Vibrational Orbit Effects

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Vibration of quadrupoles

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The vertical displacement of a beam at the IP caused by a quadrupole, located at vertical phase advance ϕ_q from the IP, vibrating with an amplitude Δy_q and an angular frequency ω_q is written as:

$$\begin{aligned}\Delta y^* &= \sum_n \sqrt{\beta^*\beta_q}\exp(-nT_0/\tau_y + i\omega_q n T_0)\sin(\phi_q + n\mu_y)k_q\Delta y_q \\ &= \sum_n \sqrt{\beta^*\beta_q}\exp(-n\alpha_y + in\mu_q)\sin(\phi_q + n\mu_y)k_q\Delta y_q\,,\end{aligned} \tag{1}$$

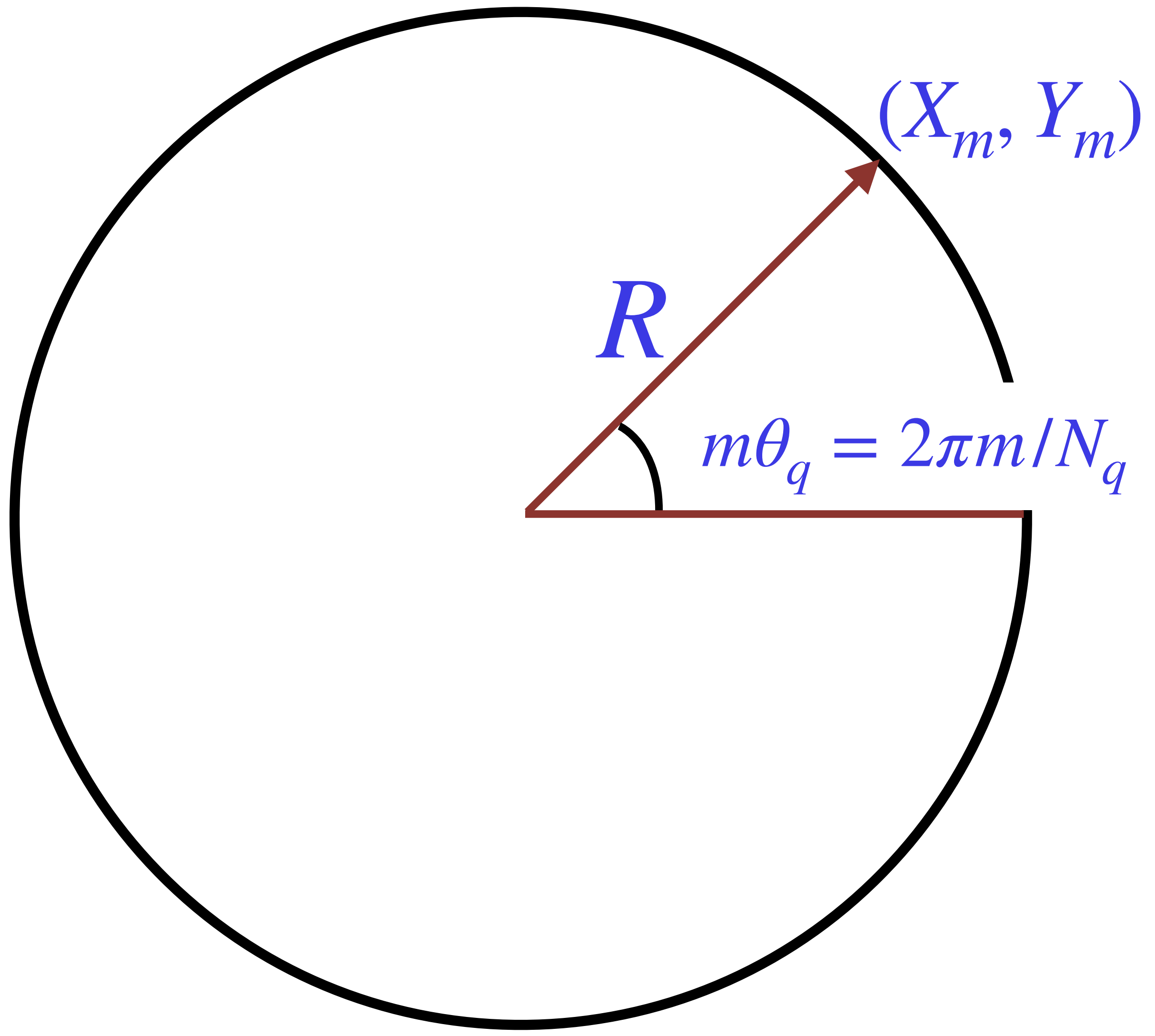
where μ_y, T_0 & τ_y, are the vertical betatron angular tune, the revolution & damping times, and $\alpha_y \equiv T_0/\tau_y$, $\mu_q \equiv \omega_q T_0$. β^*, β_q, k_q are the beta functions at the IP and the quadrupole, and the focusing strength of the quadrupole.

1.1 Vibration due to seismic motion

The vibration amplitude Δy_q can be either random at each quad, or coherent due to an external seismic wave. First let us evaluate the coherent part by assuming that the quads are distributed over the ring uniformly with the betatron phase $\phi_q = m\Delta\phi_q$, and also physically located over a ring of the radius R with a constant separation azimuthal angle θ_q, *i.e.*,

$$X_m + iY_m = R\exp(im\theta_q)\,, \tag{2}$$

where m runs over 1 through N_q, the number of quads per ring.



Resoponse to Seismic wave

Then if the quads follow the seismic wave on the ground, the displacement Δy_m of the m-th quadrupole is written as

$$\Delta y_m = u \exp\left(i(k_X X_m + k_Y Y_m - \omega_q t)\right) , \tag{3}$$

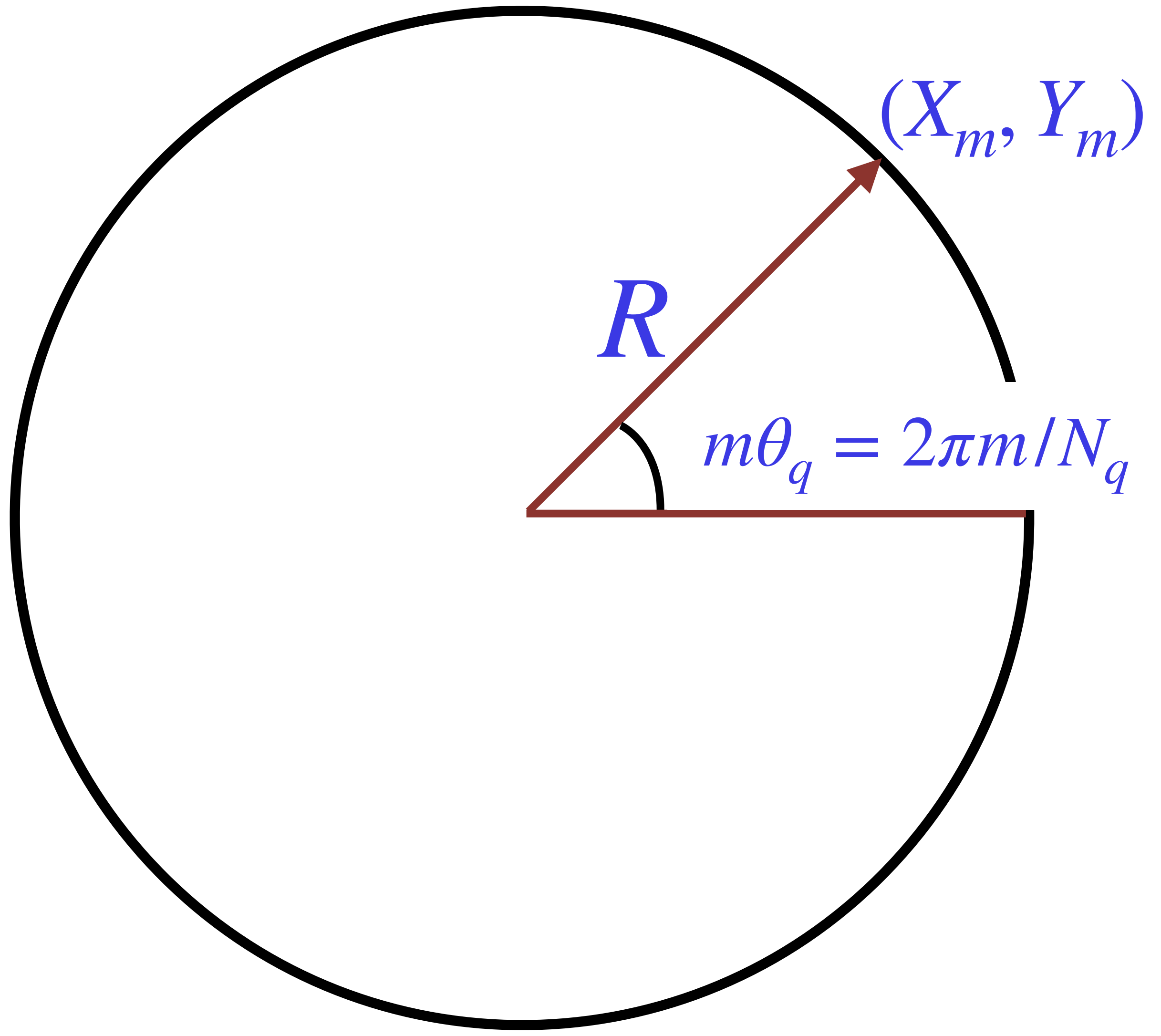
where $k_{X,Y}$ are the components of the seismic wave number vector, and u represents the amplitude. Here we just set $k_X = k$ and $k_Y = 0$ for simplicity without losing generality if the ring is nearly a circle. So we may sum up the term $\sin(\phi_q + n\mu_y)\Delta y_q$ in Eq. (1) over quadrupoles as

$$\begin{aligned} N_q d_s &= \sum_m^{N_q} \sin(\phi_q + n\mu_y)\Delta y_m \\ &= \sum_m^{N_q} \sin(m\Delta\phi_q + n\mu_y) u \exp\left(i(kR\cos m\theta_q - \omega_q t)\right) \\ &= u \sum_{\ell=-\infty}^{\infty} \sum_m^{N_q} \sin(m\Delta\phi_q + n\mu_y) J_\ell(kR) i^\ell \exp(i\ell m\theta_q - i\omega_q t) , \end{aligned} \tag{4}$$

where we have applied $\exp(ix\cos z) = \sum_\ell i^l J_\ell(x) \exp i\ell z$. Although there may be a **resonance** in Eq. (4) at $\ell \sim \pm\Delta\phi_q/\theta_q$, the index ℓ becomes too large in the case of FCC-ee Z, where $\Delta\phi_q = 83.5\,\mathrm{deg}$, $\theta_q = 360/924 \sim 0.390\,\mathrm{deg}$, and $\ell \sim 214$. As for N_q, we have taken only QD's into account here. Thus the coefficient J_ℓ becomes infinitesimal for such a large ℓ, so the resonant effect is negligible.

$$\exp(i(\boldsymbol{k}\cdot\boldsymbol{x} - \omega_q t))$$

$$\boldsymbol{k} = (k_X, k_Y) = (k, 0)$$



The term $\ell = 0$ in Eq. (4) is written as

$$d_{s0} = uJ_0(kR)\frac{\sin(\mu_y/2)\sin(n\mu_y + (\mu_y - \Delta\phi_q)/2)}{\sin(\Delta\phi_q/2)}\,. \tag{5}$$

We know $J_0(x) \leq 1$, and the rests of the rhs of Eq. (5) are not far from 1. Then we can say that the magnitude of the coherent component is smaller than the random component:

$$|d_s| \ll \sqrt{N_q}u\,. \tag{6}$$

1.2 Resonance with the betatron frequency

Next let us look at the vibration of the beam at the IP caused by the random motion of quads. Its expected value $\langle|\Delta y^*|^2\rangle$ is obtained by averaging Eq. (1) over ϕ_q as:

$$= \sum_n \sqrt{\beta^*\beta_q}\exp(-n\alpha_y + in\mu_q)\sin(\phi_q + n\mu_y)k_q\Delta y_q\,,$$

$$\begin{aligned}\langle|\Delta y^*|^2\rangle &= \frac{1}{2\pi}\int |\Delta y^*|^2 d\phi_q \\ &= \frac{\beta^*\beta_q k_q^2\langle\Delta y_q^2\rangle}{4}\frac{\exp(\alpha)(\cosh\alpha - \cos\mu_q\cos\mu_y)}{(\cosh\alpha - \cos(\mu_q - \mu_y))(\cosh\alpha - \cos(\mu_q + \mu_y))}\,,\end{aligned} \tag{7}$$

which shows resonances at $\mu_q = \pm\mu_y + 2m\pi$ with an integer m.

Resonance to the betatron frequency (cont'd)

By assuming the spectrum of $\langle \Delta y_q^2 \rangle$ is uniform around each resonance, the vibration at the IP can be evaluated as:

$$\langle |\Delta y^*|^2 \rangle = \frac{\beta^* \beta_q k_q^2}{8\alpha T_0} \sum_m S((\pm\mu_y \pm 2m\pi)/T_0)\,, \tag{8}$$

where $S(\omega)$ is the power spectrum density of $\langle \Delta y_q^2(\omega) \rangle$, and we have assumed $\cos\mu_q \cos\mu_y \sim 1/2$ and $\alpha \ll 1$.

A measurement of ground vibration tells that[1],

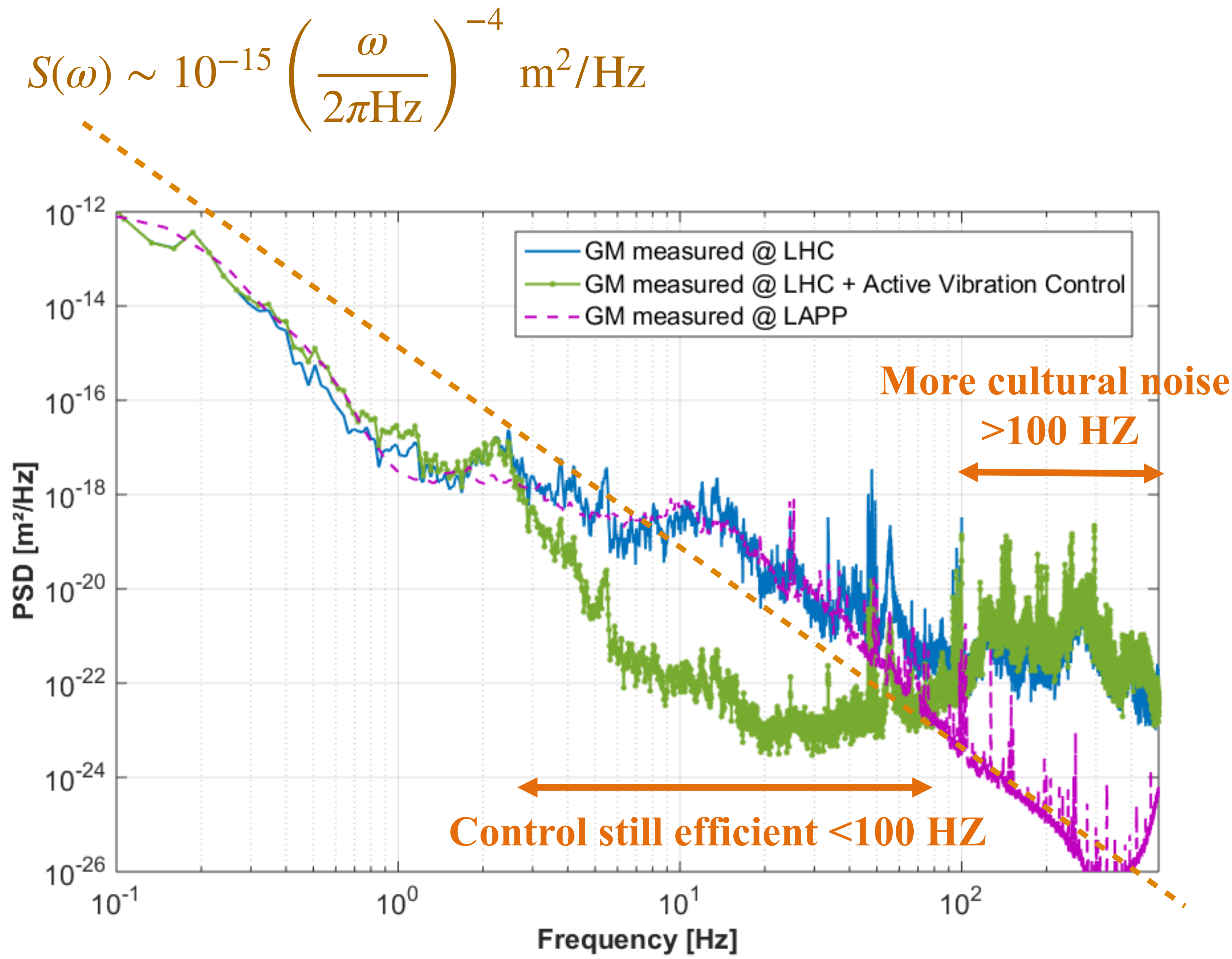
$$S(\omega) = \sigma\omega^{-4} \sim 10^{-15} \left(\frac{\omega}{2\pi \mathrm{Hz}}\right)^{-4} \mathrm{m}^2/\mathrm{Hz}\,, \tag{9}$$

with a coefficient σ, then among the resonances only the lowest one $m \sim \mu_y/2\pi$ will matter. In the case of FCC-ee, it is at

$$\omega/2\pi = \omega_r/2\pi \sim (1.2, 1.8)\,\mathrm{kHz}\,, \tag{10}$$

corresponding to $[\mu_y/2\pi] \sim (0.4, 0.6)$, resulting in

$$S(\omega_r) \sim (4.8, 0.95) \times 10^{-28}\,\mathrm{m}^2/\mathrm{Hz}\,. \tag{11}$$



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[1] https://indico.cern.ch/event/694811/contributions/2863859/attachments/1595533/2526938/2018_02_06_FCCee_MDI_workshop_Serluca.pdf

Resonance to the betatron frequency (cont'd)

Then the expected value of the orbit vibration at the IP is written as

$$\langle \Delta y^{*2} \rangle = \frac{\beta^* \beta_q k_q^2}{8\alpha T_0} \sum_m S(\omega_r) \,. \tag{12}$$

If we plugin numbers at FCC-ee Z (`FCCee_z_530_nosol_23`):

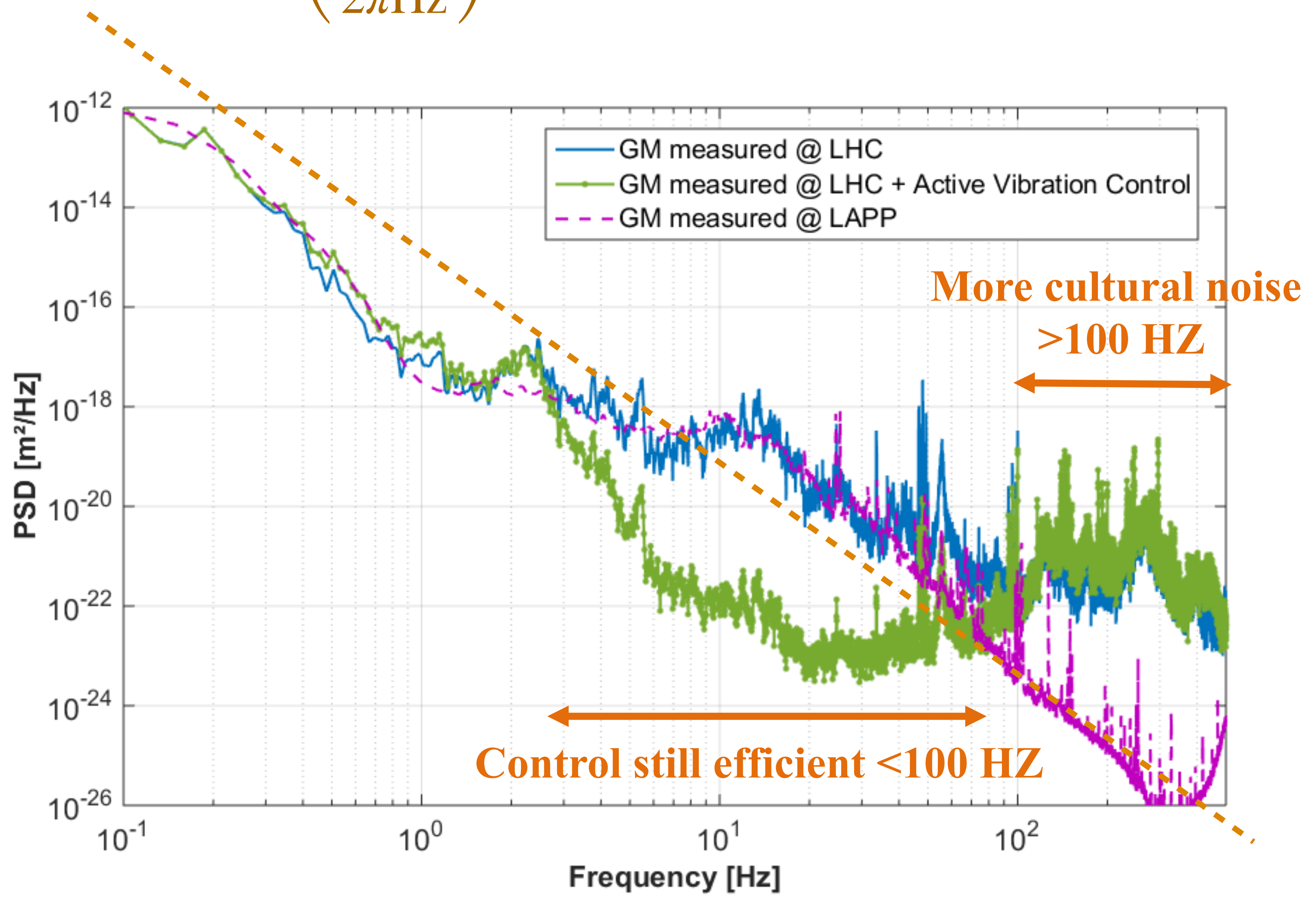
$$\begin{aligned} \beta^* &= 0.8\,\mathrm{mm}, & \langle\beta\rangle &= 436\,\mathrm{m}, \\ \langle k_q^2\rangle^{1/2} &= 0.045\,/\mathrm{m}, & \langle\beta k_q^2\rangle &= 8.5\,/\mathrm{m}, \\ \alpha &= 4.3\times 10^{-4}\,, & T_0 &= 304\,\mu\mathrm{s} \end{aligned} \tag{13}$$

into Eq, (12) and multiply the number of all quadrupoles N_q=1856, we get

$$\sqrt{\Delta y^{*2}} \sim 13.7\,\mathrm{pm}\,, \tag{14}$$

which is well smaller than the IP vertical beam size, $\sim 37\,\mathrm{nm}$.

$$S(\omega) \sim 10^{-15} \left(\frac{\omega}{2\pi\mathrm{Hz}}\right)^{-4} \mathrm{m}^2/\mathrm{Hz}$$



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1.3 Non-resonant vibration

Next let us look at the off-resonant contribution of Eq. (7). If we roughly approximate the tune-dependent term by 1, the integrated power spectrum in a range $\omega \geq \omega_c$ is given by

$$\begin{aligned} \langle \Delta y^{*2} \rangle &= \frac{N_q \beta^* \beta_q k_q^2}{4} \int_{\omega_c}^{\infty} S(\omega) \frac{d\omega}{2\pi} \\ &= \frac{N_q \beta^* \beta_q k_q^2 \sigma}{24\pi\omega_c^3} \,. \end{aligned} \tag{15}$$

In the case for the previous measurement, we estimate $\sigma \sim 1.6 \times 10^{-12}\,\mathrm{m^2/Hz}$, then

$$\sqrt{\Delta y^{*2}} \sim 32.9\,\mathrm{nm} \tag{16}$$

for $\omega_c = 2\pi \times 1\,\mathrm{Hz}$. The assumption here is that below the critical frequency ω_c, an orbit feedback suppresses the beam oscillation perfectly. Thus the expected vibration reaches to the vertical beam size at the IP. Among the vibration, the dominant contribution comes from the final quads "QC{12}*". If we exclude them, the expected vibration becomes

$$\sqrt{\Delta y^{*2}}_{\mathrm{excl.\ QC\{12\}*}} \sim 5.8\,\mathrm{nm}\,. \tag{17}$$

This value means that the contribution from other quads is small, but still not negligible. Suppressing the vibration of the final quads as well as an orbit feedback system working beyond 1 Hz will be crucial.

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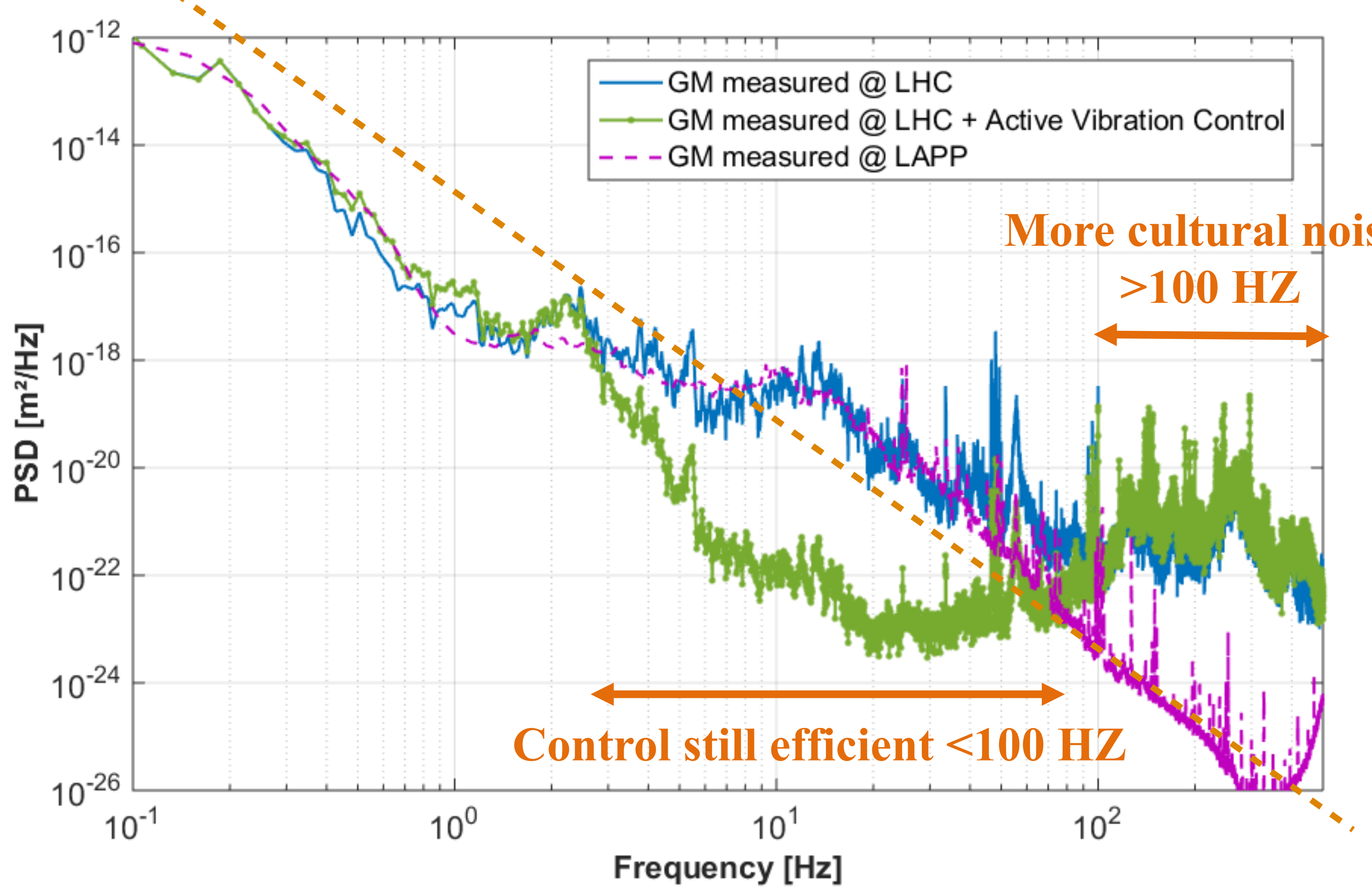
$S(\omega) \sim 10^{-15} \left(\frac{\omega}{2\pi\mathrm{Hz}}\right)^{-4}\ \mathrm{m^2/Hz}$



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1.4 Beam-beam deflection (SLC, B-factories)

If two beams have relative vertical offset at the IP by Δy^*, each beam receive a beam-beam kick at the IP:

$$\Delta p_y^* = \pm \frac{2\pi\xi_y}{\beta_y^*}\Delta y^* \,, \tag{18}$$

where ξ_y is the vertical beam-beam parameter. If we plugin the numbers for Z:

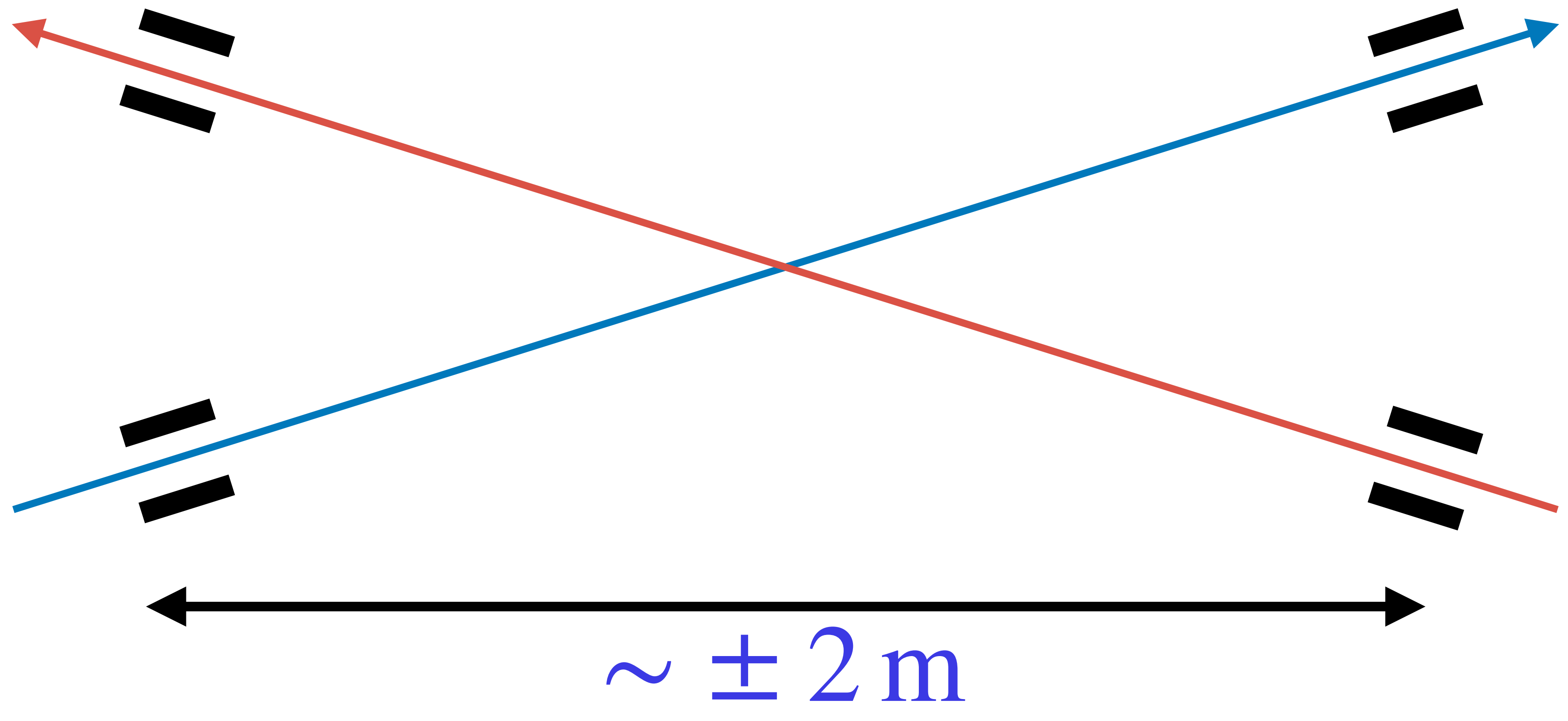
$$\xi_y = 0.135 \,, \qquad \beta_y^* = 0.8\,\mathrm{mm}, \qquad \Delta y^* = \frac{1}{10}\sigma_y^* = 3.4\,\mathrm{nm}\,, \tag{19}$$

the beam-beam kick becomes

$$\Delta p_y^* = 3.6\,\mu\mathrm{rad}\,. \tag{20}$$

If we have BPMs for both beams at $\pm 2\,\mathrm{m}$ from the IP, this kick is well larger than the resolution of the BPMs, at least for the average over the bunches.

Thus the beam-beam deflection has been the primary method to detect and correct the relative offset of two beams at the IP for a linear or double ring colliders.



By combining the readings of four BPMs at the both sides of IP for both beams, it is possible to extract the beam-beam deflection.

Maintaining the collision (2)

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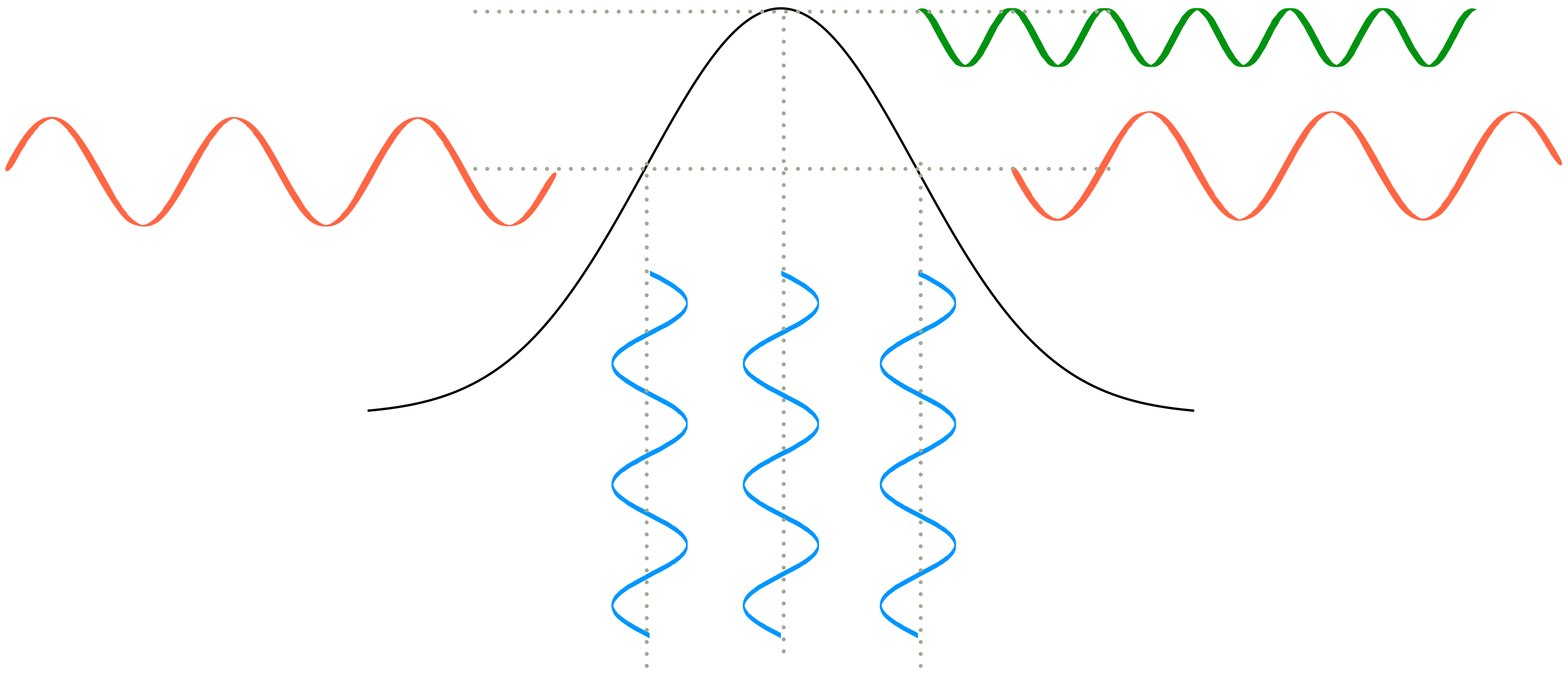
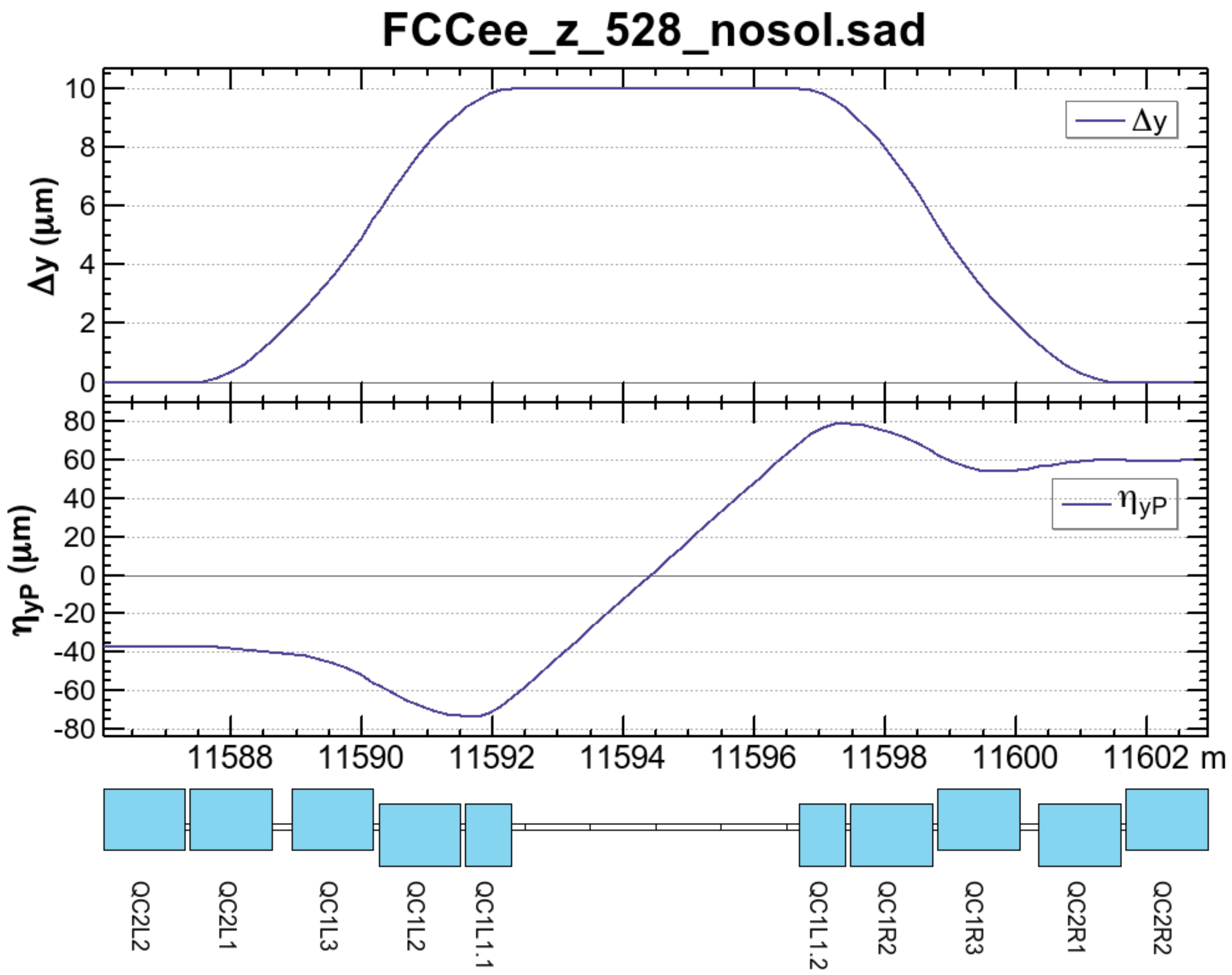
1.5 Dithering

As the horizontal beam-beam parameter is very small in low energies ($\xi_x = 0.004$ at Z), the beam-beam deflection is not appropriate for the detection of horizon offset at the IP. In such a case, a method called *dithering*, developed at PEP-II, is applicable, It shakes one beam with a single frequency, then detect the modulation of luminosity at that frequency, then by nullifying that component, the optimum offset is obtained.



An example of vertical bump at IP

- A simplest vertical bump orbit to control the IP offset can be produced by the skew dipole corrector winding of QC{12}{LR}1.
- This example does not close the dispersion.
- However, the associated vertical emittance generated by the dispersion leak is only 2.6 am by the 10 μm vertical offset at the IP. So the dispersion leak is not a practical issue.
- If this corrector is used for the IP feedback, its frequency response can be an issue, due to reduction by the beam pipe.



“latest” parameters

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Beam energy	[GeV]	45.6	80	120	182.5
Layout		PA31-1.0			
# of IPs		4			
Circumference	[km]	91.174117		91.174107	
Bending radius of arc dipole	[km]	9.937			
Energy loss / turn	[GeV]	0.0391	0.370	1.869	10.0
SR power / beam	[MW]	50			
Beam current	[mA]	1280	135	26.7	5.00
Bunches / beam		10000	880	248	40
Bunch population	$[10^{11}]$	2.43	2.91	2.04	2.37
Horizontal emittance ε_x	[nm]	0.71	2.16	0.64	1.49
Vertical emittance ε_y	[pm]	1.42	4.32	1.29	2.98
Arc cell		Long 90/90		90/90	
Momentum compaction α_p	$[10^{-6}]$	28.5		7.33	
Arc sextupole families		75		146	
$\beta^*_{x/y}$	[mm]	100 / 0.8	200 / 1.0	300 / 1.0	1000 / 1.6
Transverse tunes/IP $Q_{x/y}$		53.563 / 53.600		100.565 / 98.595	
Energy spread (SR/BS) σ_δ	[%]	0.038 / 0.132	0.069 / 0.154	0.103 / 0.185	0.157 / 0.221
Bunch length (SR/BS) σ_z	[mm]	4.38 / 15.4	3.55 / 8.01	3.34 / 6.00	1.95 / 2.75
RF voltage 400/800 MHz	[GV]	0.120 / 0	1.0 / 0	2.08 / 0	2.5 / 8.8
Harmonic number for 400 MHz		121648			
RF freuqeuncy (400 MHz)	MHz	399.994581		399.994627	
Synchrotron tune Q_s		0.0370	0.0801	0.0328	0.0826
Long. damping time	[turns]	1168	217	64.5	18.5
RF acceptance	[%]	1.6	3.4	1.9	3.0
Energy acceptance (DA)	[%]	±1.3	±1.3	±1.7	-2.8 +2.5
Beam-beam ξ_x/ξ_y [a]		0.0023 / 0.135	0.011 / 0.125	0.014 / 0.131	0.093 / 0.140
Luminosity / IP	$[10^{34}/\mathrm{cm}^2\mathrm{s}]$	182	19.4	7.26	1.25
Lifetime (q + BS + lattice)	[sec]	840	–	< 1065	< 4062
Lifetime (lum)	[sec]	1129	1070	596	744

[a] incl. hourglass.

Summary

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Tolerances for the vibration of quadrupoles are evaluated for three cases:

- A seismic wave has smaller effects than the random motion of each quadrupole with an equal amplitude.
- Resonance with the betatron frequency: weak, as the betatron frequency is in the range of kHz.
- Non-resonant, incoherent vibration of each quad produces 38 nm vertical motion at the IP for ≧1 Hz.
  - Mostly by the final quads QC{12}, but the contribution from others are not negligible.
  - Assuming each quad follows the ground motion measured at LHC & LAPP.
  - No amplification of the mechanical motion of the magnet support has been assumed.
  - Below a frequency $\lesssim$ 10 Hz, a vertical orbit feedback is stringent.
- IP vertical offset can be detected by the beam-beam deflection.
  - For horizontal except for tt, the dithering method can be applied to maximize the luminosity.
- A simple vertical bump orbit can correct the IP offset easily.
  - Frequency response can be an issue of the design of the corrector.