QED corrections for FCC-ee precision

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- Introduction
- General considerations
- LFU in $B \to K\mathit{ll}$: a case study
- Conclusions

University of Zurich UZH

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Introduction

The number of *B* and τ pairs @ FCC-ee will allow to reach incredibly small statistical errors, e.g:

- $\tau \to \mu(e)\nu\nu$ $N \sim 3\times10^{10}\times\epsilon_{eff}$ $\to$ $\sigma_{stat} \sim 10^{-5}$
- $B \to \pi\ \tau(\mu,e)\nu$ $N \sim 6\times10^{7}\times\epsilon_{eff}$ $\to$ $\sigma_{stat} \sim 10^{-4}$
- $B \to K\ \mu\mu(ee)$ $N \sim 10^{6}\times\epsilon_{eff}$ $\to$ $\sigma_{stat} \sim 10^{-3}$

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- Leptonic τ decays: Complete 2-loop QED corrections necessary if we aim to match the exp. error (*systematic err. will dominate* $\sim 10^{-4}$)

- Semileptonic B decays: QCD uncertainties are largely dominant in total rates and un-normalized distributions. But they cancel completely in clean (*and NP-sensitive*) observables such as the LFU ratios (*in both charged and neutral current transitions*) $\to$ QED dominant source of uncertainty

General considerations

QED corrections are tiny and well under control in fully inclusive rates

$$\Gamma_{\text{inlcusive}}[B \to X(\gamma)] = \Gamma_0(B \to X)[\ 1 + O(\alpha/\pi)\ ]$$

Theoretical "non-radiative" rate
(*UV-dependent quantity we would like to measure*)

few × 10^{-3}

However, what is measured are differential rates with photon cuts (θ_γ, E_γ)

$$\Gamma\,[B \to X(\gamma)]|_{\text{cuts}} = \Gamma_0(B \to X)[\ 1 + O(\ \alpha \,\text{Log}\left(\frac{m_l}{m_B}\right) \text{Log}\left(\frac{E_{cut}}{m_B}\right)\ )]$$

up to 10%
(with electrons)

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The (reported) inclusive rates are extrapolated from fit to data using MC

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QED-MC $\big\downarrow$ [e.g. PHOTOS]

$$\Gamma_{\text{inlcusive}}[B \to X(\gamma)]$$

up to 10%
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General considerations

- To which level of accuracy (and for which channels) can we trust PHOTOS & TAUOLA ?

  Z. Was *et al.* [Crakow's group]

- Is the maintenance of these tools ensured in the long term? Worth to develop alternative tools for cross checks (at least for some processes)?

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- To which level of accuracy (and for which channels) can we trust PHOTOS & TAUOLA ? Showering in QED (→ *generation of large logs*) is well understood. The two key problems are:

(I) Matching of LL result to exact $O(\alpha)$ (differential) matrix element
Necessary to achieve precision <0.1% in tree-level B & τ decays

- ✔ Available for leptonic tau decays → $O(\alpha^2 L^2)$ accuracy (TAUOLA) [My understanding discussing with Z. Was]
- ✖ Not available in semileptonic B & hadronic tau decays [*more difficult due to QED & QCD interplay*] → work in prog. in SCET, starting from $B \to l\nu(\gamma)$ [Beneke *et al.*; Neubert *et al.*]

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(II) Precise description of the "non-radiative rate" (*including long-distance effects & helicity-suppressed contributions*) → necessary to achieve precision < 1% in FCNC B decays

- ✔ $\mathcal{B}(B_s \to \mu\mu)$ [Beneke, Botbeth, Szafron, '19]
- ✔ R_K @ low q^2 [Bordone, GI, Pattori '16; GI, Naabebacsus, Zwicky '20 + Lancierini '22]
- ✖ *any other mode...*

LFU in B → K*ll*: a case study

- Analytic study of QED corrections (real & virtual) at fully differential level, using a generic hadronic form-factor

$$\bar{B}(p_B) \to \bar{K}(p_K)\, \ell^+(\ell_2)\, \ell^-(\ell_1) + \gamma(k)$$

$$\mathcal{L}_{\rm int}^{\rm EFT} = g_{\rm eff}\, L^\mu V_\mu^{\rm EFT} + {\rm h.c.}\ ,$$

$$V_\mu^{\rm EFT} = \sum_{n\geq 0} \frac{f_\pm^{(n)}(0)}{n!}(-D^2)^n[(D_\mu B^\dagger)K \mp B^\dagger(D_\mu K)] \longrightarrow f_\pm(q^2)$$

(*generic*)

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$$\bar{B}(p_B) \to \bar{K}(p_K)\,\ell^+(\ell_2)\,\ell^-(\ell_1) + \gamma(k)$$

- Proof that the dangerous $\alpha \log(m_l)$ terms cancel at the double diff. level, with a spcific choice of (*collinear-safe*) varibles

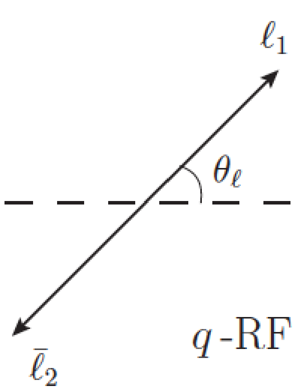
GI, Naabebacsus, Zwicky '20

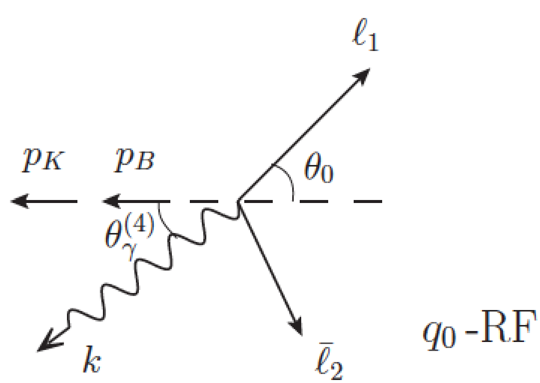
$$q_\ell^2 = (\ell_1 + \ell_2)^2, \qquad c_\ell = -\left(\frac{\vec{\ell}_1 \cdot \vec{p}_K}{|\vec{\ell}_1||\vec{p}_K|}\right)_{q-\mathrm{RF}}$$

$$\boxed{q_0^2 = (p_B - p_K)^2\,, \qquad c_0 = -\left(\frac{\vec{\ell}_1 \cdot \vec{p}_K}{|\vec{\ell}_1||\vec{p}_K|}\right)_{q_0-\mathrm{RF}}}$$

→ No LFU logs

but log(m_K) terms remain for charged mode





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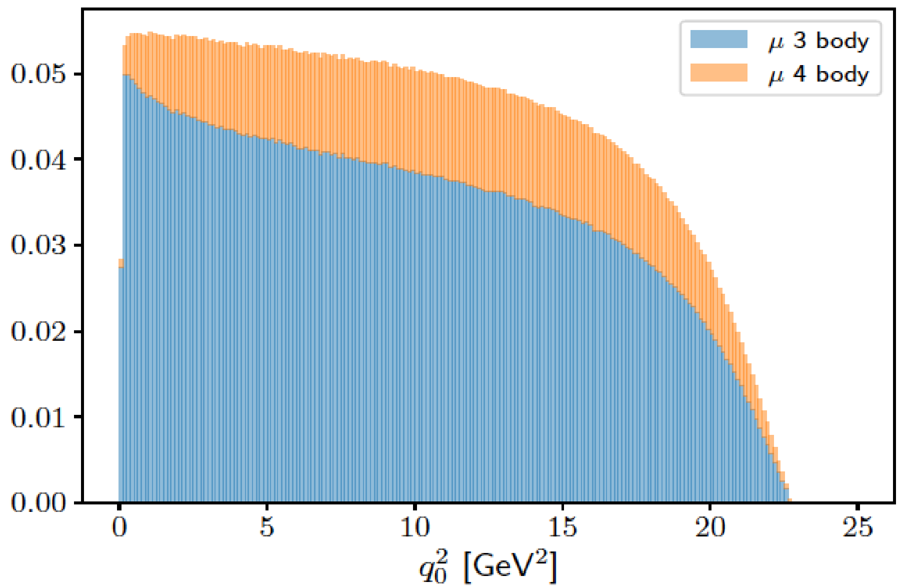
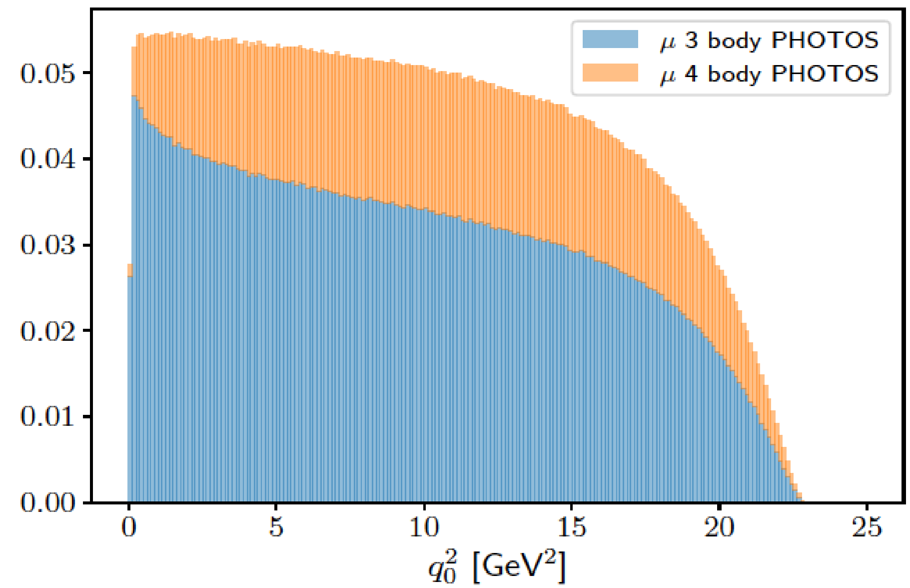
↓

- Th. basis to build a dedicated MC tool to check PHOTOS (*with arbitrary f.f. & possible LD terms*)

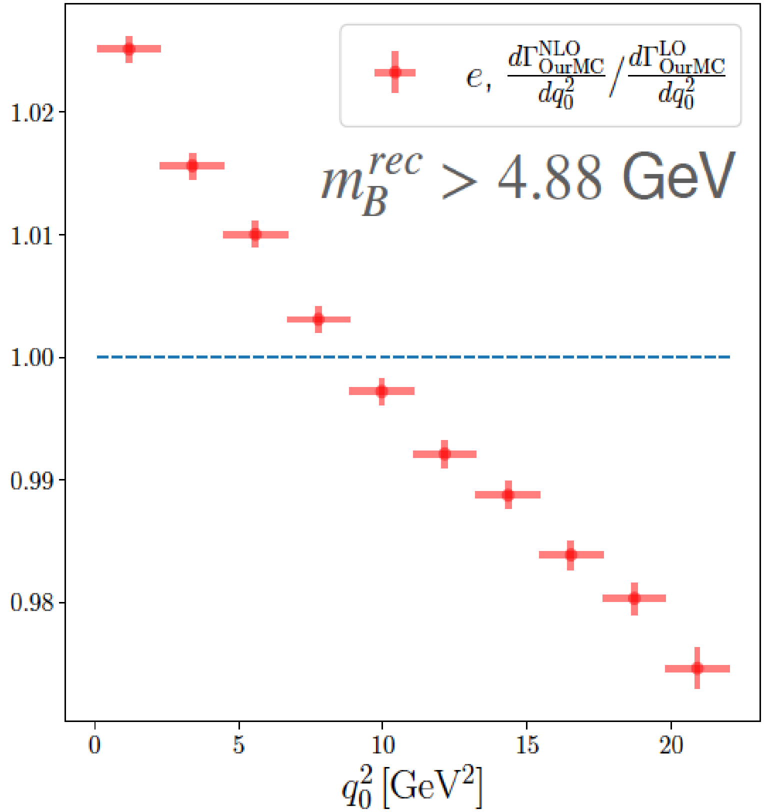
GI, Lancierini, Naabebacsus, Zwicky '22

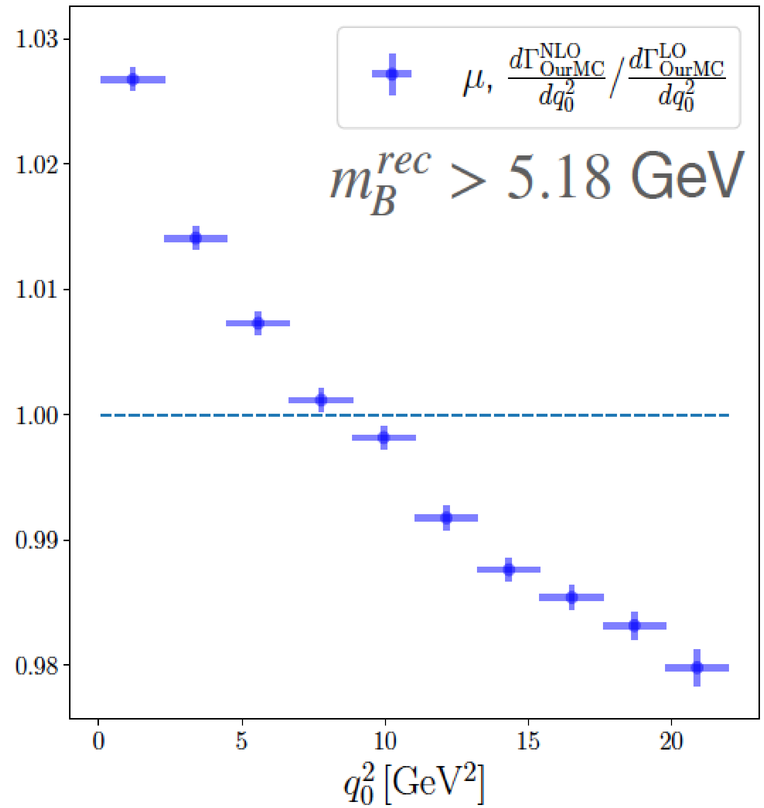




LFU in B → Kll: a case study

Relative impact of QED corrections in q_0^2, for both e and mu, with different cuts on the reconstructed B-meson mass [LHCb ref. values for R_K]:





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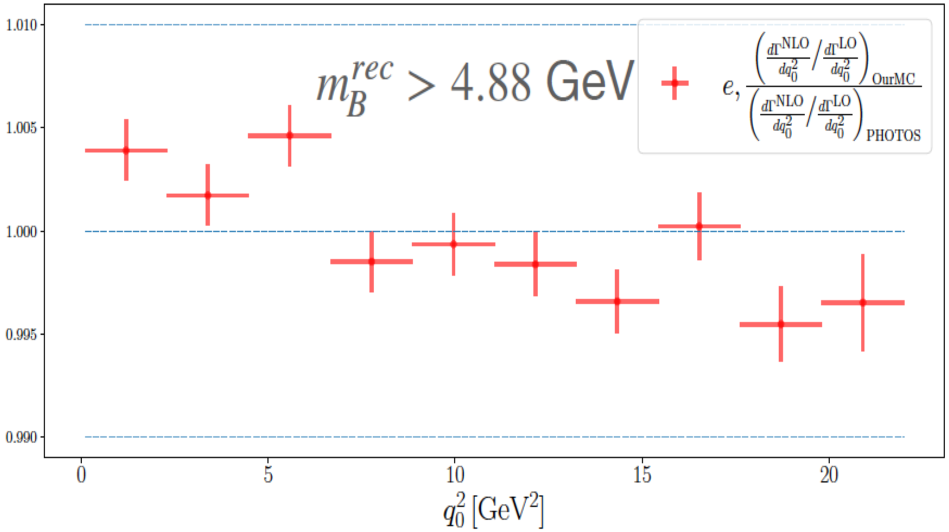
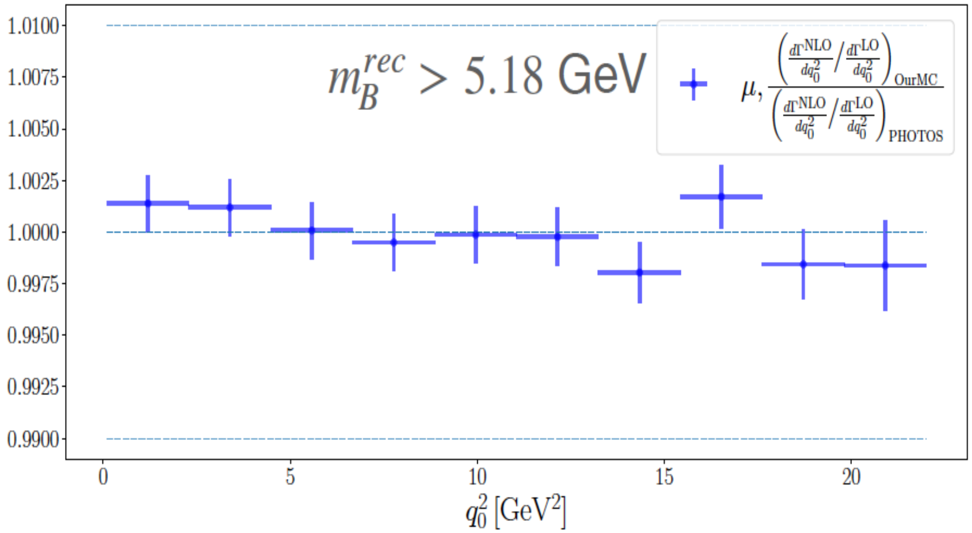
Excellent comparison with PHOTOS





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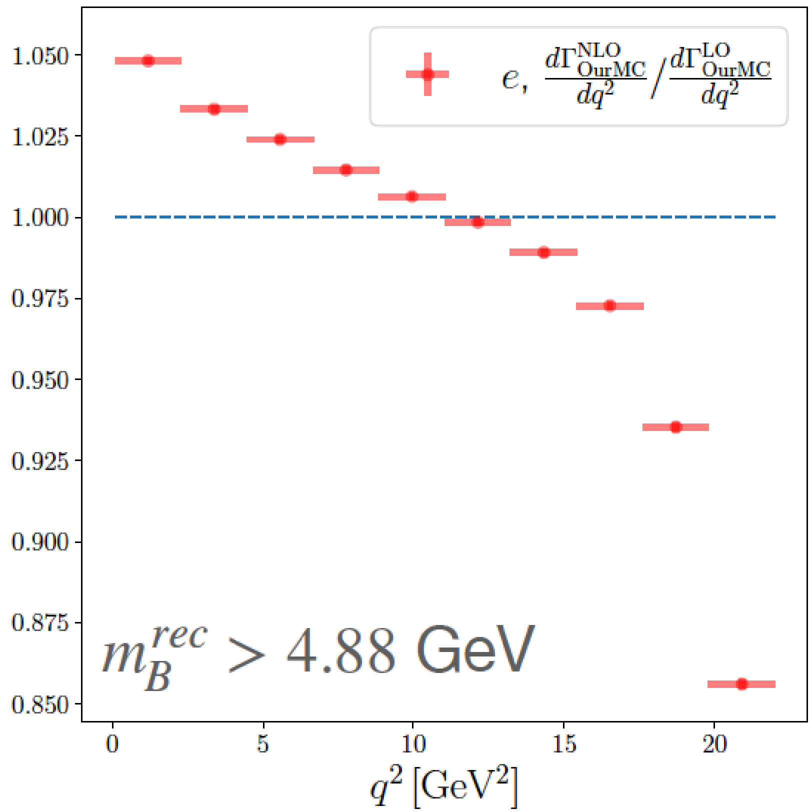
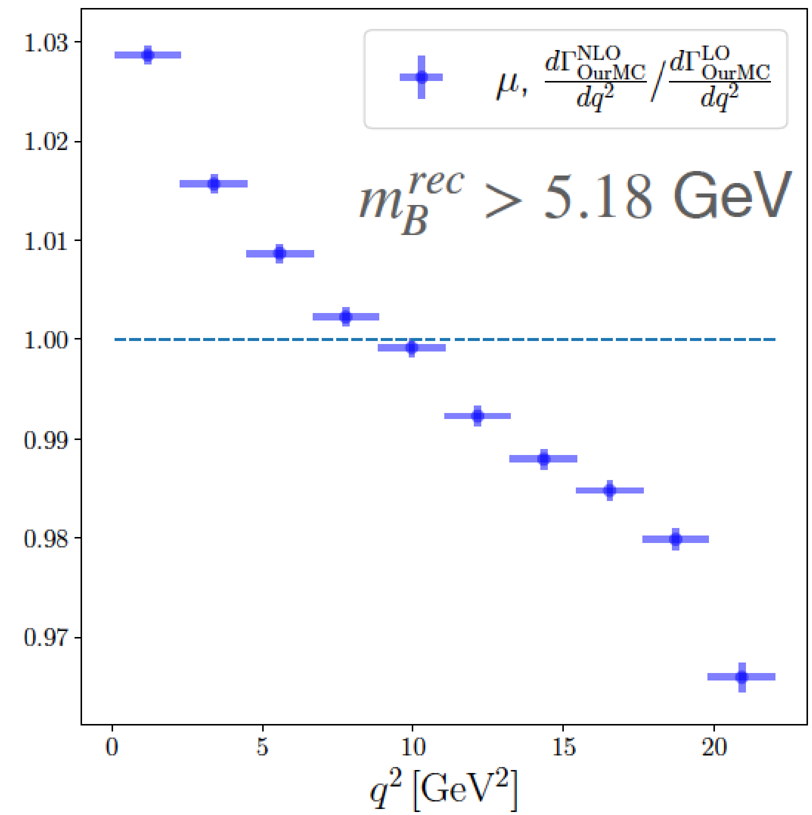
Relative impact of QED corrections in q^2, for both e and mu, with different cuts on the reconstructed B-meson mass [LHCb ref. values for R_K] → large LFU violations of QED origin





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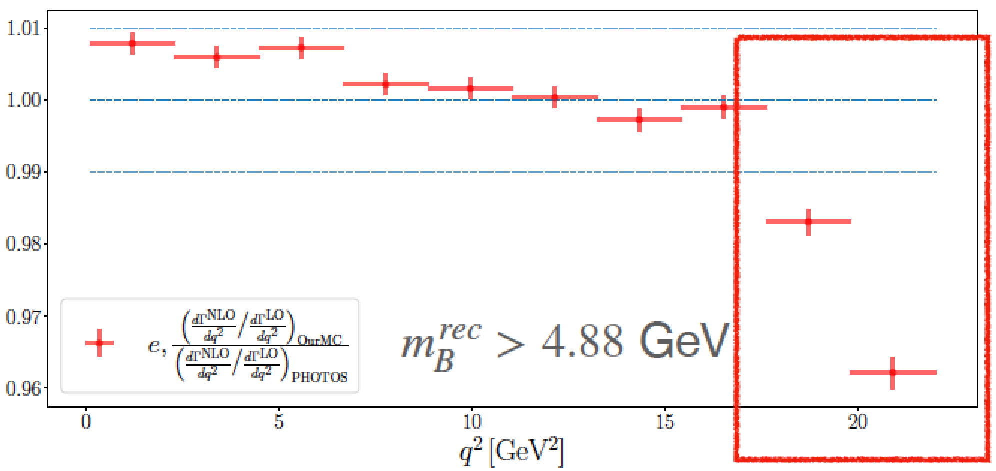
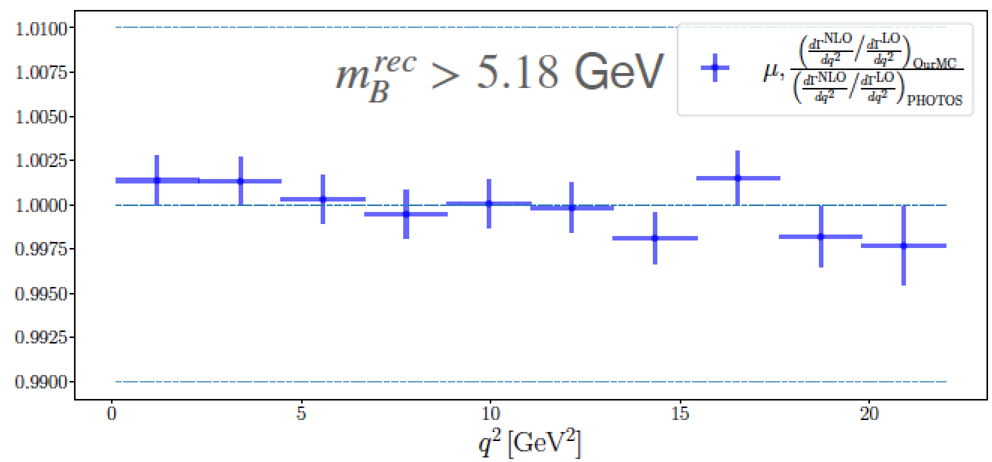
Good agreement (<1%) with PHOTOS, beside electrons @ high-q^2

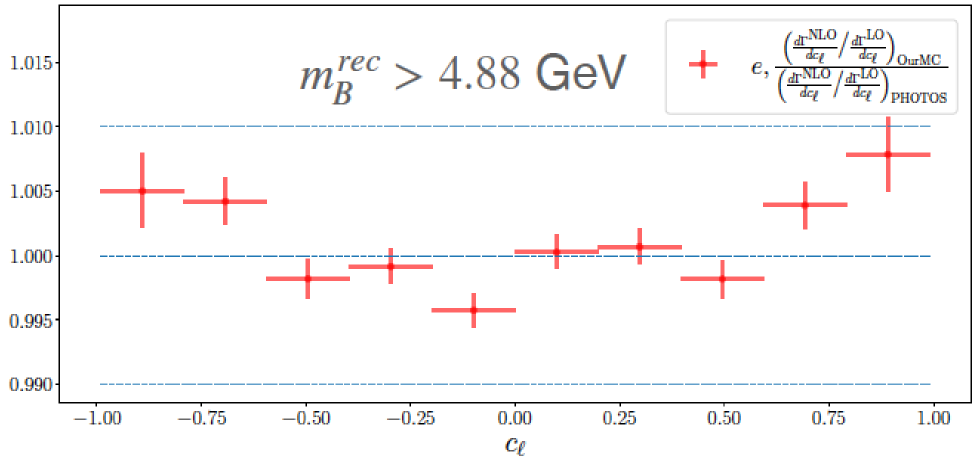


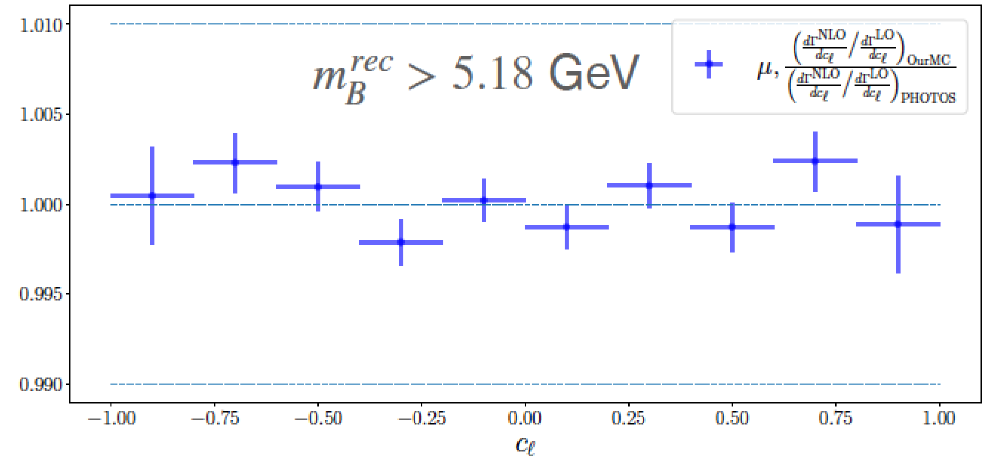


high-q^2 discrepancy well understood in terms of multi-photon emission (present only in PHOTOS)

LFU in B → K*ll*: a case study

Good comparions with PHOTOS for angular distributions:





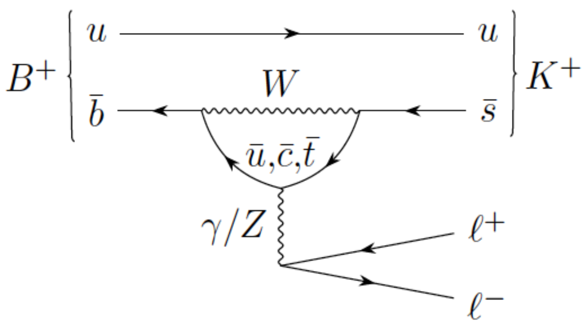
LFU in B → Kll: a case study

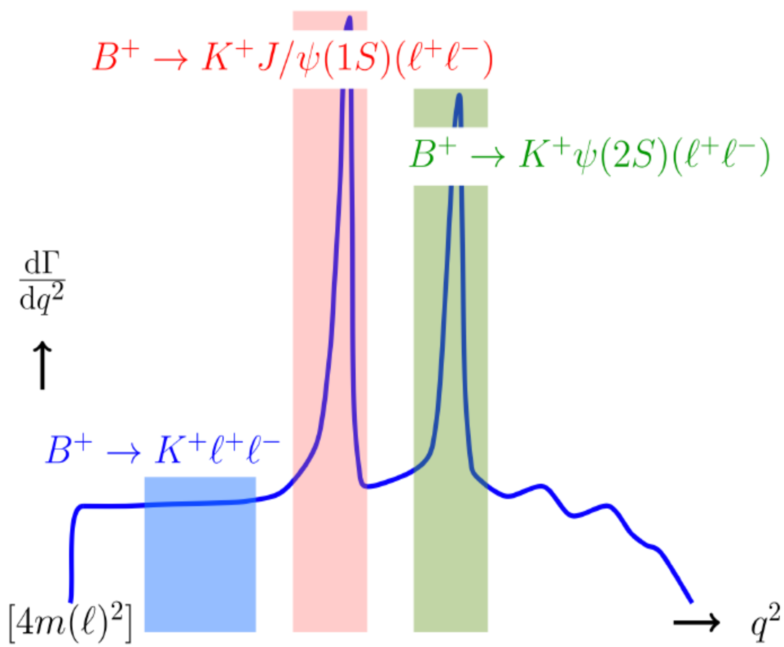
The most interesting aspect of our MC is the possibilty to check the relevance of possible SD-LD inteferece effects

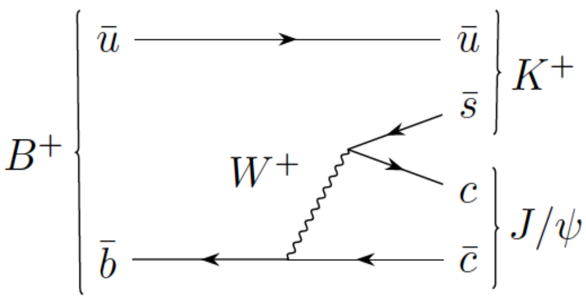
SD = Rare FCNC amplitude

LD = Narrow charmonium resonances

These effects are not inlcuded in PHOTOS ("rare modes" and "charmonium modes" simulated incoherently)







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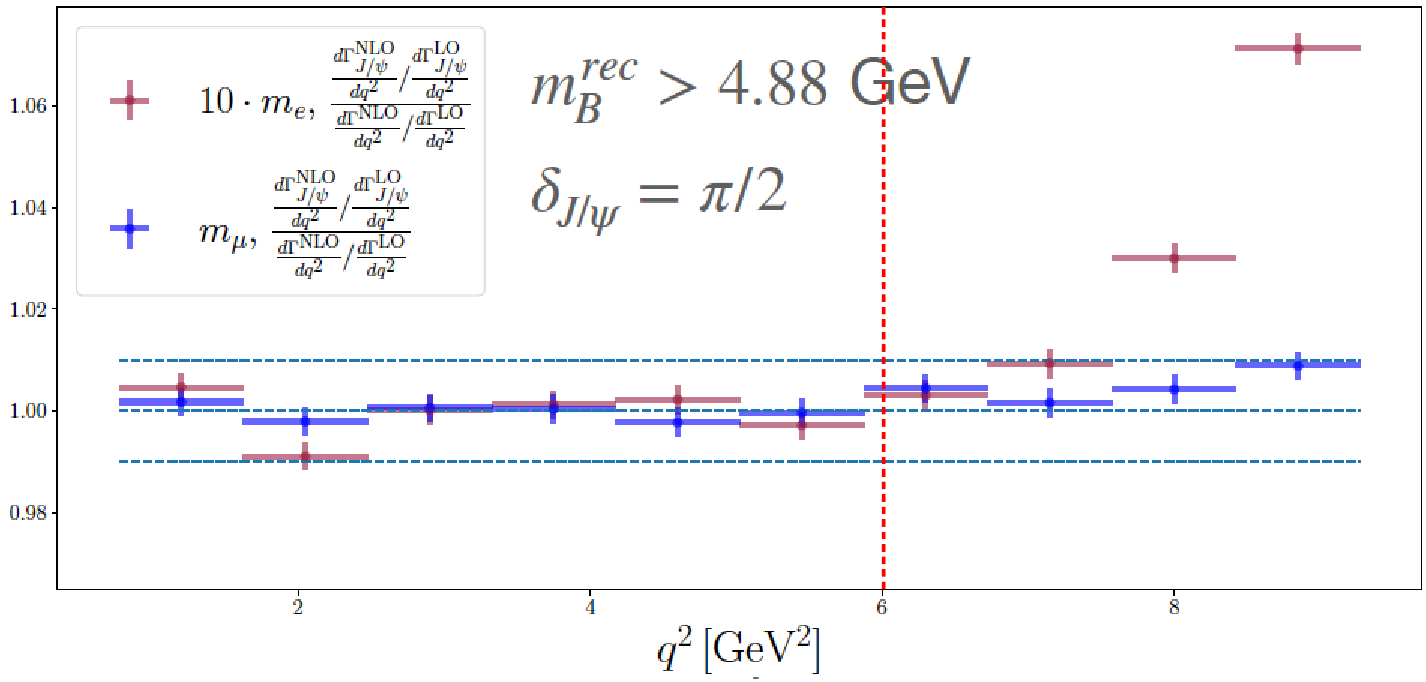

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Well within 1% for $q^2 < 6$ GeV^2

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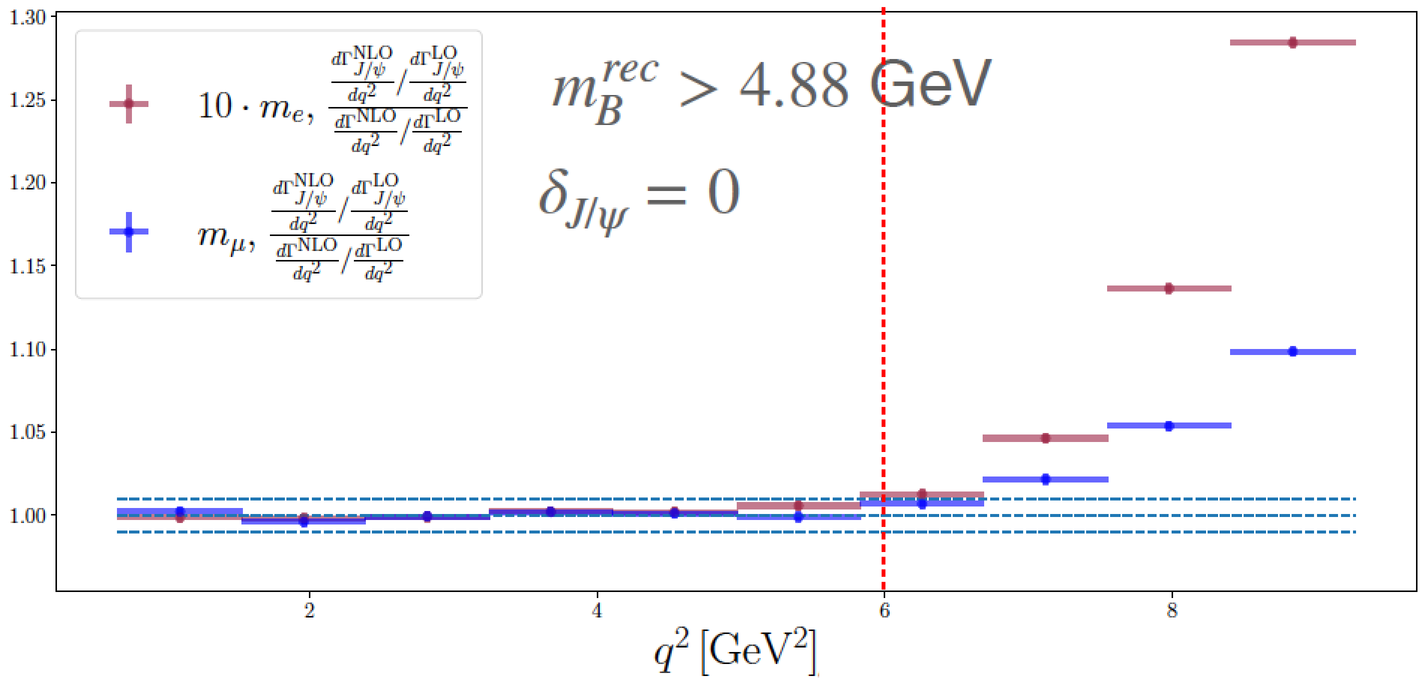
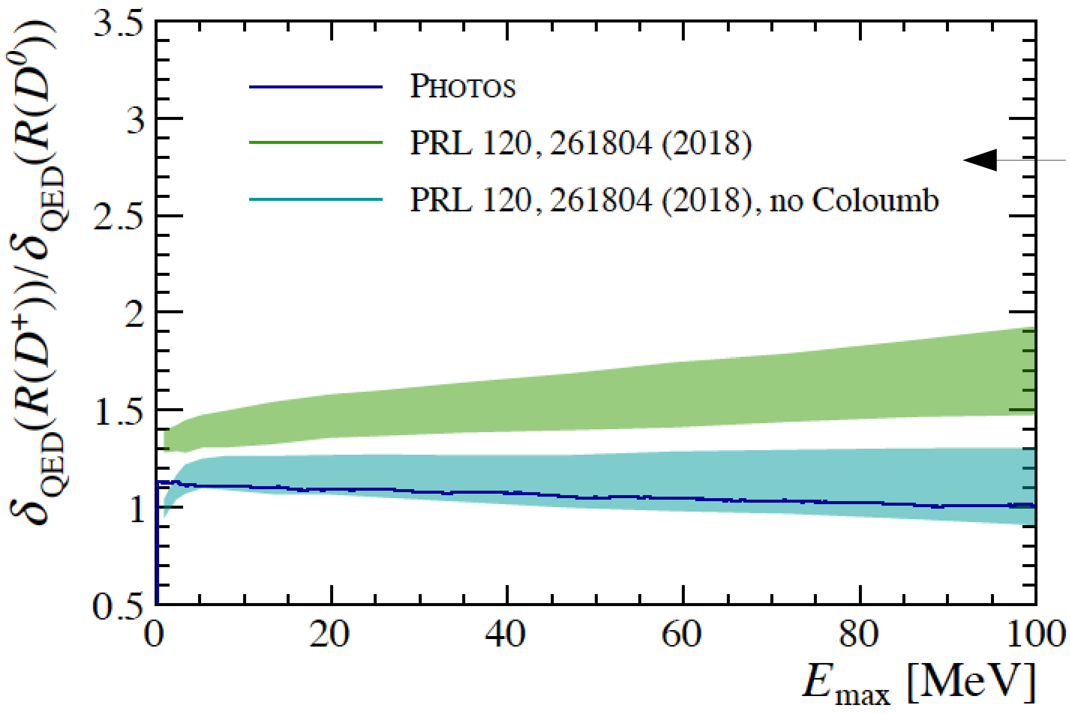


Other modes....

Cross-cheking generic tools (PHOTOS) with dedicated (*theory friendly...*) MC tools is something that should be done in several representative channles.

Impacts of radiative corrections on measurements of lepton flavour universality in $B \to D\ell\nu_\ell$ decays

Stefano Calí[a,1], Suzanne Klaver[b,1], Marcello Rotondo[c,1], Barbara Sciascia[d,1]



S. de Boer, T. Kitahara, and I. Nisandzic

Conclusions

High-precison flavor physics @ FCC-ee calls for significant steps forward in the description of QED corrections, especially in semilpetonic B decays (both charged-currents & rare modes)

A lot of work still to be done, both @ analytic level (→ exact O(α) matrix elements) and as far as simulations tools are concerend (→ *key role* !)