Corrections to AdS_5 black hole thermodynamics from higher-derivative supergravity

Alejandro Ruipérez

Università di Roma Tor Vergata & INFN Sezione di Roma Tor Vergata

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Iberian Strings

TOR VERGATA
UNIVERSITÀ DEGLI STUDI DI ROMA

INFN
Istituto Nazionale di Fisica Nucleare
SEZIONE DI ROMA TOR VERGATA

Joint work with:

D. Cassani and E. Turetta

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Introduction

- Some black holes admit a description in terms of fundamental constituents: strings and branes. Key to explain the microscopic origin of their entropies

Strominger, Vafa

- Use AdS/CFT to address the **microstate counting of AdS black holes**

Black hole	$=$	ensemble of states in quantum gravity	$\overset{\text{AdS/CFT}}{=}$	ensemble of states in the dual CFT

$$\mathcal{S}_{\text{CFT}} = \log N_{\text{micro}} = \frac{A_{\text{H}}}{4G} + \textbf{corrections}$$

- We will focus on $\text{AdS}_5/\text{CFT}_4$ and consider the **microstate counting of supersymmetric AdS_5 black holes** in minimal gauged supergravity. Dual states should exist in any holographic $\mathcal{N} = 1$ SCFT

Euclidean approach

Gibbons, Hawking

- We set ourselves in the **grand-canonical ensemble** (fixed β, Ω_i, Φ)

- The **grand-canonical partition function** $\mathcal{Z}(\beta, \Omega_i, \Phi)$ is computed by the **Euclidean path integral** with (anti-)periodic boundary conditions

$$\mathcal{Z}(\beta, \Omega_i, \Phi) = \int Dg_{\mu\nu} DA_\mu D\psi \, e^{-I[g_{\mu\nu}, A_\mu, \psi]} \overbrace{\simeq}^{\text{semiclassical approx.}} e^{-I(\beta, \Omega_i, \Phi)}$$

- $I(\beta, \Omega_i, \Phi)$ should then be identified with $\beta\times$(grand-canonical potential), leading to the **quantum statistical relation (QSR)**

$$\boxed{I = \beta E - \mathcal{S} - \beta \Omega_i J_i - \beta \Phi \, Q}$$

- By the master formula of the **AdS/CFT correspondence**:

$$\boxed{I(\beta, \Omega_i, \Phi) = -\log \mathcal{Z}_{\text{CFT}}(\beta, \Omega_i, \Phi)}$$

Review of AdS_5 black holes

We work with **minimal gauged supergravity in 5d**

$$\mathcal{L} = R + 12g^2 - \frac{1}{4}F^2 - \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\sigma\lambda}F_{\mu\nu}F_{\rho\sigma}A_\lambda$$

The **most general AdS_5 black hole** depends on **four parameters**:

$$E, J_1, J_2, Q \qquad \leftrightarrow \qquad \beta, \Omega_1, \Omega_2, \Phi$$

Chong, Cvetic, Lu, Pope

These quantities obey the **first law of black hole mechanics**

$$\mathrm{d}E = T\mathrm{d}\mathcal{S} + \Omega_1 \mathrm{d}J_1 + \Omega_2 \mathrm{d}J_2 + \Phi\,\mathrm{d}Q$$

as well as the **quantum statistical relation**

$$I = \beta E - \mathcal{S} - \beta\Omega_1 J_1 - \beta\Omega_2 J_2 - \beta\Phi\, Q$$

Reaching the BPS locus

supersymmetry $\not\Rightarrow$ **extremality**

It is crucial to reach the BPS locus following a **supersymmetric trajectory**

1. Supersymmetric solution if $E - gJ_1 - gJ_2 - \sqrt{3}\,Q = 0$
2. BPS (supersymmetric + extremal) limit: $\beta \to \infty$

Cabo-Bizet, Cassani, Martelli, Murthy

The BPS charges must satisfy an **additional constraint**:

$$\boxed{\left(\frac{2\sqrt{3}}{g}Q^* + \frac{\pi}{2Gg^3}\right)\left(\frac{4}{g^2}Q^{*\,2} - \frac{\pi}{Gg^3}(J_1^* + J_2^*)\right) = \left(\frac{2}{\sqrt{3}\,g}Q^*\right)^3 + \frac{2\pi}{Gg^3}J_1^* J_2^*}$$

In the BPS limit, the **chemical potentials** are **frozen** to the following values

$$\beta^* \to \infty\,, \qquad \Omega_1^* = \Omega_2^* = g\,, \qquad \Phi^* = \sqrt{3}$$

which coincide with the coefficients appearing in the **superalgebra**

$$\{\mathcal{Q}, \mathcal{Q}^\dagger\} \propto E - gJ_1 - gJ_2 - \sqrt{3}\,Q = 0$$

Supersymmetric thermodynamics

Let's **impose supersymmetry while keeping β finite**

$$\boxed{E - gJ_1 - gJ_2 - \sqrt{3}\,Q = 0}$$

Using this in the QSR leads to

$$I = -\mathcal{S} - \omega_1 J_1 - \omega_2 J_2 - \frac{2}{\sqrt{3}\,g}\varphi\,Q$$

where

$$\omega_1 = \beta(\Omega_1 - \Omega_1^*)\,, \qquad \omega_1 = \beta(\Omega_1 - \Omega_1^*)\,, \qquad \varphi = \frac{\sqrt{3}g}{2}\beta(\Phi - \Phi^*)$$

which are **constrained** by

$$\boxed{\omega_1 + \omega_2 - 2\varphi = 2\pi i}$$

Supersymmetric first law

$$\mathrm{d}\mathcal{S} + \omega_1\,\mathrm{d}J_1 + \omega_2\,\mathrm{d}J_2 + \frac{2}{\sqrt{3}\,g}\varphi\,\mathrm{d}Q = 0$$

The **dependence of I on β disappears** after imposing supersymmetry

Supersymmetric action and BPS entropy

Evaluating the Euclidean on-shell action of the CCLP black hole and **imposing supersymmetry**:

$$\boxed{I = \frac{16\mathtt{a}}{27}\,\frac{\varphi^3}{\omega_1\omega_2} = \frac{2\mathtt{a}}{27}\,\frac{(\omega_1+\omega_2-2\pi i)^3}{\omega_1\omega_2}}$$

$\mathtt{a}, \mathtt{c}$ are the **superconformal anomaly coefficients** ($\mathtt{c} = \mathtt{a}$ in the large N limit)

The **Legendre transform** of the supersymmetric action + non-linear constraint give the the **Bekenstein-Hawking entropy** of the black hole (more later)

Hosseini, Hristov, Zaffaroni

Cabo-Bizet, Cassani, Martelli, Murthy

$$\boxed{\mathcal{S}^* = \pi\sqrt{3{Q_R^*}^2 - 8\mathtt{a}(J_1^* + J_2^*)}}$$

Kim, Lee

CFT side of the story

The relevant CFT quantity is the **superconformal index** ($\equiv$ supersymmetric partition function on $S^1 \times S^3$) on the "second sheet"

$$\mathcal{I} = \mathrm{Tr}\,(-1)^F e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}+(\omega_1-2\pi i)\left(J_1+\frac{1}{2}Q\right)+\omega_2\left(J_1+\frac{1}{2}Q\right)}$$

This is a **refined Witten index**: only receives contributions from states annihilated by $\mathcal{Q}, \mathcal{Q}^\dagger$ Kinney, Maldacena, Minwalla, Raju; Romelsberger

The contribution of the supersymmetric black hole can be isolated by taking a **Cardy-like limit**: Cassani, Komargodsky; Choi, J. Kim, S. Kim, Nahmgoong + ...

$$\begin{aligned}\log \mathcal{I} \;=\; & -\frac{5\mathtt{a}-3\mathtt{c}}{27}\,\frac{(\omega_1+\omega_2-2\pi i)^3}{\omega_1\omega_2}+\frac{\mathtt{a}-\mathtt{c}}{3}\,\frac{(\omega_1+\omega_2-2\pi i)(\omega_1^2+\omega_2^2-4\pi^2)}{\omega_1\omega_2}\\ & +\log|\mathcal{G}_{1-\mathrm{form}}|+\text{exp-terms}\end{aligned}$$

The **leading-order contribution** in the large N limit exactly **matches the SUSY on-shell action!** Cabo-Bizet, Cassani, Martelli, Murthy

$$-\log \mathcal{I} = \frac{2\mathtt{a}}{27}\,\frac{(\omega_1+\omega_2-2\pi i)^3}{48\,\omega_1\omega_2} = I$$

Outline for the rest of the talk

The result for the index in the Cardy-like limit **holds at finite** N

$$\log \mathcal{I} = -\frac{5\mathtt{a}-3\mathtt{c}}{27}\frac{(\omega_1+\omega_2-2\pi i)^3}{\omega_1\omega_2}+\frac{\mathtt{a}-\mathtt{c}}{3}\frac{(\omega_1+\omega_2-2\pi i)(\omega_1^2+\omega_2^2-4\pi^2)}{\omega_1\omega_2}+\dots$$

Q1: How can we **match** this result from the **gravitational side**?

Q2: Does it **count** the number of **microstates of BPS black holes** beyond leading order in the large-N limit?

- Include relevant higher-derivative/quantum corrections
- Evaluate the on-shell action for the CCLP black hole and impose supersymmetry to match the result from the index
- Corrections to the entropy of BPS black holes

The four-derivative effective action

- The **goal** is to go beyond the 2∂ approximation including 4∂ terms

  EFT approach: Add **all** the possible **four-derivative terms** that are consistent with the symmetries of the two-derivative theory

- **Methodology**

  1. Start from the **off-shell** formulation of 5d **supergravity** where 4∂ supersymmetric invariants have been worked out in the literature

  $$\mathcal{L}_{\mathrm{off-shell}} = \mathcal{L}^{(2\partial)}_{\mathrm{off-shell}} + \alpha \left(\lambda_1 \mathcal{L}_{C^2} + \lambda_2 \mathcal{L}_{R^2} + \cancel{\lambda_3 \mathcal{L}_3} \right)$$

  Hanaki, Ohashi, Tachikawa
  Bergshoeff, Rosseel, Sezgin
  Ozkan, Pang

  2. **Integrate out** all the **auxiliary fields** at linear order in α

  3. Use **perturbative field redefinitions**

  $$g_{\mu\nu} \to g_{\mu\nu} + \alpha\, \Delta_{\mu\nu}\,, \qquad A_\mu \to A_\mu + \alpha\, \Delta_\mu$$

  to simplify the resulting action as much as possible

The four-derivative effective action

Cassani, AR, Turetta

- **Final result**

$$\mathcal{L} = c_0 R + 12 c_1 g^2 - \frac{c_2}{4} F^2 - \frac{c_3}{12\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda + \lambda_1 \alpha \left(\mathcal{X}_{\rm GB} - \frac{1}{2} C_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} + \frac{1}{8} F^4 - \frac{1}{2\sqrt{3}} \epsilon^{\mu\nu\rho\sigma\lambda} R_{\mu\nu\alpha\beta} R_{\rho\sigma}{}^{\alpha\beta} A_\lambda \right)$$

where $\mathcal{X}_{\rm GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$ and $c_i = 1 + \alpha g^2 \delta c_i$, with:

$$\delta c_0 = 4\lambda_2\,, \qquad \delta c_1 = -10\lambda_1 + 4\lambda_2\,, \qquad \delta c_2 = 4\lambda_1 + 4\lambda_2\,, \qquad \delta c_3 = -12\lambda_1 + 4\lambda_2$$

- **Holographic dictionary with $\mathtt{a}, \mathtt{c}$**

$$\mathtt{a} = \frac{\pi}{8 G g^3} \left(1 + 4\lambda_2 \alpha g^2 \right) \qquad \mathtt{c} = \frac{\pi}{8 G g^3} \left(1 + 4 \left(2\lambda_1 + \lambda_2 \right) \alpha g^2 \right)$$

Setting up the computation of the on-shell action at $\mathcal{O}(\alpha)$

A priori we need two ingredients to **evaluate the on-shell action at $\mathcal{O}(\alpha)$**:

1. The corrected solution
2. Boundary terms (Gibbons-Hawking terms + counterterms)

However, it is possible to show that **the two-derivative solution is enough** for evaluating the on-shell action

$$I = I^{(0)}|_{\alpha=0} + \alpha\, \cancel{\partial_\alpha I^{(0)}_{\alpha=0}} + \alpha\, I^{(1)}|_{\alpha=0} + \mathcal{O}(\alpha^2)$$

if one fixes the boundary conditions appropriately, which is automatic if we work in the **grand-canonical ensemble**

Reall, Santos

We are then left with the only task of identifying an appropriate set of **boundary terms**

Setting up the computation of the on-shell action at $\mathcal{O}(\alpha)$

Gibbons-Hawking terms

- the one associated to the GB is well known Myers; Teitelboim, Zanelli
- GH terms associated to $C_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$ and to the mixed Chern-Simons term can also be derived treating higher-derivative terms **perturbatively** and assuming **AlAdS$_5$ asymptotics** (but do not contribute)

  Grumiller, Mann, McNees; Landsteiner, Megías, Pena-Benitez; Cremonini, Liu, Szepietowski; Cassani, AR, Turetta (to appear)

Boundary counterterms

- same as in the 2∂ theory (only the GB term diverges in the 4∂ sector)

Matching the superconformal index

- We can now calculate the **on-shell action** of the CCLP black hole and impose **supersymmetry** $(\omega_1 + \omega_2 - 2\varphi = 2\pi i)$

$$I = \frac{2\pi}{27Gg^3}\left(1 - 4(3\lambda_1 - \lambda_2)\alpha g^2\right)\frac{\varphi^3}{\omega_1\omega_2} + \frac{2\pi\alpha\lambda_1}{3Gg}\frac{\varphi\left(\omega_1^2 + \omega_2^2 - 4\pi^2\right)}{\omega_1\omega_2}$$

- This fully **matches the expression for the dual index** in the Cardy-like limit once the holographic dictionary is implemented!

$$I = -\log\mathcal{I} = \frac{5\mathtt{a} - 3\mathtt{c}}{27}\frac{(\omega_1 + \omega_2 - 2\pi i)^3}{\omega_1\omega_2} - \frac{\mathtt{a} - \mathtt{c}}{3}\frac{(\omega_1 + \omega_2 - 2\pi i)(\omega_1^2 + \omega_2^2 - 4\pi^2)}{\omega_1\omega_2}$$

Cassani, AR, Turetta

Bobev, Dimitrov, Reys, Vekemans

- What about the **black hole entropy**?

The corrected BPS entropy and charges

Cassani, AR, Turetta

- **Corrected charges** from I (assuming first law and QSR):

$$E = \frac{\partial I}{\partial \beta}\,, \qquad J_1 = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_1}\,, \qquad J_2 = -\frac{1}{\beta}\frac{\partial I}{\partial \Omega_2}\,, \qquad Q = -\frac{1}{\beta}\frac{\partial I}{\partial \Phi}$$

- **Corrected BPS entropy** (from QSR and taking BPS limit):

$$\boxed{\mathcal{S}^* = \pi\sqrt{3{Q_R^*}^2 - 8\mathtt{a}(J_1^* + J_2^*) - 16\,\mathtt{a}(\mathtt{a}-\mathtt{c})\frac{(J_1^* - J_2^*)^2}{{Q_R^*}^2 - 2\mathtt{a}(J_1^* + J_2^*)}}}$$

(also Bobev, Dimitrov, Reys, Vekemans)

- **Corrected non-linear relation between the charges:**

$$\boxed{\begin{aligned}&[3Q_R^* + 4\,(2\,\mathtt{a} - \mathtt{c})]\left[3{Q_R^*}^2 - 8\mathtt{c}\,(J_1^* + J_2^*)\right]\\ &= {Q_R^*}^3 + 16\,(3\mathtt{c} - 2\mathtt{a})\,J_1^* J_2^* + 64\mathtt{a}\,(\mathtt{a} - \mathtt{c})\frac{(Q_R^* + \mathtt{a})(J_1^* - J_2^*)^2}{{Q_R^*}^2 - 2\mathtt{a}(J_1^* + J_2^*)}\,.\end{aligned}}$$

Two more methods to compute the **BPS entropy**

- By means of **Wald's formula**: $\mathcal{S} = -2\pi \int_\Sigma \mathrm{d}^3x \sqrt{\gamma}\, \frac{\delta S}{e\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma}$
  - Corrected solution needed but **near-horizon geometry** is enough
  - Restrict to $J_1 = J_2$ case (**Gutowski-Reall black hole**) for simplicity

- As a **constrained Legendre transform** of the index/BPS on-shell action:

$$\mathcal{S} = \ \mathrm{ext}_{\{\omega_1,\omega_2,\varphi,\Lambda\}} \left[-I - \omega_1 J_1 - \omega_2 J_2 - \varphi\, Q_R - \Lambda\,(\omega_1+\omega_2-2\varphi-2\pi i)\right]$$

$$-\frac{\partial I}{\partial \omega_i} = J_i + \Lambda\,, \qquad -\frac{\partial I}{\partial \varphi} = Q_R - 2\Lambda\,, \qquad \omega_1+\omega_2-2\varphi = 2\pi i$$

The three methods give rise to the **same answer**!

Cassani, AR, Turetta

Our main results

- Construction of a **four-derivative extension of 5d minimal gauged supergravity** that captures the corrections to the index

- We have evaluated the **on-shell action** of the CCLP black hole **at linear order** in the corrections, showing that it **matches the CFT prediction** when **supersymmetry** is imposed

$$\boxed{I = -\log \mathcal{I}}$$

- We have computed the **corrections to the BPS entropy** using different methods, showing that all of them agree

- We have shown that the **index counts** the number of black hole **microstates** beyond leading order in the large-N limit approx.

Open questions and future directions

- Better understanding of the BPS limit

- What information can be extracted from the near-horizon?

Cassani, AR, Turetta (to appear)

- Extension to matter-coupled supergravities. Multi-charge case.

- Thermodynamics of near-BPS AdS_5 black holes. Corrections to the mass gap? CFT description?

Boruch, Heydemann, Iliesiu, Turiaci

- Possible stringy origins of the gravitational EFT?

Bilal, Chu
Liu, Minasian

Thank you!

Legendre transform of the superconformal index

$$\mathcal{S} = \text{ext}_{\{\omega_1,\omega_2,\varphi,\Lambda\}} \left[-I - \omega_1 J_1 - \omega_2 J_2 - \varphi\, Q_R - \Lambda\,(\omega_1 + \omega_2 - 2\varphi - 2\pi i)\right]$$

$$-\frac{\partial I}{\partial \omega_i} = J_i + \Lambda\,, \qquad -\frac{\partial I}{\partial \varphi} = Q_R - 2\Lambda\,, \qquad \omega_1 + \omega_2 - 2\varphi = 2\pi i$$

- By virtue of Euler's theorem on homogeneous functions, $\mathcal{S} = 2\pi i \Lambda_{\text{ext}}$
- Entropy must be real $\Rightarrow$ Λ must be purely imaginary

$$\Lambda^3 + p_2 \Lambda^2 + p_1 \Lambda + p_0 + \frac{p_{-1}}{\Lambda - \frac{Q_R}{2}} = 0$$

- **Factorization condition** $(\Lambda^2 + X)(\text{rest}) = 0$ is **equivalent to the non-linear constraint** among the charges
- Corrections to BPS entropy

$$S = 2\pi\sqrt{X} = \pi\sqrt{3{Q_R^*}^2 - 8\mathtt{a}(J_1^* + J_2^*) - 16\,\mathtt{a}(\mathtt{a} - \mathtt{c})\frac{(J_1^* - J_2^*)^2}{{Q_R^*}^2 - 2\mathtt{a}(J_1^* + J_2^*)}}$$

Cassani, AR, Turetta

Wald entropy from the near-horizon geometry

- We want to compute the **BPS entropy** using **Wald's formula**
- **Corrected solution needed** but the **near-horizon geometry** is enough
- Focus on the $b = a$ case (**Gutowski-Reall black hole**), much simpler

$$
\begin{aligned}
\mathrm{d}s^2 &= v_1 \left(-\varrho^2 \mathrm{d}t^2 + \frac{\mathrm{d}\varrho^2}{\varrho^2} \right) + \frac{v_2}{4} \left[\sigma_1^2 + \sigma_2^2 + v_3 \left(\sigma_3 + w\, \varrho\, \mathrm{d}t \right)^2 \right] \\
A &= e\, \varrho\, \mathrm{d}t + p\, \left(\sigma_3 + w\, \varrho\, \mathrm{d}t \right)
\end{aligned}
$$

where

$$
v_i = v_i^{\mathrm{GR}} + \alpha\, \delta v_i\,, \quad w = w^{\mathrm{GR}} + \alpha\, \delta w\,, \quad e = e^{\mathrm{GR}} + \alpha\, \delta e\,, \quad p = p^{\mathrm{GR}} + \alpha\, \delta p
$$

Solving the corrected EOMs boils down to a linear system of algebraic eqs

$$
\mathcal{M} \mathcal{X} = \mathcal{N}\,, \qquad \mathcal{X} = \mathcal{X}^H + \mathcal{X}^P
$$

- $\mathcal{X}^H$ is the **homogeneous solution**, fixed by **boundary conditions**
- $\mathcal{X}^P$ is a **particular solution**, it contains the **“new physics”**

Wald entropy

$$\boxed{\mathcal{S} = -2\pi \int_{\Sigma} \mathrm{d}^3x \sqrt{\gamma}\, \frac{\delta S}{e\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma}}$$

Using the **corrected near-horizon solution** and expressing the result in terms of the charges, one gets

$$\boxed{\mathcal{S}^* = \pi\sqrt{3{Q_R^*}^2 - 16\mathtt{a}J^*}}$$

which agrees with the expression obtained from the on-shell action and from the Legendre transform of the superconformal index

The corrections are encoded in the coefficients of the superconformal anomaly $\mathtt{a}, \mathtt{c}$

Cassani, AR, Turetta