Inflation with Massive Spin-2 Ghosts

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Outline

- Motivations
  - Quadratic gravity and spin-2 ghosts
  - Inflation and scale invariance
- Our “minimal” model
  - Gravitational DOFs
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- Predictions
  - Dynamical scale generation
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The ghost problem in quadratic gravity

[2208.12832]

The most straightforward way to make GR renormalizable is to add quadratic powers of curvature tensors to the action.

$$S_{\rm EH} = \int \mathrm{d}^4x\sqrt{-g}M_{\rm pl}^2 R \qquad\qquad S_{\rm QG} = \int \mathrm{d}^4x\sqrt{-g}\Big(-\beta\phi^2 R + \gamma R^2 - \kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\Big)$$

dimensionful

dimensionless

Four metric derivatives means a spin-2 double pole and the classical Ostrogradsky instability.

$$\Delta_{hh} \sim \frac{1}{p^2} - \frac{1}{(p^2 - m_{\rm gh}^2)} \quad \Rightarrow \quad \mathcal{H} \sim \pi_+^2 - \pi_-^2 + \cdots$$

unbounded Hamiltonian

After standard quantization, this leads to negative norm states and issues with unitarity.

$$\big[\hat{a}_+(\mathbf{p}),\, \hat{a}_+^\dagger(\mathbf{q})\big] = \delta^3(\mathbf{p}-\mathbf{q})$$
$$\big[\hat{a}_-(\mathbf{p}),\, \hat{a}_-^\dagger(\mathbf{q})\big] = -\delta^3(\mathbf{p}-\mathbf{q})$$

$$\Rightarrow \quad \sum_n \big|\,\langle n|S|\alpha\rangle\,\big|^2 \neq 1$$

breakdown of probability interpretation

The ghost problem in quadratic gravity

Nice solutions are possible…

- Remove ghosts from asymptotic spectrum Lee-Wick-style
  - Quantize ghosts as “fakeons” that don’t appear by definition [Anselmi 1801.00915]
  - Demonstrate ghosts are unstable with nice decay products [Donoghue, Menezes 1908.02416]
- Use alternative quantization procedures
  - Define generalized QM norm [Salvio 1907.00983]
  - Employ (non-Hermitian) *PT*-symmetric QFT [Bender, Mannheim 0706.0207]

In any case, unitarity is satisfied below the ghost mass energy threshold.

The subspace of the total Fock space spanned by $|p_T\rangle \,, \quad p_T^2 < m_{\rm gh}^2$ is necessarily positive-definite.

Slow-roll inflation

[1807.08749]

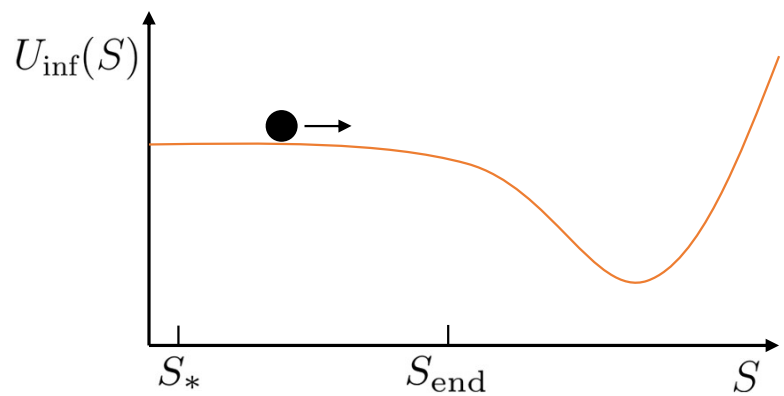
The horizon and flatness problems may be resolved by an early phase of exponential expansion.

$$H = \sqrt{\frac{U}{3M_{\rm Pl}^2}} \approx \text{const.} \qquad \Delta a \sim \exp\left(\int_{t_*}^{t_{\rm end}} {\rm d}t H\right) = e^{N_e} \qquad N_e \overset{!}{\approx} 50 - 60$$

A scalar (inflaton) with a sufficiently flat potential that rolls to a true vacuum achieves this nicely.

$$N_e = \frac{1}{M_{\rm Pl}^2} \int_{S_*}^{S_{\rm end}} {\rm d}S \frac{U_{\rm inf}}{U'_{\rm inf}}$$

$*$ CMB horizon exit



$$\epsilon = \frac{M_{\rm Pl}^2}{2} \left(\frac{U'_{\rm inf}}{U_{\rm inf}}\right)^2 \Bigg|_{S=S_*} \qquad \eta = M_{\rm Pl}^2 \left(\frac{U''_{\rm inf}}{U_{\rm inf}}\right) \Bigg|_{S=S_*}$$

Slow-roll parameters determine the observable scalar spectral index and tensor-to-scalar ratio.

$$n_s = 1 - 6\epsilon + 2\eta \qquad r = 16\epsilon$$

$n_s \lesssim 1$ hints at scale invariance, so let us marry these two concepts!

The model

Consider a Jordan frame action that couples QG to an inflaton with quartic self-interaction.

$$S_{\rm J} = \int {\rm d}^4x\sqrt{-g}\left(\frac{1}{2}\nabla_\mu S\nabla^\mu S - \frac{\beta}{2}S^2R - \frac{\lambda}{4}S^4 + \gamma R^2 - \kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right)$$

Expand around classical backgrounds and York-decompose the graviton DOFs:

$$S \to S_{\rm cl} + \hat{S} \qquad h_{\mu\nu} = \tilde{h}_{\mu\nu} + \partial_\mu V_\nu + \partial_\nu V_\mu + \left(\partial_\mu\partial_\nu - \frac{1}{4}\eta_{\mu\nu}\Box\right)a + \frac{1}{4}\eta_{\mu\nu}h_\rho{}^\rho$$

$$g_{\mu\nu} \to \eta_{\mu\nu} + h_{\mu\nu} \qquad \phi = h_\mu{}^\mu - \Box\, a$$

The vector and one scalar modes drop out from the free action after integrating by parts.

$$S_{\rm J}^{\rm (free)} = \int {\rm d}^4x\left[\frac{1}{2}\hat{S}\left(\Box - m_S^2\right)\hat{S} + \frac{9\gamma}{16}\phi\,\Box\left(\Box - m_\phi^2\right)\phi - \hat{S}\left(\frac{3}{4}\beta S_{\rm cl}\Box\right)\phi - \frac{\kappa}{2}\delta_{\mu\nu\rho\sigma}\tilde{h}^{\mu\nu}\Box\left(\Box - m_{\rm gh}^2\right)\tilde{h}^{\rho\sigma}\right]$$

$$m_\phi^2 = \frac{\beta}{12\gamma}S_{\rm cl}^2$$

$$m_S^2 = 3\lambda S_{\rm cl}^2$$

$$m_{\rm gh}^2 = \frac{\beta}{4\kappa}S_{\rm cl}^2$$

Effective potential

The one-loop effective potential is calculated from the Hessian matrix of the free action.

$$U_{\text{1-loop}}(S) = -\frac{i}{2}\Big(\mathrm{Tr}\big[\ln(\Box - m_+^2)\big] + \mathrm{Tr}\big[\ln(\Box - m_-^2)\big] + \mathrm{Tr}\big[\ln\big(\delta_{\mu\nu\rho\sigma}(\Box - m_{\text{gh}}^2)\big)\big]\Big) + \cdots$$

$$m_\pm^2 = \frac{1}{2}\left(m_S^2 + (1+6\beta)m_\phi^2\right) \pm \frac{1}{2}\sqrt{\left(m_S^2 + (1+6\beta)m_\phi^2\right)^2 - 4m_S^2 m_\phi^2}$$

The traces may be rewritten in momentum space and evaluated using dim. reg. under $\overline{\text{MS}}$.

$$U_{\text{scal}}(S) = \sum_{j=\pm} \frac{1}{64\pi^2} m_j^4 \left[\ln\left(\frac{m_j^2}{\mu^2}\right) - \frac{3}{2}\right] \qquad U_{\text{gh}}(S) = \frac{5}{64\pi^2} m_{\text{gh}}^4 \left[\ln\left(\frac{m_{\text{gh}}^2}{\mu^2}\right) - \frac{1}{10}\right]$$

renorm. scale

$$U_{\text{eff}}(S) = U_{\text{scal}}(S) + U_{\text{gh}}(S) + \frac{\lambda}{4}S^4 + U_0$$

Using the mass definitions, the potential may then be rewritten in terms of a single log.

$$U_{\text{eff}}(S) = \left[C_1 + C_2 \ln\left(\frac{S^2}{\mu^2}\right)\right] S^4 + U_0$$

Dynamical scale generation

Including the CW contributions gives us the one-loop Jordan frame action.

$$S_J^{(\mathrm{eff})} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}S\Box S - \frac{\beta}{2}S^2R + \gamma R^2 - \kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - U_{\mathrm{eff}}(S)\right)$$

This potential has a non-zero minimum, indicating spontaneous breakdown of scale symmetry.

$$\left.\frac{\partial U_{\mathrm{eff}}(S)}{\partial S}\right|_{S=v_S} = 0 \qquad\qquad v_S = \mu\exp\left(-\frac{1}{4} - \frac{C_1}{2C_2}\right)$$

We may define the zero-point energy and identify the Planck scale in terms of this VEV.

$$U_{\mathrm{eff}}(v_S) = 0 \quad \Longrightarrow \quad U_0 = \frac{\mu^4}{2}C_2\exp\left(-1 - \frac{2C_1}{C_2}\right)$$

$$\left.-\frac{1}{2}\beta S^2R\right|_{S=v_S} = -\frac{1}{2}M_{\mathrm{Pl}}^2R \quad \Longrightarrow \quad M_{\mathrm{Pl}}^2 = \beta v_S^2$$

Inflationary predictions

[2012.09706]
[2012.11608]

The inflationary potential is identified after eliminating R^2 with an auxiliary field and Weyl-rescaling to the Einstein frame.

$$S_E^{(\mathrm{inf})} = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2}F(S)^2 S\Box S - \frac{1}{2}M_{\mathrm{Pl}}^2 R - \kappa C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} - U_{\mathrm{inf}}(S,\phi)\right)$$

We can exploit the valley structure to eliminate the scalaron and compute slow-roll parameters.

$$m_\phi^2 \gg H^2, \quad \left.\frac{\partial U_{\mathrm{inf}}(S,\phi)}{\partial\phi}\right|_{\phi=\phi(S)} = 0 \quad \Rightarrow \quad U_{\mathrm{inf}}(S,\phi(S)) = U_{\mathrm{inf}}(S) = \frac{M_{\mathrm{Pl}}^4\, U_{\mathrm{eff}}(S)}{\beta^2 S^4 + 16\gamma\, U_{\mathrm{eff}}(S)}$$

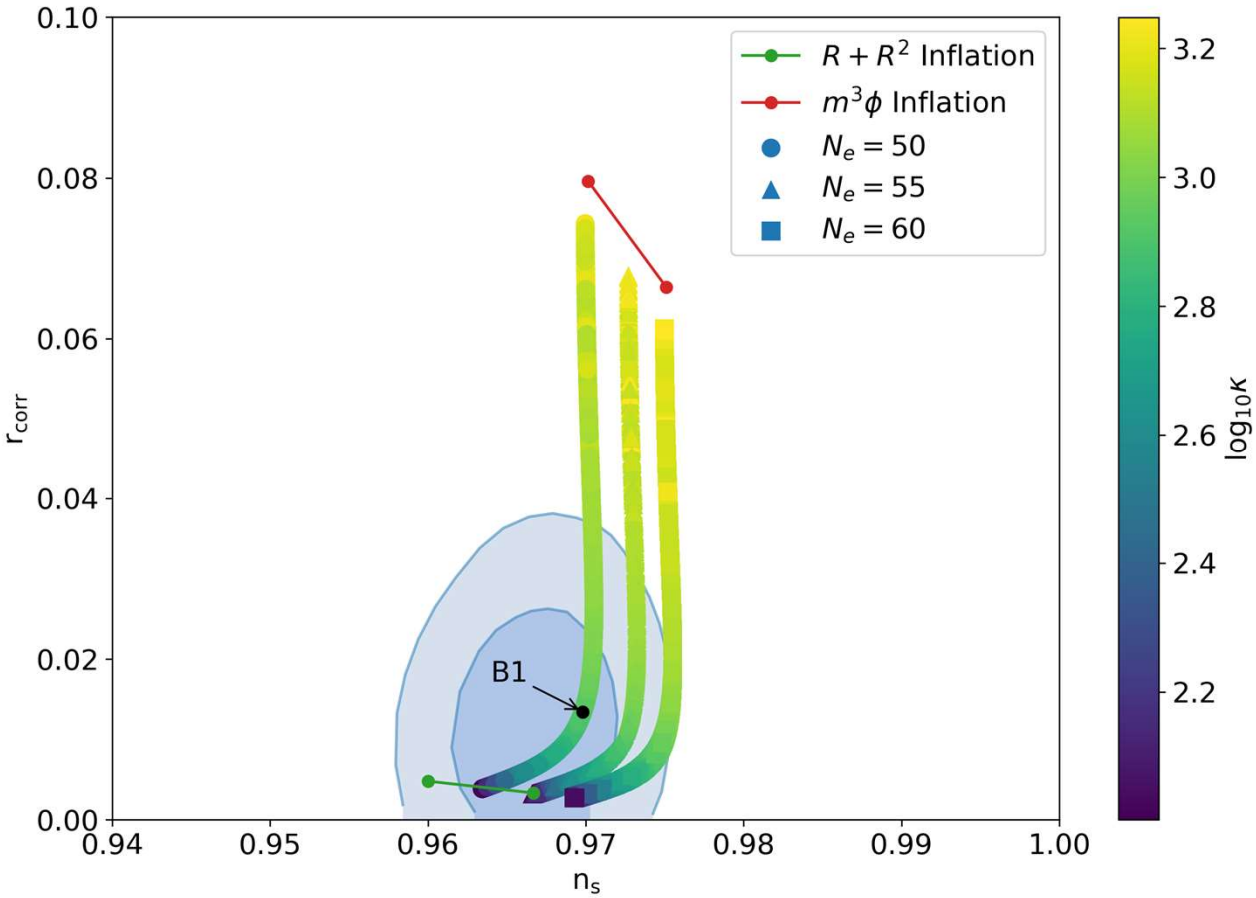
We also account for corrections to r that come from the Weyl curvature term.

$$r_{\mathrm{corr}} = r\left(1 + \frac{2H^2}{m_{\mathrm{gh}}^2}\right)^{-1} \simeq r\left(1 + \frac{2U_{\mathrm{inf}}(S_*)}{3M_{\mathrm{Pl}}^2 m_{\mathrm{gh}}^2(S_*)}\right)^{-1}$$

Inflationary predictions

[1807.06211]

Predicted scalar spectral index and tensor-to-scalar ratio for varying e-folds and Weyl coupling constant. Results sit nicely between Starobinksy (green) and linear inflation (red).



Parameter space restricted by Planck scalar power spectrum amplitude.

$$\ln\left(10^{10} A_s\right) = 3.044 \pm 0.014$$

$$A_s = \frac{U_{\rm inf}}{24\pi^2 \epsilon M_{\rm Pl}^4}$$

Benchmark point (B1) corresponds to

$$\lambda = 0.005 \qquad \kappa = 837$$

$$\beta = 5.62 \times 10^2 \qquad \gamma = 1.22 \times 10^8$$

where the mass values are given by

$$m_\phi^{\rm B1}(S = v_S) \simeq 6.35 \times 10^{13}\,{\rm GeV}$$

$$m_{\rm gh}^{\rm B1}(S = v_S) \simeq 4.21 \times 10^{16}\,{\rm GeV}$$

Summary

- Quadratic gravity is theoretically appealing because, and in spite of, fourth-order metric derivatives.
  - The ghost problem needs addressing, but there are a few promising solutions.
  - Spin-2 ghosts only cause issues at high energies and should not be ignored in general.

- Ghost contributions to the CW potential have important effects.
  - When paired with a scalar, we encounter spontaneous breaking of scale symmetry.
  - This breaking dynamically generates the Planck scale and a nice inflationary potential.

- We achieve inflationary predictions that sit perfectly inside experimental constraints with a very minimal field content.

Thank you for your attention!

Additional details

One-loop CW potential:

$$U_{(\text{1-loop})}(S) = -\frac{i}{2}\ln\left[\text{Det}\left(\frac{\delta^2 S_{\text{T}}^{(\text{quad})}}{\delta\psi\delta\psi}\right)\right] = -\frac{i}{2}\ln(\text{Det}M) - \frac{i}{2}\,\text{Tr}\big[\ln\big(\delta_{\mu\nu\rho\sigma}\Box\left(-\Box + m_{\text{gh}}^2\right)\big)\big]$$

$$M = \begin{pmatrix} \frac{9\gamma}{8}\left(\Box^2 - m_\phi^2\Box\right) & -\frac{3}{4}\beta S\Box \\ -\frac{3}{4}\beta S\Box & \Box - m_S^2 \end{pmatrix} \qquad \ln(\text{Det}M) = \text{Tr}\big[\ln\big(\Box - m_+^2\big)\big] + \text{Tr}\big[\ln\big(\Box - m_-^2\big)\big] + \cdots$$

$$\tilde{h}^{\mu\nu}\delta_{\mu\nu\rho\sigma}\tilde{h}^{\rho\sigma} = \tilde{h}^{\mu\nu}P^{(2)}_{\mu\nu\rho\sigma}\tilde{h}^{\rho\sigma} \qquad \text{Tr}\big(P^{(2)}_{\mu\nu\rho\sigma}\big) = \delta^{\mu\nu\rho\sigma}P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}(d+1)(d-2)$$

$$P^{(2)}_{\mu\nu\rho\sigma} = \frac{1}{2}\big(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}\big) - \frac{1}{d-1}\theta_{\mu\nu}\theta_{\rho\sigma} \qquad \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

Einstein frame:

$$F(S) = \frac{1}{(1+4A)B}\left[(1+4A)B + \frac{3}{2}M_{\text{Pl}}^2\Big((1+4A)B' + 4A'B\Big)^2\right]^{1/2} \qquad A(S) = \frac{4\gamma\, U_{\text{inf}}(S)}{B(S)^2 M_{\text{Pl}}^4} \qquad B(S) = \frac{\beta S^2}{M_{\text{Pl}}^2}$$

More plots

Predicted inflationary observables for varying κ and γ .