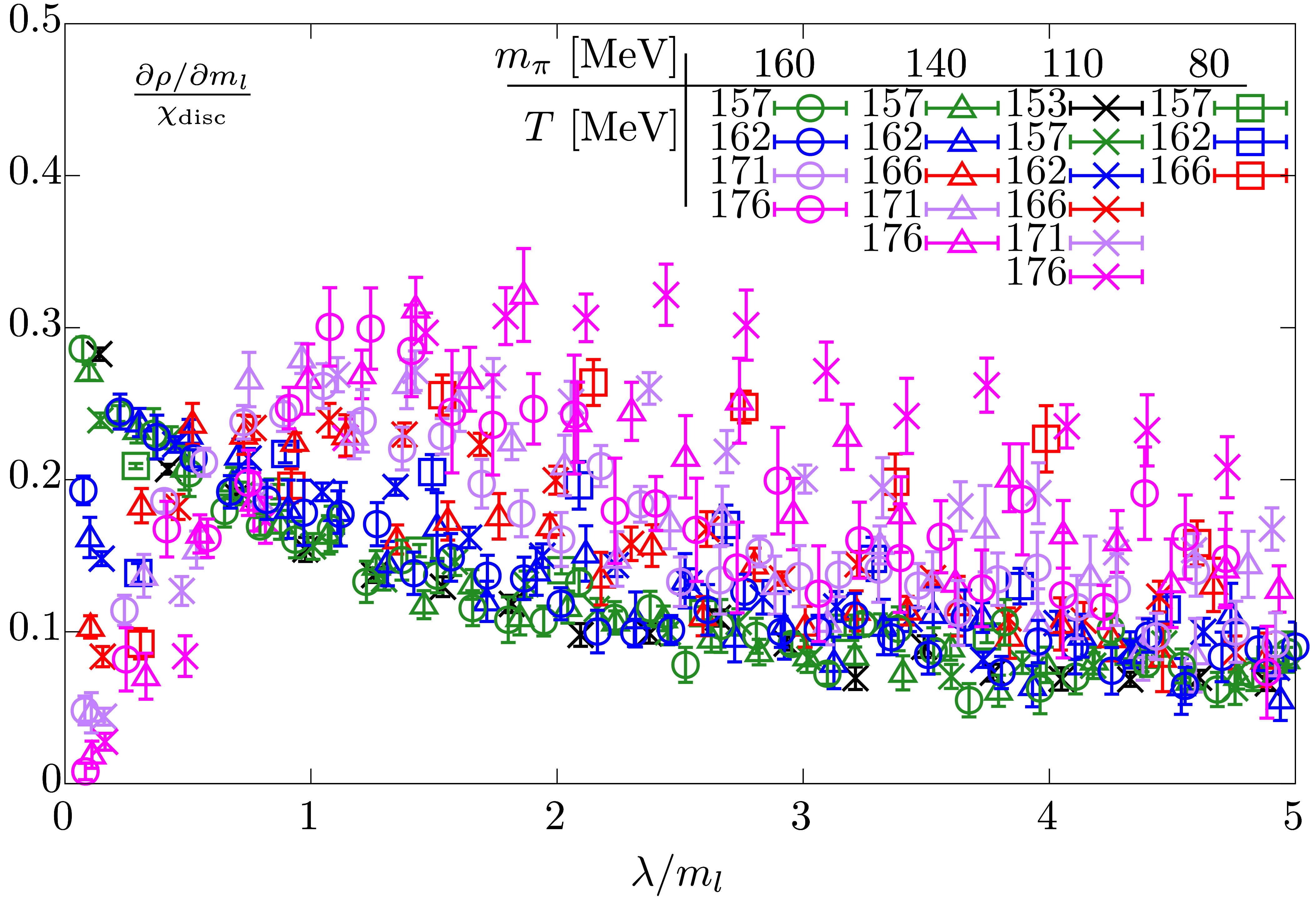
$\tilde{T}_c^0 \equiv T_c^0(N_\tau = 8) \approx 145.6$ MeV



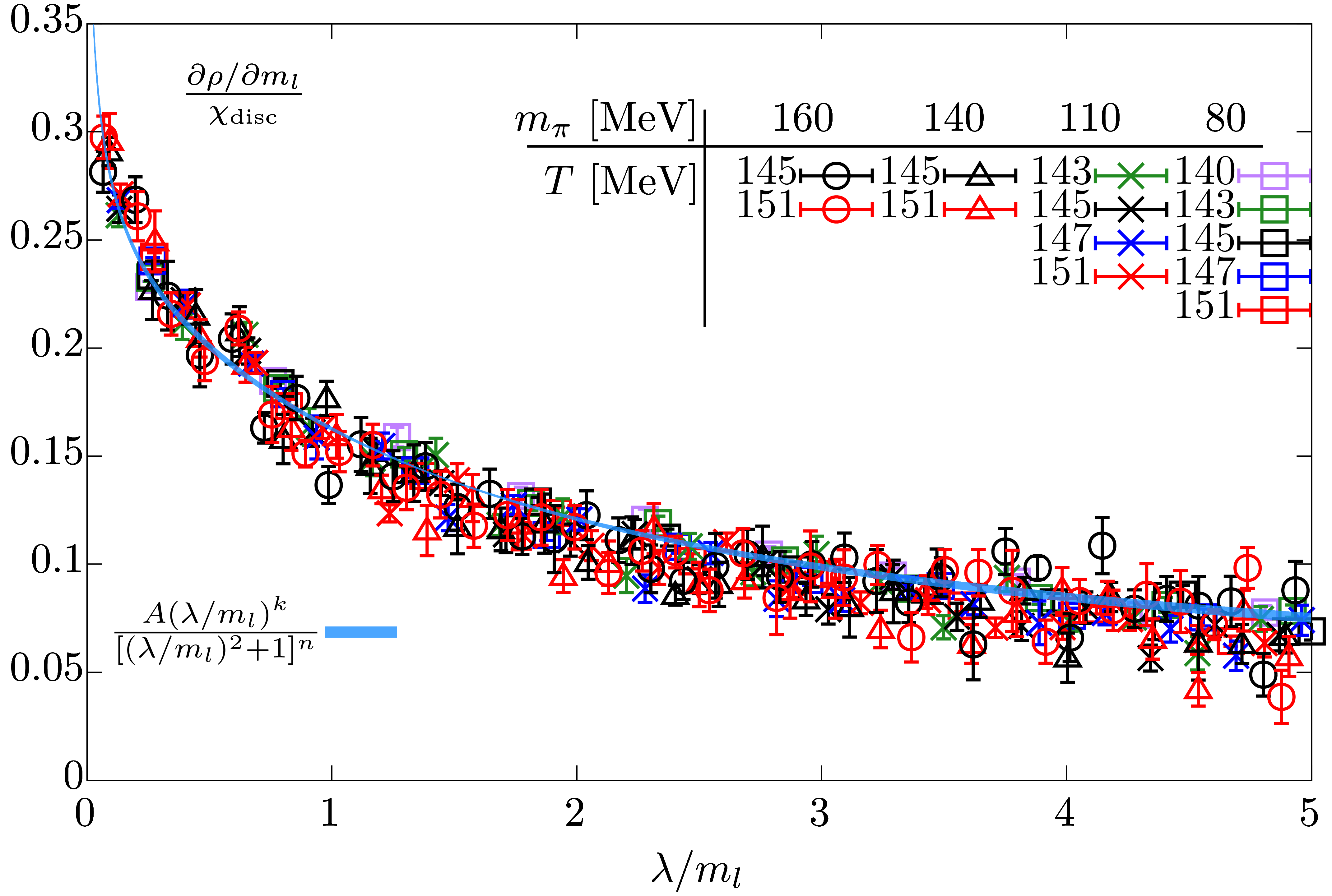
HTD, Wei-Ping Huang, Min Lin et al., PoS LATTICE2021 (2022) 591

Scaling behavior in $\partial\rho/\partial m_l/\chi_{\mathrm{disc}}$ v.s. λ/m_l near $\mathrm{T_c}$

No scaling at $T \in [1.08, 1.2]\tilde{T}_c^0$



Scaling at $T \in [0.96, 1.04]\tilde{T}_c^0$

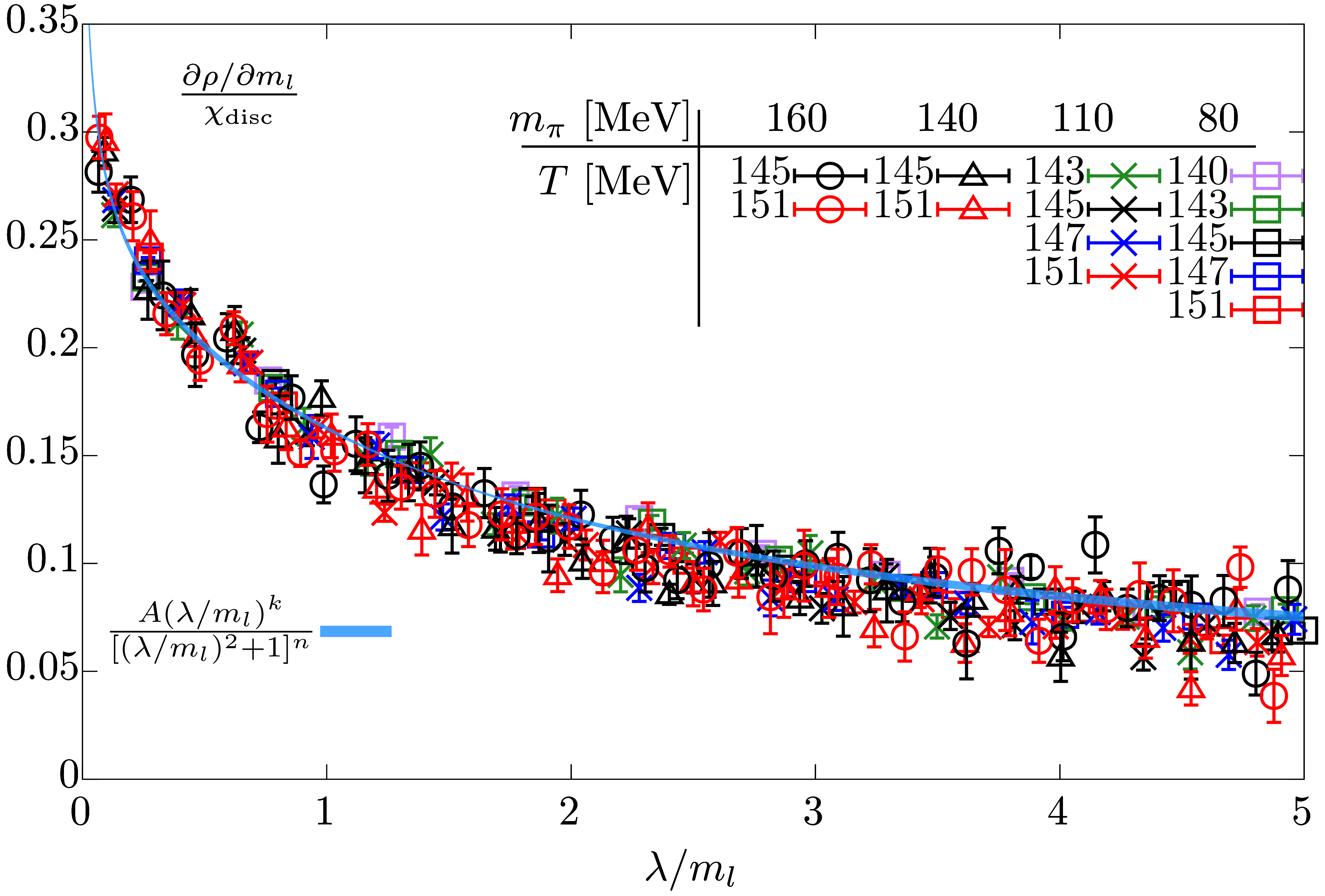


Wei-Ping Huang et al., work in progress,
HTD, Wei-Ping Huang, Min Lin et al., PoS LATTICE2021 (2022) 591

$$\partial\rho/\partial m_l = f\left(\lambda/m_l\right) \times \chi_{disc}$$

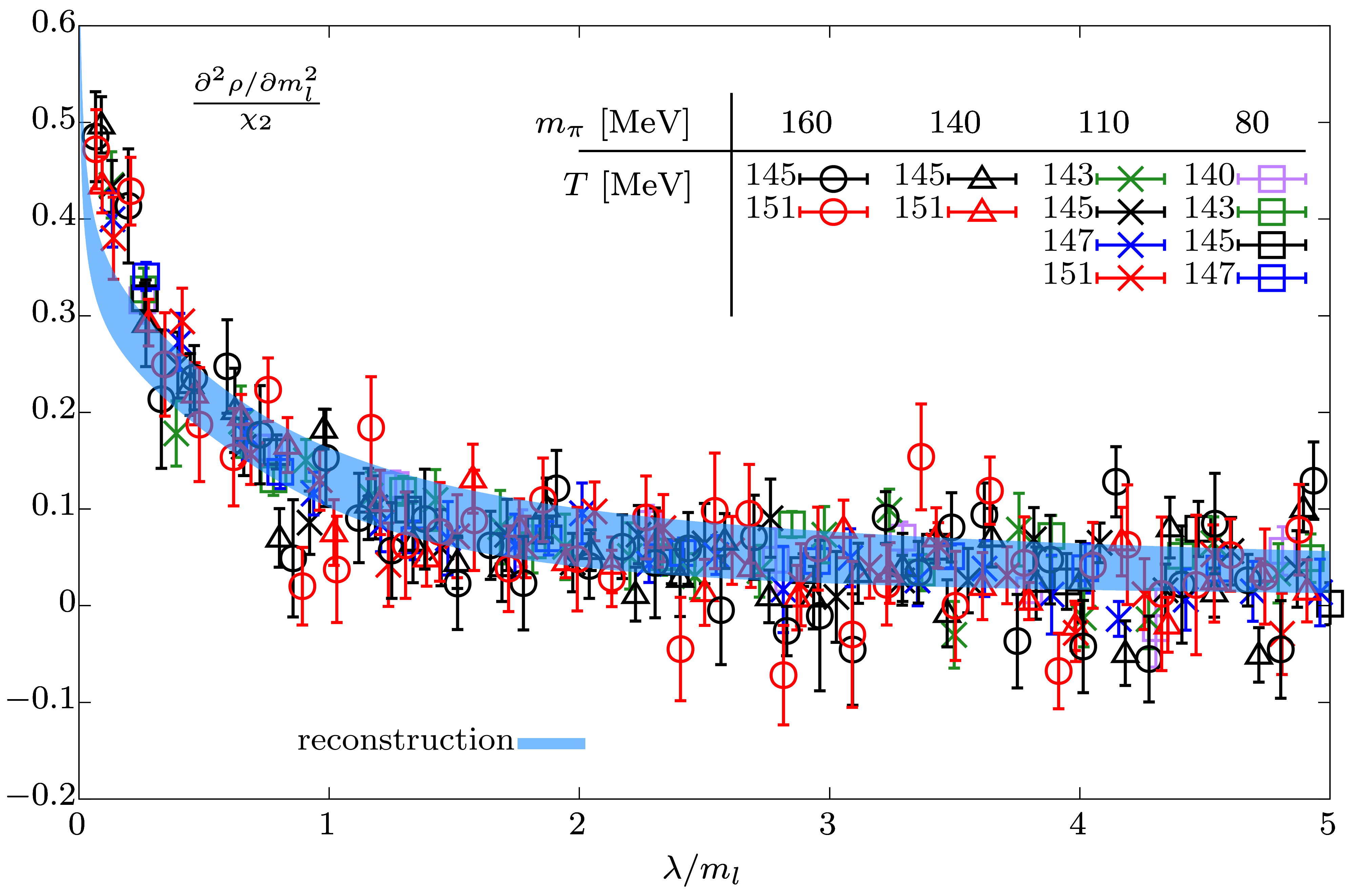
Criticality of QCD in correlated Dirac Eigenvalues

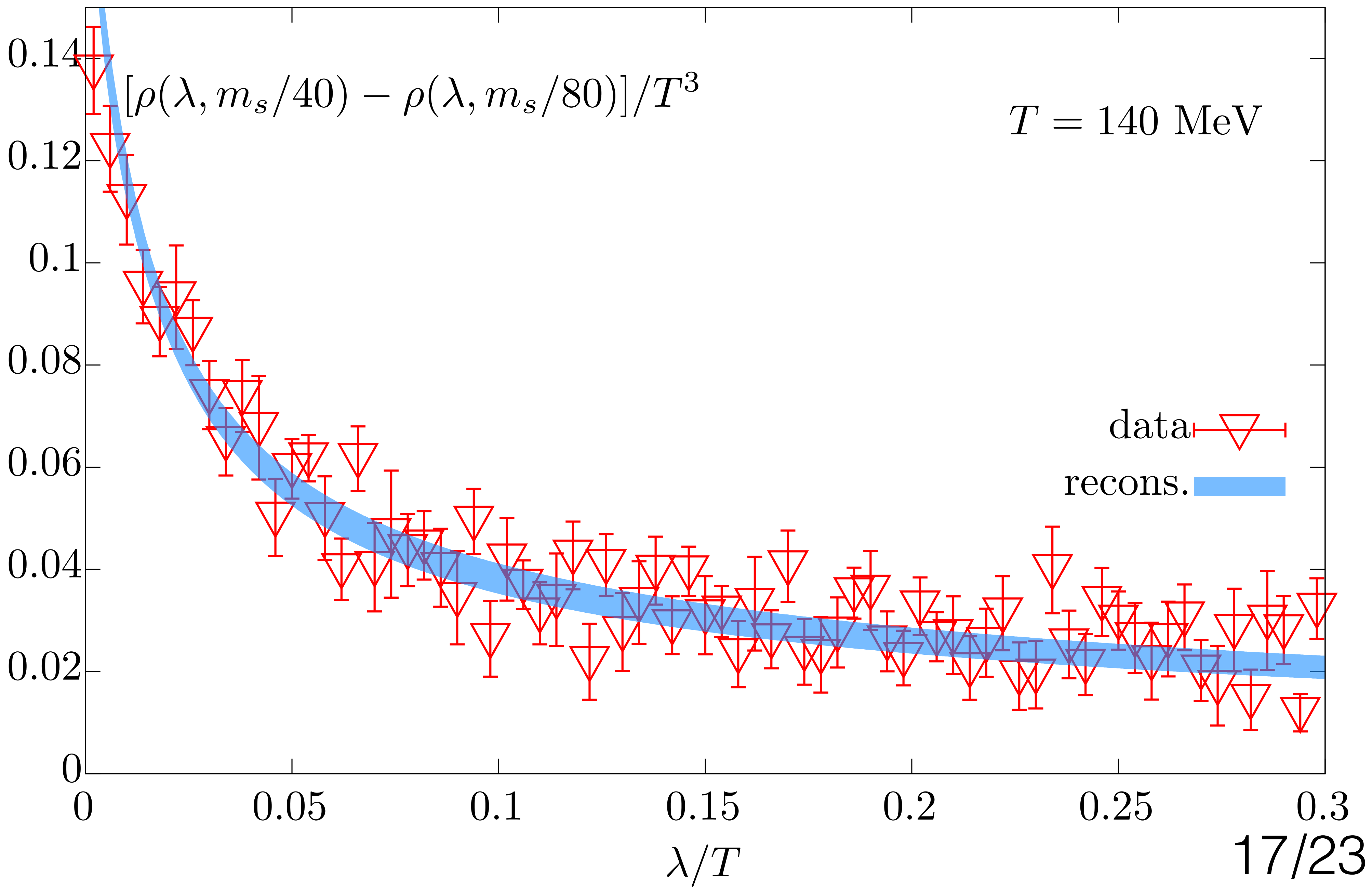
$$\partial\rho/\partial m_l = f\left(\lambda/m_l\right) \times \chi_{disc}$$



Wei-Ping Huang et al., work in progress

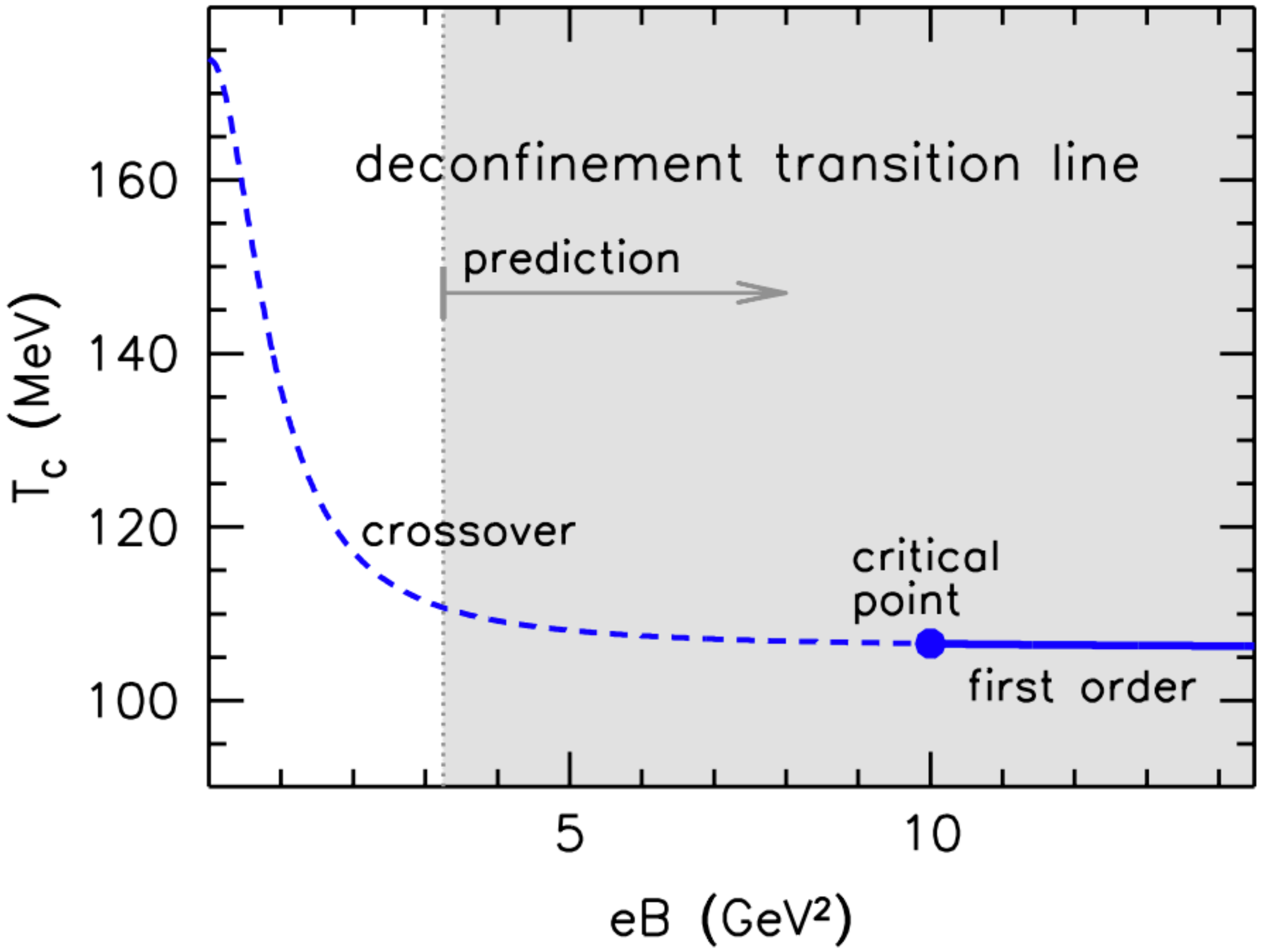
Reproduction of $\partial^2\rho/\partial m_l^2/\chi_2$ & ρ without fits!





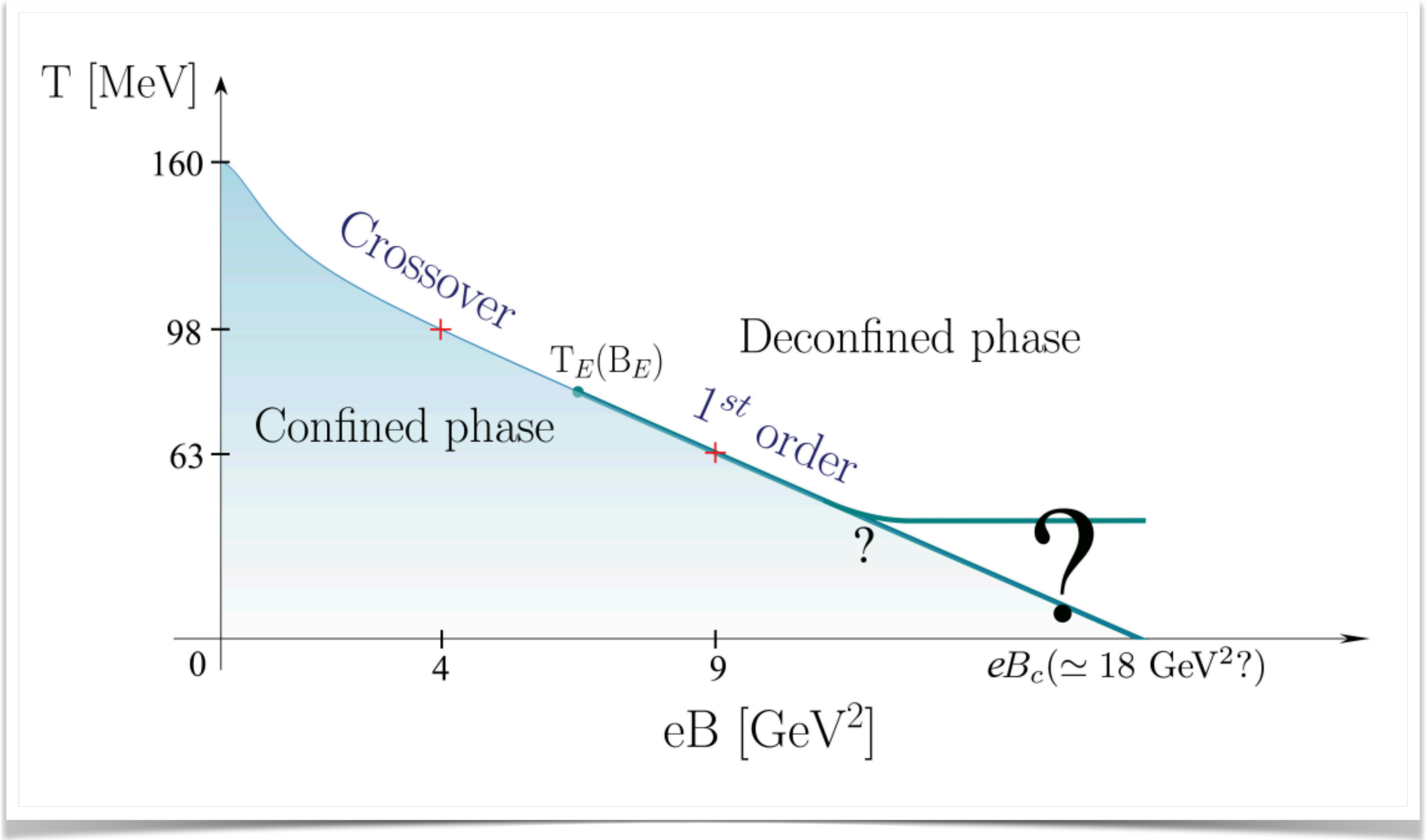
A plausible CEP in T-eB plane

Prediction of
a Critical End Point in the T-eB plane



G. Endrodi, JHEP 07 (2015) 173

Observation of a 1st order phase transition at eB=9 GeV²



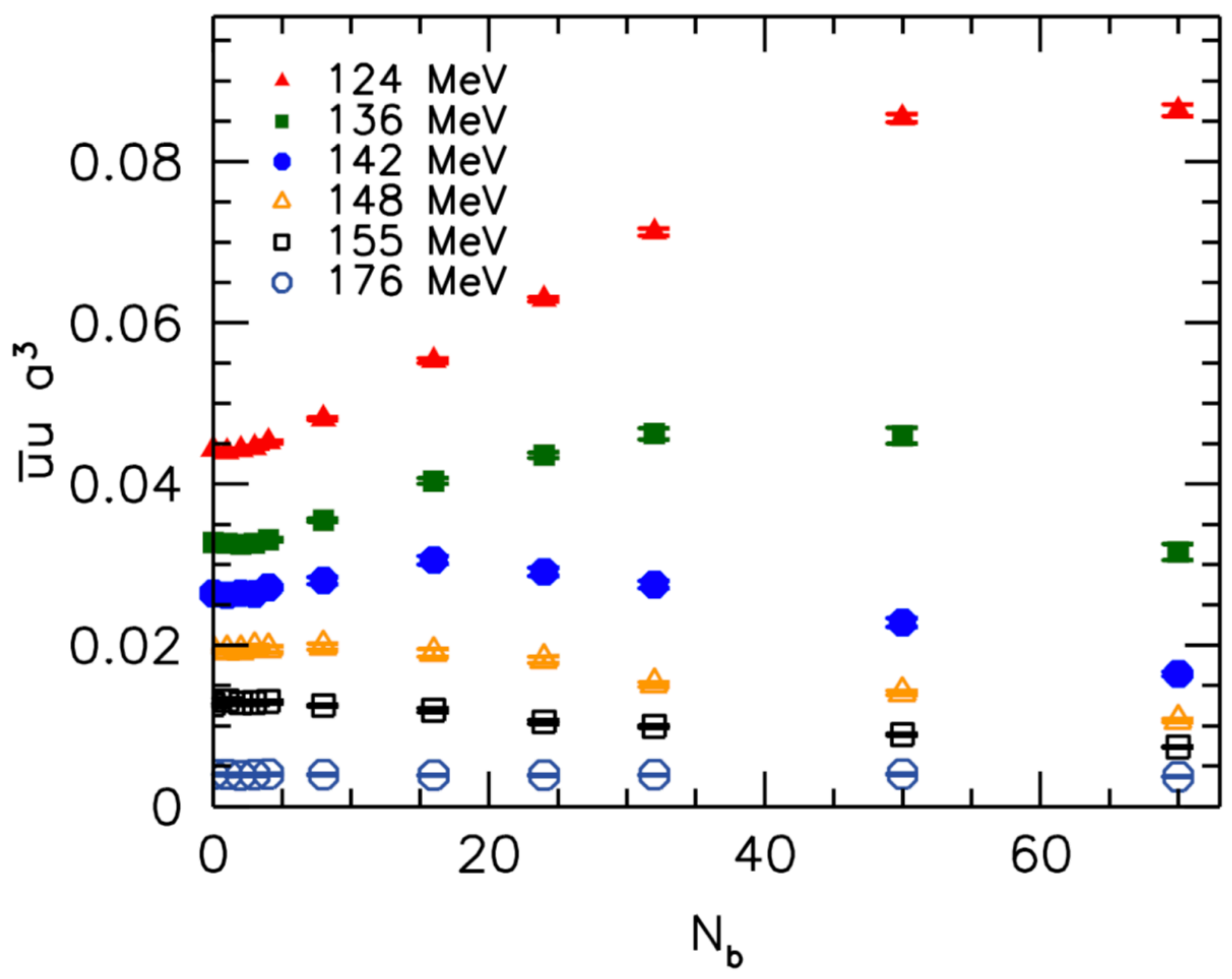
M. D'Elia et al., Phys.Rev.D 105 (2022) 3, 034511

QCD transition in strong magnetic fields

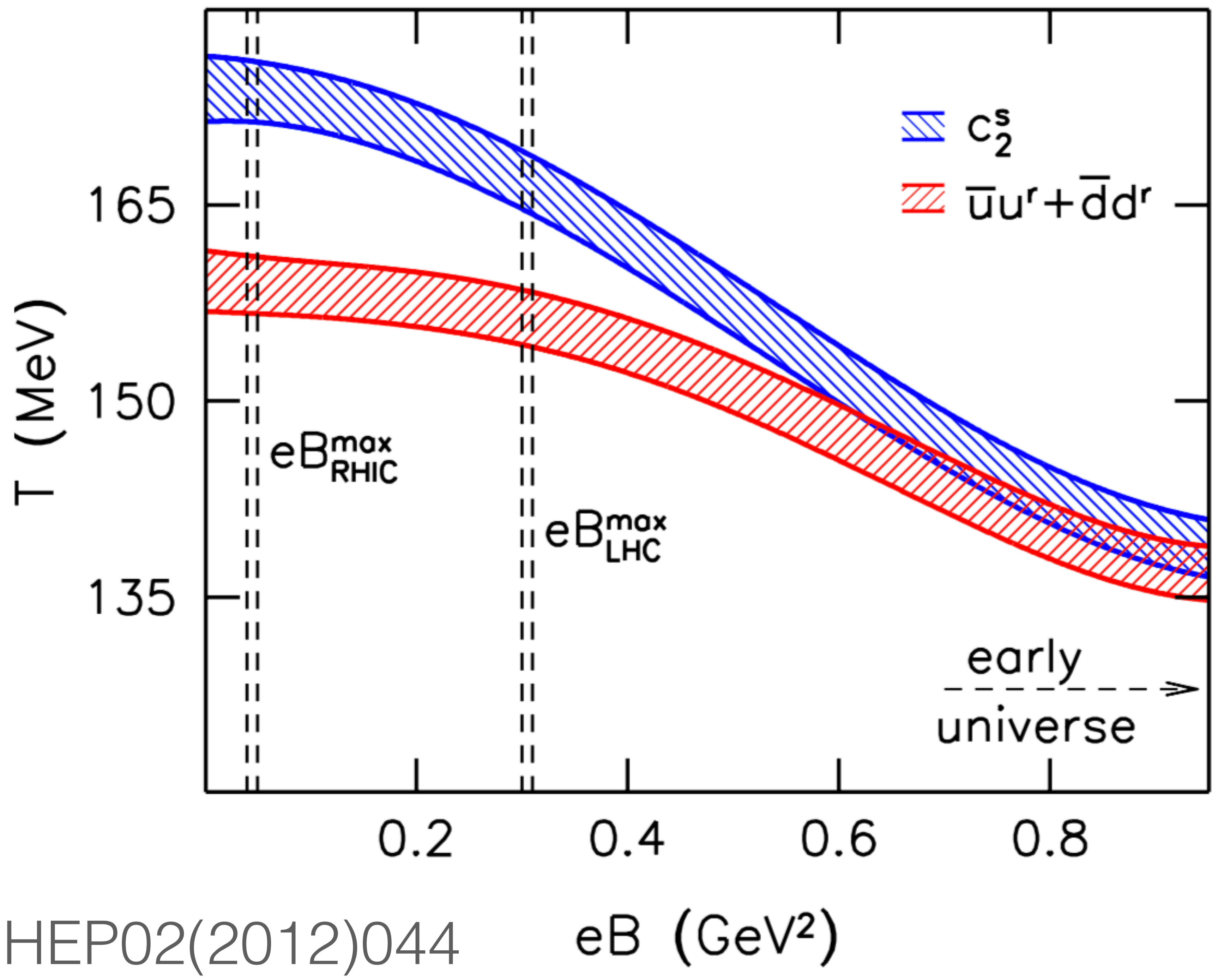
Inverse magnetic catalyses and reduction of T_{pc}

Continuum extrapolated lattice QCD results with physical pion mass

Inverse magnetic catalysis (IMC)

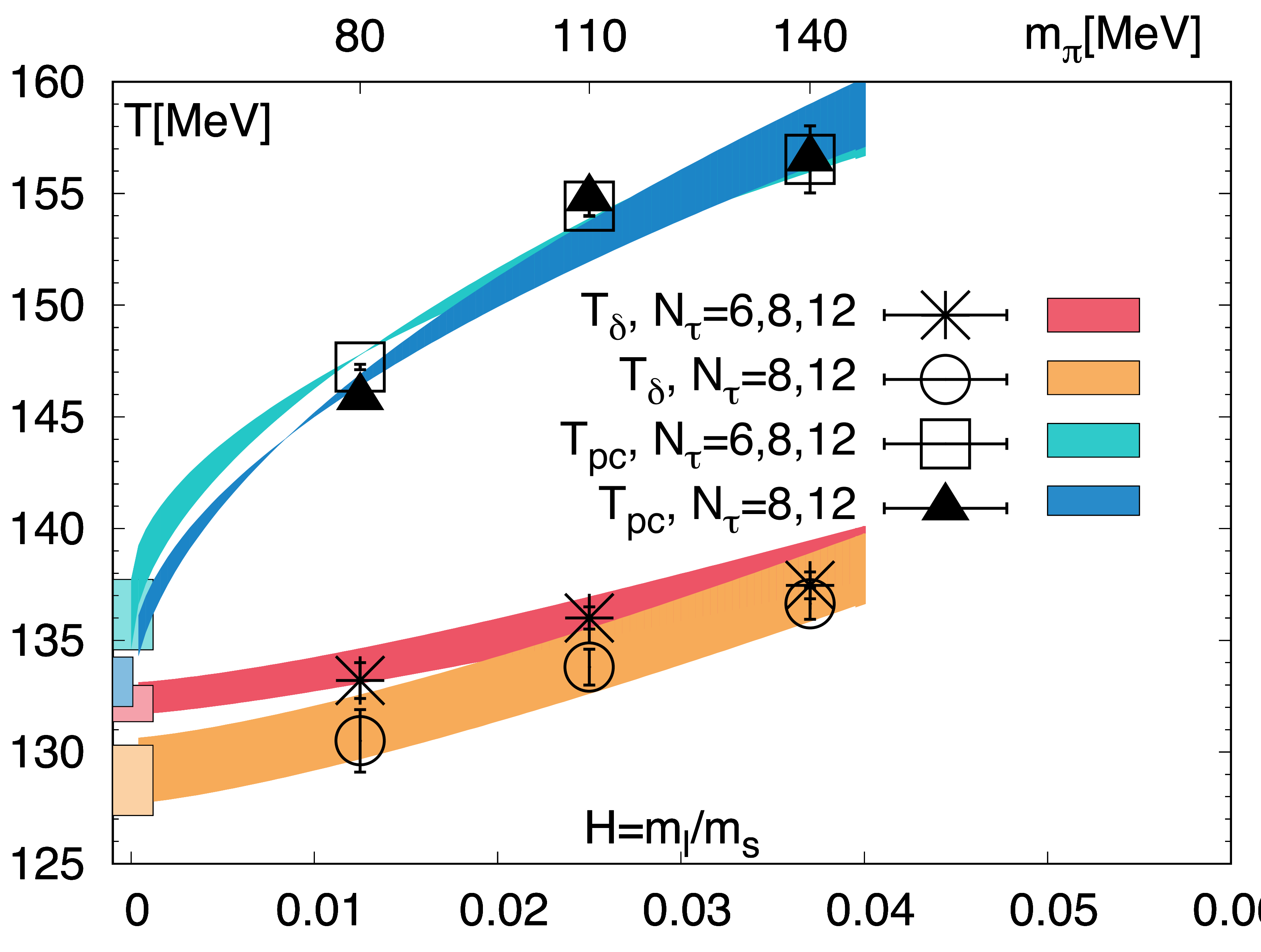


$eB\uparrow\ T_{pc}\downarrow$



Bali et al., JHEP02(2012)044

eB=0, N_f=2+1 QCD



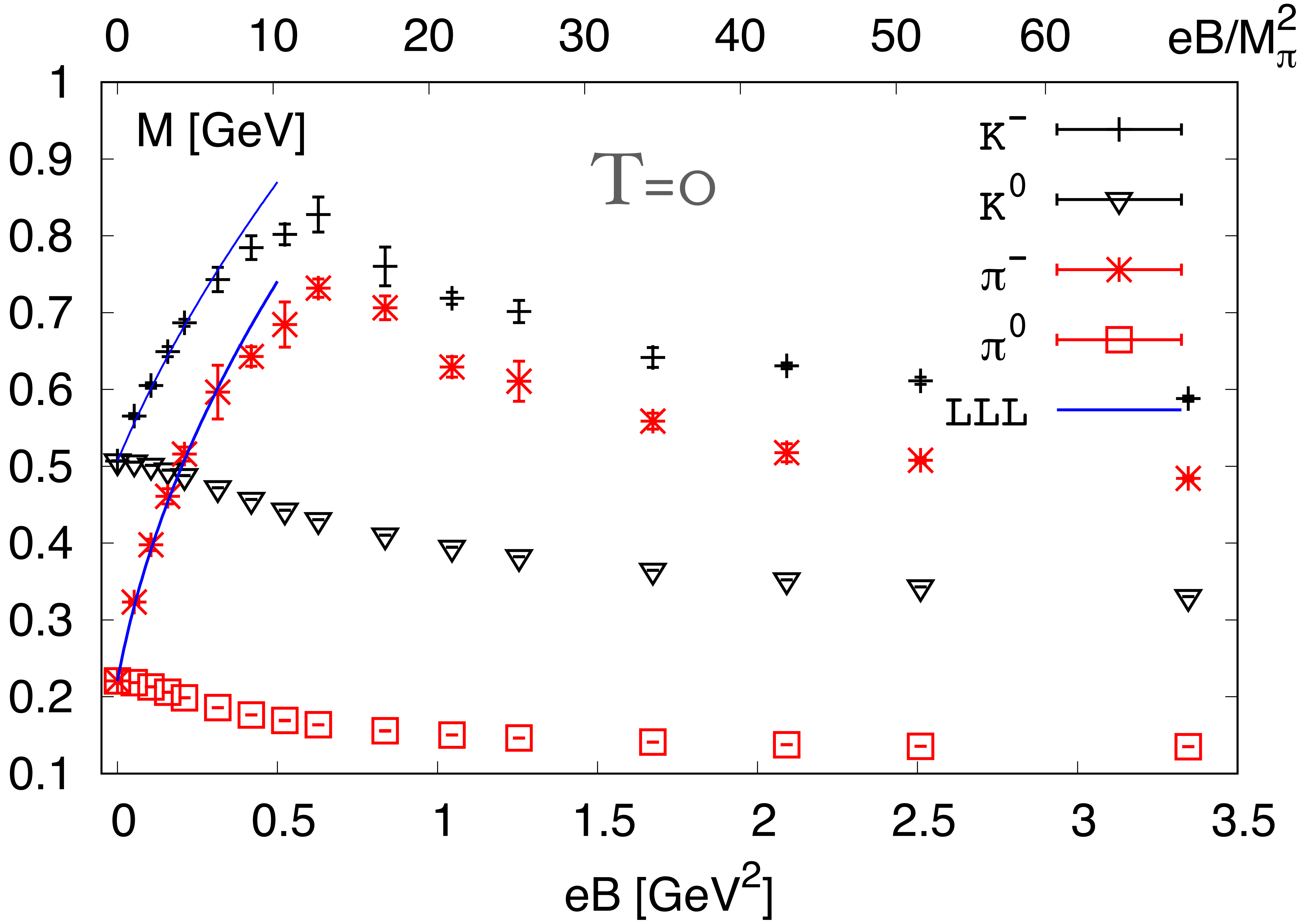
HTD, P. Hegde, O. Kaczmarek et al.[HotQCD], Phys. Rev. Lett. 123 (2019) 062002
HTD, arXiv:2002.11957

Reduction of T_{pc} always associated with IMC? Not necessarily!

Role of hadrons?

Masses of $\pi^{0,\pm}$ and $K^{0,\pm}$ and energy density

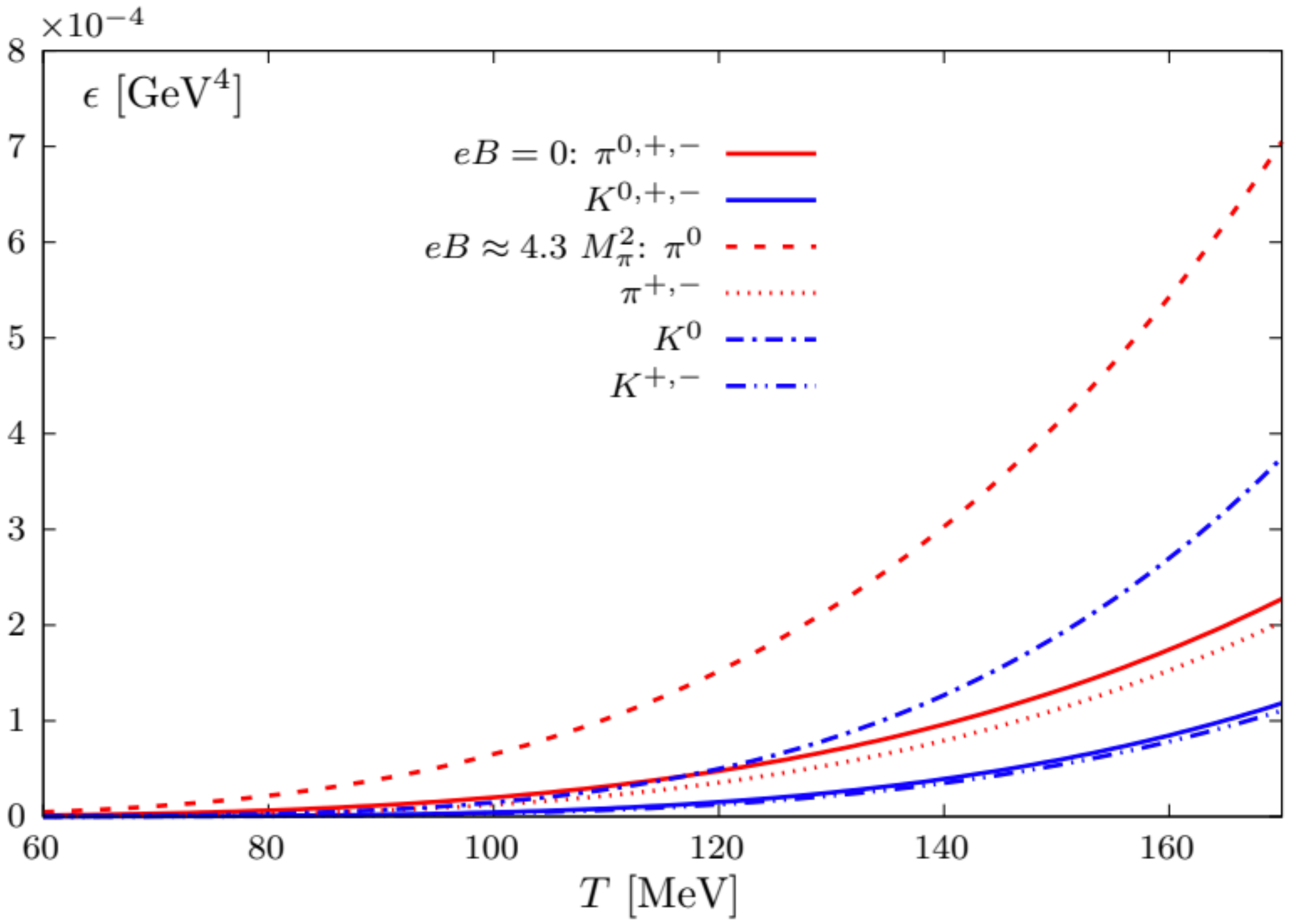
$N_{f=2+1}$ QCD, $M_\pi(eB=0) \approx 220$ MeV,
$32^3 \times 96$ lattices with $a^{-1} \approx 1.7$ GeV and HISQ action



HTD, S.-T. Li, A. Tomiya, X.-D. Wang, Y. Zhang, PRD 126 (2021) 082001

See quenched LQCD results in Bali et al., PRD 97 (2018) 034505, Luschevskaya et al., NPB 898 (2015) 627

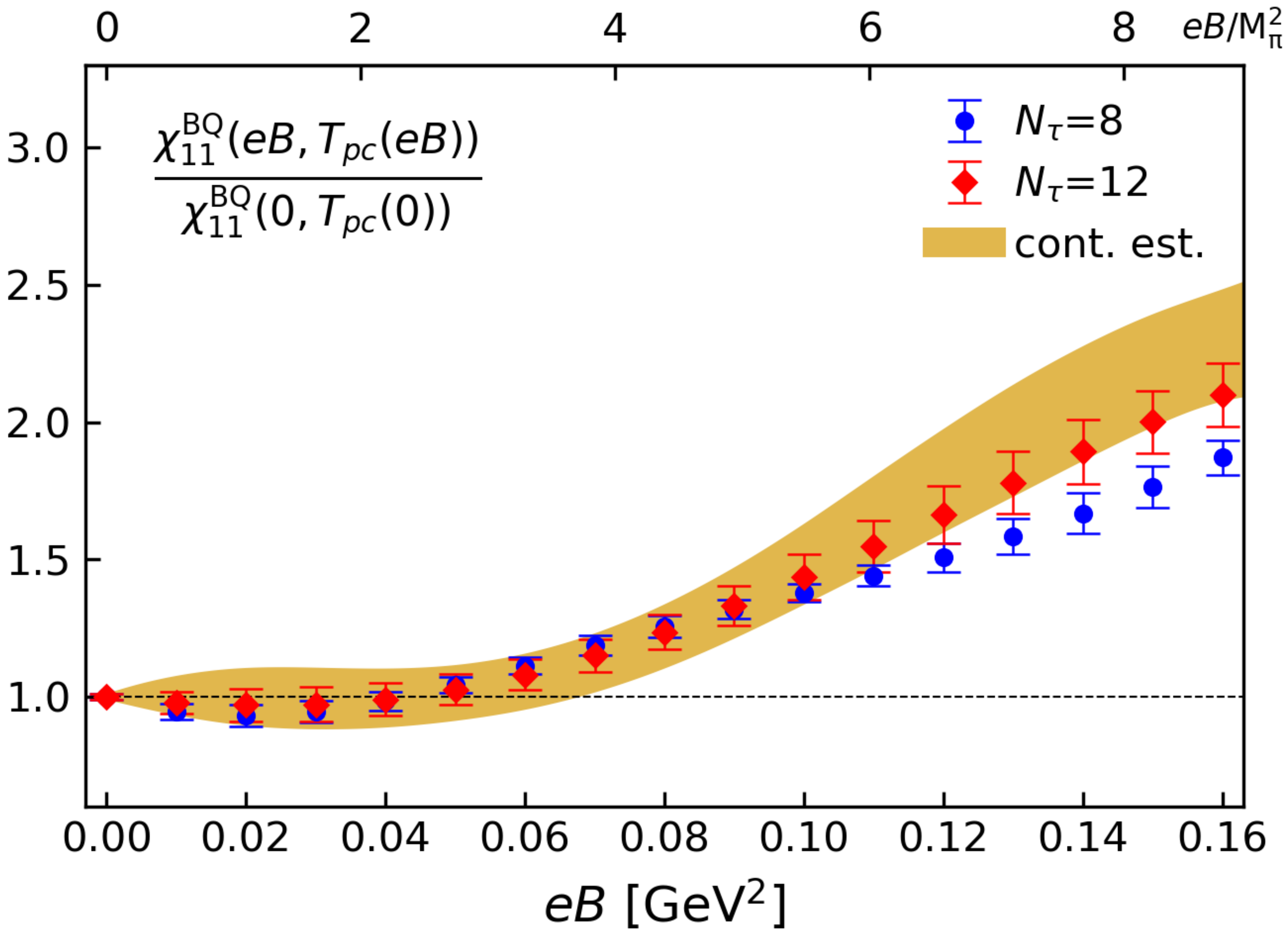
Energy density in Hadron resonance gas model



HTD, S.-T. Li, Q. Shi, A. Tomiya, X.-D. Wang, Y. Zhang, arXiv: 2011.04870

Ratio X(*eB*)/X(*eB*=0) for 2nd order off-diagonal fluctuations

N_f=2+1 QCD, $M_\pi(eB=0) \approx 135$ MeV, $T_{pc}(eB=0) \approx 157$ MeV, $32^3 \times 8$ and $48^3 \times 12$ lattices with HISQ action



Central Collisions
Smaller eB

Peripheral Collisions
Larger eB

X(eB)/X(eB=0) : Rcp like observable

At $eB \simeq M_\pi^2$: deviation from unity is mild

At $eB \simeq 8M_\pi^2$: ~2 !

Note: $T_{pc}(eB \simeq 10M_\pi^2)/T_{pc}(eB=0) \sim 99\,\%$

HTD, S.-T. Li, J.-H. Liu and X.-D. Wang, QM2022, arXiv:2208.07285

Ratio X(*eB*)/X(*eB*=0) for 2nd order off-diagonal fluctuations

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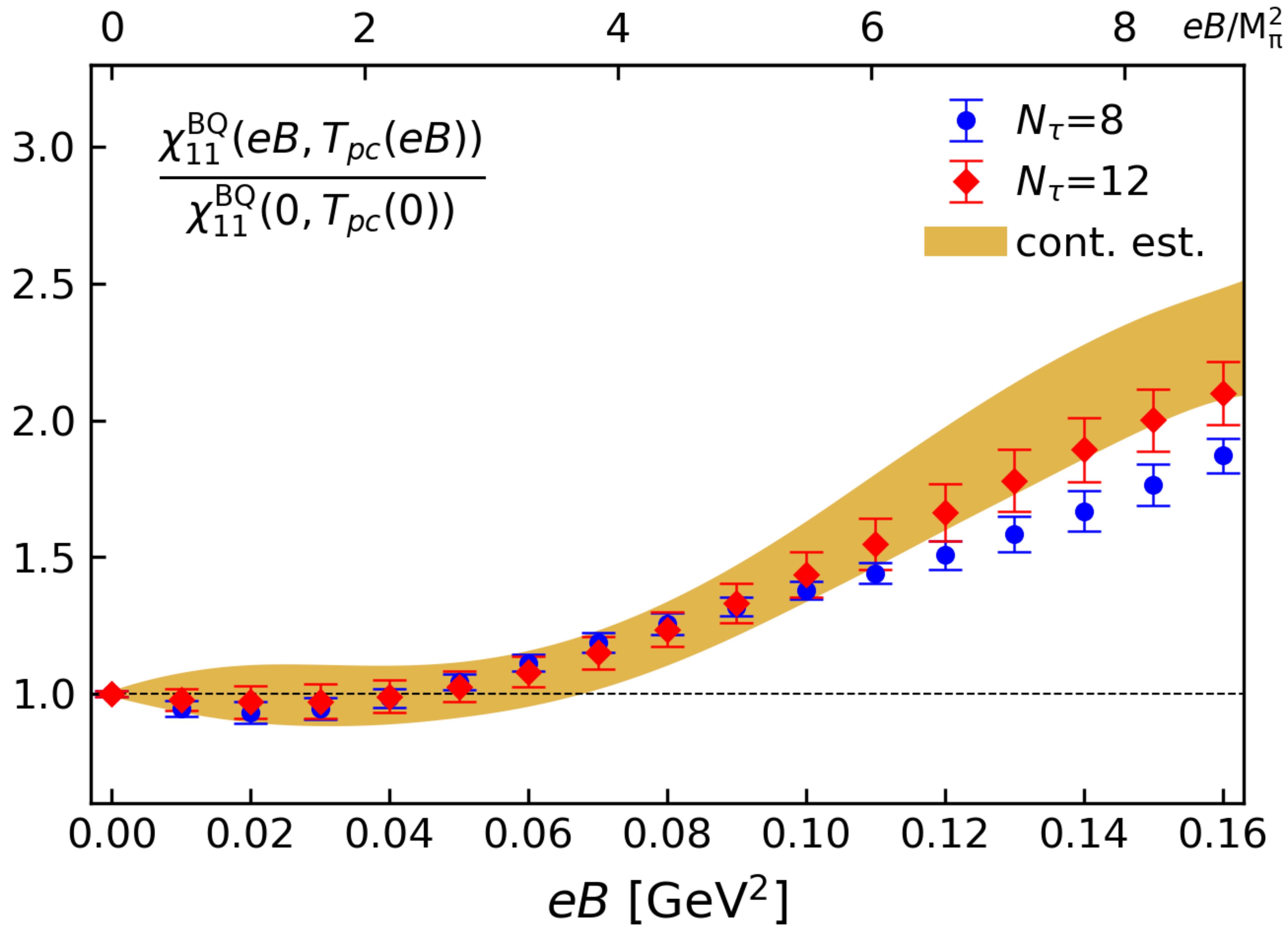
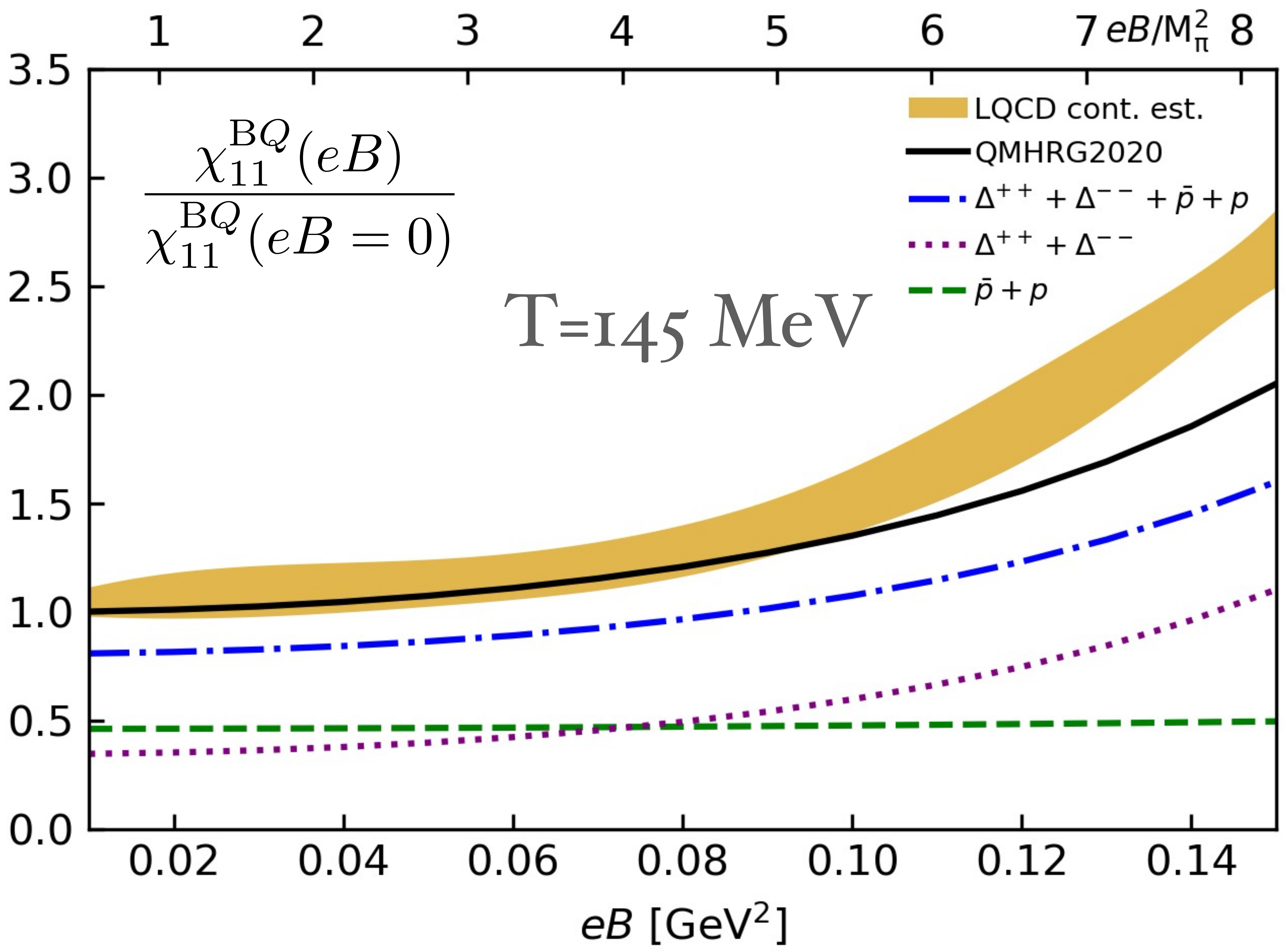




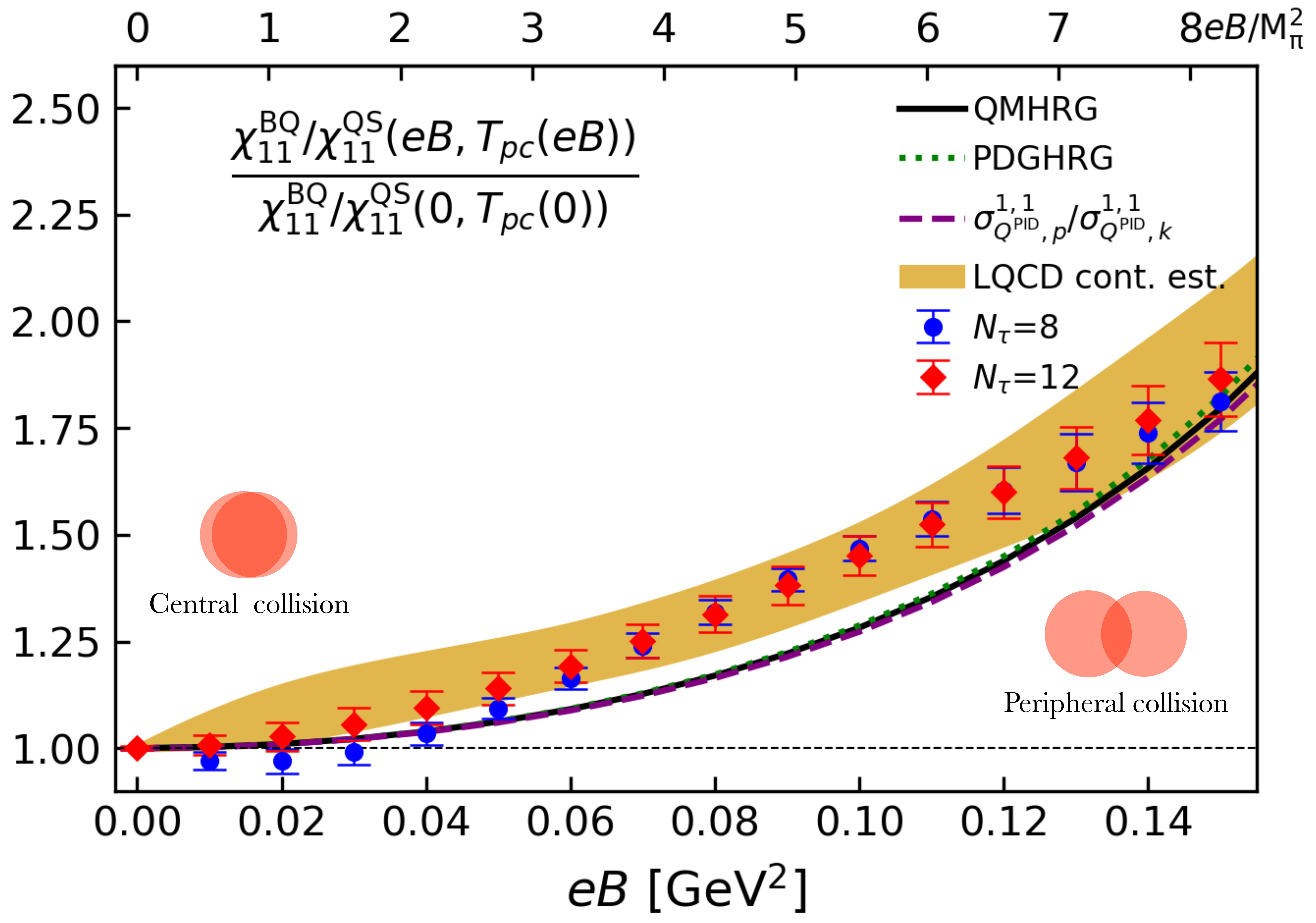
Central Collisions
Smaller eB

Peripheral Collisions
Larger eB

HTD, S.-T. Li, J.-H. Liu and X.-D. Wang, QM2022, arXiv:2208.07285

QCD benchmarks for the manifestation of *eB* in conserved charge fluctuations

See Jun-Hong Liu's talk on Fri.



Jun-Hong Liu et al., work in progress

Summary & Outlook

- Most relevant criticality at $\mu_B = 0$ to thermodynamics at CEP:
2nd O(4) phase transition $\Longrightarrow T_c^{CEP} < T_c^0 \approx 132\text{MeV}$

- Criticality in Dirac eigenvalue correlation: Microscopic manifestation of the criticality in a narrow T window

- QCD benchmarks for 2nd off-diagonal fluctuations in a background magnetic field: possibility to detect the existence of a magnetic field in HIC

- Search for criticality in the T-eB plane