Quantum cosmology, eternal inflation and swampland conjectures

GF, A.Vilenkin arxiv:2302.04962 (JCAP)

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Quantum cosmology

- *Did our universe have a beginning?*
- Inflationary spacetimes are geodesically past-incomplete!
- *How did the universe come to be and what was its initial state?*
- Quantum cosmology $\rightarrow$ Creation of the universe from "nothing"
- The initial state is determined by the "wavefunction of the universe"

Quantum cosmology

- Solution to the WDW equation: $\hat{H}\Psi = 0$ with appropriate boundary conditions

- Developed approaches: Hartle-Hawking "No boundary" state and Vilenkin's "tunneling" wavefunction

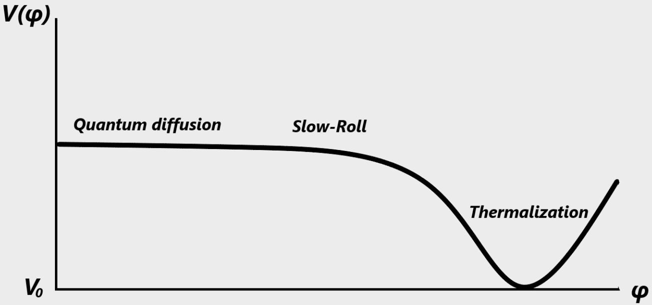
- Slow-roll conditions $|V'|/V \ll 1, \quad |V''|/V \ll 1$

- "No boundary" "Tunneling"

$$P_{HH} \propto e^{+\frac{2}{3V(\phi)}} \qquad P_T \propto e^{-\frac{2}{3V(\phi)}}$$

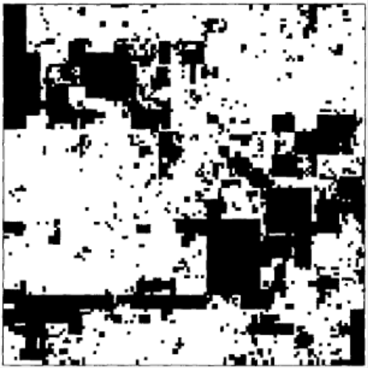
Eternal inflation

- A natural consequence of successful inflationary scenarios is self-reproduction



Eternal inflation

- Spacetime is a self similar fractal: Highly inhomogeneous on superhorizon scales



Aryal, Vilenkin (1987)

Swampland conjectures

- A class of low-energy EFT are not UV complete in string theory $\rightarrow$ swampland
- Refined swampland conjectures:

$$|V'| \geq cV, \quad \text{or} \quad V'' \leq -\tilde{c}V$$

- In stark contradiction with slow-roll inflation!
- Motivation: consider quantum cosmology away from the slow-roll regime

The model

- Closed FRW universe with minimally coupled scalar field ϕ:

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{2} - \frac{1}{2}(\nabla\phi)^2 - V(\phi) \right)$$

- Hilltop potential $V(\phi)$:

$$V(\phi) = 3H^2 \left(1 - \frac{1}{6}\mu^2\phi^2\right) \quad , \quad \mu = \frac{m}{H}$$

- Compact metric with scale factor $a = a(t)$:

$$ds^2 = -dt^2 + a(t)^2 d\Omega_3^2$$

Nucleation probability

- To lowest order the nucleation probability is given by the instanton:

$$P_{nuc} \propto e^{-|S_E|}$$

- For slow-roll conditions, flat hilltop $\mu \ll 1 \rightarrow$ homogeneous deSitter instanton $\phi = 0$

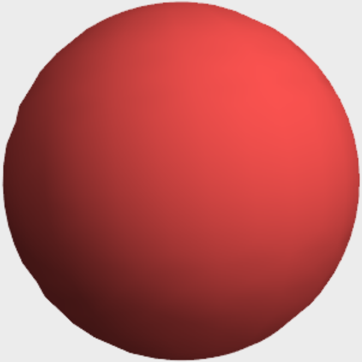
$$S_E^{(dS)} = -\frac{8\pi^2}{H^2}$$

- As the hilltop steepens $\mu \gtrsim 2 \rightarrow$ domain wall instanton $\phi \approx A \sin \zeta$

$$|S_E^{(wall)}| < |S_E^{(dS)}|$$

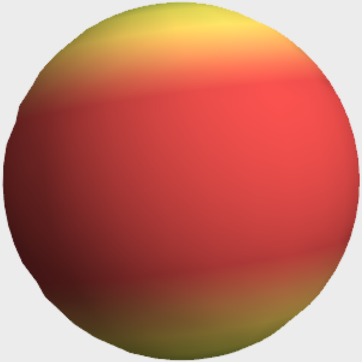
deSitter instanton

- Homogeneous deSitter for $\mu \ll 1$



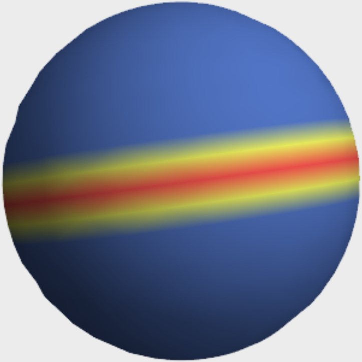
Thick domain wall instanton

- Appearance of domain wall for $\mu \approx 2$



Thin domain wall instanton

- Thin wall for $\mu \gg 1$



Quantization

- Expand scalar field ϕ in inhomogeneous modes ϕ_n:

$$\phi = (2\pi^2)^{1/2} \sum_{n,l,m} \phi_{nlm}(t) Q^n_{lm}(\mathbf{x})$$

- Quantize the geometry and matter fields:

$$\Pi_a \to -i\frac{\partial}{\partial a},\ \Pi_{\phi_n} \to -i\frac{\partial}{\partial \phi_n}$$

- Impose Hamiltonian constraint:

$$\hat{H}\Psi = 0$$

Wheeler-De Witt equation

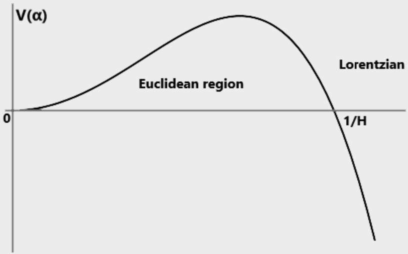
- Wheeler-DeWitt equation:

$$\left[\frac{1}{24\pi^2}\frac{\partial^2}{\partial a^2} - 6\pi^2 V(a) - \sum_n \mathcal{H}_n\right]\Psi(a,\phi_n) = 0$$

$$V(a) = a^2\left(1 - H^2 a^2\right)$$

$$\mathcal{H}_n = \frac{1}{2a^2}\frac{\partial^2}{\partial \phi_n^2} - \frac{1}{2a^2}\omega_n^2\phi_n^2$$

$$\omega_n^2 = (n^2-1)a^4 - m^2 a^6$$



Tunneling boundary conditions

- General solution in the ansatz:

$$\Psi \propto \exp\left[-12\pi^2 S_0(a) - \tfrac{1}{2} R_n(a)\phi_n^2\right]$$

- Boundary condition: Expanding universe at $Ha \gg 1$:

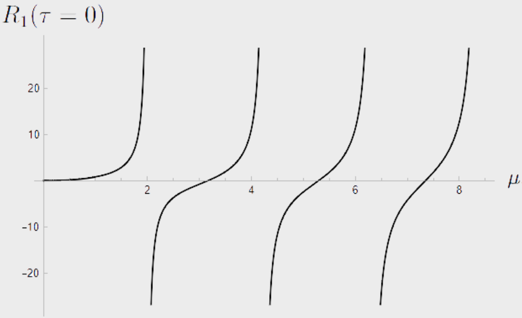
$$\Pi_a \Psi \approx -i\tfrac{\partial \Psi}{\partial a} \rightarrow \ldots \rightarrow \dot{a} \approx +Ha$$

- Regularity condition: Scalar field fluctuations are suppressed:

$$R_n^{\pm}(a) \geq 0\ , \quad Re\{R_n(a)\} \geq 0.$$

Violation of Regularity

- Homogeneous mode becomes unstable for $\mu > 2$:



- Higher order modes interfere with pockets of regularity

Fokker-Planck approximation

- Dynamics of stochastic eternal inflation for $\mu \ll 1$:

$$\frac{\partial \rho}{\partial t} = \frac{H^3}{8\pi^2}\frac{\partial^2 \rho}{\partial \phi^2} + \frac{1}{3H}\frac{\partial}{\partial \phi}\left(\rho \frac{\partial V}{\partial \phi}\right)$$

- Diffusion equation for the probability distribution $\rho(\phi, t)$
- Random walk due to quantum fluctuations: $\frac{H^3}{8\pi^2}\frac{\partial^2 \rho}{\partial \phi^2}$
- Classical drift due to the slope of the potential: $\frac{1}{3H}\frac{\partial}{\partial \phi}\left(\rho \frac{\partial V}{\partial \phi}\right)$

Volume of EI region

- Quantum fluctuations dominate for $|\phi| \ll \phi_q \equiv H^2/m$
- Volume of eternally inflating region:

$$\mathcal{V}_{EIR}(t) \sim \mathcal{V}_0 \exp(3Ht) \int_{-\kappa\phi_q}^{\kappa\phi_q} \rho(\phi, t) d\phi$$

- EIR is a self similar fractal of dimension:

$$\mathcal{V}_{EIR}(t) \propto \exp(dHt) \quad , \quad d = 3 - \mu^2/3$$

Quantum cosmology

- Wavefunction yields probability distribution for ϕ_n at constant a:

$$dP(\phi_n, t) = \prod_n d\phi_n \rho_n(\phi_n)$$

- Gaussians with mode variances given by:

$$\sigma_n^2(t) = \frac{1}{2Re(R_n(t))}$$

- Total field variance:

$$\sigma^2 = \langle \phi^2 \rangle = \sum_n n^2 \sigma_n^2$$

- EIR is a fractal with dimension:

$$\mathcal{V}_{EIR} \propto e^{(3-\gamma)Ht} \equiv e^{dHt} \quad , \quad d = 3 - \gamma = \frac{9}{2} - \left(\frac{9}{4} + \mu^2\right)^{1/2}$$

Conclusions

- Determined the initial state of a spherical universe without slow-roll conditions
- For $\mu < 2$ stochastic eternal inflation of $d > 2$
- For $\mu > 2$ formation of domain wall
- The two instantons are a continuation of eachother!
- Future work: explore the evolution of a domain wall universe

Thank you!