Probing Minimal Grand Unification through Gravitational Waves

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Outline

- Yukawa Sector of Minimal $SO(10)$ Unification
- Symmetry breaking and Gravitational Waves
- Proton decay and Coupling Unification

Searching for a Minimal Yukawa Sector

- Most constructions: complex 10_H Babu, Mohapatra 1992

→ Reducing number of parameters: $SO(10) \times U(1)_{PQ}$

❊ Our proposal: only $SO(10)$ gauge symmetry Babu, Bajc, Saad 2016

→ No new fermions beyond the three families of chiral 16s.

→ Non-supersymmetric framework.

Proposal: Minimal Yukawa Sector

- Fermion bilinear: $16 \times 16 = 10_s + 120_a + 126_s$
- 10 and 120 are real representations of $SO(10)$
- 126 is complex representation of $SO(10)$
- The most general Yukawa sector

$$\mathcal{L}_{yuk} = 16_F (Y_{10}^i 10_H^i + Y_{120}^j 120_H^j + Y_{126}^k \overline{126}_H^k) 16_F$$

$i = 1, 2, ..n_{10}$, $j = 1, 2, ..n_{120}$ and $k = 1, 2, ..n_{126}$

※ $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$ Babu, Bajc, Saad 2016

Fermion masses

$$M_U = \overbrace{D}^{10_H} + \overbrace{S}^{\overline{126}_H} + \overbrace{A}^{120_H}$$
$$M_D = D + r_1 S + e^{i\phi} A$$
$$M_E = D - 3 r_1 S + r_2 A$$
$$M_{\nu_D} = D - 3S + r_2^* e^{i\phi} A$$
$$M_{\nu_R} = \underbrace{c_R S}_{\mathrm{diag}(M_1, M_2, M_3)}$$

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}.$$

Babu, Bajc, Saad 2016

Fermion Fit

Observables (Δm^2_{ij} in eV^2)	Values at M_Z scale Input	Best Fit: NO	Best Fit: IO	Values at M_{GUT} scale NO	IO
$y_u/10^{-6}$	6.65±2.25	6.65	6.71	2.85	2.87
$y_c/10^{-3}$	3.60±0.11	3.60	3.60	1.54	1.54
y_t	0.986±0.0086	0.986	0.986	0.48	0.48
$y_d/10^{-5}$	1.645±0.165	1.645	1.675	0.73	0.74
$y_s/10^{-4}$	3.125±0.165	3.125	3.146	1.38	1.39
$y_b/10^{-2}$	1.639±0.015	1.639	1.639	0.637	0.637
$y_e/10^{-6}$	2.7947±0.02794	2.7947	2.7899	2.8873	2.8817
$y_\mu/10^{-4}$	5.8998±0.05899	5.8998	5.9021	5.924	5.894
$y_\tau/10^{-2}$	1.0029±0.01002	1.0029	1.0012	0.985	0.989
$\theta_{12}^{\mathrm{CKM}}/10^{-2}$	22.735±0.072	22.735	22.739	22.73	22.74
$\theta_{23}^{\mathrm{CKM}}/10^{-2}$	4.208±0.064	4.208	4.204	4.79	4.79
$\theta_{13}^{\mathrm{CKM}}/10^{-3}$	3.64±0.13	3.64	3.64	4.15	4.15
δ^{CKM}	1.208±0.054	1.208	1.204	1.207	1.204
$\Delta m^2_{21}/10^{-5}$	7.425±0.205	7.425	7.433	9.714	950.84
$\Delta m^2_{31}/10^{-3}$ (NO)	2.515±0.028	2.515	-	12.909	-
$\Delta m^2_{32}/10^{-3}$ (IO)	-2.498±0.028	-	-2.497	-	-12.515
$\sin^2\theta_{12}$	0.3045±0.0125	0.3045	0.3053	0.308	0.177
$\sin^2\theta_{23}$ (NO)*	0.5705±0.0205	0.5726	-	0.484	-
$\sin^2\theta_{23}$ (IO)*	0.576±0.019	-	0.5819	-	0.542
$\sin^2\theta_{13}$ (NO)	0.02223±0.00065	0.02223	-	0.007	-
$\sin^2\theta_{13}$ (IO)	0.02239±0.00063	-	0.02238	-	0.0223
χ^2	-	3×10^{-8}	$2.77^\dagger$	-	-

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Features: M_R

Quantity	Best fit prediction	
	NO	IO
(m_1, m_2, m_3)	$(0.00014, 0.0086, 0.0501)$ eV	$(0.04922, 0.04997, 0.00038)$ eV
$(\sum_i m_i, m_\beta, m_{\beta\beta})$	$(0.0589, 0.0088, 0.0016)$ eV	$(0.099, 0.041, 0.033)$ eV
$(\delta, \varphi_1, \varphi_2)$	$(326.4, 109.0, 94.7)^\circ$	$(209.5, 164.4, 343.4)^\circ$
(M_1, M_2, M_3)	$(2.13 \times 10^5, 6.46 \times 10^{11}, 2.28 \times 10^{14})$ GeV	$(1.31 \times 10^4, 6.42 \times 10^{11}, 2.37 \times 10^{14})$ GeV

- $M_3 \sim 10^{14}$ GeV (seesaw scale)
- $M_2 \sim \frac{m_c}{m_t} M_3 \sim 10^{11}$ GeV
- $M_{2,3} \gg M_1 \sim 10^5$ GeV
- $M_1^{\mathrm{IO}} \sim M^{\mathrm{NO}}/10$

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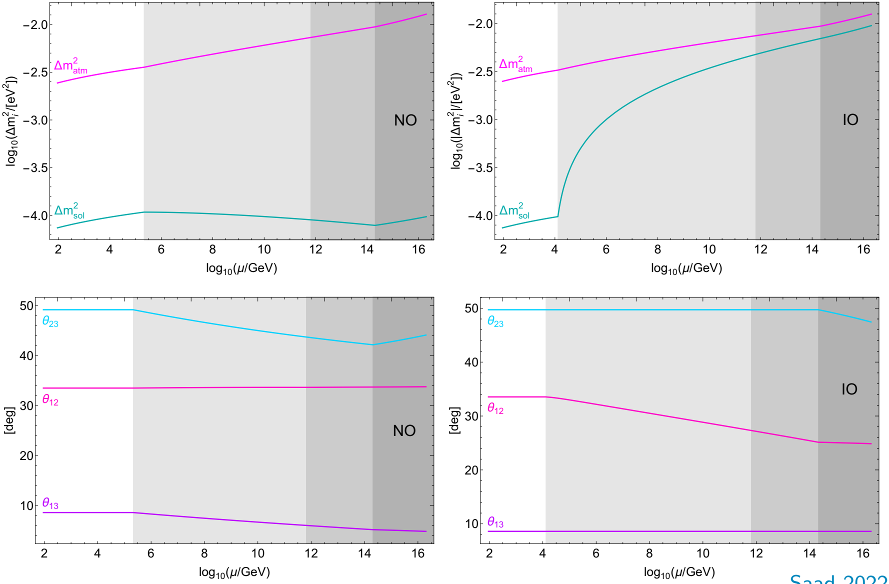
Importance of RGE running



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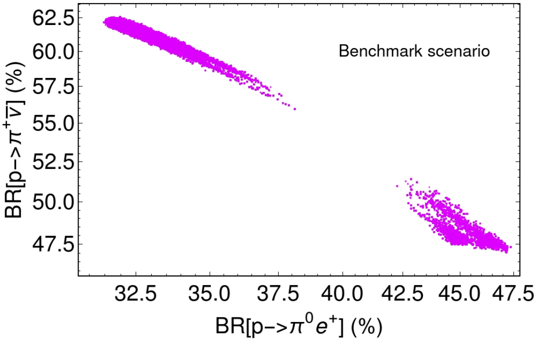
Features: Proton decay correlations

p **decay modes**	**BR[%] predictions**
$(p \to e^+\pi^0, p \to e^+K^0, p \to e^+\eta)$	$(32.4, 0.52, 0.03)$
$(p \to \mu^+\pi^0, p \to \mu^+K^0, p \to \mu^+\eta)$	$(0.42, 4.3, 0.002)$
$(p \to \overline{\nu}\pi^+, p \to \overline{\nu}K^+)$	$(61.5, 0.81)$



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Pulsar timing data

- NANOGrav : 12.5 yrs of pulsar timing data 2020

→ strong evidence for a stochastic common-spectrum process

→ Interpreted as a GW signal: $A \sim \mathcal{O}(10^{-15})$@ $f \sim 1/yr$

★ Cosmic string model gives an excellent fit Ellis, Lewicki 2020

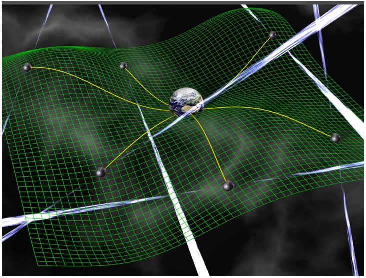
→ $G\mu = (2 \times 10^{-11} - 3 \times 10^{-10})$@ 95% C.L.

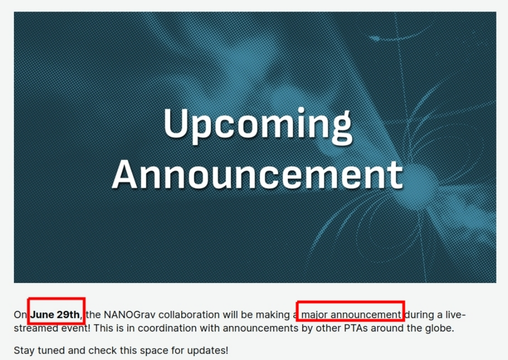
❄ Similar hints from PPTA 2021, EPTA 2021, and IPTA 2022

❄ GW energy density Fu et. al. 2022

$$\left(\Omega_{\rm GW}(f)h^2\right)_{\rm PTA} \approx 2.02 \cdot 10^{-10} \left(\frac{A}{10^{-15}}\right)^2 \times \left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma}$$

New results upcoming





Higgs sector

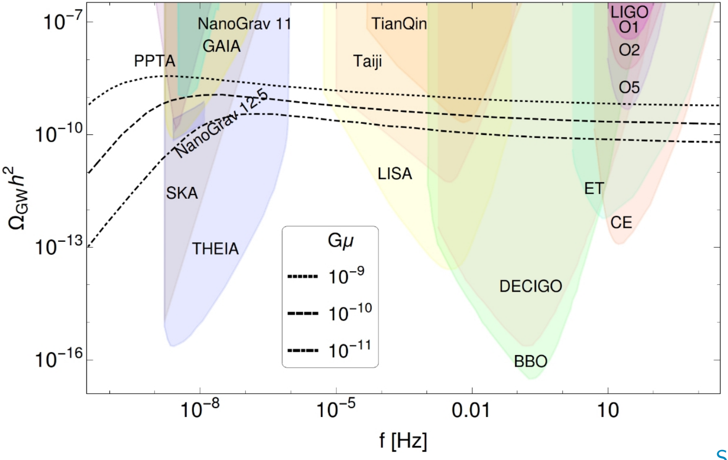
- Minimal scenario to provide cosmic string network

$$
\begin{aligned}
SO(10) &\xrightarrow[54_H]{M_X} SU(4)_C \times SU(2)_L \times SU(2)_R \times D \\
&\xrightarrow[45_H]{M_I} \boxed{SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}} \\
&\xrightarrow[126_H]{M_{II}} \boxed{SU(3)_C \times SU(2)_L \times U(1)_Y} \\
&\xrightarrow[10_H + 126_H]{M_{EW}} SU(3)_C \times U(1)_{em}
\end{aligned}
$$

the breaking by 126_H leaves a remnant Z_2 symmetry (not broken by tensor representations)

GW from stable cosmic string network

$$G\mu = 4.22 \times 10^{-38} v_R^2$$



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String scale and fermion mass fit

- CMB: $G\mu < 1.1 \times 10^{-7}$ Charnock et. al. 2016 $\Rightarrow M_{II} \lesssim 2 \times 10^{15}$ GeV
- LIGO: 2021 $G\mu \lesssim 10^{-8}$ ($f \sim \mathcal{O}(10)$ Hz) $\Rightarrow M_{II} \lesssim 5 \times 10^{14}$ GeV

✲ Neutrino mass fit: $M_3 \lesssim 10^{15}$ GeV

✲ Up-quark mass fit: $M_3 \gtrsim 2 \times 10^{13}$ GeV

✲ Valid: $M_3 = [2 \times 10^{13} - 10^{15}]$ GeV

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String scale and fermion mass fit

- $S = v_{126}^u \underbrace{Y_{126}}_{\text{diagonal}}$
- 10_H: $M_U, M_D \propto \underbrace{D}_{\text{no extra factor}}$
- $m_b \sim D_{33} + r_1 S_{33}$
- $m_t \sim S_{33} \gg D_{33} + r_1 S_{33}$
- $M_3 = v_R (Y_{126})_{33}$

$$v_R = \begin{cases} v_R^{\min} = 0.5 \times M_3 \,, & (Y_{126})_{33}^{max} = 2 \\ v_R^{\max} = 2.05 \times M_3 \,, & (Y_{126})_{33}^{min} = 0.48 \end{cases}$$

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Consistent $SO(10)$ and GW fits

- excellent fit for:

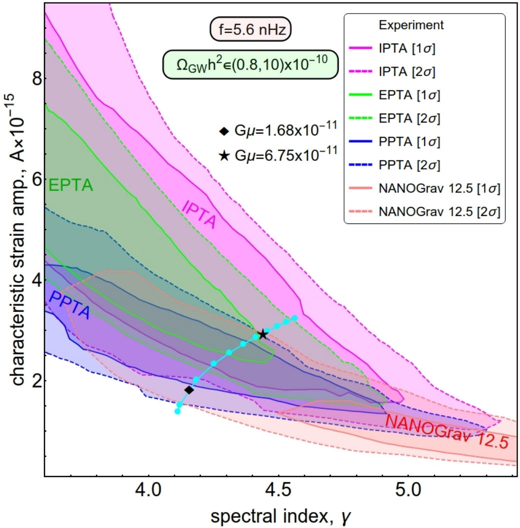
$\rightarrow$ $\Omega_{\rm GW} h^2 \in (2,6) \times 10^{-10}$ @ 2σ CL

- Corresponds to

$\rightarrow$ $G\mu \in (4.9, 6.9) \times 10^{-11}$

- Restricted seesaw scale:

$\rightarrow$ $v_R \in (3.4, 4.1) \times 10^{13}$ GeV



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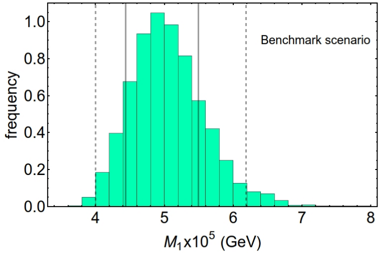
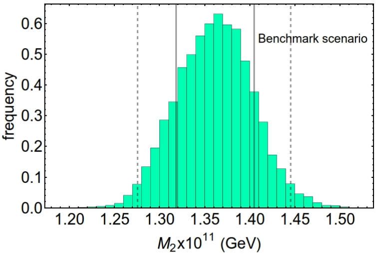
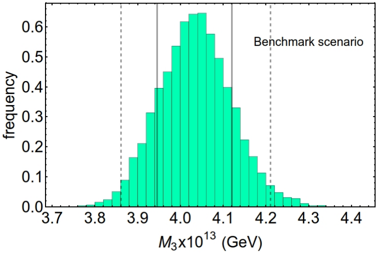
Consistent $SO(10)$ and GW fits







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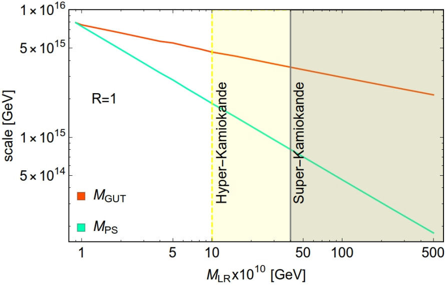
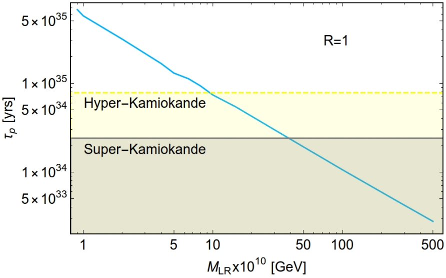
GW from $SO(10)$: consistent with proton decay?





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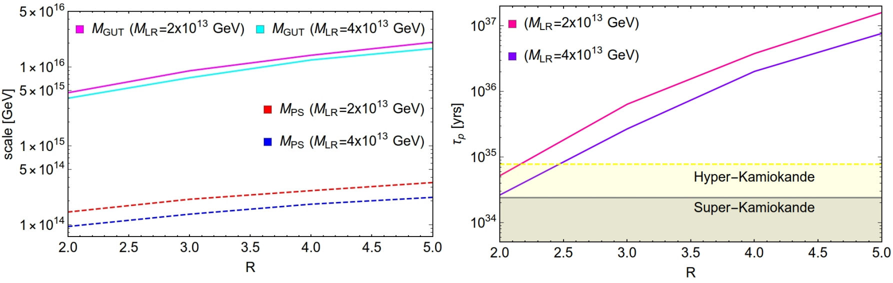
Compatibility with a small threshold correction



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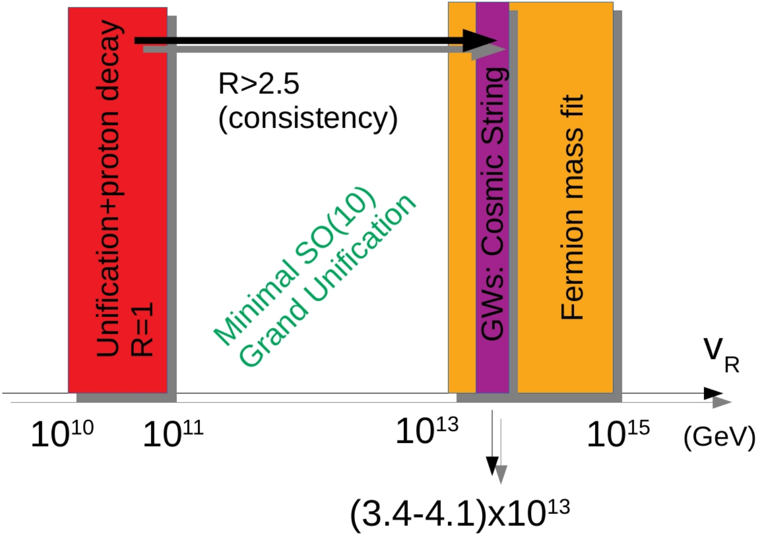
Upshot



Unification+proton decay
R=1
R>2.5
(consistency)
Minimal SO(10)
Grand Unification
GWs: Cosmic String
Fermion mass fit
V_R
10^10
10^11
10^13
10^15
(GeV)
(3.4-4.1)x10^13

Summary

- Minimal Yukawa sector: $\{n_{10}, n_{120}, n_{126}\} = \{1, 1, 1\}$
- Minimal SSB sector (cosmic string): $45_H + 54_H$
- Dominant modes: $p \to \overline{\nu}\pi^+$ and $p \to e^+\pi^0$
- Fermion mass fit: $M_3 = [2 \times 10^{13} - 10^{15}]$ GeV
- $M_2 \sim \frac{m_c}{m_t} M_3 \sim 10^{11}$ GeV
- $M_{2,3} \gg M_1 \sim 10^5$ GeV, $M_1^{\mathrm{IO}} \sim M^{\mathrm{NO}}/10$
- GW/PTAs: $v_R \in (3.4, 4.1) \times 10^{13}$ GeV
- Fully testable in a number of gravitational wave observatories

THANK YOU!

Explaining features

$$\frac{|S|}{v} \sim \begin{pmatrix} 4.5\times 10^{-10} & 0. & 0. \\ 0. & 1.3\times 10^{-3} & 0. \\ 0. & 0. & 4.8\times 10^{-1} \end{pmatrix}$$

$$\frac{|D|}{v} \sim \begin{pmatrix} 3.0\times 10^{-6} & 2.8\times 10^{-5} & 1.7\times 10^{-4} \\ 2.8\times 10^{-5} & 2.4\times 10^{-4} & 2.7\times 10^{-3} \\ 17\times 10^{-4} & 2.7\times 10^{-3} & 2.6\times 10^{-3} \end{pmatrix}$$

$$\frac{|A|}{v} \sim \begin{pmatrix} 0 & 2.3\times 10^{-5} & 1.2\times 10^{-4} \\ 2.3\times 10^{-5} & 0 & 2.5\times 10^{-3} \\ 1.2\times 10^{-4} & 2.5\times 10^{-3} & 0 \end{pmatrix}$$

Explaining features

$$\frac{|S|}{v} \sim \begin{pmatrix} 4.5\times10^{-10} & 0. & 0. \\ 0. & 1.3\times10^{-3} & 0. \\ 0. & 0. & 4.8\times10^{-1} \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix}$$

$$\frac{|D|}{v} \sim \begin{pmatrix} 3.0\times10^{-6} & & \\ & 2.4\times10^{-4} & \\ & & 2.6\times10^{-3} \end{pmatrix} \sim \begin{pmatrix} y_{d,e,u} & 0 & 0 \\ 0 & y_{s,\mu} & 0 \\ 0 & 0 & y_b \end{pmatrix}$$

$$M_U \sim S, \qquad M_{D,E} \sim D, \qquad M_2 \sim \frac{y_c}{y_t} M_3, \qquad M_1 \ll M_{2,3}$$

Explaining features

- At the GUT scale $m_t/m_b \sim 75$
- $y_t \sim S_{33}/v$
- $y_b \sim (D_{33} + r_1 S_{33})/v$
- $(D_{33} + r_1 S_{33})/S_{33} \sim 10^{-2}$
- $D_{33}, r_1 S_{33} \ll S_{33} \sim m_t$
- $M_3 = v_R (Y_{126})_{33}$ determines String scale

Explaining features

$$\frac{|M_{\nu_D}|}{v} \sim (D - 3S)/v$$

$$= \begin{pmatrix} 3.1 \times 10^{-6} & 2.8 \times 10^{-5} & 1.7 \times 10^{-4} \\ 2.8 \times 10^{-5} & & 2.73348 \times 10^{-3} \\ 1.7 \times 10^{-4} & 2.73348 \times 10^{-3} & \end{pmatrix}$$

$$- \begin{pmatrix} 0 & 0 & 0 \\ 0 & 4 \times 10^{-3} & 0 \\ 0 & 0 & 1.4 \end{pmatrix}$$

Explaining features

$$M_N = -M_{\nu_D} \begin{pmatrix} M_1^{-1} & & \\ & M_2^{-1} & \\ & & M_3^{-1} \end{pmatrix} M_{\nu_D}^T$$

$$\sim \begin{pmatrix} \frac{10^{-7}}{M_1} + 10^{-15} & \frac{10^{-7}}{M_1} + 10^{-14} & \frac{10^{-6}}{M_1} + 10^{-13} \\ \frac{10^{-7}}{M_1} + 10^{-14} & \frac{10^{-7}}{M_1} + 10^{-12} & \frac{10^{-6}}{M_1} + 10^{-12} \\ \frac{10^{-6}}{M_1} + 10^{-13} & \frac{10^{-6}}{M_1} + 10^{-12} & \frac{10^{-5}}{M_1} + 10^{-10} \end{pmatrix}$$

Works if $M_1 \sim 10^5$ GeV !
(IO works if first column and row have significant entries:
$M^{\mathrm{IO}} \sim M^{\mathrm{NO}}/10$)