Probing CP violation in semi-leptonic b-decays through time-evolution

Contribution to FCC Flavour Physics Programme Workshop
Based on: arXiv:2008.08000 and arXiv:2208.10880

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Motivation

We are interested in CP-violation on semileptonic b-decays ($b \to s\ell\ell$ and $b \to s\nu\bar{\nu}$)

- FCNC processes, which are loop supressed in the SM (potential sensitivity to NP)
- Hints of NP in $b \to s\ell\ell$
  - LFU Ratios:

$$R_{K^{(*)}} = \frac{\Gamma(B \to K^{(*)}\mu^+\mu^-)}{\Gamma(B \to K^{(*)}e^+e^-)}$$

  - Tension wrt the SM of $\sim 3\sigma$ for R_K and R_{K^*}
  - Set of deviations in branching fractions and optimized angular observables
- Constraining CP-phases of potential NP (Wilson coefficients) in semileptonic decays.

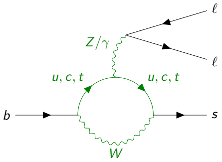
$b \to s\ell\ell$ and $b \to s\nu\bar{\nu}$ Effective Hamiltonians

Local operator effective theory at scales below the electroweak scale

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i \mathcal{C}_i \, \mathcal{O}_i$$

Short distance dynamics

Long distance structure



$$\mathcal{C}_i = \mathcal{C}_i^{NP} + \mathcal{C}_i^{SM}$$

Operators relevant for this transition

$b \to s\ell\ell$

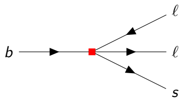
$$\mathcal{O}_{9^{(\prime)}} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \ell)$$

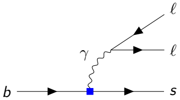
$$\mathcal{O}_{10^{(\prime)}} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_{7^{(\prime)}} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_{R(L)} b) F^{\mu\nu}$$

$b \to s\nu\bar{\nu}$

$$\mathcal{O}^{\nu}_{L(R)} = \frac{e^2}{16\pi^2} (\bar{s}\gamma_\mu P_{L(R)} b)(\bar{\nu}\gamma^\mu (1-\gamma_5)\nu)$$





From $b \to s\ell\ell$ to $b \to s\nu\bar{\nu}$

$\mathcal{C}_9 \to C_L^\nu$ $\quad$ $\mathcal{C}_{10} \to -C_L^\nu$ $\quad$ $\mathcal{C}_9' \to C_R^\nu$ $\quad$ $\mathcal{C}_{10}' \to -C_R^\nu$ $\quad$ $\mathcal{C}_7 \to 0$ $\quad$ $\mathcal{C}_7' \to 0$

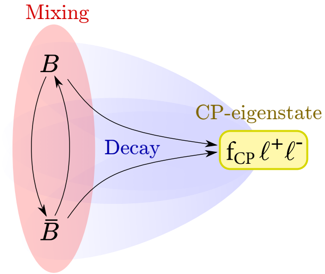
Constrain CP-phases through time-evolution

- Direct CP-asymmetries probe only the interplay of strong and weak phases
- To cleanly constrain CP-phases of NP we can look at the interaction of $B\bar{B}$ mixing and weak phases

$$B \to f_{\rm CP}\, \ell\bar{\ell} \qquad \text{vs} \qquad \bar{B} \to f_{\rm CP}\, \ell\bar{\ell}$$

- We will look at the following cases:
  - P vs V meson
  - Charged vs neutral leptons

$$f_{\rm CP} = K_S,\, K_S\pi^0,\, \phi \qquad\qquad \ell\bar{\ell} = \ell^+\ell^-,\, \nu\bar{\nu}$$



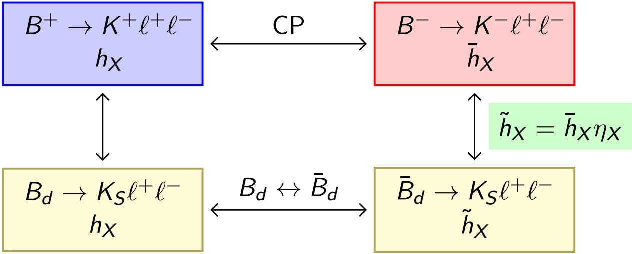
$B_d \to K_S \ell^+\ell^-$	$B_d \to K_S \nu\bar{\nu}$
$B_d \to K^{*0}(\to K_S\pi^0)\ell^+\ell^-$	$B_d \to K^{*0}(\to K_S\pi^0)\nu\bar{\nu}$
$B_s \to \phi\ell^+\ell^-$	$B_s \to \phi\nu\bar{\nu}$

$B_d \to K_S \ell\ell$

$B \to K\ell\ell$: Charged vs Neutral mode

$$\frac{d^2\Gamma(B^+ \to K^+\ell^+\ell^-)}{dq^2\, d\cos\theta_\ell} = G_0(q^2) + G_1(q^2)\cos\theta_\ell + G_2(q^2)\frac{1}{2}(3\cos^2\theta_\ell - 1)$$

$$G_2 = -\frac{4\beta_\ell^2}{3}\left(|h_V|^2 + |h_A|^2 - 2\,|h_T|^2 - 4\,|h_{T_t}|^2\right) \qquad \bar{h}_A \propto (\mathcal{C}_{10} + \mathcal{C}_{10'}) f_+(q^2)$$



h_X: Transversity amplitudes $\quad \eta_X$: CP-parity associated to h_X

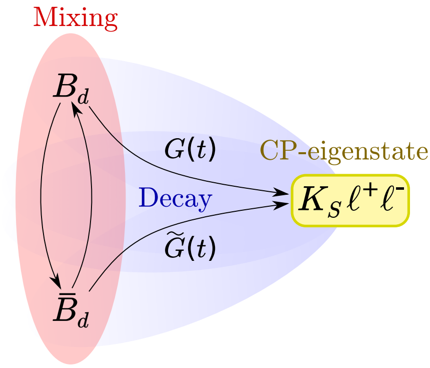
$$\eta_{V,A,P,T_t} = -1 \quad \text{and} \quad \eta_{S,T} = 1 \quad \Longrightarrow \quad \tilde{h}_X^{\text{SM}} = -\bar{h}_X^{\text{SM}}$$

[Dunietz et al '01, Descotes-Genon et al '15]

Time dependent analysis of $B_d \to K_S \ell\ell$ [2008.08000]

$$G_i(t) + \widetilde{G}_i(t) = e^{-\Gamma t}\left[(G_i + \widetilde{G}_i) \cosh\left(\frac{\Delta\Gamma t}{2}\right) - h_i \sinh\left(\frac{\Delta\Gamma t}{2}\right) \right]$$

$$G_i(t) - \widetilde{G}_i(t) = e^{-\Gamma t}\left[(G_i - \widetilde{G}_i) \cos(\Delta m t) - s_i \sin(\Delta m t) \right]$$



- 6 new observables

$$\sigma_i \equiv \frac{s_i}{\Gamma_\ell} \qquad \theta_i \equiv \frac{h_i}{\Gamma_\ell}$$

- $y = \Delta\Gamma_{B_d}/2\Gamma$ very small $\Rightarrow$ Only 3 observables (σ_i) accessible in $B_d \to K_S \ell^+ \ell^-$
- Interplay between mixing and decay $\Rightarrow$ Access weak phases!

$$s_2 = -\frac{8\beta_\ell^2}{3} \mathrm{Im}\left[e^{i\phi}\left[\tilde{h}_V h_V^* + \tilde{h}_A h_A^* - 2\tilde{h}_T h_T^* - 4\tilde{h}_{T_t} h_{T_t}^* \right]\right]$$

$$G_2 = -\frac{4\beta_\ell^2}{3}\left(|h_V|^2 + |h_A|^2 - 2\,|h_T|^2 - 4\,|h_{T_t}|^2 \right)$$

Time-integrated observables: Coherent vs incoherent production

- Time integration different for hadronic machines (incoherent production) and B-factories (coherent production).
  - Incoherent: $t \in [0, \infty)$ $\Rightarrow$time since b-quarks have been produced
  - Coherent: $t \in (-\infty, \infty)$ $\Rightarrow$time difference between B and $\bar{B}$ decay

$$\langle G_i - \widetilde{G}_i \rangle_{\text{Hadronic}} = \frac{1}{\Gamma}\left[\frac{1}{1+x^2} \times (G_i - \widetilde{G}_i) - \frac{x}{1+x^2} \times s_i\right]$$

$$\langle G_i - \widetilde{G}_i \rangle_{\text{B-factory}} = \frac{2}{\Gamma}\frac{1}{1+x^2}[G_i - \widetilde{G}_i]$$

- Hadronic machines involve an additional term compared to the B-factories ($x = \delta m/\Gamma$ $\Rightarrow$mixing parameter).

Time dependent analysis of $B_d \to K_S \ell^+ \ell^-$ [2008.08000]

- Requires flavour tagging
- **Accurately** computed in the SM and on V, A scenarios (independent of form factors and $c\bar{c}$ contributions!)

$$\sigma_2 \approx \sigma_0 = -\frac{\sin\phi}{2}$$

- In practice, **big sensitivity to complex phases** and independent of real SM-like NP

Observable	SM	$\mathcal{C}_{9\mu}^{\rm NP} = -1.12$	$\mathcal{C}_{9\mu}^{\rm NP} = -1.12 + i1.00$
σ_0	0.368(5)	0.368(5)	0.273(6)
σ_2	−0.359(5)	−0.359(5)	−0.266(6)

$B \to K\nu\bar{\nu}$ and $B \to K^*\nu\bar{\nu}$

CP-phases in $B \to K\nu\bar{\nu}$ and $B \to K^*\nu\bar{\nu}$

No available angular distribution for $B \to K\nu\bar{\nu}$ since ν goes undetected, one angle available in the case of $B \to K^*(\to K_S\pi^0)\nu\bar{\nu}$ (Longitudinal polarization fraction)

$$\mathcal{B}(B^+ \to K^+\nu\bar{\nu}) \propto \frac{1}{3}\sum_\nu (1-2\eta_\nu)\epsilon_\nu^2$$

$$\mathcal{B}(B^+ \to K^{*+}\nu\bar{\nu}) \propto \frac{1}{3}\sum_\nu (1+1.31\eta_\nu)\epsilon_\nu^2$$

$$\langle F_L(B^+ \to K^{*+}\nu\bar{\nu})\rangle \propto \frac{\sum_\nu (1+2\eta_\nu)\epsilon_\nu^2}{\sum_\nu (1+1.31\eta_\nu)\epsilon_\nu^2},$$

$$\epsilon_\nu = \frac{\sqrt{|C_L^\nu|^2 + |C_R^\nu|^2}}{|C_{\mathrm{SM}}^\nu|}, \; \eta_\nu = \frac{-\mathrm{Re}(C_L^\nu C_R^{\nu *})}{|C_L^\nu|^2 + |C_R^\nu|^2},$$

- CP-phases only accesible when right handed currents are present
- In absence of C_R^ν ($C_R^{\nu,\mathrm{SM}} = 0$) $\Rightarrow$Time-evolution observables (mixing + decay)

Time-dependent observables: $B \to K_S \nu\bar{\nu}$

- They can probe CP-phases when only left handed NP is present
- 1 amplitude $\bar{h}^{\nu}_V = -\bar{h}^{\nu}_A \propto V_{tb} V^*_{ts} (C^{\nu}_L + C^{\nu}_R) f_+$
- 3 observables available $\mathcal{B}^{\nu}$, $A^{\mathrm{CP},\nu}$, s^{ν}_0

$$\begin{aligned} \mathcal{B}^{\nu} &\propto |V_{tb} V^*_{ts}|^2 \, |(C^{\nu}_L + C^{\nu}_R)|^2 \, |f_+|^2 \\ s^{\nu}_0 &\propto \mathrm{Im}[(V_{tb} V^*_{ts})^2 e^{i\phi} (C^{\nu}_L + C^{\nu}_R)]^2 \, |f_+|^2 \end{aligned}$$

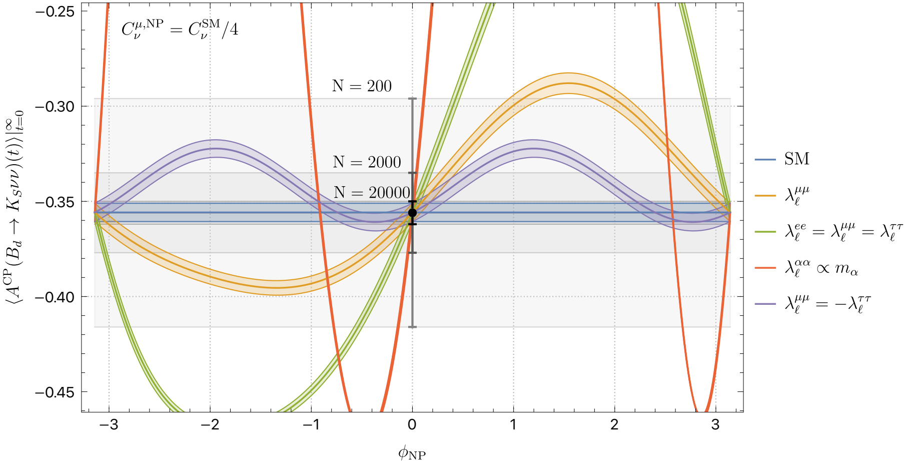
$$G^{\nu}_0(t) - \bar{G}^{\nu}_0(t) = e^{-\Gamma t} [(G^{\nu}_0 - \bar{G}^{\nu}_0) \cos(x \Gamma t) - s^{\nu}_0 \sin(x \Gamma t)] \, .$$

- Form factor dependence naturally cancels in the ratio $s_0 / \mathcal{B}$!

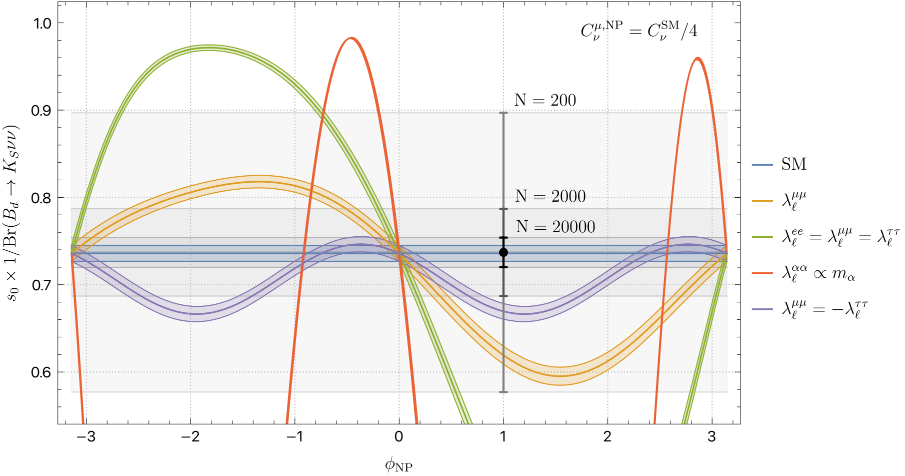
NP benchmark + future prospects at Belle II and FCCee

- Simplified projections for the future experimental uncertanties (no background, no detector acceptance, no systematics)
  - 200 events (Belle II $50\mathrm{ab}^1$)
  - 2000 events
  - 20000 events (FCCee, CEPC)
- Sum over neutrinos ⇒lepton flavour structure
- Comparaison with NP in C_L^ν (25% of SM) with several lepton flavour structures
  - Effect only on muon neutrinos
  - Democratic (Same effect for each family)
  - Hierarchical (Effects proportional to the masses of the charged leptons)
  - Anomaly free (Effect on muons opposed to tau)

NP benchmark + future prospects at Belle II and FCCee



NP benchmark + future prospects at Belle II and FCCee



Time-dependent observables: $B \to K^*\nu\bar{\nu}$

$$\left.\frac{d\Gamma(B_d \to K^{*0}\nu\bar{\nu})}{dq^2 d\cos\theta_M}\right|_{\rm no\ mixing} = \sum_\nu \left[\frac{3}{2} J_{1c}^\nu \cos^2\theta_M + J_{1s}^\nu \sin^2\theta_M\right],$$

$$J_{1c}^\nu \to |A_0^{L\nu}|^2, \qquad J_{1s}^\nu \to \frac{3}{4}\left[|A_\perp^{L\nu}|^2 + |A_\parallel^{L\nu}|^2\right].$$

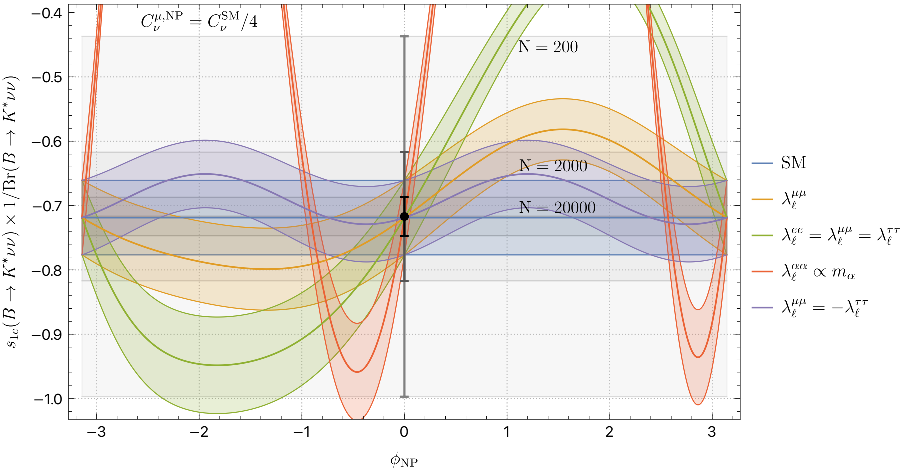
$$J_i^\nu(t) - \bar{J}_i^\nu(t) = e^{-\Gamma t}[(J_i^\nu - \bar{J}_i^\nu)\cos(x\Gamma t) - s_i^\nu \sin(x\Gamma t)].$$

- 3 amplitudes combining 3 different form factors
- 6 observables available $\mathcal{B}^\nu$, F_L^ν, $A^{\mathrm{CP},\nu}$, $A_{F_L}^\nu$, s_{1c}^ν, s_{2c}^ν
- Hadronic uncertainties do not cancel trivially

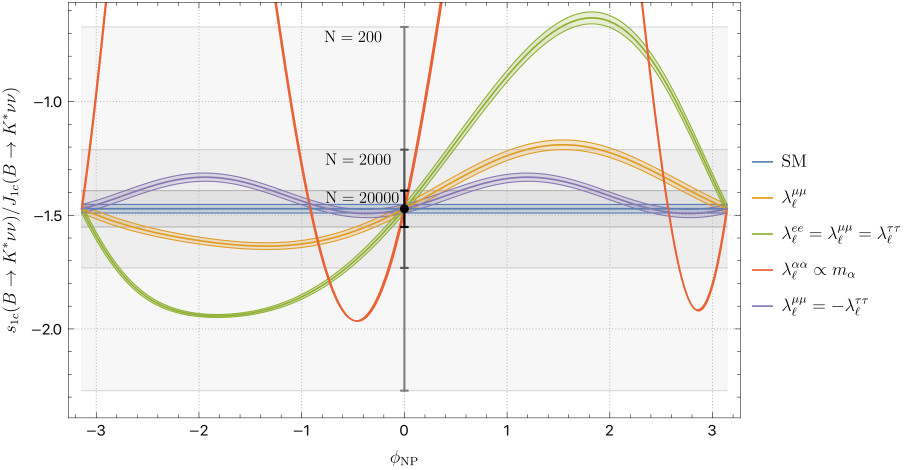
Optimized observables

Certain combinations allow for cancellation of hadronic uncertainties
$\langle A_{F_L}\rangle/\langle F_L\rangle$, s_{1c}/J_{1c}

NP benchmark + future prospects at Belle II and FCCee



NP benchmark + future prospects at Belle II and FCCee



Optimized!

Conclusions

- A $B_d \to K_S \ell^+ \ell^-$ time dependent analysis can help to constrain NP complex phases in $b \to s\ell\ell$
  - LHCb, BelleII? Precision at FCCee/CEPC?
- Similarly in $B_d \to K^{(*)} \nu\bar{\nu}$ a time-dependent analysis might (if no right handed currents are present) be the only way of accessing CP-phases
  - BelleII, FCCee/CEPC?
- Time dependent observables can be clean probes as their form factor dependence can be cancelled in optimized observables.
- Similar arguments can be made in the case of B_s decays.

Thank You!

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Back up

NP from an EFT Lagrangian

Same flavour structure for all operators

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}_{\text{SM}} - \frac{1}{v^2} \overbrace{\lambda^q_{ij} \; \lambda^\ell_{\alpha\beta}} \Big[C_T \left(\bar{Q}^i_L \gamma_\mu \sigma^a Q^j_L\right)\left(\bar{L}^\alpha_L \gamma^\mu \sigma^a L^\beta_L\right) + C_S \left(\bar{Q}^i_L \gamma_\mu Q^j_L\right)\left(\bar{L}^\alpha_L \gamma^\mu L^\beta_L\right)$$
$$+ C'_{RL} \left(\bar{d}^i_R \gamma_\mu d^j_R\right)\left(\bar{L}^\alpha_L \gamma^\mu L^\beta_L\right) + C'_{LR} \left(\bar{Q}^i_L \gamma_\mu Q^j_L\right)\left(\bar{\ell}^\alpha_R \gamma^\mu \ell^\beta_R\right) + C'_{RR} \left(\bar{d}^i_R \gamma_\mu d^j_R\right)\left(\bar{\ell}^\alpha_R \gamma^\mu \ell^\beta_R\right)\Big]$$

Quark Sector: $U(2)_q$ and Minimal Flavour Violation

We classify the NP flavour structure in terms of an approximate $U(2)_{q=Q,D}$ flavour symmetry.

$$\begin{array}{c} \mathbf{q} \equiv (q^1_L, q^2_L) \sim (\mathbf{2}, \mathbf{1}) \\ \mathbf{d} \equiv (d^1_R, d^2_R) \sim (\mathbf{1}, \mathbf{2}) \\ d^3_R, q^3_L \sim (\mathbf{1}, \mathbf{1}) \end{array} \qquad \lambda^q = \underbrace{\begin{pmatrix} \lambda^q_{22} & 0 & 0 \\ 0 & \lambda^q_{22} & 0 \\ 0 & 0 & \lambda^q_{33} \end{pmatrix}}_{\text{leading term } U(2)_q \text{ limit}} + \underbrace{\mathcal{O}(U(2)_q \text{ breaking})}_{\substack{\text{Non diagonal terms} \\ \text{Aligned with SM Yukawas (MFV)}}}$$

Lepton Sector: $U(1)^3_\ell$ symmetry

We consider $\lambda^\ell_{i\neq j} \simeq 0$ in order to fulfil LFV limits

1. The simplest: $\lambda^\ell_{\mu\mu} \neq 0$; $\lambda^\ell_{ee} = \lambda^\ell_{\tau\tau} = 0$
2. The anomaly-free assignment: $\lambda^\ell_{\mu\mu} = -\lambda^\ell_{\tau\tau}$; $\lambda^\ell_{ee} = 0$
3. The hierarchical charge scenario: $\lambda^\ell_{ee} \ll \lambda^\ell_{\mu\mu} \ll \lambda^\ell_{\tau\tau}$
4. The democratic scenario $\lambda^\ell_{ee} = \lambda^\ell_{\mu\mu} = \lambda^\ell_{\tau\tau}$

Weak Effective Theory

[Buchalla, Buras,...]

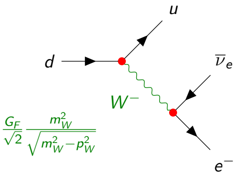
Local operator effective theory at scales below the electroweak scale

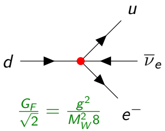
$$\mathcal{H}_{\text{eff}} \propto \sum_i \mathcal{C}_i \, \mathcal{O}_i$$

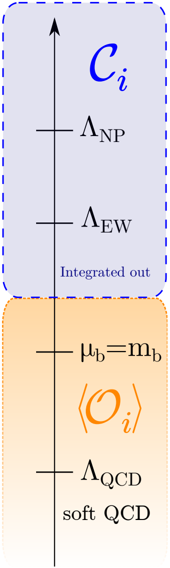
Short distance dynamics

Long distance structure

- As in Fermi Theory: non-local high energy processes $\Rightarrow$ local operators ($\mathcal{O}_i$) + Wilson coefficients ($\mathcal{C}_i$).
- Factorisation of scales $\mathcal{C}_i$ vs $\mathcal{O}_i$
- $\mathcal{C}_i$ accurately computed:
  - Matching at EW scale
  - Running to m_b scale (RGE, anomalous dimension and resummation)
- Model-independent NP framework through shifts of WC







Relating (G)MFV Lagrangian with WET

$$\mathcal{B}(B \to K\nu\bar{\nu}) = (4.5 \pm 0.7) \times 10^{-6} \frac{1}{3} \sum_\nu (1 - 2\eta_\nu)\epsilon_\nu^2 ,$$
$$\mathcal{B}(B \to K^*\nu\bar{\nu}) = (6.8 \pm 1.1) \times 10^{-6} \frac{1}{3} \sum_\nu (1 + 1.31\eta_\nu)\epsilon_\nu^2 ,$$
$$\mathcal{B}(B \to X_s\nu\bar{\nu}) = (2.7 \pm 0.2) \times 10^{-5} \frac{1}{3} \sum_\nu (1 + 0.09\eta_\nu)\epsilon_\nu^2 ,$$

[0902.0160]

where $\langle F_L \rangle$ is the longitudinal K^* polarisation fraction in $B \to K^*\nu\bar{\nu}$ decays. For each flavour of neutrino $\nu = \nu_e, \nu_\mu, \nu_\tau$, the two NP parameters can in turn be expressed as

$$\epsilon_\nu = \frac{\sqrt{|C_L^\nu|^2 + |C_R^\nu|^2}}{|C_{\rm SM}^\nu|} , \ \eta_\nu = \frac{-{\rm Re}(C_L^\nu C_R^{\nu *})}{|C_L^\nu|^2 + |C_R^\nu|^2} ,$$

where $C_{L,R}^\nu = C_{L,R}^{\nu,{\rm SM}} + C_{L,R}^{\nu,{\rm NP}}$ and $C_L^{\nu,{\rm SM}} = -6.38$ and $C_R^{\nu,{\rm SM}} = 0$ at $\mu = m_b$. Including leading $U(2)_q$ breaking effects we can write again

$$C_L^{\nu_\alpha,{\rm NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\alpha\alpha}^\ell [V_{ts}^* \theta_q e^{-i\phi_q} + r_{23}][C_S - C_T] ,$$
$$C_R^{\nu_\alpha,{\rm NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\alpha\alpha}^\ell r_{23} C_{RL}' ,$$

with $\alpha = e, \mu, \tau$.

Relating (G)MFV Lagrangian with WET

Linear order in $U(2)_q$ breaking

$$
b \to s\ell\ell \begin{cases}
C_9^{\mu,\mathrm{NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\mu\mu}^\ell [V_{ts}^* \theta_q e^{-i\phi_q} + r_{23}] \times (C_T + C_S + C'_{LR}) \\
C_{10}^{\mu,\mathrm{NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\mu\mu}^\ell [V_{ts}^* \theta_q e^{-i\phi_q} + r_{23}] \times (-C_T - C_S + C'_{LR}) \,, \\
C_{9'}^{\mu,\mathrm{NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\mu\mu}^\ell r_{23} (C'_{RR} + C'_{RL}) \\
C_{10'}^{\mu,\mathrm{NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\mu\mu}^\ell r_{23} (C'_{RR} - C'_{RL})
\end{cases}
$$

$$
b \to s\nu\bar{\nu} \begin{cases}
C_L^{\nu_\alpha,\mathrm{NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\alpha\alpha}^\ell [V_{ts}^* \theta_q e^{-i\phi_q} + r_{23}][C_S - C_T] \\
C_R^{\nu_\alpha,\mathrm{NP}} = -\frac{\pi}{\alpha_{em} V_{tb} V_{ts}^*} \lambda_{33}^q \lambda_{\alpha\alpha}^\ell r_{23} C'_{RL}
\end{cases}
$$

$$
s \to d\nu\bar{\nu} \begin{cases}
C_{sd}^{\nu_\alpha,\mathrm{NP}} = \frac{\pi s_W^2}{\alpha_{em}} \lambda_{33}^q \lambda_{\alpha\alpha}^\ell [\theta_q^2 V_{ts} V_{td}^* (C_S - C_T) \\
\quad + \theta_q (V_{ts} e^{i\phi_q} r_{13}^* + V_{td}^* e^{-i\phi_q} r_{23}) (C_S - C_T) \\
\quad + r_{12} (C_S - C_T + C'_{RL})]
\end{cases}
$$

$B \to K^*\nu\bar{\nu}$

- 3 amplitudes combining 3 different form factors
- 6 observables available $\mathcal{B}^\nu$,F_L^ν, $A^{\mathrm{CP},\,\nu}$, $A^\nu_{F_L}$, s^ν_{1c}, s^ν_{2c}

$$\bar{A}^L_\perp \propto V_{tb}V^*_{ts}(C^\nu_L + C^\nu_R)V(q^2) \qquad \bar{A}^L_\parallel \propto V_{tb}V^*_{ts}(C^\nu_L - C^\nu_R)A_1(q^2)$$

$$\bar{A}^L_0 \propto V_{tb}V^*_{ts}(C^\nu_L - C^\nu_R)\left[A_1(q^2) - \beta(m_B, m_{K*}, q^2)A_2(q^2)\right]$$

$$\left.\frac{d\Gamma(B_d \to K^{*0}\nu\bar{\nu})}{dq^2 d\cos\theta_M}\right|_{\text{no mixing}} = \sum_\nu \left[\frac{3}{2}J^\nu_{1c}\cos^2\theta_M + J^\nu_{1s}\sin^2\theta_M\right],$$

where

$$J^\nu_{1c} \to |A^{L\nu}_0|^2\,, \qquad J^\nu_{1s} \to \frac{3}{4}\left[|A^{L\nu}_\perp|^2 + |A^{L\nu}_\parallel|^2\right].$$

$$J^\nu_i(t) - \bar{J}^\nu_i(t) = e^{-\Gamma t}[(J^\nu_i - \bar{J}^\nu_i)\cos(x\Gamma t) - s^\nu_i \sin(x\Gamma t)]\,.$$