Modular Average and Weyl Anomaly in Two-Dimensional Schwarzian Theory

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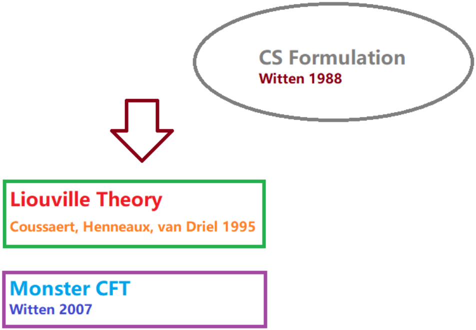
History


CS Formulation
Witten 1988
Liouville Theory
Coussaert, Henneaux, van Driel 1995
Monster CFT
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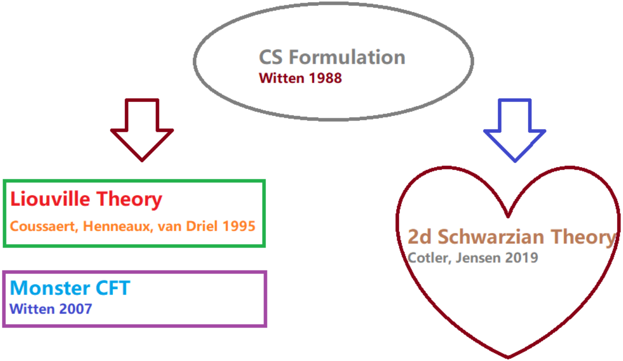
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2d Schwarzian Theory
Cotler, Jensen 2019

Boundary Theory for Torus

- two-dimensional Schwarzian theory [Cotler, Jensen 2019]

$$
\begin{aligned}
& S_{\mathrm{Gb}} \\
= \quad & \frac{k}{2\pi}\int dt d\theta\,\left(\frac{3}{2}\frac{(D_-\partial_\theta\mathcal{F})(\partial_\theta^2\mathcal{F})}{(\partial_\theta\mathcal{F})^2}-\frac{D_-\partial_\theta^2\mathcal{F}}{\partial_\theta\mathcal{F}}\right) \\
& -\frac{k}{2\pi}\int dt d\theta\,\left(\frac{3}{2}\frac{(D_+\partial_\theta\bar{\mathcal{F}})(\partial_\theta^2\bar{\mathcal{F}})}{(\partial_\theta\bar{F})^2}-\frac{D_+\partial_\theta^2\bar{\mathcal{F}}}{\partial_\theta\bar{\mathcal{F}}}\right), \quad (1)
\end{aligned}
$$

where

$$
\mathcal{F}\equiv\frac{F}{E_\theta^+}, \qquad \bar{\mathcal{F}}\equiv\frac{\bar{F}}{E_\theta^-}; \qquad (2)
$$

$$
D_+=\frac{1}{2}\partial_t-\frac{1}{2}\frac{E_t^-}{E_\theta^-}\partial_\theta, \qquad D_-=\frac{1}{2}\partial_t-\frac{1}{2}\frac{E_t^+}{E_\theta^+}\partial_\theta \qquad (3)
$$

- measure $\int d\mathcal{F}d\bar{\mathcal{F}}\,\left(1/(\partial_\theta\mathcal{F}\partial_\theta\bar{\mathcal{F}})\right)$

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$$-\frac{k}{4\pi}\int dtd\theta\ \mathrm{Tr}(A_\theta A_\theta + \bar{A}_\theta \bar{A}_\theta) + \frac{k}{8\pi}\int dtd\theta\ (A_\theta^2 A_\theta^2 + \bar{A}_\theta^2 \bar{A}_\theta^2) \quad (4)$$

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- implement the averaging of a modular group to compute the Rényi-2 mutual information for disjoint two-intervals
- non-perturbative effect kills the phase transition

Rényi-2 Mutual Information

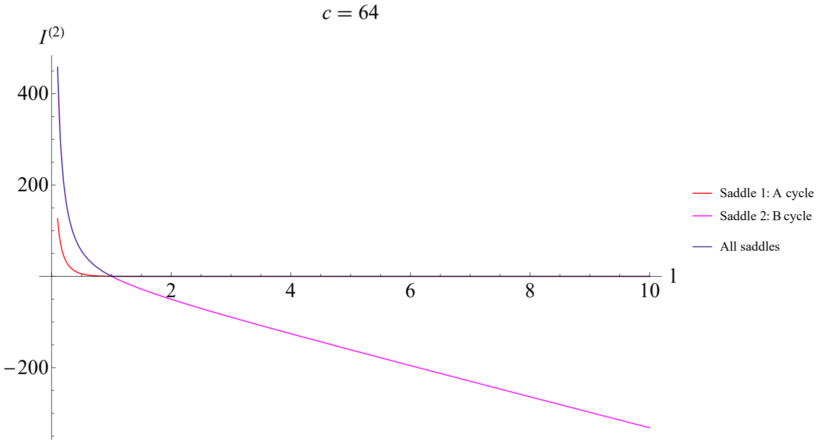

Figure: We show the Rényi-2 mutual information from summing all saddle points, A-cycle, and B-cycle with $c = 64$. Not all saddle points decay to zero when $l \equiv -i\tau \to \infty$, where τ is a complex structure (like B-cycle).

First-Order Derivative

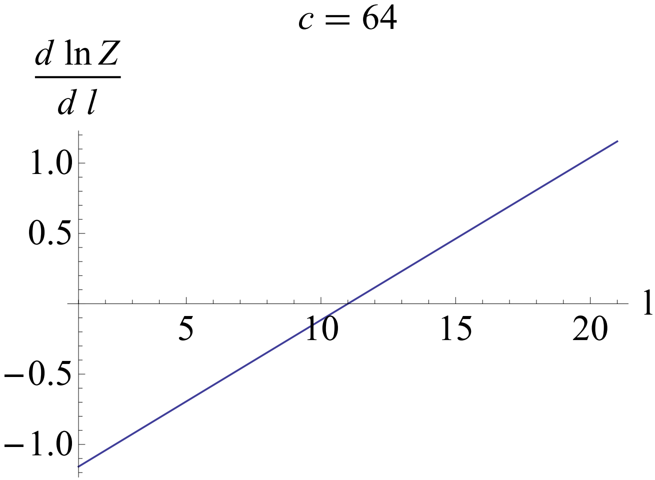

Figure: The first-order derivative of the logarithmic partition function is a continuous function for l.

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- implement the averaging of a modular group to compute the Rényi-2 mutual information for disjoint two-intervals
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- boundary theory=resummation of perturbative gravity

Thank you!