Trans-IR flows to black hole singularities

Ayan Kumar Patra, IFT, Madrid

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In collaboration with E.Caceres, A.Kundu, S.Shashi

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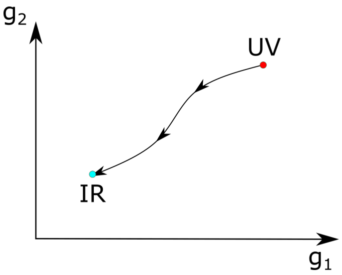
Outline

- Introduction & Motivation
- Holographic a-function
- Thermal a-function
- Free Kasner Flows
- Quantum Information and trans-IR flow
- Conclusion & Future directions

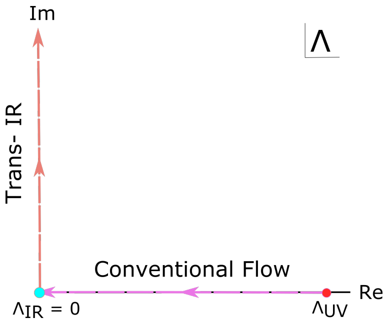
RG Flow:-Interpolation between theories at different energy scales—

- whose fixed points are the CFTs
- Flows from high energy(UV) fixed point to low energy(IR) fixed point

View as a curve in the space of couplings parametrized by energy scale Λ.



Trans IR Flows:- We will consider systems with $\Lambda_{IR} = 0$.



Trans-IR flow is accessed by imaginary values of Λ.

Holographic RG Flow:- In gauge/gravity duality, adding matter to the bulk is dual to deforming the boundary CFT, *e.g* scalar field ϕ is dual to operator $\mathcal{O}$

$$\int d^{d+1}x\sqrt{-g}(\nabla_\mu\phi\nabla^\mu\phi + V(\phi)) \leftrightarrow \int d^dx\phi_0\mathcal{O} \tag{1}$$

Relevant deformations trigger RG flows.

- Energy scale in dual field theory is realized as the bulk radial direction.
- Radial direction becomes timelike implies that interior is trans-IR.

(Frenkel, Hartnoll, Kruthoff, Shi 2020)

Introduction & Motivation

(I) We extend Holographic RG flow to the interior of a black hole by analytically continuing the radial direction.

(II) In QFT, one may analytically continue a nontrivial renormalization group (RG) flow beyond its infrared (IR) fixed point to a "trans-IR" section of the flow by complexifying the energy.

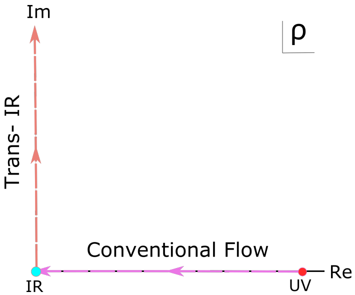
(III) One particularly interesting physical quantity in Einstein gravity that monotonically decreases along the holographic flow is the so called 'a'-function. We find such a monotonically decreasing function along the entire flow including the trans-IR section.

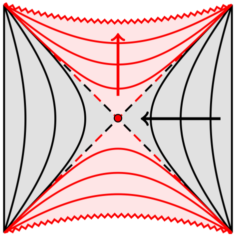
We ask three specific questions:

(Q1) Do trans-IR flows obey a monotonicity condition?

(Q2) What characterizes the endpoint of trans-IR flows?

(Q3) How does the trans-IR flow relate to quantum information?





Holographic a-function

(I) We start with Einstein gravity in $(d+1)$ dimensions coupled to various scalar fields,

$$S = \frac{1}{2\ell_p^{d-1}} \int d^{d+1}x \sqrt{-g}(R + \mathcal{L}_{\text{matter}})$$

The potential has number of extrema with $\mathcal{L}_{\text{crit}} = -\frac{d(d-1)\alpha_i}{L^2}$.

(II) At those stationary points the solution is AdS_{d+1}.

(III) Consider a solution where the scalar sits at a UV fixed point near the boundary and makes transition to an IR fixed point in the interior. This transition can be thought of as the holographic version of RG Flow.

(IV) From the field theory perspective, boundary CFT flows from UV fixed point to IR fixed point. (Myers, Sinha 2010)

Holographic a-function

(V) One usually considers the domain wall ansatz for such HRG flow,

$$ds^2 = e^{2A(\rho)}(-dt^2 + d\vec{x}^2_{d-1}) + d\rho^2$$

(VI) One defines,

$$a(\rho) = \frac{\pi^{\frac{1}{2}}}{\Gamma\left(\frac{d}{2}\right)\ell_p^{d-1}}\left(\frac{1}{A'(\rho)}\right)^{d-1}$$

(VII) By using Null energy condition, $k^\mu k^\nu T_{\mu\nu} \geq 0$, one can show that $a'(\rho) \geq 0$

$$\Downarrow$$

a is a monotonic function in the radial direction: $a_{\text{UV}} \geq a_{\text{IR}}$

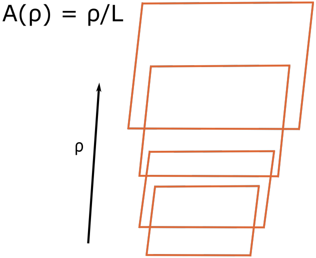
(Myers, Sinha 2010)

Holographic a-function

Intuitive Picture:-

- Take the domain wall ansatz, $ds^2 = e^{2A(\rho)}(-dt^2 + d\vec{x}^2_{d-1}) + d\rho^2$.

$$a(\rho) = \frac{\pi^{\frac{1}{2}}}{\Gamma\left(\frac{d}{2}\right)\ell_p^{d-1}}\left(\frac{1}{A'(\rho)}\right)^{d-1}$$



- Minkowski foliation of the AdS-spacetime.
- The holographic 'a' function measures the numbers of d.o.f at each slice.

Thermal a-function

(I) We propose a thermal analogue of the above a-function,

$$a_T(\rho) = \frac{\pi^{\frac{1}{2}}}{\Gamma\left(\frac{d}{2}\right)\ell_p^{d-1}}\left(\frac{f(\rho)}{A'(\rho)}\right)^{d-1}$$

We choose the metric ansatz to be

$$ds^2 = e^{2A(\rho)}(-f(\rho)^2 dt^2 + d\vec{x}_{d-1}^2) + d\rho^2$$

We set the location of the horizon at $\rho = 0$ and the boundary at $\rho = \infty$.

(II) One interesting fact is that $a_T(\rho)$ is same for the vacuum AdS_{d+1} and the AdS-Schwarzschild - a consequence of both geometries being dual to states of the same UV CFT.

(III) Using Null energy condition one can show that $a'(\rho) \geq 0$ for $\rho \geq 0$. Now one analytically continue the radial direction $\rho \to ik$, $t \to t_I - \text{Sgn}(t_I)\frac{i\gamma}{2T}$, to access the black hole interior.

(IV) To show monotonicity in the interior region, one considers a different metric ansatz,

$$ds^2 = \frac{1}{r^2}\left[-f(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{f(r)} + d\vec{x}^2\right] \tag{2}$$

(Hartnoll et al. 2004.01192)

Using the null energy condition, we find $\frac{da_T}{dr} \leq 0$ when $r \geq r_h$. In summary,

$$\frac{da_T}{d\rho} \geq 0 : \text{ UV} \rightarrow \text{ IR} \tag{3}$$

$$\frac{da_T}{dk} \leq 0 : \text{ Trans-IR} \tag{4}$$

Hence, a_T is a monotonic function of the entire flow. **(Q1)**

Free Kasner Flows

- We take $(d+1)$-dimensional Einstein gravity ($d \geq 2$) with negative cosmological constant $\Lambda = -d(d-1)/2$ (setting the AdS radius to 1) and coupled to a scalar field ϕ with potential $V(\phi) = m^2\phi^2$. With $16\pi G_{d+1} = 1$, the action is,

$$I = \int d^{d+1}x\sqrt{-g}\left(R + d(d-1) - \frac{1}{2}\left[\nabla^\alpha\phi\nabla_\alpha\phi + V(\phi)\right]\right) \qquad (5)$$

- EOM:-

$$G_{\mu\nu} - \frac{d(d-1)}{2}g_{\mu\nu} = \frac{1}{4}\left[2\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\left(\nabla^\alpha\phi\nabla_\alpha\phi + m^2\phi^2\right)\right], \qquad (6)$$

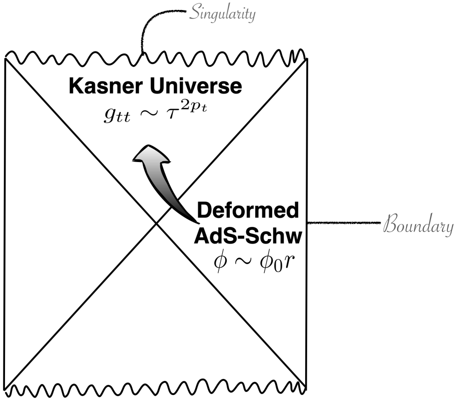
$$(\Box - m^2)\phi = 0, \qquad (7)$$

Free Kasner Flows

- We take solutions of the form

$$ds^2 = \frac{1}{r^2}\left[-f(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{f(r)} + d\vec{x}^2\right] \quad (8)$$

(Hartnoll et al. 2004.01192)



Plugging this ansatz into the Klein-Gordon equation and combining the result with the *tt* and *rr* components of the Einstein + scalar equations yields a set of ODEs,

$$\phi'' + \left(\frac{f'}{f} - \frac{d-1}{r} - \frac{\chi'}{2}\right)\phi' + \frac{\Delta(d-\Delta)}{r^2 f}\phi = 0, \tag{9}$$

$$\chi' - \frac{2f'}{f} - \frac{\Delta(d-\Delta)\phi^2}{(d-1)rf} - \frac{2d}{rf} + \frac{2d}{r} = 0, \tag{10}$$

$$\chi' - \frac{r}{d-1}\left(\phi'\right)^2 = 0, \tag{11}$$

Temperature of the black hole-

$$T = \frac{|f'_+|e^{-\chi_+/2}}{4\pi}. \tag{12}$$

Near Singularity & Near Boundary Expressions:-

	Near-boundary ($r \to 0$)	Near-singularity ($r \to \infty$)
$\phi(r)$ $(\Delta \neq d/2)$	$\phi_0 r^{d-\Delta} + \frac{\langle \mathcal{O} \rangle}{2\Delta - d} r^{\Delta}$	$\sqrt{2(d-1)(\rho-d)} \log r$
$\phi(r)$ $(\Delta = d/2)$	$\phi_0 r^{d/2} \log r$	$\sqrt{2(d-1)(\rho-d)} \log r$
$\chi(r)$ $(\Delta \neq d/2)$	$\frac{d-\Delta}{2(d-1)} \phi_0^2 r^{2(d-\Delta)} + \frac{2\Delta(d-\Delta)}{d(d-1)(2\Delta-d)} \phi_0 \langle \mathcal{O} \rangle r^d + \frac{\Delta}{2(d-1)(2\Delta-d)^2} \langle \mathcal{O} \rangle^2 r^{2\Delta}$	$2(\rho-d) \log r + \chi_1$
$\chi(r)$ $(\Delta = d/2)$	$\frac{\phi_0^2}{4d(d-1)} r^d \left[2 + 2d \log r + d^2 (\log r)^2\right]$	$2(\rho-d) \log r + \chi_1$
$f(r)$	$e^{\chi(r)} \left(1 - \langle T_{tt} \rangle r^d\right)$	$-f_1 r^{\rho}$

Define,

$$r = \tau^{-2/\rho}, \quad \rho = d + \left(\frac{d-1}{2}\right) c^2 \tag{13}$$

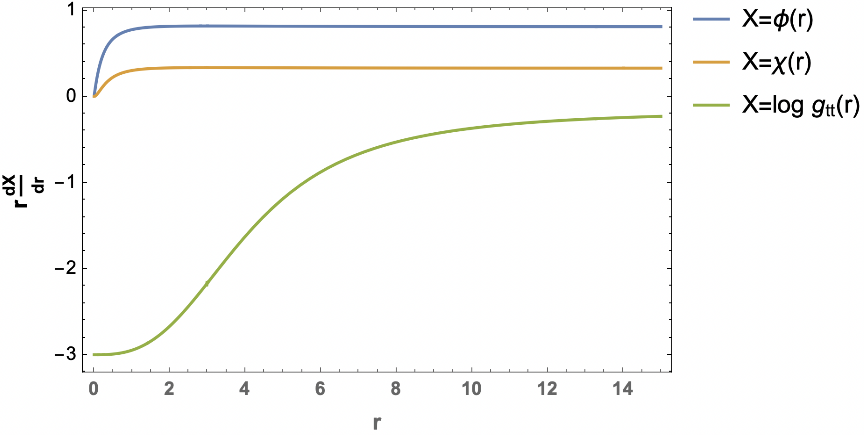
Kasner Metric:-

$$ds^2 \sim -d\tau^2 + \tau^{2p_t} dt^2 + \tau^{2p_x} d\vec{x}^2, \quad \phi(\tau) \sim -\sqrt{2} p_\phi \log \tau, \tag{14}$$

Kasner Exponents:-

$$p_t = 1 - \frac{2(d-1)}{\rho}, \quad p_x = \frac{2}{\rho}, \quad p_\phi = \frac{2\sqrt{(d-1)(\rho-d)}}{\rho}. \tag{15}$$

Free Kasner Flows



$d = 3,\ \Delta = 2,\ r_h = 3$

Free Kasner Flows

Thermal a-function for the metric ansatz (2) is,

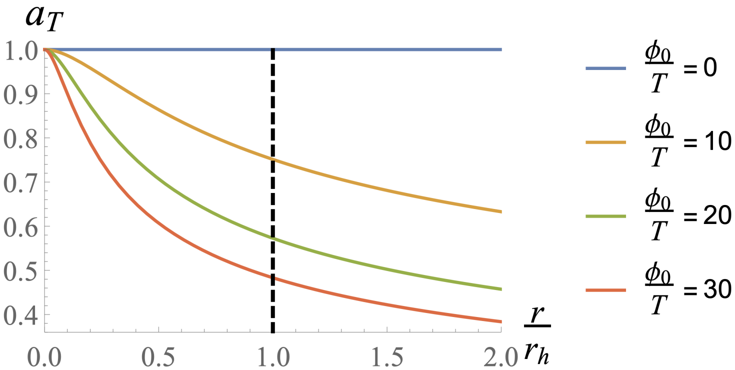
$$a_T(r) = \frac{\pi^{\frac{d}{2}}}{\Gamma\left(\frac{d}{2}\right)\ell_P^{d-1}} e^{-\frac{(d-1)\chi(r)}{2}}$$

In the Kasner regime, the χ function blows up -

$$\chi(r) = (d-1)c^2 \log(r) + \chi_1$$

So, the a-function vanishes at the singularity!!!

It might indicate a total loss of the d.o.f in the CFT. **(Q2)**



a_T
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.0
0.5
1.0
1.5
2.0
$\frac{r}{r_h}$
$\frac{\phi_0}{T} = 0$
$\frac{\phi_0}{T} = 10$
$\frac{\phi_0}{T} = 20$
$\frac{\phi_0}{T} = 30$

Quantum Information and trans-IR flow

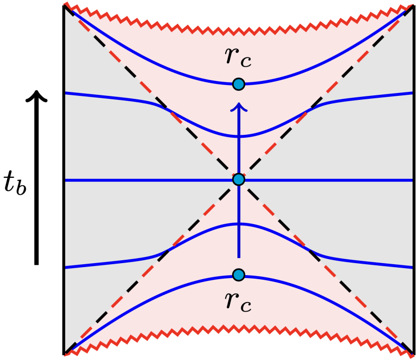
(Q3) How does the boundary observables relate to or probe the trans-IR flows?

Bulk quantities describes boundary observables,

- **Geodesic approximation (GA)**: 2-point functions are approximated by the spacelike geodesic lengths. [Balasubramanian, Ross 2000]
- **RT/HRT proposal**: Entanglement entropy encoded by minimal codimension-2 bulk surfaces. [Ryu, Takayanagi 2006] [Hubeny, Rangamani, Takayanagi 2007]
- **Complexity = Volume (CV)**: Holographic complexity is related to the volume of the maximal codimension-1 bulk slice. [Susskind 2014] [Susskind, Stanford 2014]
- **Complexity = Action (CA)**: Holographic complexity is given by the gravitational action evaluated on Wheeler-deWitt patch. [Brown, Roberts, Susskind, Swingle, Zhao 2016]

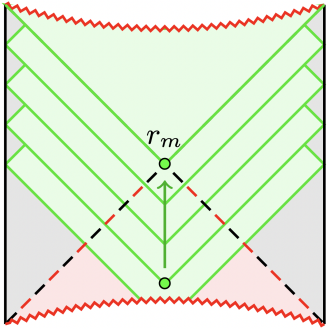
Quantum Information and trans-IR flow

Higher dimensional surfaces stuck at some critical radius r_c and thus not a good probe for the entire trans-IR flow.



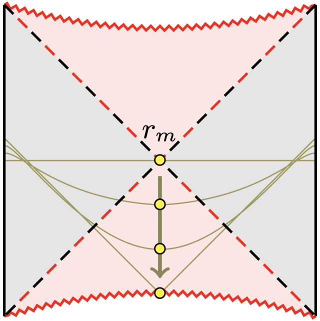
(a) Complexity = Volume

At the late times, $t_b \to \infty$: $\frac{d\mathcal{C}_V}{dt_b} \sim a_T(r_c)^{\frac{1}{d-1}}$



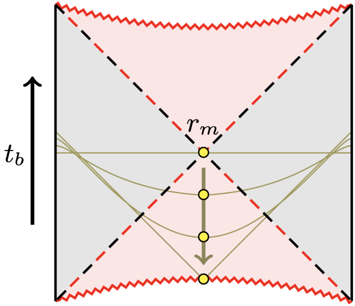
(b) Complexity = Action

Null joints exist for $t_b \geq t_c$.

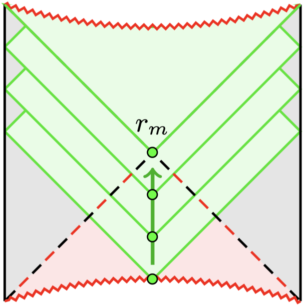


(c) Spacelike geodesics

Spacelike geodesics probe the trans-IR, $0 \leq t_b \leq t_c$



(a) Early time: $0 \leq t_b \leq t_c$



(b) Late time: $t_b \geq t_c$

One can say that null joints are a good probe to the trans-IR flow for $t_b \geq t_c$ and space like geodesics are a good probe when $0 \leq t_b \leq t_c$.
(Q3)

Conclusion & Future directions

(I) We have constructed a candidate for the a-function which is monotonic along the entire flow.

(II) From holographic point of view, this thermal a-function vanishes at the singularity indicating a total loss of d.o.f.

(III) We find an existence of a complementarity between the space-like geodesics and null joint of the WDW patch. In other words, they probe the trans-IR regime in complementary fashion.

Conclusion & Future directions

Interesting future directions:-

- Understanding the trans-IR flow from boundary field theory point of view.
- Studying a_T in more examples of flows (higher- order scalar interactions, vectors, etc.).
- Analogous extension of a_T to higher-curvature theories.
- Study the near singularity behaviour of the thermal 'a' function(or the trans-IR flows) for more general types of singularity for example Belinskii-Khalatnikov-Lifshitz (BKL).

Thank You!