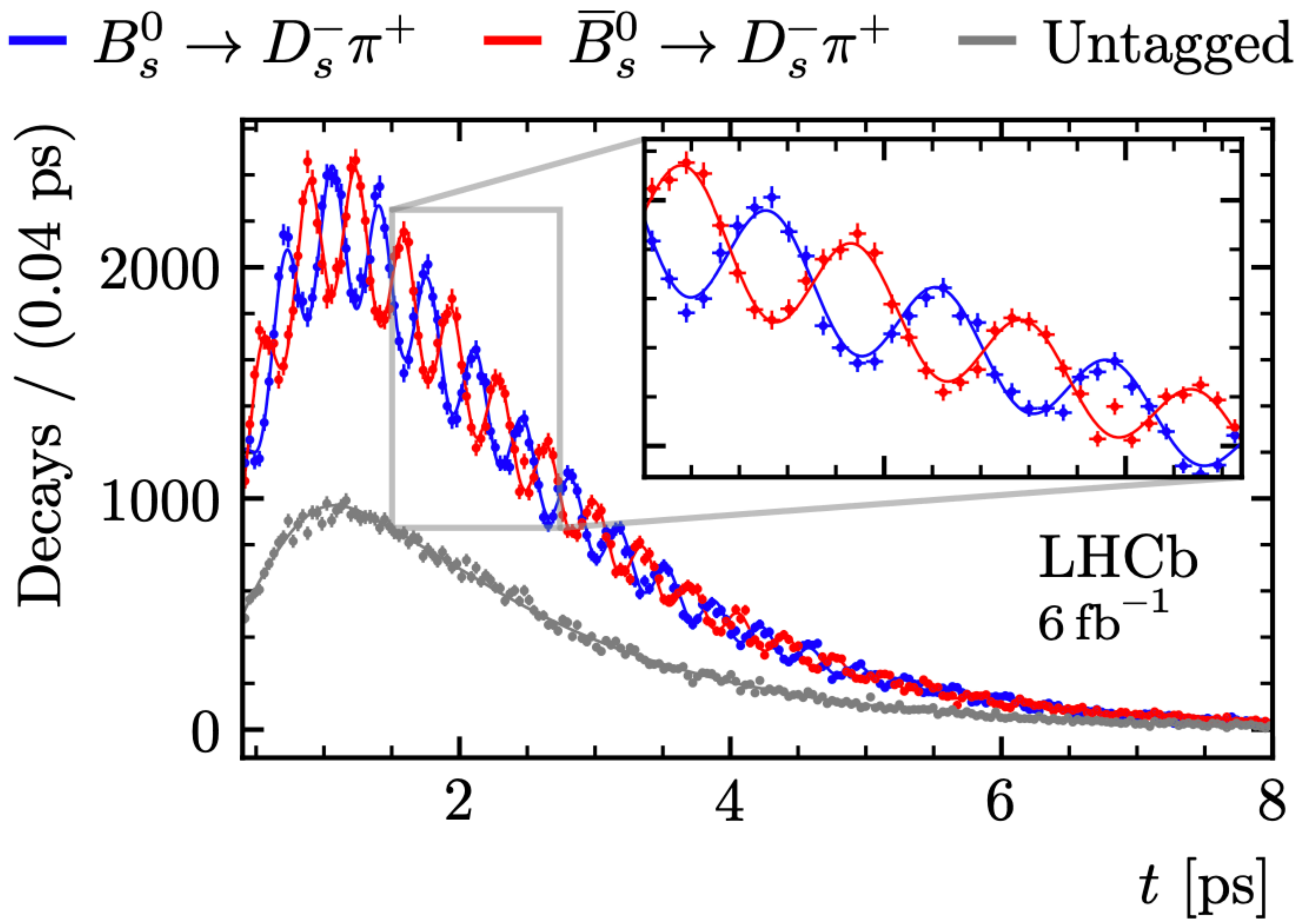
Ultimate theory precision of meson mixing observables



FCC Flavour physics programme

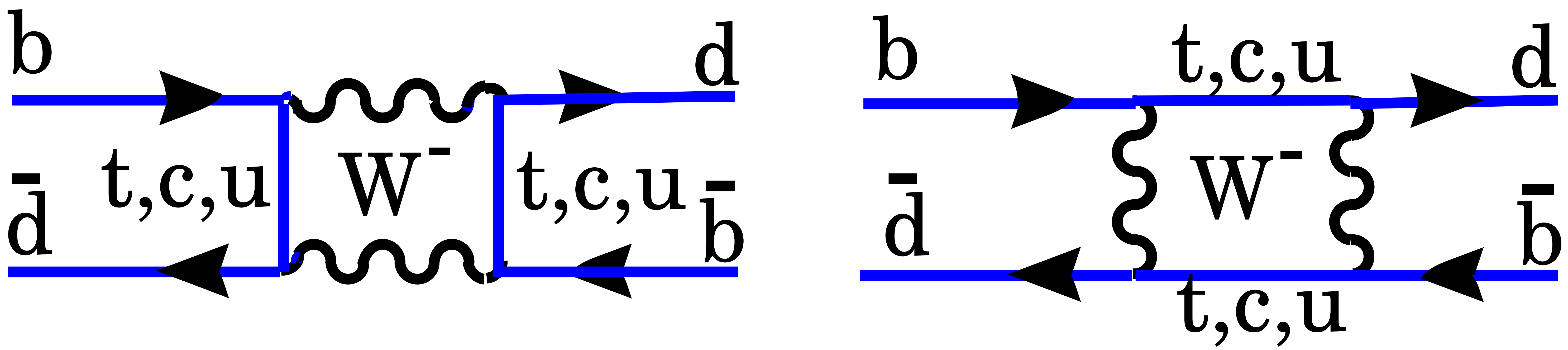
CERN, 12.9.2022

Alexander Lenz
Siegen University

Outline

- B-mixing
- Mass differences ΔM_q
- Decay rate differences $\Delta\Gamma_q$ and semileptonic CP asymmetries a_{sl}^q
- One observable to find them (= BSM) all!

B-MIXING



$|M_{12}|$, $|\Gamma_{12}|$ and $\phi_{12} =$ arg$(-M_{12}/\Gamma_{12})$ can be related to three observables:

- **Mass difference:** $\Delta M := M_H - M_L \approx 2|M_{12}|$ **(off-shell)**
  $|M_{12}|$: heavy internal particles: t, SUSY, ...
- **Decay rate difference:** $\Delta\Gamma := \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi_{12}$ **(on-shell)**
  $|\Gamma_{12}|$: light internal particles: u, c, ... (almost) no NP!!!
- **Flavor specific/semi-leptonic CP asymmetries:** e.g. $B_q \to Xl\nu$ (*semi-leptonic*)

$$a_{sl} \quad \equiv \quad a_{fs} = \frac{\Gamma(\overline{B}_q(t) \to f) - \Gamma(B_q(t) \to \overline{f})}{\Gamma(\overline{B}_q(t) \to f) + \Gamma(B_q(t) \to \overline{f})} = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_{12}$$

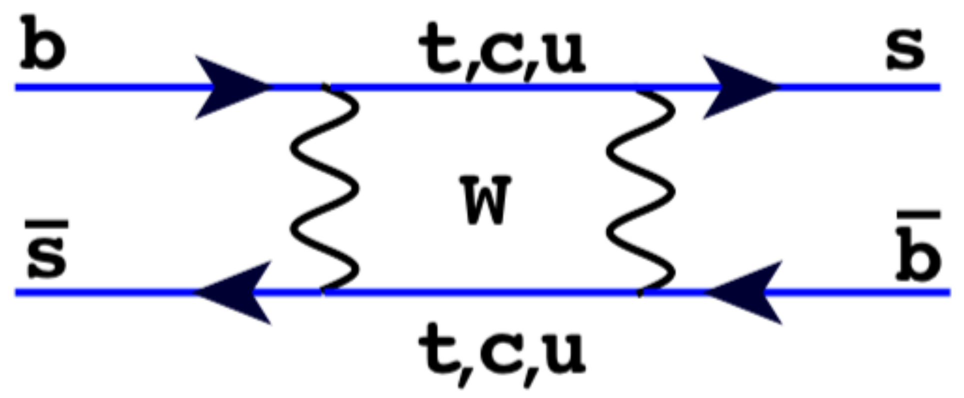
Mass difference ΔM_q

Experiment: HFLAV 2022

$\Delta m_s = 17.765 \pm 0.006\ \mathrm{ps}^{-1}$

$\Delta m_d = 0.5065 \pm 0.0019\ \mathrm{ps}^{-1}$

Theory



CKM $\lambda_t = V_{tb} V_{ts}^*$

Inami-Lim

Buras Jamin Weisz

$$M_{12}^s = \frac{G_F^2}{12\pi^2} \lambda_t^2 M_W^2 S_0(x_t) B f_{B_s}^2 M_{B_s} \hat{\eta}_B$$

In the SM one operator:

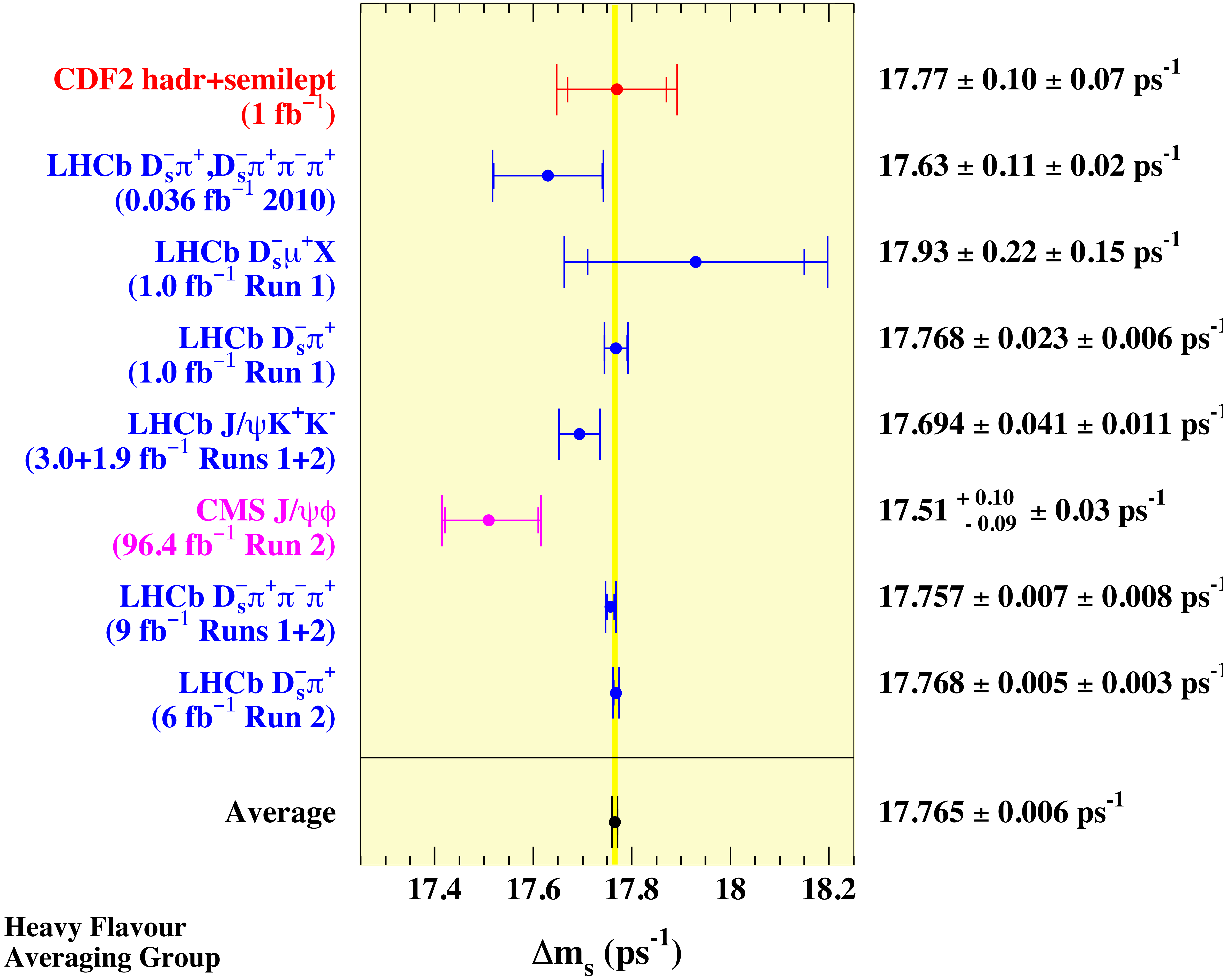
$$Q = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) b^\alpha \times \bar{s}^\beta \gamma^\mu (1-\gamma_5) b^\beta$$

$$\langle Q \rangle \equiv \langle B_s^0 | Q | \bar{B}_s^0 \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B(\mu)$$

Non-perturbative theory input:

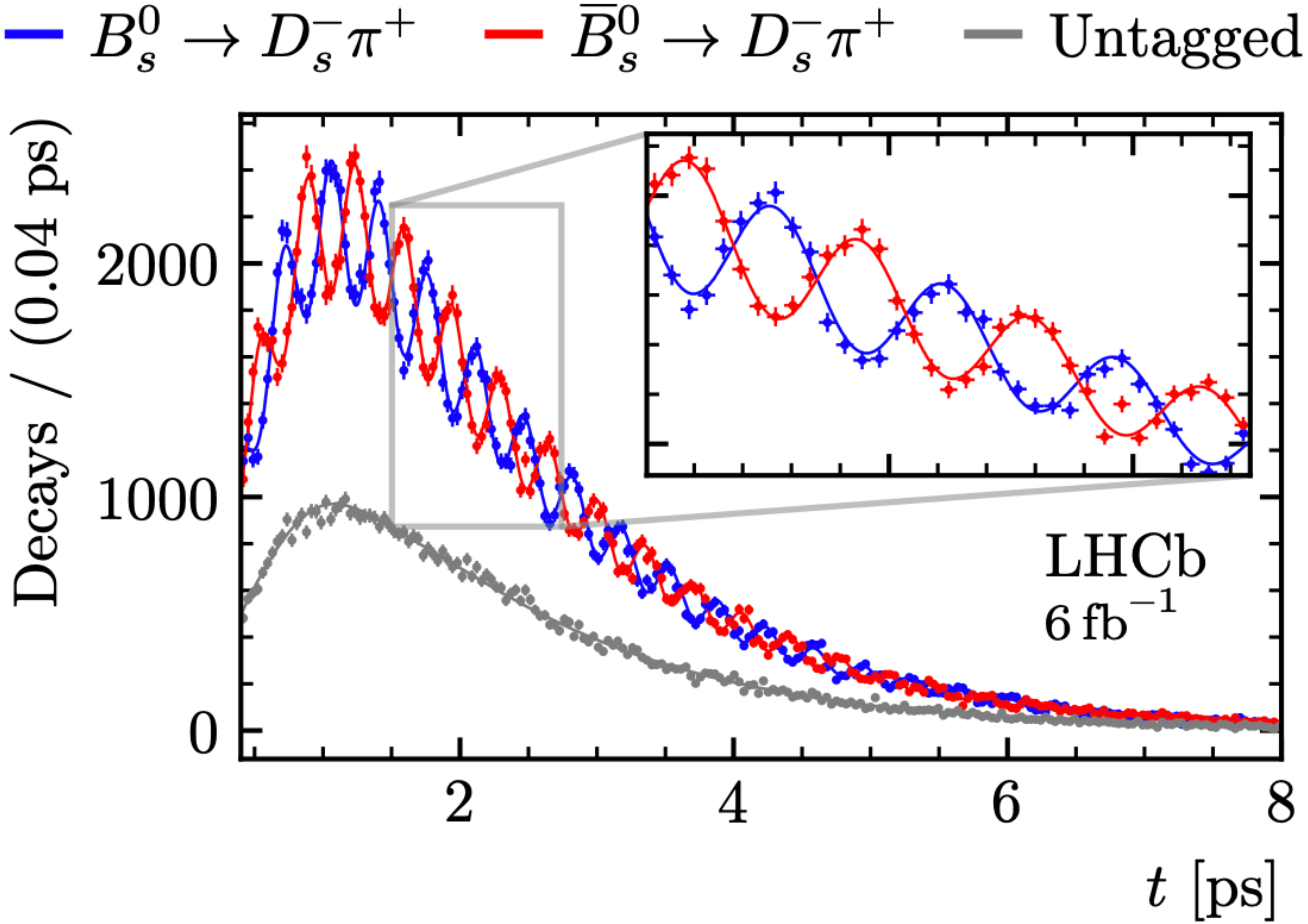
1) **Lattice:** ETM, FNAL-MILC, RBC-UKQCD, HPQCD
2) **Sum rules:** Siegen, Durham

Mass difference ΔM_q



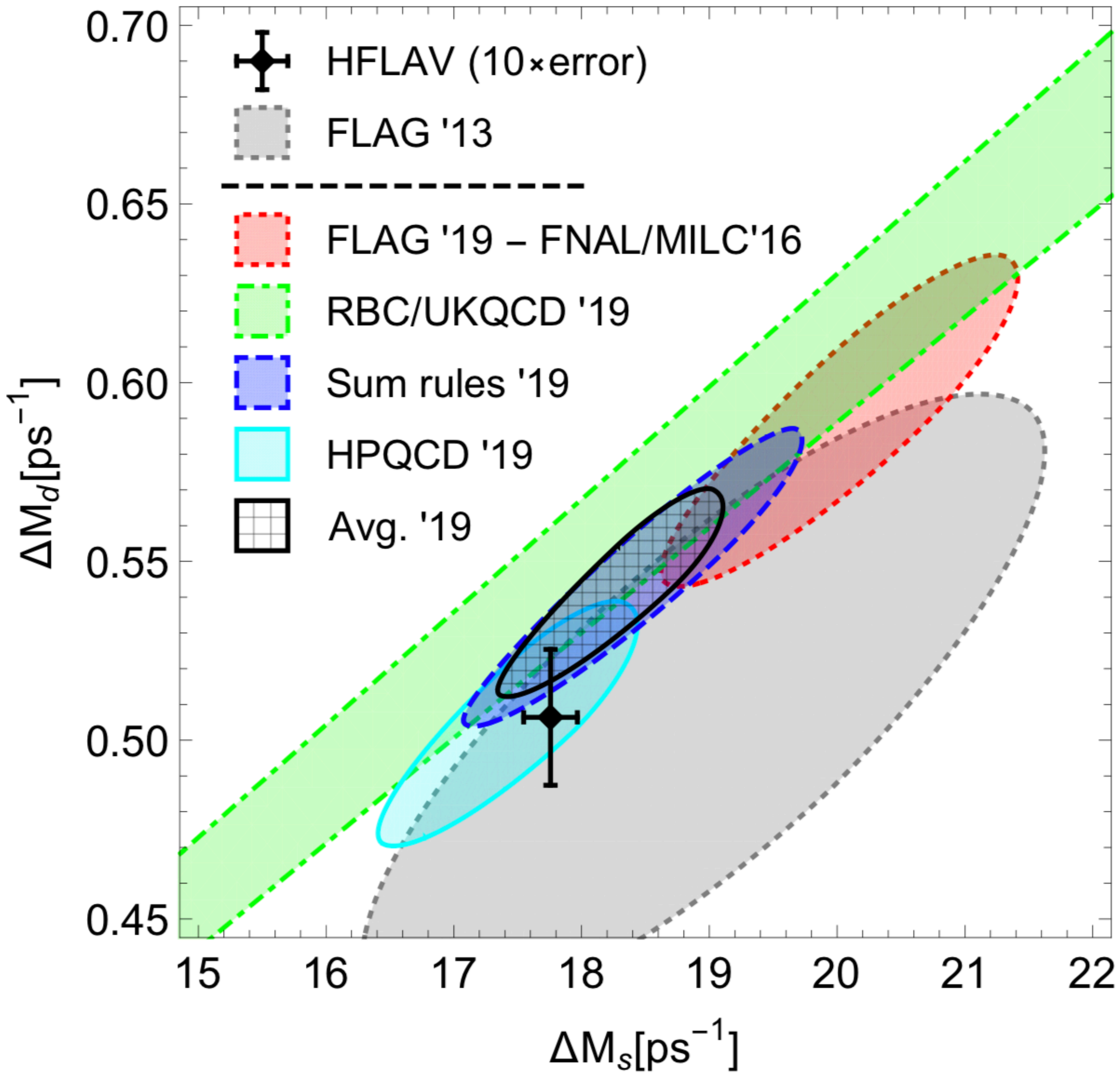
Heavy Flavour
Averaging Group

Mass difference ΔM_q



LHCb
2104.04421

Mass difference ΔM_q



Why is this interesting?

1. **Interesting SM test per se - QCD/BSM**
2. **Determination of SM parameter**
3. **Many BSM models predict large effects in ΔM_q**

Active field:

- Flag 19: mostly FNAL-MILC (2/16)
- RBC-UK: 12-18
- Sum rules: Durham 4/19 (based on Siegen 16-18, Durham 17)
- HPQCD: 07/19

Averages of lattice and sum rules
Di Luzio, Kirk, AL, Rauh
1909.11087

$$\Delta M_d^{\text{Average 2019}} = \left(0.533^{+0.022}_{-0.036}\right) \text{ps}^{-1} = \left(1.05^{+0.04}_{-0.07}\right) \Delta M_d^{\text{exp}} ,$$
$$\Delta M_s^{\text{Average 2019}} = \left(18.4^{+0.7}_{-1.2}\right) \text{ps}^{-1} = \left(1.04^{+0.04}_{-0.07}\right) \Delta M_s^{\text{exp}} ,$$

Mass difference ΔM_q

Theory error budget

AL, Tetlalmatzi-Xolocotzi 1912.07621

- Non-perturbative averages of lattice and sum rules, Di Luzio, Kirk, AL, Rauh, 1909.11087
- CKM fitter input from 12/2019

$$\Delta M_s^{\rm SM} = (18.77 \pm 0.86)\,{\rm ps}^{-1}\,,$$
$$\Delta M_d^{\rm SM} = (0.543 \pm 0.029)\,{\rm ps}^{-1}\,,$$

$\Delta M_s^{\rm SM}$	This work	ABL 2015	LN 2011	LN 2006
Central Value	$18.77\,{\rm ps}^{-1}$	$18.3\,{\rm ps}^{-1}$	$17.3\,{\rm ps}^{-1}$	$19.3\,{\rm ps}^{-1}$
$f_{B_s}\sqrt{B_1^s}$	3.1%	13.9%	13.5%	34.1%
V_{cb}	3.4%	4.9%	3.4%	4.9%
$\bar{m}_t(\bar{m}_t)$	0.3%	0.7%	1.1%	1.8%
$\Lambda_5^{\rm QCD}$	0.2%	0.1%	0.4%	2.0%
γ	0.1%	0.1%	0.3%	1.0%
$\lvert V_{ub}/V_{cb}\rvert$	$< 0.1\%$	0.1%	0.2%	0.5%
$\overline{m}_b$	$< 0.1\%$	$< 0.1\%$	0.1%	$- - -$
Total	4.6%	14.8%	14.0%	34.6%

Huge improvement/no improvement

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BUT

		δM_s
FNAL16	$f_{B_s}\sqrt{\hat{B}} = 274(8)\,{\rm MeV}\ ({\rm N_f} = 2+1)\,,$	5.8%
HPQCD19	$f_{B_s}\sqrt{\hat{B}} = 256.1(5.7)\,{\rm MeV}\ ({\rm N_f} = 2+1+1)\,.$	4.4%
	Average lattice /sum rule	3.1%

Mass difference ΔM_q

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Lattice predictions cover the range
250.4…282 MeV => 266.2(15.8) MeV **11.9%**

Mass difference ΔM_q

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vs. projections: 1812.07638

2035: $f_{B_s}, \hat{B} \sim$ **0.5%** **1%**

a long, long way to go

Mass difference ΔM_q

Theory error budget **AL, Tetlalmatzi-Xolocotzi 1912.07621**

Assume lattice can do $\pm 1\,\%$

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Within the SM

$$V_{tb}V_{ts}^* = -c_{12}\frac{\sqrt{1-|V_{ub}|^2-V_{cb}^2}}{\sqrt{1-|V_{ub}|^2}}V_{cb} - s_{12}\frac{1-|V_{ub}|^2-V_{cb}^2}{\sqrt{1-|V_{ub}|^2}}V_{ub}$$

$$s_{12} = \frac{\frac{V_{us}}{V_{ud}}}{\sqrt{1+\frac{V_{us}^2}{V_{ud}^2}}}\,, \qquad c_{12} = \frac{1}{\sqrt{1+\frac{V_{us}^2}{V_{ud}^2}}}\,, \qquad V_{ub} = |V_{ub}|e^{-i\gamma}$$

δM_s

$|V_{cb}|^{\rm incl.,2022} = (42.16 \pm 0.51)\cdot 10^{-3}$ 2.4%

Bordone, Capdevilla, Gambino 2107.00604

$|V_{cb}|^{\rm excl.,PDG} = (39.5 \pm 0.9)\cdot 10^{-3}$ 4.6%

$|V_{cb}|^{\Delta M_q} = (41.6 \pm 0.7)\cdot 10^{-3}$ 3.4%

King, Kirk, AL, Rauh 1911.07856

Mass difference ΔM_q

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King, Kirk, AL, Rauh 1911.07856

Inclusive and exclusive cover the range

$$(38.6\ldots 42.67)\cdot 10^{-3}$$
$$=(40.7 \pm 2.0)\cdot 10^{-3}$$ 10%

For $\pm 1\ \%$ we need $\delta V_{cb} \approx 0.2\cdot 10^{-3}$

Mass difference ΔM_q

Theory error budget AL, Tetlalmatzi-Xolocotzi 1912.07621

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For $\delta M_s \pm 1\,\%$ we need $\delta V_{cb} \approx 0.2 \cdot 10^{-3}$

- **Inclusive fits in** Bordone, Capdevilla, Gambino 2107.00604 **and** Bernlochner, Fael, Olschewsky, Person, van Toner, Vos, Welsch 2205.10274 **agree for V_{cb}, but disagree for the matrix element of the Darwin operator => room for improvement (see also** AL, Piscopo, Rusov, 2208.02643**)**

- **Exclusive V_{cb} determination up to 1.4% at Belle II (**SNOWMASS, 2207.06307**)**

- **Even higher precision from several 10^8 on-shell $W^+ \to c\bar{b}$ decays at FCC-ee (**Monteil, Wilkinson 2106.01259, Monteil, AL 2207.11055**)**

  $\delta V_{cb} \approx 0.16 \cdot 10^{-3}$ Monteil, 12.9.2022 flavour@FCCee

Mass difference ΔM_q

Theory error budget **AL, Tetlalmatzi-Xolocotzi 1912.07621**

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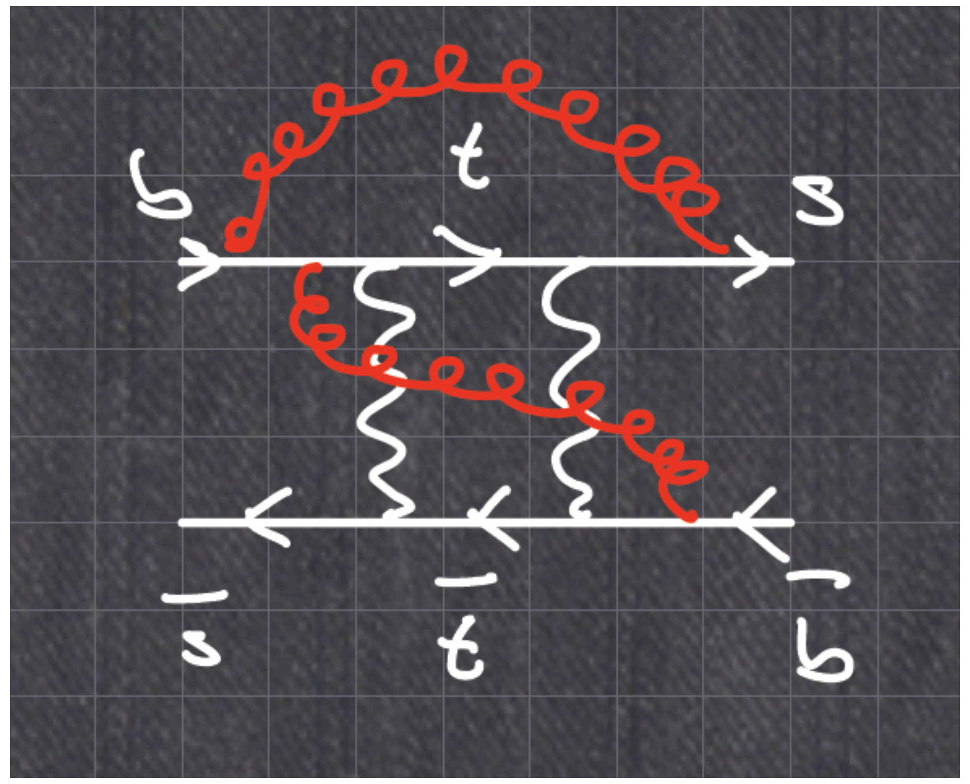
3-loop QCD corrections



2-loop QCD corrections: **Buras, Jamin, Weisz 1990**

$$1 \to \eta_B \approx 0.84$$

expect an effect of $\pm 0.16\,\alpha_s/\pi = \pm\,1\,\%$

Gorbahn, Stamou,...? > 2035

Mass difference ΔM_q

2035:

- Lattice values for dim 6 matrix elements converge
- V_{cb} inclusive vs exclusive converges and direct measurement at FCC-ee
- 3-loop corrections known and confirmed

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- Lattice values for dim 6 matrix elements converge
- V_{cb} inclusive vs exclusive converges and direct measurement as FCC-ee
- 3-loop corrections known and confirmed

$$\Delta M_s^{\mathrm{SM},2035} = (19.20 \pm 0.29)\,\mathrm{ps}^{-1}$$

$$\Delta M_s^{\mathrm{EXP},2035} = (17.750 \pm 0.002)\,\mathrm{ps}^{-1}$$

Discovery of BSM with 5 standard deviations

Decay rate difference $\Delta\Gamma_s$

Calculation is more difficult than mass difference - use Heavy Quark Expansion

$$\Gamma_{12} = 16\pi^2 \left(\tilde{\Gamma}_6 \frac{\langle \tilde{\mathcal{O}}_6 \rangle}{m_b^3} + \tilde{\Gamma}_7 \frac{\langle \tilde{\mathcal{O}}_7 \rangle}{m_b^4} + \ldots \right)$$

Each term can be split up into a perturbative part and non-perturbative matrix elements

$$\Gamma_i = \Gamma_i^{(0)} + \frac{\alpha_s}{4\pi}\Gamma_i^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \Gamma_i^{(2)} + \ldots$$

$$R_2 = \frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_\rho \gamma^\mu (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta \gamma_\mu (1-\gamma^5) s^\beta)$$

$$R_3 = \frac{1}{m_b^2}(\bar{b}^\alpha \overleftarrow{D}_\rho (1-\gamma^5) D^\rho s^\alpha)(\bar{b}^\beta (1-\gamma^5) s^\beta)$$

Sum rules and lattice 1909.11087

Status of theory predictions

known

HP-QCD 1910.00970

Q_1, Q_2, Q_3 Q_4, Q_5, R_2, R_3

Obs.	$\tilde{\Gamma}_6^{(0)}$	$\tilde{\Gamma}_6^{(1)}$	$\tilde{\Gamma}_6^{(2)}$	$\langle \mathcal{O}^{d=6} \rangle$	$\tilde{\Gamma}_7^{(0)}$	$\tilde{\Gamma}_7^{(1)}$	$\langle \mathcal{O}^{d=7} \rangle$	$\sum$
Γ_{12}^s	$++$	$++$	$+$	$+++$	$++$	0	$+$	$11 + (***)$
Γ_{12}^d	$++$	$++$	$+$	$+++$	$++$	0	$+$	$11 + (***)$

NNLO-QCD Gerlach, Nierste, Shtabovenko, Steinhauser 2205.07907

Decay rate difference $\Delta\Gamma_s$

In the ratio Γ_{12}/M_{12} theory uncertainties are cancelling

$$\mathrm{Re}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) = -\frac{\Delta\Gamma_s}{\Delta M_s}, \qquad \mathrm{Im}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right) = a_{fs}^s .$$

$$-\frac{\Gamma_{12}^s}{M_{12}^s} = \frac{\lambda_c^2\Gamma_{12}^{s,cc} + 2\lambda_c\lambda_u\Gamma_{12}^{s,uc} + \lambda_u^2\Gamma_{12}^{s,uu}}{\lambda_t^2\tilde{M}_{12}^s} = \frac{\Gamma_{12}^{s,cc}}{\tilde{M}_{12}^s} + 2\frac{\lambda_u}{\lambda_t}\frac{\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}}{\tilde{M}_{12}^s} + \left(\frac{\lambda_u}{\lambda_t}\right)^2\frac{\Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc} + \Gamma_{12}^{s,uu}}{\tilde{M}_{12}^s}$$

$$\frac{V_{ub}V_{ud}}{V_{tb}V_{td}} = \lambda^{0.8} \qquad \frac{V_{ub}V_{us}}{V_{tb}V_{ts}} = \lambda^{2.8}$$

- No CKM dependence!
- No GIM suppression!
- No imaginary part!
- Small $\approx \mathcal{O}(5\cdot 10^{-3})$
- Leading contribution to $\Delta\Gamma/\Delta M$

- CKM suppression
- GIM suppression
- Imaginary part via CKM
- Leading contribution to a_{fs}
- Tiny contribution to $\Delta\Gamma/\Delta M$

- Stronger CKM suppression
- Very strong GIM suppression
- Imaginary part via CKM
- Subleading contribution to a_{fs} and sub-subleading contribution to $\Delta\Gamma/\Delta M$

$$\frac{\Delta\Gamma_s}{\Delta M_s} = \left(4.33 \pm 0.78\,(1/m_b)^{+0.23}_{-0.44}\,(\mu)^{+0.09}_{-0.19}\,(\mu,1/m_b) \pm 0.12\,(B) \pm 0.05\,(para.)\right)$$

NNLO-QCD: Gerlach, Nierste, Shtabovenko,Steinhauser 2205.07907

Decay rate difference $\Delta\Gamma_s$

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$

$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

- **Decay constants cancel completely**
- **Bag parameter cancel largely**
- **V_{cb} dependence cancels!**

SM predictions (AL, Tetlalmatzi-Xolocotzi 1912.07621, Gerlach, Nierste, Shtabovenko, Steinhauser 2205.07907)

$$\Delta\Gamma_s^{\text{SM2019}} = (0.091 \pm 0.013)\,\text{ps}^{-1}$$

$$\Delta\Gamma_s^{\text{SM2022}} = (0.076 \pm 0.017)\,\text{ps}^{-1}$$

$$\Delta\Gamma_s^{\text{HFLAV2022}} = (0.084 \pm 0.005)\,\text{ps}^{-1}$$

- **Good agreement**
- **Experiment about 3 times more precise**

$$\Delta\Gamma_d^{\text{SM 2019}} = (2.6 \pm 0.4) \cdot 10^{-3}\,\text{ps}^{-1}$$

$$\Delta\Gamma_d^{\text{HFLAV2021}} = (0.7 \pm 6.6) \cdot 10^{-3}\,\text{ps}^{-1}$$

- **Might solve the D0 di-muon asymmetry**
- **Experimental number needed**

$\Delta\Gamma_s^{\text{SM}}/\Delta M_s^{\text{SM}}$	this work	ABL 2015	LN 2011	LN 2006
Central Value	$48.2 \cdot 10^{-4}$	$48.1 \cdot 10^{-4}$	$50.4 \cdot 10^{-4}$	$49.7 \cdot 10^{-4}$
$B_{R_2}^s$	10.9%	14.8%	17.2%	15.7%
μ	6.6%	8.4%	7.8%	9.1%
$B_{R_0}^s$	3.2%	2.1%	3.4%	3.0%
B_3^s	2.2%	2.1%	4.8%	3.1%
$\bar{z}$	0.9%	1.1%	1.5%	1.9%
m_b	0.9%	0.8%	1.4%	1.0%
$B_{R_3}^s$	0.5%	0.2%	0.2%	– – –
$B_{\tilde{R}_3}^s$	–	0.6%	0.5%	– – – –
$\bar{m}_t(\bar{m}_t)$	0.3%	0.7%	1.1%	1.8%
m_s	0.3%	0.1%	1.0%	0.1%
Λ_5^{QCD}	0.2%	0.2%	0.8%	0.1%
$B_{\tilde{R}_1}^s$	0.2%	0.7%	1.9%	– – –
$B_{R_1}^s$	0.1%	0.5%	0.8%	– – –
γ	< 0.1%	0.0%	0.0%	0.1%
$\lvert V_{ub}/V_{cb}\rvert$	< 0.1%	0.0%	0.0%	0.1%
V_{cb}	< 0.1%	0.0%	0.0%	0.0%
Total	13.4%	17.3%	20.1%	18.9%

- **Strong test of HQE**
- **Violation of Quark hadron duality must be small**

Decay rate difference $\Delta\Gamma_s$

2035:

- Lattice and sum rule values for dim 7 matrix elements
- Better understanding of quark masses
- α_s/m_b corrections determined
- Lattice values for dim 6 matrix elements converge

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$$\Delta\Gamma_s^{\mathrm{SM2035}} = (0.085 \pm 0.005)\,\mathrm{ps}^{-1}$$

$$\Delta\Gamma_s^{\mathrm{HFLAV2035}} = (0.080 \pm 0.002)\,\mathrm{ps}^{-1}$$

Amazing confirmation of HQE framework

Semi-leptonic CP asymmetries

Relation to experiment

$$\Re\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = -\frac{\Delta\Gamma_s}{\Delta M_q}$$

CP violating!

$$\Im\left(\frac{\Gamma_{12}^q}{M_{12}^q}\right) = a_{sl}^q$$

- **Decay constants cancel completely**
- **Bag parameter cancel largely**

SM predictions (AL, Tetlalmatzi-Xolocotzi 1912.07621)

$$a_{fs}^{s,\,\mathrm{SM}\,2019} = (2.06 \pm 0.18)\cdot 10^{-5} \qquad a_{fs}^{s,\,\mathrm{HFLAV}\,2019} = (-60 \pm 280)\cdot 10^{-5}$$

$$a_{fs}^{d,\,\mathrm{SM}\,2019} = -(4.73 \pm 0.42)\cdot 10^{-4} \qquad a_{fs}^{d,\,\mathrm{HFLAV}\,2019} = (-21 \pm 17)\cdot 10^{-4}$$

- **Very sensitive to BSM effects!**
- **Experimental number needed**

$$a_{fs}^q = 480 \cdot 10^{-5} \sin\phi_{12}^q$$

Semi-leptonic CP asymmetries

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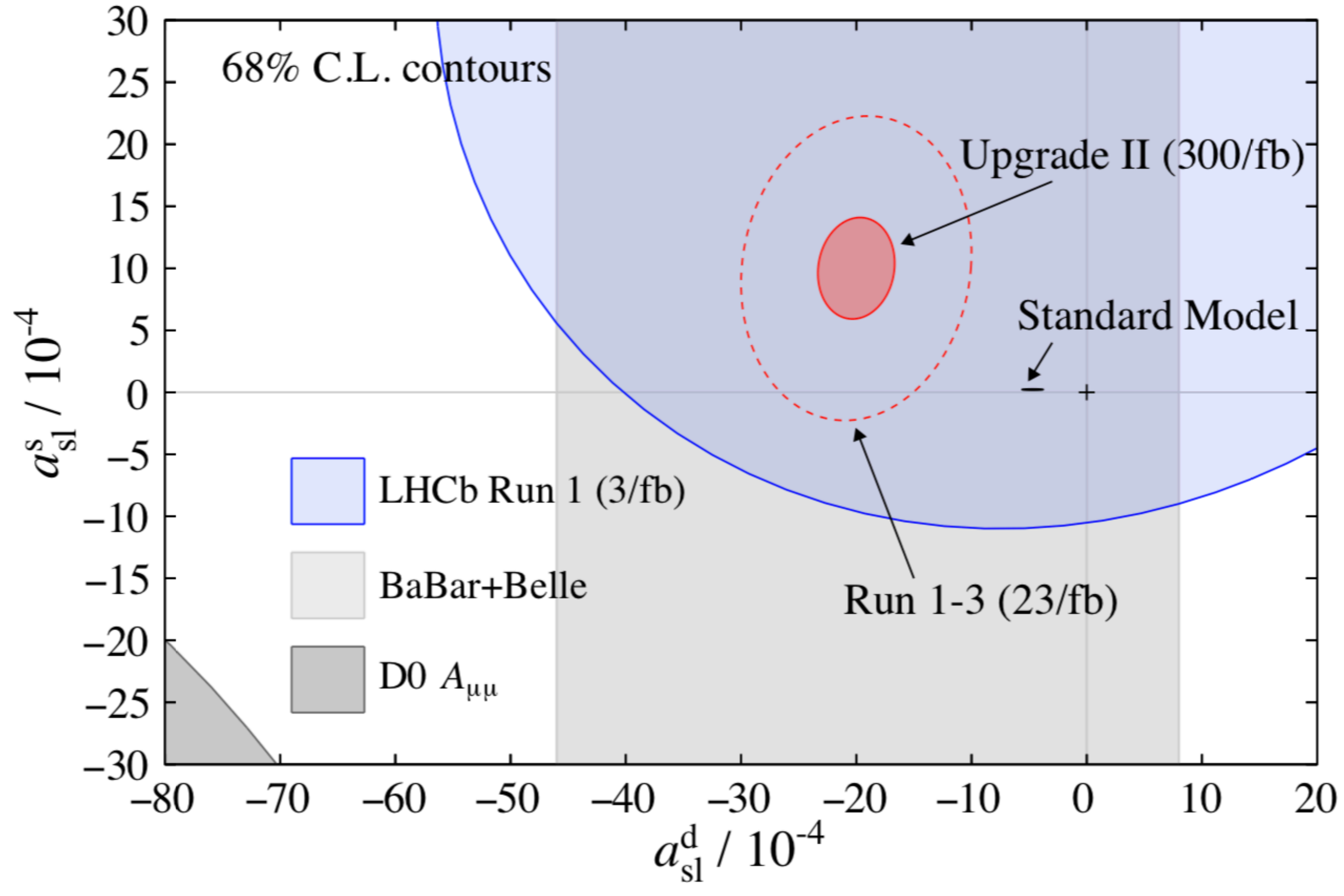

$\delta a_{sl}^s \approx 3 \cdot 10^{-5}$ **Monteil, 12.9.2022 flavour@FCCee! (p.23)**

As soon as Exp gets close to SM:

PHYSICAL REVIEW D **102,** 093002 (2020)

Renormalization scale setting for D-meson mixing

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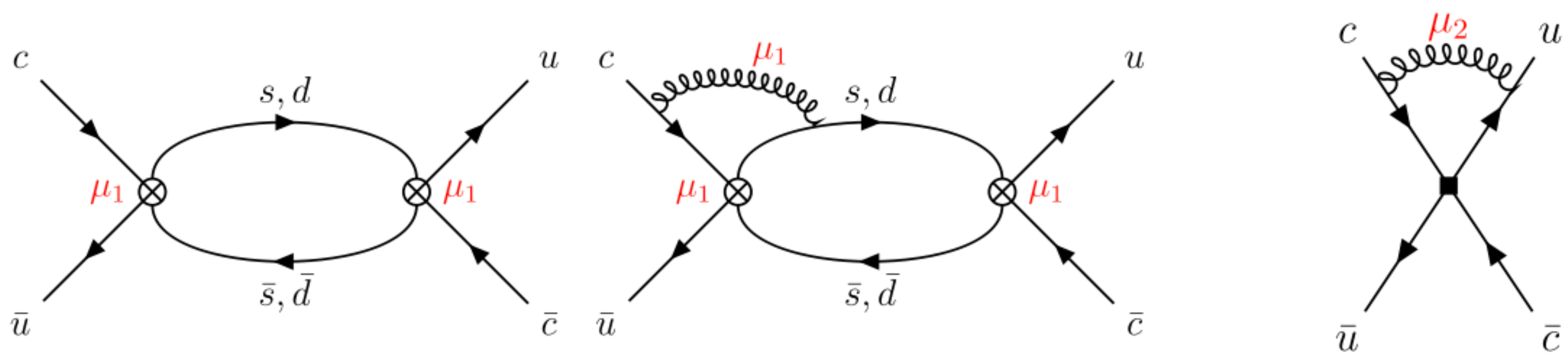
(Received 20 July 2020; accepted 7 October 2020; published 13 November 2020)

A naive application of the heavy quark expansion (HQE) yields theory estimates for the decay rate of neutral *D* mesons that are 4 orders of magnitude below the experimental determination. It is well known that this huge suppression results from severe Glashow-Iliopoulos-Maiani cancellations. We find that this mismatch can be solved by individually choosing the renormalization scale of the different internal quark contributions. For *b* and *c* hadron lifetimes, as well as for the decay rate difference of neutral *B* mesons, the effect of our scale setting procedure lies within the previously quoted theory uncertainties, while we get enlarged theory uncertainties for the semileptonic *CP* asymmetries in the *B* system.

$$\Gamma_{cc}(\mu - 2\epsilon)$$

$$\Gamma_{uc}(\mu - \epsilon)$$

$$\Gamma_{uu}(\mu)$$



ϵ (GeV)	Γ_{12}^s/M_{12}^s	Γ_{12}^d/M_{12}^d
0.	**−0.00499+0.000022*I***	**−0.00497−0.00050*I***
0.2.	**−0.00494+0.000023*I***	**−0.00492−0.00053*I***
0.5.	**−0.00484** + 0.000026*I*	**−0.00482** − 0.00059*I*
1.0	**−0.00447** + 0.000037*I*	**−0.00448** − 0.00084*I*
1.5.	−0.00287 + 0.000091*I*	−0.00309 − 0.0021*I*

Semi-leptonic CP asymmetries

2035:
- Better understanding of GIM cancellations
- NNLO analysis
- Better understanding of quark masses
- Better knowledge of CKM elements
- Lattice and sum rule values for dim 7 matrix elements
- α_s/m_b corrections determined

$a_{\rm sl}^{s,\rm SM}$	this work	ABL 2015	LN 2011	LN 2006
Central Value	$2.06 \cdot 10^{-5}$	$2.22 \cdot 10^{-5}$	$1.90 \cdot 10^{-5}$	$2.06 \cdot 10^{-5}$
μ	6.7%	9.5%	8.9%	12.7%
$\bar{z}$	4.0%	4.6%	7.9%	9.3%
$\lvert V_{ub}/V_{cb}\rvert$	2.6%	5.0%	11.6%	19.5%
$B_{R_3}^s$	2.3%	1.1%	1.2%	1.1%
$B_{\tilde{R}_3}^s$	-	2.6%	2.8%	2.5%
m_b	1.3%	1.0%	2.0%	3.7%
γ	1.1%	1.3%	3.1%	11.3%
$B_{R_2}^s$	0.8%	0.1%	0.1%	– – –
$\Lambda_5^{\rm QCD}$	0.6%	0.5%	1.8%	0.7%
$\bar{m}_t(\bar{m}_t)$	0.3%	0.7%	1.1%	1.8%
B_3^s	0.3%	0.3%	0.6%	0.4%
$B_{R_0}^s$	0.3%	0.2%	0.3%	– – –
m_s	$< 0.1\%$	0.1%	0.1%	0.1%
$B_{\tilde{R}_1}^s$	$< 0.1\%$	0.5%	0.2%	– – –
$B_{R_1}^s$	$< 0.1\%$	$< 0.1\%$	0.0%	– – –
V_{cb}	$< 0.1\%$	0.0%	0.0%	0.0%
Total	8.8%	12.2%	17.3%	27.9%

$$a_{fs}^{s,\mathrm{SM2035}} = (2.0 \pm 0.2) \cdot 10^{-5} \qquad a_{fs}^{s,\mathrm{HFLAV2035}} = (-60 \pm 30) \cdot 10^{-5}$$

$$a_{fs}^{d,\mathrm{SM2035}} = -(4.7 \pm 0.4) \cdot 10^{-4} \qquad a_{fs}^{d,\mathrm{HFLAV2035}} = (-21.0 \pm 3.0) \cdot 10^{-4}$$

$$a_{fs}^{d,\,\mathrm{HFLAV\,2019}} = (-21 \pm 17) \cdot 10^{-4}$$

Discovery of BSM with more than 5 standard deviations

$$a_{fs}^{s,\,\mathrm{HFLAV\,2019}} = (-60 \pm 280) \cdot 10^{-5}$$

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$\lvert V_{ub}/V_{cb} \rvert$	2.6%	5.0%	11.6%	19.5%
$B^s_{R_3}$	2.3%	1.1%	1.2%	1.1%
$B^s_{\tilde{R}_3}$	-	2.6%	2.8%	2.5%
m_b	1.3%	1.0%	2.0%	3.7%
γ	1.1%	1.3%	3.1%	11.3%
$B^s_{R_2}$	0.8%	0.1%	0.1%	− − −
$\Lambda_5^{\rm QCD}$	0.6%	0.5%	1.8%	0.7%
$\bar{m}_t(\bar{m}_t)$	0.3%	0.7%	1.1%	1.8%
B^s_3	0.3%	0.3%	0.6%	0.4%
$B^s_{R_0}$	0.3%	0.2%	0.3%	− − −
m_s	< 0.1%	0.1%	0.1%	0.1%
$B^s_{\tilde{R}_1}$	< 0.1%	0.5%	0.2%	− − −
$B^s_{R_1}$	< 0.1%	< 0.1%	0.0%	− − −
V_{cb}	< 0.1%	0.0%	0.0%	0.0%
Total	8.8%	12.2%	17.3%	27.9%

$\delta a_{sl}^{s} \approx 3 \cdot 10^{-5}$ **Monteil, 12.9.2022 flavour@FCCee! (p.23)**

$$a_{fs}^{s,\rm SM2035} = (2.0 \pm 0.2) \cdot 10^{-5} \qquad a_{fs}^{s,\rm HFLAV2035} = (-60 \pm 3) \cdot 10^{-5}$$

$$a_{fs}^{d,\rm SM2035} = -(4.7 \pm 0.4) \cdot 10^{-4} \qquad a_{fs}^{d,\rm HFLAV2035} = (-21.0 \pm 0.3) \cdot 10^{-4}$$

Discovery of BSM with more than 20 standard deviations

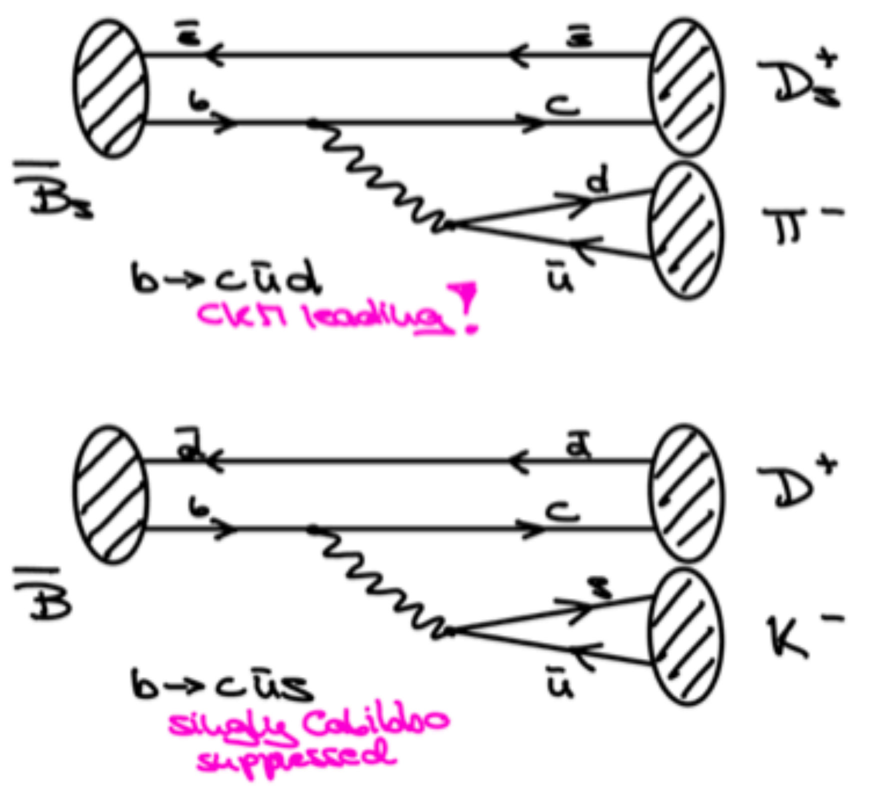
$$a_{fs}^{d,\,\rm HFLAV\,2019} = (-21 \pm 17) \cdot 10^{-4}$$

$$a_{fs}^{s,\,\rm HFLAV\,2019} = (-60 \pm 280) \cdot 10^{-5}$$

Flavour-specific CP asymmetries

3 σ to 9 σ deviation of experiment from QCDf predictions with standard error estimates

Colour-allowed Tree-level Decays

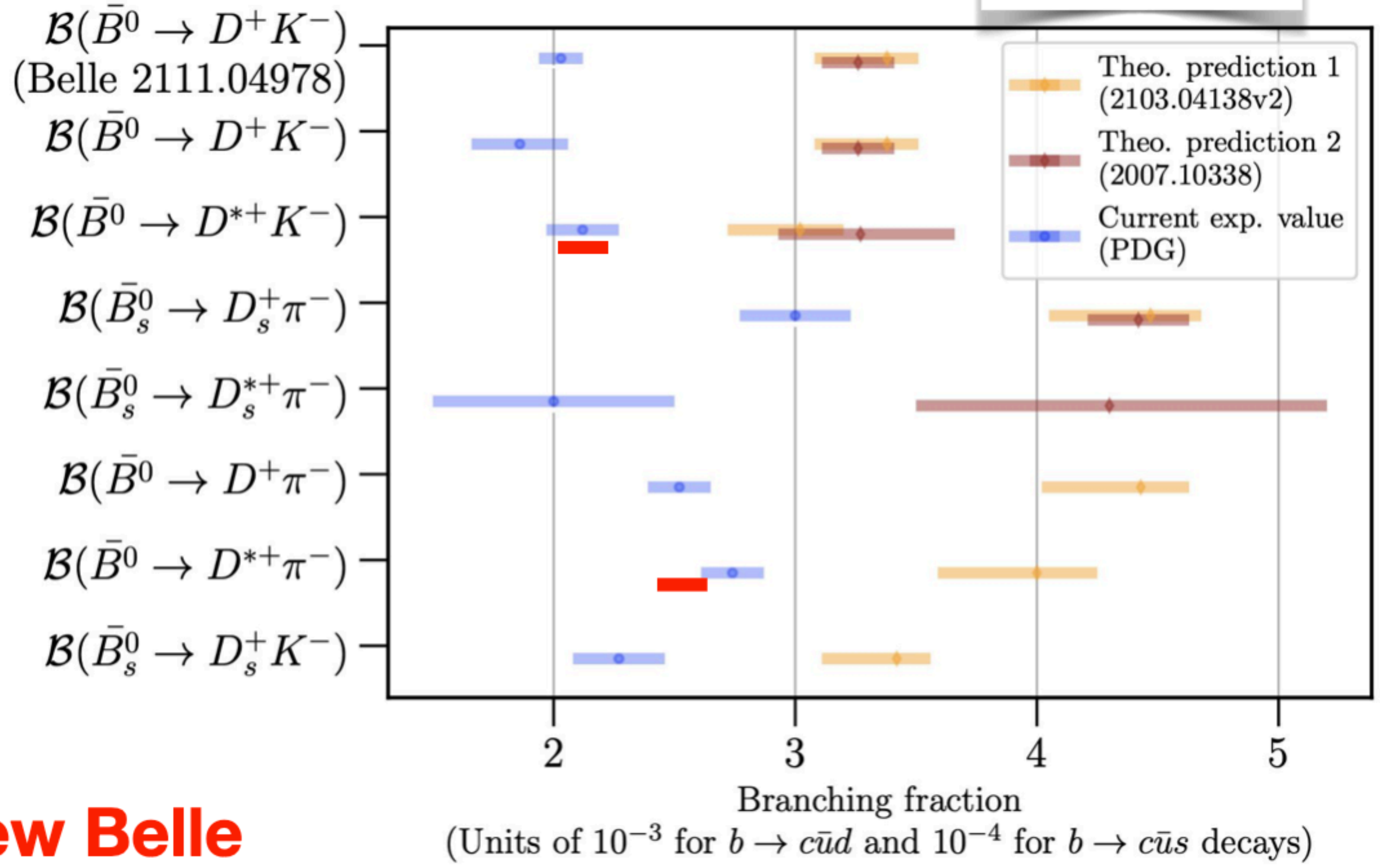


- CKM leading decays
- The are no annihilation, penguins,…
- QCDf should work at its best!

Beneke, Buchalla, Neubert, Sachrajda 1999…

$$\langle D_q^{(*)+} L^- | \mathcal{Q}_i | \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

N. Skidmore



— New Belle

Flavour-specific CP asymmetries

Gershon, AL, Rusov, Skidmore
2111.04478

- a^q_{fs} is typically measured with semi-leptonic B_q decays
- One could also use the flavour specific $\bar{B}_s \to D_s^+ \pi^-$ decay
- Assume: there is new physics in these decays, potentially CP violating
- Derive CP asymmetry

$\approx 2r \sin\phi \sin\varphi < 0.40$

$$A^q_{\rm fs} = \frac{a^q_{\rm fs} - 2r\sin\phi\sin\varphi + 2a^q_{\rm fs} r\cos\phi\cos\varphi + a^q_{\rm fs} r^2}{1 + 2r\cos\phi\cos\varphi + r^2 - 2a^q_{\rm fs} r\sin\phi\sin\varphi} \approx a^q_{\rm fs} - A^q_{\rm dir}$$

Constrained by semi-leptonic Measurements

Significant exp. deviation of A^q_{fs} from a^q_{sl}
= unambiguous and theory independent signal for BSM

$\delta a^s_{sl} \approx 3 \cdot 10^{-5}$ Monteil, 12.9.2022 flavour@FCCee! (p.23)

Conclusion

2035:

- Non-perturbative improvements (lattice, sum rules)
- perturbative improvements
- better understanding of Quark masses
- Determination of CKM elements

Discovery potential for BSM effects with more than 20 sigma in ΔM, $\Delta\Gamma$ and a_{sl} possible

Interesting additional Null-test

$$a^s_{fs}(\bar{B}_s \to D_s^+ \pi^-) - a^s_{sl}$$

Fleischer, Vos 1606.06042
Gershon, AL, Rusov, Skidmore 2111.04478