The correspondence between rotating black holes and fundamental strings

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Work to appear, with

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Black holes are massive, large, highly degenerate objects

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Strings have very massive, large, highly degenerate states

Black hole entropy (Bekenstein)

$$S_{BH} \propto M^2$$

String degeneracy (Hagedorn)

$$S_{st} \propto M$$

Different?

Black hole entropy (Bekenstein)

$$S_{BH} \propto M^2$$

String degeneracy (Hagedorn)

$$S_{st} \propto M$$

In what units?

Gravitational units

$$S_{BH} = \frac{M^2}{M_P^2}$$

String units

$$S_{st} = \frac{M}{M_s}$$

$$M_s = g_s M_P$$

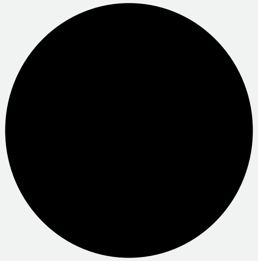
perturbatively, $M_s \ll M_P$

Black holes are strongly gravitating

Strings are weakly gravitating

$$\text{curvature} \sim \frac{1}{(GM)^2}$$

$$\sim \left(\frac{M_s}{g_s^2 M}\right)^2 \frac{1}{\ell_s^2}$$



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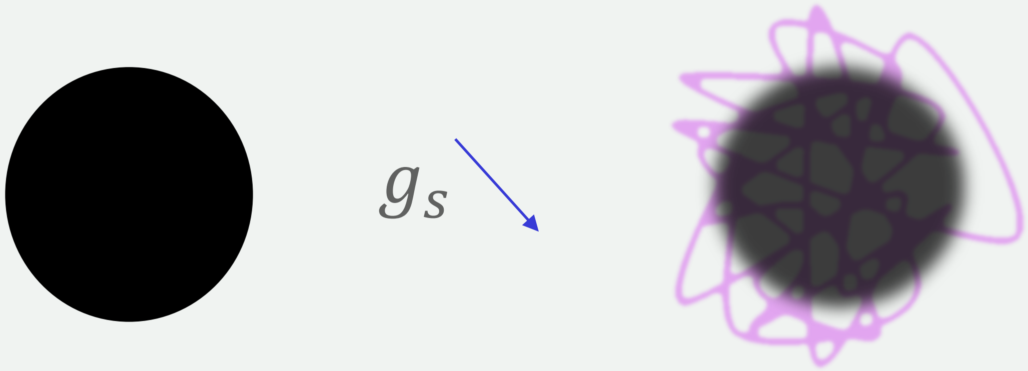

$$g_s^2 \sim \frac{M_s}{M}$$

$$\text{curvature} \sim \frac{1}{(GM)^2}$$

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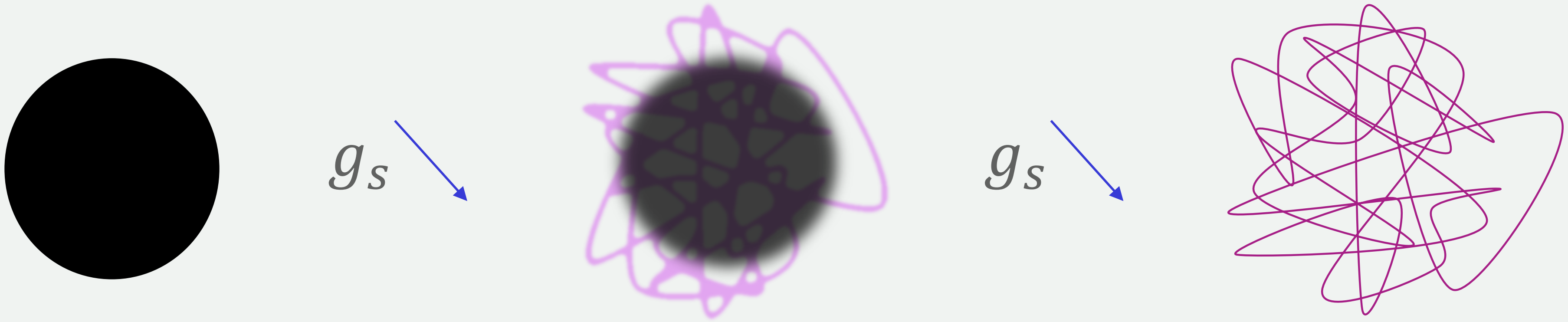


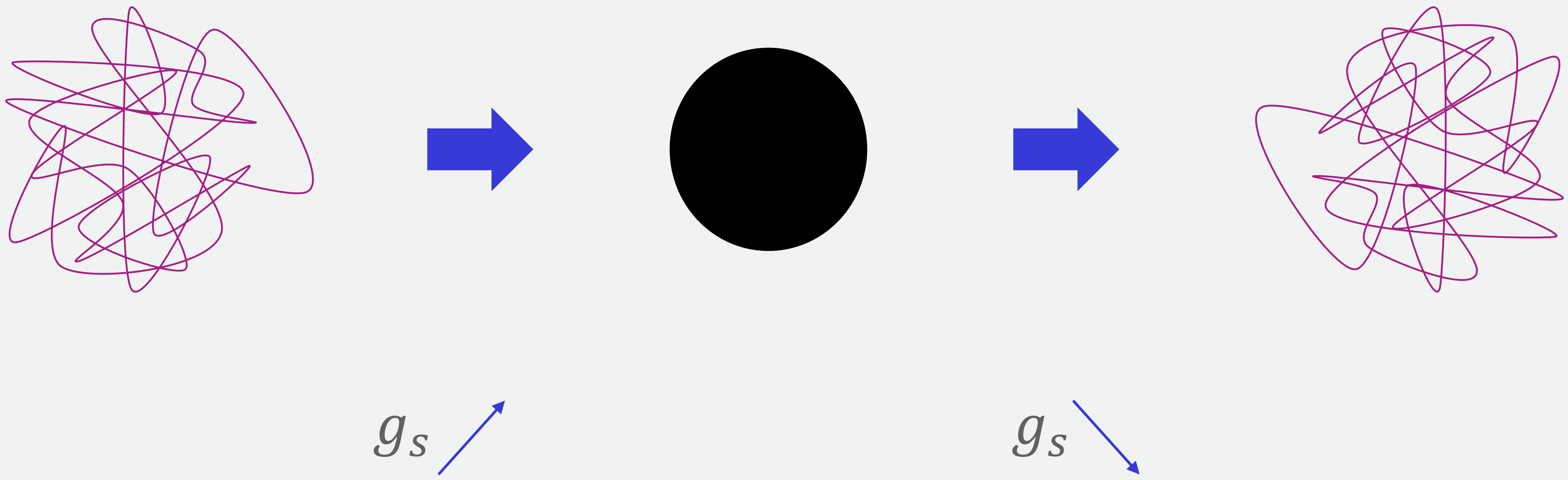
$$g_s^2 \sim \frac{M_s}{M}$$

Correspondence

Tune the coupling g_s up and down

Switch between massive string and black hole

Veneziano, Susskind, Horowitz, Polchinski, Damour...



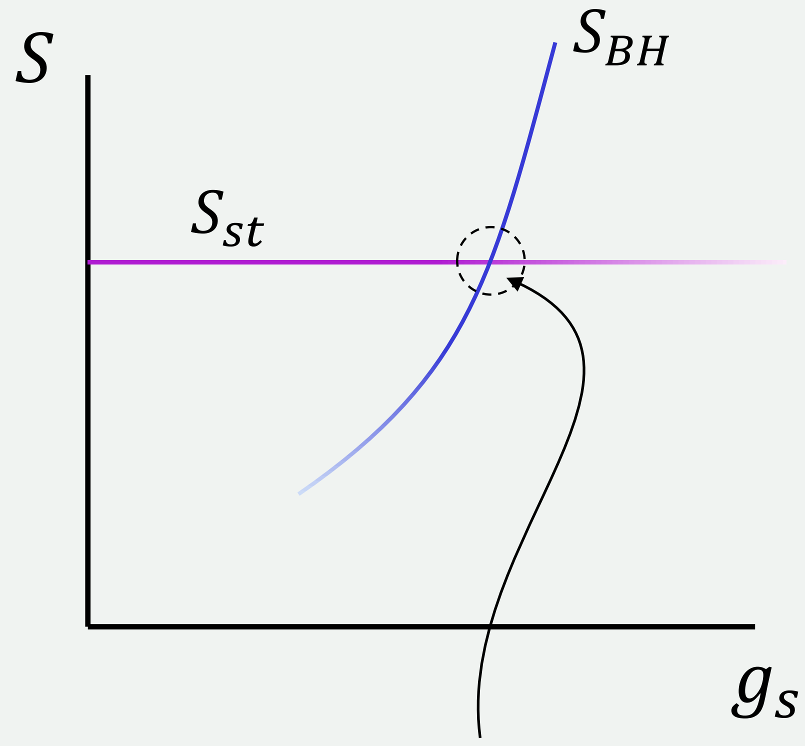
g_s
g_s

$$M_s = g_s M_P$$

fix mass in string units, $\frac{M}{M_s}$

$$S_{BH} = g_s^2 \frac{M^2}{M_s^2}$$

$$S_{st} = \frac{M}{M_s}$$



Correspondence

Parametric match of entropy

$$S_{BH}\Big|_{corr} \sim S_{st}$$

$$g_s^2\Big|_{corr} = \frac{M_s}{M} \ll 1$$

good

50 years back

Clues and puzzles

Kerr bound on black holes

$$J \leq M^2$$

Regge bound on strings

$$J \leq M^2$$

black holes = strings?

$$J \leq M^2$$

Misleading!

$$J \leq M^2$$

What units?

$$J \leq M^2$$

gravitational units

$$J \leq \frac{M^2}{M_P^2}$$

string units

$$J \leq \frac{M^2}{M_s^2}$$

$$M_S = g_S M_P \ll M_P$$

$$J_{Kerr} = \frac{M^2}{M_P^2} = g_s^2 \frac{M^2}{M_s^2} \ll J_{Regge} = \frac{M^2}{M_s^2}$$

Puzzles

$$J_{Kerr} \ll J_{Regge}$$

Massive string states with high enough spin

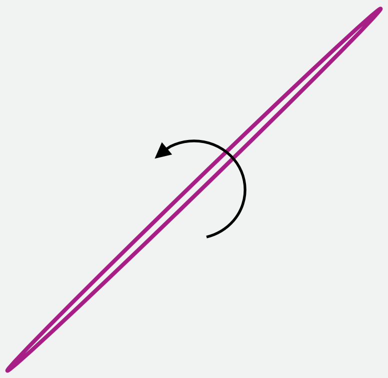
$$J_{Kerr} < J \leq J_{Regge}$$

don't have black hole counterparts

Strings with $J = J_{Regge}$

do not look like black holes at all

Non-degenerate, rigidly rotating bars



$J = M^2 L_s^2$

Black holes: $J \leq J_{Kerr} = \frac{M^2}{M_P^2}$

No problem

There exist string states with the same mass, spin, and (parametric) degeneracy

But $J \leq GM^2$ is a bound for 4D black holes

In $D \geq 5$ $\exists$ black holes with arbitrarily large spins

But Regge bound $J \leq \frac{M^2}{M_s^2}$ is for strings in any D

Ultraspinning black holes with

$$J \gg J_{Regge}$$

don’t have string counterparts

Start with fastly spinning string

As g_s grows, what does it turn into?

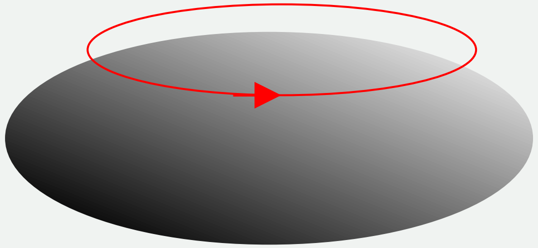
Start with fastly spinning black hole

As g_s decreases, what does it turn into?

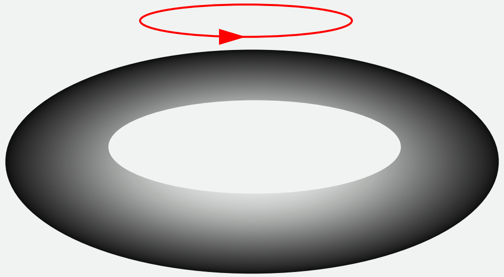
Rotating black holes in higher D

$D \geq 5$ with rotation in a single plane

$\exists$ black holes/rings with arbitrarily large J



Myers+Perry 1986



RE+Reall 2001

Two length scales

$$\ell_M = (GM)^{\frac{1}{D-3}}$$

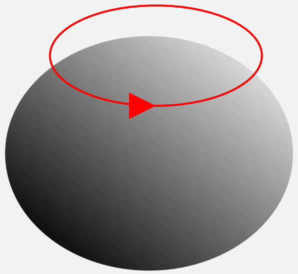
$$\ell_J = \frac{J}{M}$$

Ultraspinnning: $\ell_J \gg \ell_M$

$\ell_J \lesssim \ell_M$

unique

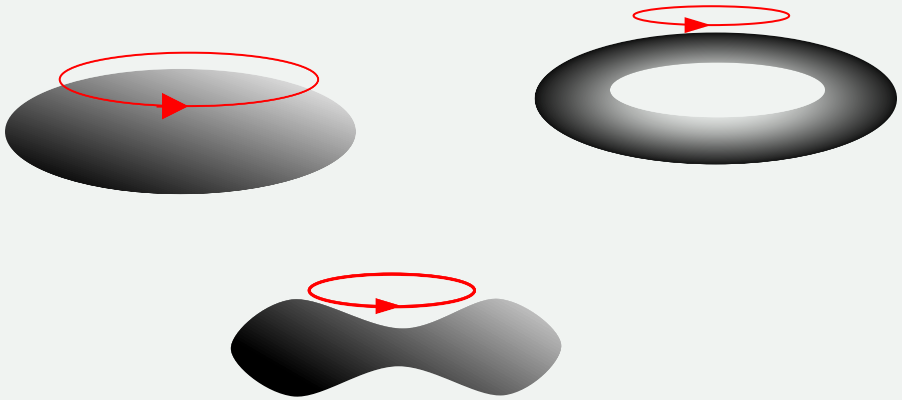
stable



$\ell_J \gtrsim \ell_M$

many

unstable to fragmentation



Dynamical “Kerr bound”

Only stable solution is M-P plump black hole

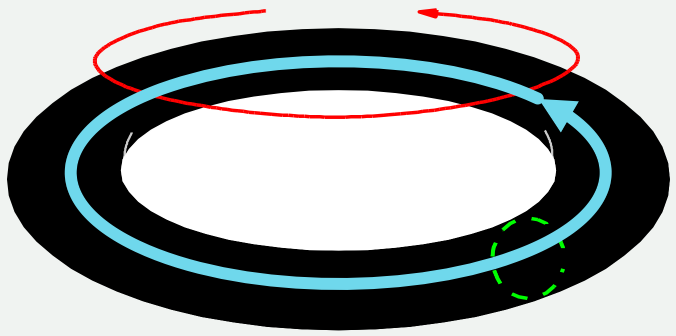
with $\ell_J \lesssim \ell_M$

$$J \lesssim M(GM)^{\frac{1}{D-3}}$$

Dipole rings

No charge, but a dipole

Can be stable



Ring of black fundamental string

String configurations

String balls

Classical Nambu-Goto (conformal gauge w/ $X^0 = p^0\tau$)

$$X^i = \frac{1}{2}(A^i(\tau - \sigma) + B^i(\tau + \sigma))$$

A^i, B^i arbitrary up to $|\partial_\sigma A^i|^2 = |\partial_\sigma B^i|^2 = 1$

String balls

$$X^i = \frac{1}{2}\left(A^i(\tau - \sigma) + B^i(\tau + \sigma)\right)$$

A^i, B^i arbitrary up to $\left|\partial_\sigma A^i\right|^2 = \left|\partial_\sigma B^i\right|^2 = 1$

A^i, B^i: random walk

Mañes 2004

String balls

Quantum: Light-cone gauge

$$X^i = X_L^i + X_R^i$$

$$X_L^i = x^i + i \sum_{n \neq 0} \frac{1}{n} \alpha_n^i e^{-2in(\tau - \sigma)}$$

String balls

$$\alpha^{i_1}_{-n_1}\alpha^{i_2}_{-n_2}\ \alpha^{i_3}_{-n_3}\dots|0\rangle \qquad\qquad N = \sum_i n_i$$

(similar for X^i_R, plus constraint $N_L = N_R$)

$$\langle (X^i)^2\rangle = \sqrt{N} = M$$

Random walk, $\sqrt{N}$ steps

Mitchell+Turok 1987

Degeneracy $S \sim \sqrt{N} = M$

Add rotation

Russo+Susskind 1994

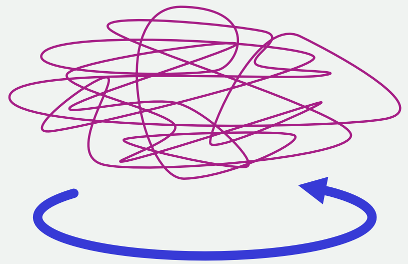
$$S \sim \sqrt{N - J} = \sqrt{M^2 - J}$$

Large when away from Regge bound

Shape

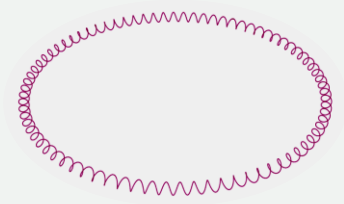
Rotating strings pancake along rotation plane

$$\ell_{par}^2 \sim \sqrt{N} \; > \; \ell_{perp}^2 \sim \sqrt{N - J}$$



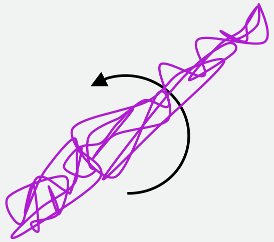
Other rotating strings

Plasmid strings



Blanco-Pillado+RE+Iglesias 2007

String bars

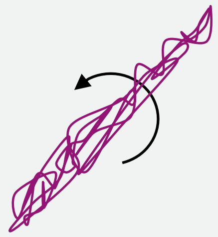


can construct both classical and quantum

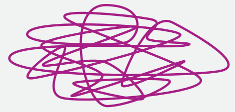
When interactions are turned on, $g_s > 0$, they all become unstable

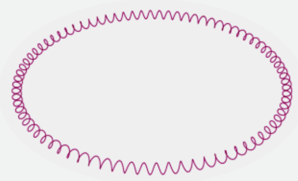
But some are short-lived, some longer-lived

Short-lived: gravitational antenas

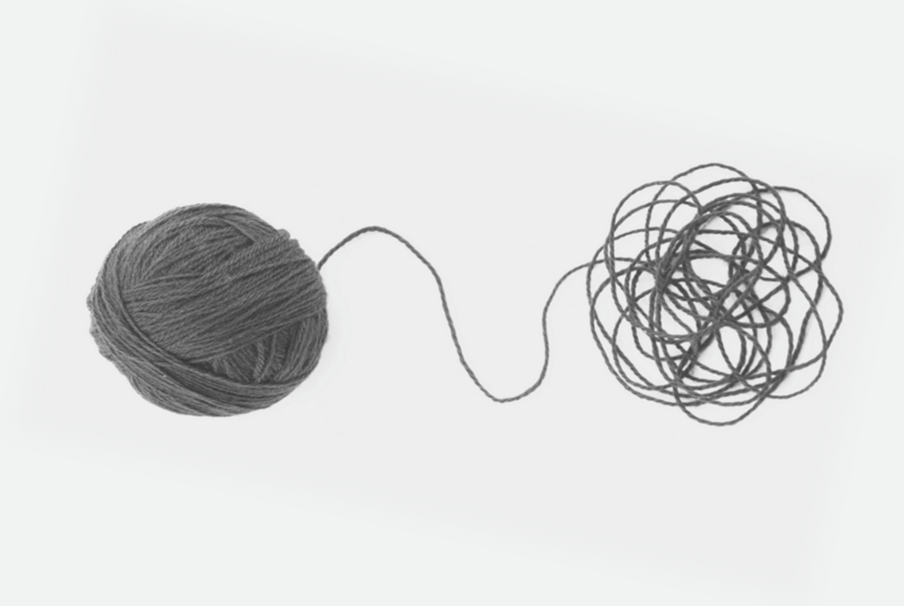


Longer-lived: ~thermal radiation





This allows to establish a correspondence across J as we vary g_s



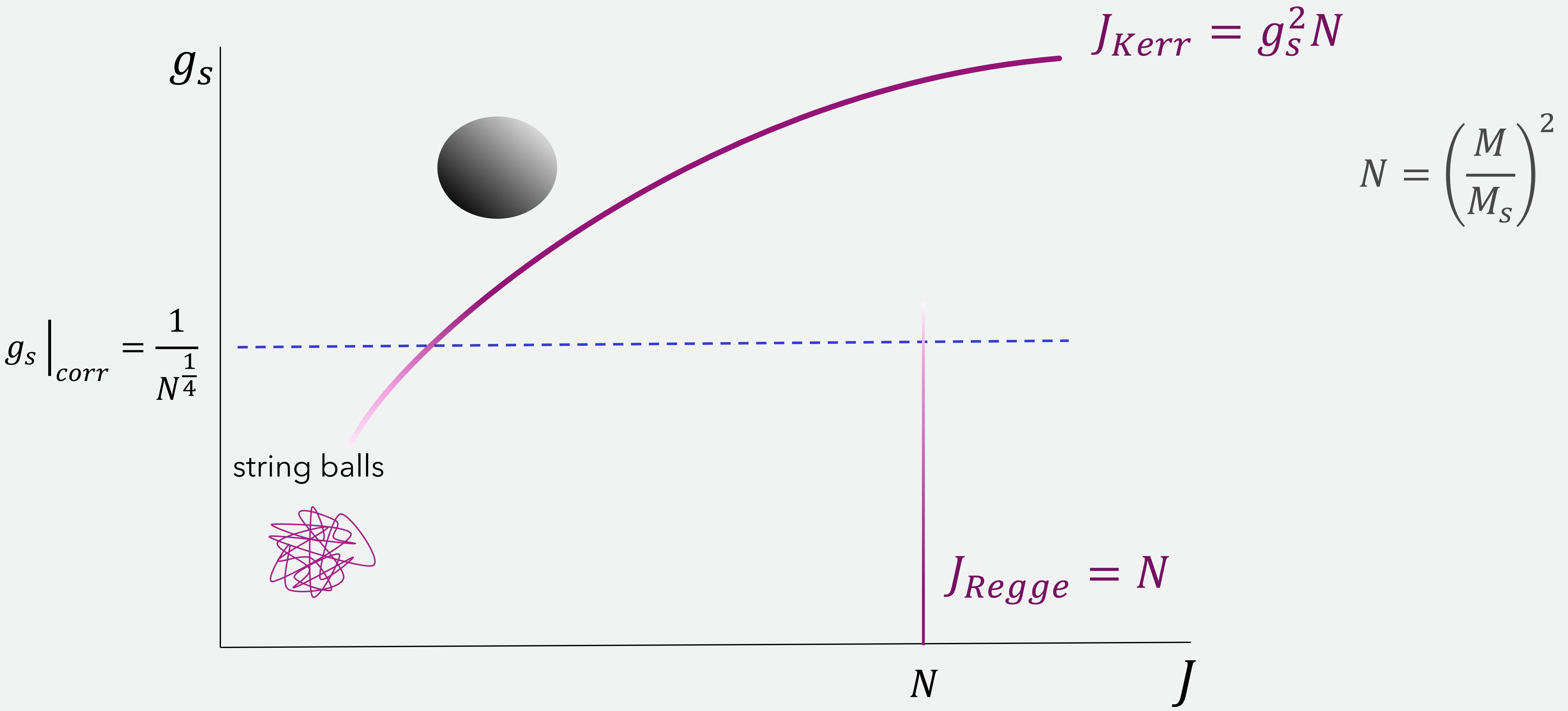
Notice that

$$\ell_M = (GM)^{\frac{1}{D-3}} : \text{grows with } g_s$$

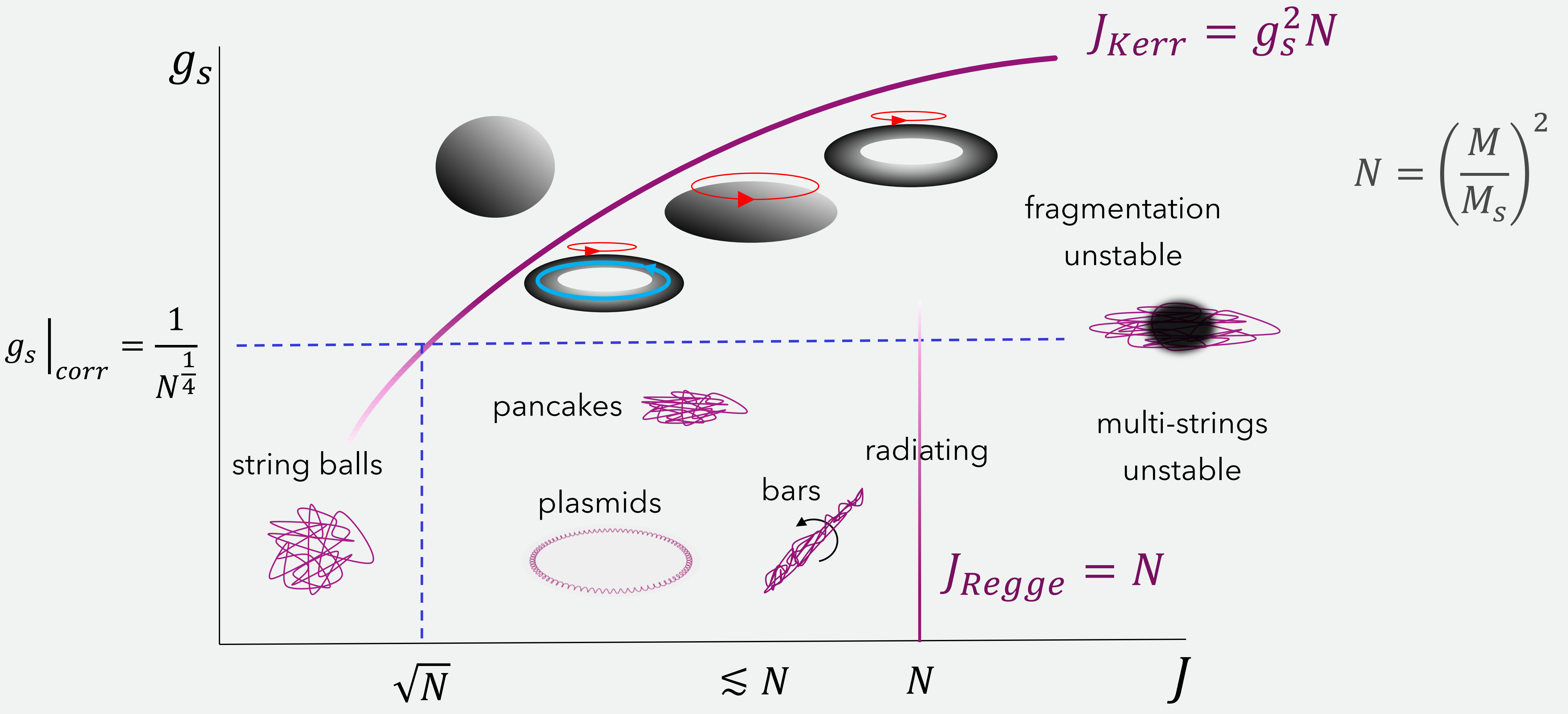
$$\ell_J = \frac{J}{M} : \text{indep of } g_s$$

$\ell_M > \ell_J$ for large enough g_s

When gravitational force is strong enough, always Kerr-like black hole



$J_{Kerr} = g_s^2 N$
g_s
$N = \left(\frac{M}{M_s}\right)^2$
$g_s\big|_{corr} = \frac{1}{N^{\frac{1}{4}}}$
string balls
$J_{Regge} = N$
N
J



$J_{Kerr} = g_s^2 N$
g_s
$N = \left(\frac{M}{M_s}\right)^2$
fragmentation
unstable
$g_s\big|_{corr} = \frac{1}{N^{\frac{1}{4}}}$
pancakes
multi-strings
unstable
radiating
string balls
bars
plasmids
$J_{Regge} = N$
$\sqrt{N}$
$\lesssim N$
N
J

There remains an unclear range in 4D

Possibly some mixed string-black hole system

Near extremality the description changes

Final remarks

Fundamental strings are not really fundamental

Theories of quantum gravity formulated in spacetime variables are only valid perturbatively

This is the case for perturbative quantum gravity, and also for perturbative string theory

Non-perturbative theories of quantum gravity w/ black holes are holographic

Strings appear as effective objects

Very effective

Fundamental strings allow to go as close as possible to black holes

in a quantum theory of gravity with spacetime variables

Correspondence describes the transition between descriptions

Must understand how it works

Rotation:

First step ✓

More work ahead

Nejc Čeplak

Andrea Puhm

Marija Tomašević

Thank you

Coming up soon

Eurostrings 2023

Gijón

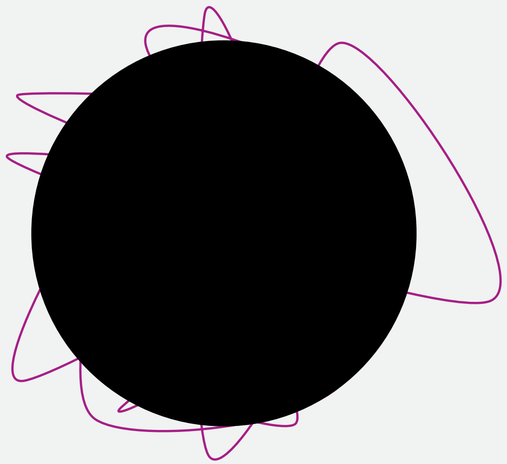
24 - 28 April 2023

Gravity: New perspectives from strings & higher dimensions

Benasque

9 - 22 July 2023

Extra slides



Near extremality the description changes

Near-extremal with charge:

Gravitational decoupling of throat

Relevant degrees of freedom are not long massive strings, but short strings moving along brane: CFT

Near extremality the description changes

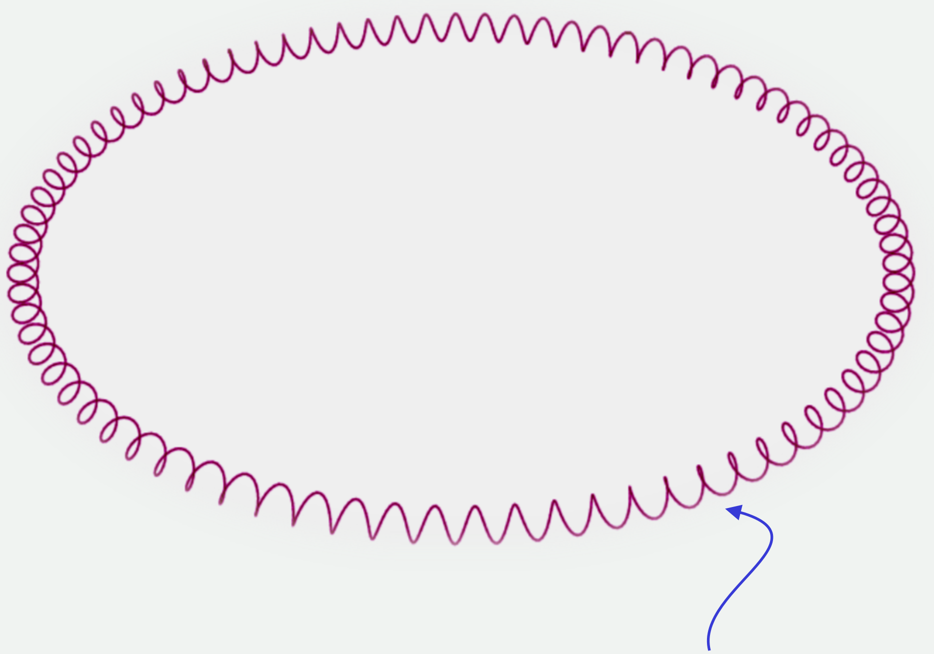
Near-extremal rotating:

Gravitational decoupling of throat

Relevant degrees of freedom are not long massive strings, but

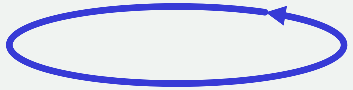
short strings moving along brane: Kerr/CFT?

Plasmid strings



arbitrary wiggly profile

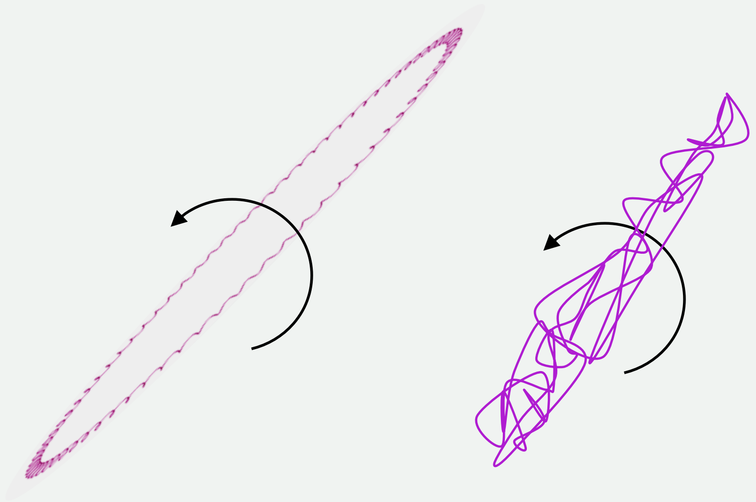
fundamental string dipole



$$J = \frac{1}{4} M^2 \,, \qquad S \sim \sqrt{J} \sim M$$

Blanco-Pillado+RE+Iglesias 2007

String bars



$$J < M^2 \,, \qquad S \sim \sqrt{M^2 - J}$$

limit to Regge-bound strings

Unstable black holes break apart: multi-string configurations

This may happen in stages: curvature on horizon can be very inhomogeneous