Normalizing Flows at the LHC Preparing for the Future — ML in HEP, HEPHY/OeAW Vienna —

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Dr. Claudius Krause

CV Research Experience Dr. rer. nat (2013–2016) I MU Munich Effective Field Theories Postdoc (2016–2018) (conceptual & applied) IFIC Valencia Electroweak Phase Transition Feodor-Lynen Fellow (2018–2020) Beyond the SM Fermilab Machine Learning Applications to Postdoc (2020–2022) simulation & data analyses Rutgers University **E** Talks Community / Organizer Seminars (22) "Fast Calorimeter Challenge 2022" recent: ITP, LBNL, NIKHEF • "Multibosons At The Energy Invited (26) Frontier" workshop at Fermilab recent: IAIFI, Snowmass CSS Pheno-Seminar at Rutgers

• Contributed (21) recent: MODE, DPF, ML4Jets

• HEP Journal Club at Fermilab

Open Questions in High-Energy Physics.

What's not in the Standard Model

- The Nature of Dark Matter
- Neutrino masses
- The Baryon Asymmetry of the Universe
- Dark Energy / Inflation

Experimental Anomalies

- Flavor Observables
- (g − 2)_µ
- Hubble constant H₀

Theoretical Problems

- The Hierarchy Problem
- Origin of Flavor
- Unification of Forces
- Quantum Gravity

Currently explored in Experiments

- Higgs & electroweak Sector of the SM
- Neutrino masses and Hierarchy
- Strong Dynamics
- New Particles & Interactions

We will have a lot more data in the near future.



https://lhc-commissioning.web.cern.ch/schedule/HL-LHC-plots.htm





Machine Learning and LHC Event Generation, A. Butter, CK et al. [2203.07460]





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- (A lot of) high-precision simulations.
- Analyzing high-dimensional data: Simulation-based Inference and data-driven Anomaly Searches.



Machine Learning and LHC Event Generation, A. Butter, CK et al. [2203.07460]

- (A lot of) high-precision simulations.
- Analyzing high-dimensional data: Simulation-based Inference and data-driven Anomaly Searches.

ML has impacted every aspect of the simulation chain, with one class of models being very powerful: **Normalizing Flows**

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Normalizing Flows learn a change-of-coordinates efficiently.



Normalizing Flows learn a change-of-coordinates efficiently.



Having access to the log-likelihood (LL) allows several training options:

- \Rightarrow Based on samples: via maximizing LL(samples).
- \Rightarrow Based on target function f(x): via matching p(x) to f(x).

NFs can also be used for inference: learn p(parameters|data).

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Normalizing Flows attack Bottlenecks in the Analysis Chain





Normalizing Flows increase the Sensitivity in our Analyses



(My Contributions to) Normalizing Flows at the LHC



- NFs learn the parameters θ of a series of easy transformations. Dinh et al. [arXiv:1410.8516], Rezende/Mohamed [arXiv:1505.05770]
- Each transformation is 1d & has an analytic Jacobian and inverse.
 - ⇒ We use Rational Quadratic Splines Durkan et al. [arXiv:1906.04032], Gregory/Delbourgo [IMA J. of Num. An., '82]
- Require a triangular Jacobian for faster evaluation.

 \Rightarrow The parameters θ depend only on a subset of all other coordinates.

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https://engineering.papercup.com/posts/normalizing-flows-part-2/

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Autoregressive Blocks (MAF/IAF)

- Coordinates are transformed autoregressivly $\Rightarrow \theta_{x_i}(x_{j < i})$
- + Are very powerful.
- Have a fast and a slow direction.

Bipartite Blocks (Coupling Layers)

• Coordinates are split in 2 sets, transforming each other \Rightarrow

$$\theta_{x\in A}(x\in B)$$
 & $\theta_{x\in B}(x\in A)$

- + Are equally fast in both directions.
- Are not as expressive.

(My Contributions to) Normalizing Flows at the LHC



Phase Space integration uses Importance Sampling.

$$I = \int_{0}^{1} f(\vec{x}) d\vec{x} \qquad \xrightarrow{\text{MC}} \quad \frac{1}{N} \sum_{i} f(\vec{x}_{i}) \qquad \vec{x}_{i} \dots \text{uniform}, \quad \sigma_{\text{MC}}(I) \sim \frac{1}{\sqrt{N}}$$
$$= \int_{0}^{1} \frac{f(\vec{x})}{q(\vec{x})} q(\vec{x}) d\vec{x} \qquad \xrightarrow{\text{MC}} \quad \frac{1}{N} \sum_{i} \frac{f(\vec{x}_{i})}{q(\vec{x}_{i})} \qquad \vec{x}_{i} \dots q(\vec{x}),$$
$$\text{In the limit } q(\vec{x}) \propto f(\vec{x}), \text{ we get } \sigma_{\text{IS}}(I) = 0$$

We therefore have to find a $q(\vec{x})$ that approximates the shape of $f(\vec{x})$.

 \Rightarrow Once found, we can use it for event generation, *i.e.* sampling p_i, ϑ_i , and φ_i according to $d\sigma(p_i, \vartheta_i, \varphi_i)$

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$$\begin{split} I &= \int_{0}^{1} f(\vec{x}) \ d\vec{x} & \xrightarrow{\text{MC}} \quad \frac{1}{N} \sum_{i} f(\vec{x}_{i}) & \vec{x}_{i} \dots \text{uniform}, \quad \sigma_{\text{MC}}(I) \sim \frac{1}{\sqrt{N}} \\ &= \int_{0}^{1} \frac{f(\vec{x})}{q(\vec{x})} \ q(\vec{x}) d\vec{x} & \xrightarrow{\text{MC}} \quad \frac{1}{N} \sum_{i} \frac{f(\vec{x}_{i})}{q(\vec{x}_{i})} & \vec{x}_{i} \dots q(\vec{x}), \\ & \text{In the limit } q(\vec{x}) \propto f(\vec{x}), \text{ we get } \sigma_{\text{IS}}(I) = 0 \end{split}$$

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We need both samples x and their probability q(x). \Rightarrow We use a bipartite, coupling-layer-based Flow.

i-flow: Numerical Integration with Normalizing Flows.



Statistical Divergences are used as loss functions:

• Kullback-Leibler (KL) divergence:

$$D_{KL} = \int p(x) \log \left[\frac{p(x)}{q(x)} dx \right] \approx \frac{1}{N} \sum \frac{p(x_i)}{q(x_i)} \log \frac{p(x_i)}{q(x_i)}, \quad x_i \dots q(x)$$

Sherpa needs a high-dimensional integrator.

Sherpa is a Monte Carlo event generator for the Simulation of High-Energy Reactions of PArticles. We use Sherpa to

- compute the matrix element of the process.
- map the unit-hypercube of our integration domain to momenta and angles. To improve efficiency, Sherpa uses a recursive multichannel algorithm.

$$\Rightarrow n_{dim} = \underbrace{3n_{final} - 4}_{\text{kinematics}} + \underbrace{n_{final} - 1}_{\text{multichannel}}$$

https://sherpa.hepforge.org/

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Figure of merit: Unweighting efficiency

- Unweighting: we need to accept/reject each event with probability ^{f(xi)}/_{max f(x)}. The kept events are unweighted and reproduce the shape of f(x).
- The unweighting efficiency is the fraction of events that "survives" this procedure.



An easy example: $e^+e^- \rightarrow 3j$.



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High Multiplicities are difficult to learn in this setup.

unweighting efficiency		LO QCD			
$\langle w \rangle / w_{ m max}$		<i>n</i> =0	n = 1	<i>n</i> =2	n =3
$W^+ + n$ jets	Sherpa	$2.8\cdot 10^{-1}$	$3.8\cdot10^{-2}$	$7.5 \cdot 10^{-3}$	$1.5 \cdot 10^{-3}$
	i-flow	$6.1\cdot10^{-1}$	$1.2\cdot10^{-1}$	$1.0\cdot10^{-2}$	$1.8 \cdot 10^{-3}$
	Gain	2.2	3.3	1.4	1.2
$W^- + n$ jets	Sherpa	$2.9\cdot10^{-1}$	$4.0 \cdot 10^{-2}$	$7.7 \cdot 10^{-3}$	$2.0 \cdot 10^{-3}$
	i-flow	$7.0 \cdot 10^{-1}$	$1.5 \cdot 10^{-1}$	$1.1\cdot10^{-2}$	$2.2 \cdot 10^{-3}$
	Gain	2.4	3.3	1.4	1.1
Z + n jets	Sherpa	$3.1\cdot10^{-1}$	$3.6\cdot10^{-2}$	$1.5\cdot10^{-2}$	$4.7 \cdot 10^{-3}$
	i-flow	$3.8\cdot10^{-1}$	$1.0\cdot 10^{-1}$	$1.4\cdot10^{-2}$	$2.4 \cdot 10^{-3}$
	Gain	1.2	2.9	0.91	0.51
	C.	Gao, S. Höche,	J. Isaacson, CK ,	H. Schulz [arXiv:	2001.10028, PRD]

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Improvements:

- make channel number a conditional variable and learn it separately.
- re-use matrix elements multiple times.
- introduce learnable soft permutations, use VEGAS for base dist.

A. Butter, T. Heimel, J. Isaacson, CK, F. Maltoni, O. Mattelaer, T. Plehn, R. Winterhalder [in preparation]

(My Contributions to) Normalizing Flows at the LHC



We use the same calorimeter geometry as $\operatorname{CaloGAN}$

- We consider a toy calorimeter inspired by the ATLAS ECal: flat alternating layers of lead and LAr
- They form three instrumented layers of dimension 3×96 , 12×12 , and 12×6



CaloGAN: Paganini, de Oliveira, Nachman [1705.02355, PRL; 1712.10321, PRD]

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- The GEANT4 configuration of CALOGAN is available at $_{\rm https://github.com/hep-lbdl/CaloGAN}$
- We produce our own dataset: available at [DOI: 10.5281/zenodo.5904188]
- Showers of e^+, γ , and π^+ (100k each)
- All are centered and perpendicular
- $E_{\rm inc}$ is uniform in [1, 100] GeV and given in addition to the energy deposits per voxel:



CaloGAN: Paganini, de Oliveira, Nachman [1705.02355, PRL; 1712.10321, PRD]

CALOFLOW uses a 2-step approach to learn $p(\vec{\mathcal{I}}|E_{inc})$.

Flow I

- learns $p_1(E_0, E_1, E_2 | E_{inc})$
- is optimized using the log-likelihood.

Flow II

- learns $p_2(\vec{\mathcal{I}}|E_0, E_1, E_2, E_{inc})$ of normalized showers
- in CALOFLOW v1 (2106.05285 called "teacher"):
 - Masked Autoregressive Flow trained with log-likelihood
 - Slow in sampling ($\approx 500 \times$ slower than $\rm CALOGAN)$
- in CALOFLOW v2 (2110.11377 called "student"):
 - Inverse Autoregressive Flow trained with Probability Density Distillation from teacher (log-likelihood prohibitive)

van den Oord et al. [1711.10433]

- i.e. matching IAF parameters to frozen MAF
- Fast in sampling ($\approx 500 \times$ faster than $\rm CALOFLOW$ v1)

A Classifier provides the "ultimate metric".

According to the Neyman-Pearson Lemma we have:

- The likelihood ratio is the most powerful test statistic to distinguish the two samples.
- A powerful classifier trained to distinguish the samples should therefore learn (something monotonically related to) this.
- If this classifier is confused, we conclude $p_{\text{GEANT4}}(x) = p_{\text{generated}}(x)$
- \Rightarrow This captures the full 504-dim. space.

- ? But why wasn't this used before?
- \Rightarrow Previous deep generative models were separable to almost 100%!

DCTRGAN: Diefenbacher et al. [2009.03796, JINST]

$\operatorname{CaloFLOW}$ passes the "ultimate metric" test.

According to the Neyman-Pearson Lemma we have: $p_{\text{GEANT4}}(x) = p_{\text{generated}}(x)$ if a classifier cannot distinguish data from generated samples.

AUC		DNN based classifier				
		Geant 4 vs . CaloGAN	GEANT 4 vs. (teacher) CALOFLOW v1	GEANT 4 vs. (student) CALOFLOW v2		
e ⁺	unnorm.	1.000(0)	0.859(10)	0.786(7)		
	norm.	1.000(0)	0.870(2)	0.824(4)		
γ	unnorm.	1.000(0)	0.756(48)	0.758(14)		
	norm.	1.000(0)	0.796(2)	0.760(3)		
<i>π</i> ⁺	unnorm.	1.000(0)	0.649(3)	0.729(2)		
7	norm.	1.000(0)	0.755(3)	0.807(1)		
AUC $(\in [0.5, 1])$: Area Under the ROC Curve, smaller is better, i.e. more confused						

Sampling Speed: The Student beats the Teacher!

		CaloFlow*		CaloGAN*	${ m Geant4}^\dagger$	
		teacher	student			
	training	22+82 min	+ 480 min	210 min	0 min	
	generation time per shower	36.2 ms	0.08 ms	0.07 ms	1772 ms	
*: on our TITAN V GPU, [†] : on the CPU of CaloGAN: Paganini, de Oliveira, Nachman [1712.10321, PRD]						



CALOFLOW: Comparing Shower Averages: e^+







CALOFLOW: histograms: e^+



CALOFLOW: histograms: e^+



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Going the next step: towards deployment in FastSimulation

We have a rapidly evolving field: need a survey of current approaches on a common dataset!



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(My Contributions to) Normalizing Flows at the LHC



Bump Hunts have few model assumptions.



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Background Signal

Signal

Background

0.8

Sidebands (SB)

8

10

Simulation-based approaches are model-dependent.

Simulation-based approaches:

• fully supervised:

train classifier on simulated signal and background

- depends on quality of simulation
- high signal model dependence
- provides upper limit on all approaches
- idealized anomaly detector:

train classifier on data and simulated background

- depends on quality of simulation
- still background model dependent
- provides upper limit on data-driven anomaly detection

Data-driven approaches are background model-independent.



Data-driven approaches are background model-independent.



Data-driven approaches are background model-independent.

Classifying Anomalies THrough Outer Density Estimation (CATHODE):

- train "outer" density estimator
 p_{data}(x|m_{JJ} ∈ SB)
- sample "artificial" events from $p_{outer}(x|m_{JJ} \in SR)$
- can also oversample
- train a classifier on these samples vs data



\Rightarrow combines the best of CWoLa-Hunting and ANODE!

A. Hallin, J. Isaacson, G. Kasieczka, **CK**, B. Nachman, T. Quadfasel, M. Schlaffer, D. Shih, M. Sommerhalder [2109.00546, PRD]

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CATHODE outperforms other anomaly detectors.

Results:

- showing SIC = TPR/ \sqrt{FPR}
- CATHODE approaches idealized AD
- outperforms ANODE (only 1 density estimator)
- outperforms CWoLa (robust against correlations)

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A. Hallin, CK et al. [2109.00546, PRD]



\Rightarrow These strategies are now being explored in ATLAS and CMS.

ATLAS [2005.02983, PRL]

Normalizing Flows at the LHC Preparing for the Future

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- Understanding everything based on 1st principles suffers from computational bottlenecks that can be tackled with ML, and especially Normalizing Flows.

Normalizing Flows at the LHC Preparing for the Future

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- Understanding everything based on 1st principles suffers from computational bottlenecks that can be tackled with ML, and especially Normalizing Flows.



Normalizing Flows at the LHC Preparing for the Future

- \Rightarrow With more efficiency and more sensitivity, we will be able to use the LHC to its full potential.
- ⇒ Normalizing Flows density estimators and generative models will help with this endeavor.



The Baryon Asymmetry of the Universe?

Backup

Taming Jacobians 1: with Autoregressive Blocks



 Masked Autoregressive Flow (MAF) is slow in sampling and fast in inference. Papamakarios et al. [arXiv:1705.07057]

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Adding Noise is important for the sampling quality.



• The log-likelihood is less noisy, but smaller. Yet, the quality of the samples is much better!

• This is due to a "wider" mapping of space and less overfitting.

CALOFLOW: Flow I+II histograms: e^+



CALOFLOW: Flow II histograms: e^+



Nearest Neighbors: e^+ (student)



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Normalizing Flows at the LHC

Comparing Shower Averages: γ







Flow I histograms: γ



Flow I+II histograms: γ



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Flow II histograms: γ



Nearest Neighbors: γ (student)



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Comparing Shower Averages: π^+







CALOFLOW: Flow I histograms: π^+



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