A non-relativistic vivisection of 11-dimensional supergravity

Chris Blair
IFT-UAM/CSIC

Iberian Strings 2023
Universidad de Murcia, 12 Jan 2023

Based on:

arXiv:2104.07579 with D. Gallegos, N. Zinnato

Work in progress with E. Bergshoeff, J. Lahnsteiner, J. Rosseel

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My focus is supergravity in 11 dimensions

Unique maximal supergravity in maximal dimension (11)

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or $E^a{}_\mu$

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$$S = \int \sqrt{-G}(R - \tfrac{1}{2}F^2) + \tfrac{1}{6}C \wedge F \wedge F + \text{fermionic}$$

$$\delta_\epsilon {E^a}_\mu = \bar{\epsilon}\gamma^a \Psi_\mu \quad \delta_\epsilon C_{\mu\nu\rho} = 3\bar{\epsilon}\gamma_{[\mu\nu}\Psi_{\rho]} \quad \delta_\epsilon \Psi_\mu = D_\mu \epsilon + \tfrac{1}{24}(\gamma_\mu \not{F} - 3\not{F}\gamma_\mu)\epsilon$$

Dimensional reductions: maximal SUGRA in lower dimensions

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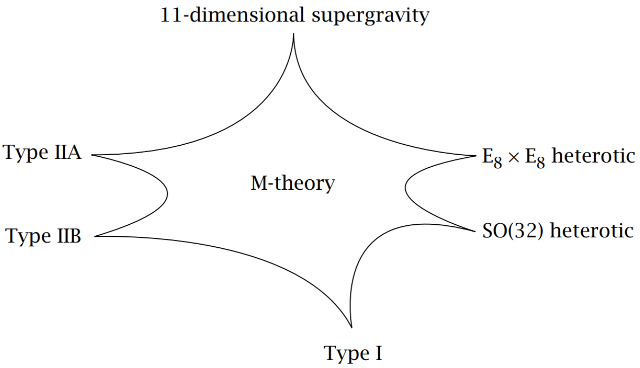
Dimensional reductions: maximal SUGRA in lower dimensions

11-d SUGRA is the low energy limit of M-theory



A portrait of the theory as a young M

Other limits give other ways of exploring M-theory

This talk: non-relativistic string/brane limits

Decoupling limits $\rightarrow$ strings/branes with non-relativistic target space, spectrum, symmetries

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Why study?
– Quantum Gravity (aka M-theory) via its non-relativistic corner
– non-rel versions of AdS/CFT (previous talk)

– related by duality to other limits of M-theory (early 2000s) e.g. DLCQ, non-commutative open string/brane theories
– curved non-Lorentzian (Newton-Cartan) geometries (recently)

Non-relativistic limit of strings

Following [Gomis, Ooguri]: $A, B = 0, 1$ and $a, b = 2, \ldots, 8$

$$\mathrm{d}s^2 = c^2 \eta_{AB} \mathrm{d}X^A \mathrm{d}X^B + \delta_{ab} \mathrm{d}X^a \mathrm{d}X^b \quad B = -\frac{1}{2} c^2 \epsilon_{AB} \mathrm{d}X^A \wedge \mathrm{d}X^B$$

Worldsheet action:

$$S = \int d^2\sigma \, c^2 \left(\eta_{AB} \partial_\alpha X^A \partial^\alpha X^B - \epsilon_{AB} \epsilon^{\alpha\beta} \partial_\alpha X^A \partial_\beta X^B \right) + \partial_\alpha X^a \partial^\alpha X_a$$

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$$\leftrightarrow \int d^2\sigma \, \frac{1}{2c^2} \lambda_\alpha{}^A \lambda^\alpha{}_A + \lambda_{\alpha A} (\eta^{AB} \epsilon_{BC} \partial^\alpha X^C - \epsilon^{\alpha\beta} \partial_\beta X^A) + \partial_\alpha X^a \partial^\alpha X_a$$

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$$= \int d^2\sigma \, \lambda_+ \partial_- X^+ + \lambda_- \partial_+ X^- + \partial_\alpha X^a \partial^\alpha X_a$$

lightcone coordinates in both worldsheet and target space: X^A become chiral/antichiral

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lightcone coordinates in both worldsheet and target space: X^A become chiral/antichiral

– non-relativistic spectrum $E \sim P^2$

– boost invariant $\delta X^a = \Lambda_A{}^a X^A$, $\delta X^A = 0$

$(x' = x + vt)$ $\qquad \delta\lambda_\pm = -\Lambda_\pm{}^a \partial_\pm X_a$

Non-relativistic limit of membranes

Similar limits for any $p - 1$ brane using associated p-form potential

M-theory: membrane non-relativistic limit (also possible: five-brane)

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Curved geometry version: $A = 0, 1, 2$ and $a = 3, \ldots, 10$

$$G_{\mu\nu} = c^2 \tau^A{}_\mu \tau^B{}_\nu \eta_{AB} + c^{-1} e^a{}_\mu e^b{}_\nu \delta_{ab}$$

$$C_{\mu\nu\rho} = -c^3 \epsilon_{ABC} \tau^A{}_\mu \tau^B{}_\nu \tau^C{}_\rho + c_{\mu\nu\rho}$$

Two types of (orthogonal) vielbeins: longitudinal ('time') $\tau^A{}_\mu$
transverse ('space') $e^a{}_\mu$

$$\tau^A{}_\mu \tau^\mu{}_B = \delta^A_B \quad e^a{}_\mu e^\mu{}_b = \delta^a_b \quad \tau^A{}_\mu \tau^\nu{}_A + e^a{}_\mu e^\nu{}_a = \delta^\nu_\mu$$

$$\tau^A{}_\mu e^\mu{}_a = 0 = \tau^\mu{}_A e^a{}_\mu$$

This gives a 'membrane Newton-Cartan geometry'

$$G_{\mu\nu} = c^2 \tau^A{}_\mu \tau^B{}_\nu \eta_{AB} + c^{-1} e^a{}_\mu e^b{}_\nu \delta_{ab} \quad C_{\mu\nu\rho} = -c^3 \epsilon_{ABC} \tau^A{}_\mu \tau^B{}_\nu \tau^C{}_\rho + c_{\mu\nu\rho}$$

Geometric objects:

– Intrinsic torsion: $T_{\mu\nu}{}^A = 2\partial_{[\mu}\tau_{\nu]}{}^A$

– Field strength: $F_{\mu\nu\rho\sigma} = f_{\mu\nu\rho\sigma} - c^3 6 T_{[\mu\nu}{}^A \tau^B{}_\rho \tau^C{}_{\sigma]} \quad f_{\mu\nu\rho\sigma} = 4\partial_{[\mu} c_{\nu\rho\sigma]}$

– Connection s.t. $\nabla\tau^A = 0 = \nabla(e^\mu{}_a e^{\nu a})$
$\rightarrow$ curvature $\bar{R}$, torsion $\Gamma_{[\mu\nu]}{}^\rho \sim \tau^\rho{}_A T_{\mu\nu}{}^A$

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– Measure $\Omega = c\sqrt{-G}$

– Define $f_{abcd} \equiv e^\mu{}_a e^\nu{}_b e^\rho{}_c e^\sigma{}_d f_{\mu\nu\rho\sigma}$, $f_{abcA} \equiv e^\mu{}_a e^\nu{}_b e^\rho{}_c \tau^\sigma{}_A f_{\mu\nu\rho\sigma}$, etc.

Expand 11-d SUGRA (bosonic part)

$$G_{\mu\nu} = c^2 \tau^A{}_\mu \tau^B{}_\nu \eta_{AB} + c^{-1} e^a{}_\mu e^b{}_\nu \delta_{ab} \quad C_{\mu\nu\rho} = -c^3 \epsilon_{ABC} \tau^A{}_\mu \tau^B{}_\nu \tau^C{}_\rho + c_{\mu\nu\rho}$$

Expanding the action

$$S = \int \sqrt{-G} R - \tfrac{1}{2} F \wedge \star F - \tfrac{1}{6} C \wedge F \wedge F$$

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$$\begin{aligned} S &= \int \sqrt{-G} R - \tfrac{1}{2} F \wedge \star F - \tfrac{1}{6} C \wedge F \wedge F \\ &= c^3 \int -\tfrac{\Omega}{4!} f^{(-)abcd} f^{(-)}_{abcd} \\ &\quad + c^0 \int \Omega(\bar{R} - T^{aAB} T_{a(AB)} + \tfrac{3}{2} T^{aA}{}_A T_{aB}{}^B - \tfrac{1}{12} f^{abcA} f_{abcA} \\ &\qquad + \tfrac{1}{4} \epsilon_{ABC} f^{ABab} T_{ab}{}^C) + \tfrac{1}{6} c_3 \wedge f_4 \wedge f_4 \\ &\quad + O(c^{-3}) \end{aligned}$$

$$f_{\mu\nu\rho\sigma} = 4\partial_{[\mu} c_{\nu\rho\sigma]}$$

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At $O(c^3)$:
– T^2 terms from R and F^2 cancel
– Non-zero term from F^2 and CFF involving (anti-)self-dual field strengths $f^{(\pm)}_{abcd} = \frac{1}{2}(f_{abcd} \pm \frac{1}{4!} \epsilon_{abcd}{}^{efgh} f_{efgh})$

Possible divergences can be removed using Lagrange multiplier trick

Introduce Lagrange multiplier field λ_{abcd} to remove c^3 term

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S &\supset c^3 \int -\tfrac{\Omega}{4!} f^{(-)abcd} f^{(-)}_{abcd} \\
&\longleftrightarrow -c^0 \int \Omega \tfrac{2}{4!} \lambda_{abcd} f^{(-)abcd} + c^{-3} \int \Omega \tfrac{1}{4!} \lambda_{abcd} \lambda^{abcd}
\end{aligned}
$$

eom sets $\lambda_{abcd} = c^3 f^{(-)}_{abcd}$

Take the non-relativistic limit

For $c^3 \to \infty$ get [CB, Gallegos, Zinnato]

$$S_{\text{non-rel}} = \int \Omega\Big(R - T^{aAB} T_{a(AB)} + \tfrac{3}{2} T^{aA}{}_{A} T_{aB}{}^{B} - \tfrac{1}{12} f^{abcA} f_{abcA} + \tfrac{1}{4}\epsilon_{ABC} f^{ABab} T_{ab}{}^{C} - \tfrac{2}{4!}\lambda_{abcd} f^{(-)abcd}\Big) + \tfrac{1}{6} c_3 \wedge f_4 \wedge f_4$$

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Non-relativistic symmetries:

– local rotations $\mathrm{SO}(1,2)$ on A, B and $\mathrm{SO}(8)$ on a, b

– boosts $(x' = x + vt)$

$\delta\tau^{A}{}_{\mu} = 0$, $\delta e^{a}{}_{\mu} = \Lambda^{a}{}_{A}\tau^{A}{}_{\mu}$, $\delta c_{\mu\nu\rho} = -3\epsilon_{ABC}\Lambda_{a}{}^{A}e^{a}{}_{[\mu}\tau^{B}{}_{\nu}\tau^{C}{}_{\rho]}$

$\delta\lambda_{abcd} =$ a.s.d. part of $4\Lambda^{A}{}_{[a}f_{|A|bcd]}$

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– emergent dilatations local parameter α

$\delta\tau^{A}{}_{\mu} = +\alpha\tau^{A}{}_{\mu},\ \delta e^{a}{}_{\mu} = -\tfrac{1}{2}\alpha e^{a}{}_{\mu},\ \delta c_{\mu\nu\rho} = 0,\ \delta\lambda_{abcd} = -\alpha\lambda_{abcd}$

Now include SUSY

Non-relativistic expansion of all 11-d fields:

$$E^{\hat{a}}{}_{\mu} = (c\tau^{A}{}_{\mu}\,, c^{-1/2} e^{a}{}_{\mu}) \quad C_{\mu\nu\rho} = -c^{3}\epsilon_{ABC}\tau^{A}{}_{\mu}\tau^{B}{}_{\nu}\tau^{C}{}_{\rho} + c_{\mu\nu\rho}$$

$$\Psi_{\mu} = c^{-1}\psi_{+\mu} + c^{1/2}\psi_{-\mu} \quad \Pi_{\pm}\psi_{\pm\mu} = \psi_{\pm\mu} \quad \Pi_{\pm} \equiv \tfrac{1}{2}(1 \pm \gamma_{012})$$

Action with $\psi\psi$ terms: Lagrange multiplier trick still works

SUSY transformations of bosons are finite for $c \to \infty$

$$\delta_Q \tau^{A}{}_{\mu} = \bar{\epsilon}_{-}\gamma^{A}\psi_{-\mu} \quad \delta_Q e^{a}{}_{\mu} = \bar{\epsilon}_{+}\gamma^{a}\psi_{-\mu} + \bar{\epsilon}_{-}\gamma^{a}\psi_{+\mu}$$

$$\begin{aligned}\delta_Q c_{\mu\nu\rho} &= 6\bar{\epsilon}_{+}\epsilon_{ABC}\gamma^{A}\psi_{+[\mu}\tau^{B}{}_{\nu}\tau^{C}_{\rho]} + 3\bar{\epsilon}_{-}\gamma_{ab}\psi_{-[\mu}e^{a}{}_{\nu}e^{b}{}_{\rho]} \\ &\quad + 6\left(\bar{\epsilon}_{+}\gamma_{Aa}\psi_{-[\mu}\tau^{A}{}_{\nu}e^{a}{}_{\rho]} + \bar{\epsilon}_{-}\gamma_{Aa}\psi_{+[\mu}\tau^{A}{}_{\nu}e^{a}{}_{\rho]}\right)\end{aligned}$$

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$\to$ have to impose geometric constraints s.t. these vanish:

$$T_{ab}{}^{A} = 0 \quad T_{a}{}^{\{AB\}} = 0 \quad f^{(+)}{}_{abcd} = 0 \quad f_{Aabc} = 0$$

more precisely 'super-covariant' versions of these tensors

Now include SUSY

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Constraints ensure SUSY invariance of non-rel action

Presence of c^3 terms reveals emergent fermionic shift symmetry

c.f. $\mathcal{N} = 1$ in 10d [Bergshoeff, Lahnsteiner, Romano, Rosseel, Simsek]

Fermionic symmetries have an explanation in expansion

Expand action $S = c^3 S_3 + c^0 S_0 + c^{-3} S_{-3} + \dots$

Expand SUSY transformation $\delta = c^3 \delta_3 + c^0 \delta_0 + c^{-3} \delta_{-3} + \dots$

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$\Rightarrow$ form of δ_3 gives emergent fermionic shift symmetry

$$\delta_S \psi_{+\mu} = \tau^A{}_\mu \rho_{A+} - \frac{1}{2} e^a{}_\mu \gamma_a \eta_- \quad \delta_S \psi_{-\mu} = \tau^A{}_\mu \gamma_A \eta_-$$

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If $S_3 = 0$, $\delta S = 0$ then $\boxed{\delta_3 S_0 = 0 \quad \delta_0 S_0 + \delta_3 S_{-3} = 0}$ true before $c \to \infty$

$\Rightarrow$ form of δ_3 gives emergent fermionic shift symmetry

$$\delta_S \psi_{+\mu} = \tau^A{}_\mu \rho_{A+} - \tfrac{1}{2} e^a{}_\mu \gamma_a \eta_- \quad \delta_S \psi_{-\mu} = \tau^A{}_\mu \gamma_A \eta_-$$

$\Rightarrow$ After limit (no S_{-3}) need $\delta_3 \equiv 0$ for $\delta_0 S_0 = 0$

Fermionic symmetries have an explanation in expansion

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$$\delta_Q \psi_{-\mu} = \nabla_\mu \epsilon_- + \tfrac{1}{24} e^e{}_\mu \gamma_e \not{f}^{(-)} \epsilon_+ \quad \delta_Q \psi_{+\mu} = \nabla_\mu \epsilon_+ - \tfrac{1}{12} \tau^A{}_\mu \gamma_A \not{\chi} \epsilon_+ - \tfrac{1}{8} e^e{}_\mu \not{\chi} \gamma_e \epsilon_-$$

$$\nabla_\mu \epsilon_- \equiv (\partial_\mu + \tfrac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} + \tfrac{1}{4} \omega_\mu{}^{AB} \gamma_{AB} - \tfrac{1}{2} b_\mu) \epsilon_-$$

$$\nabla_\mu \epsilon_+ \equiv (\partial_\mu + \tfrac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} + \tfrac{1}{4} \omega_\mu{}^{AB} \gamma_{AB} + b_\mu) \epsilon_+ + \tfrac{1}{2} \omega^{Aa} \gamma_{Aa} \epsilon_-$$

What have we found?

– Non-relativistic 11-dimensional supermultiplet

$$\tau^A{}_\mu \quad e^a{}_\mu \quad c_{\mu\nu\rho} \quad \lambda_{abcd} \quad \psi_{+\mu} \quad \psi_{-\mu}$$

together with constraints (vanishing of geometric tensors)

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– Vary constraints $\to$ more constraints... closure? Solve by gauge fixing? (in progress)

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– Repeat for M5 non-relativistic limit using dual six-form?

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Thanks for listening!

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