Dispersive determination of Higgs boson mass

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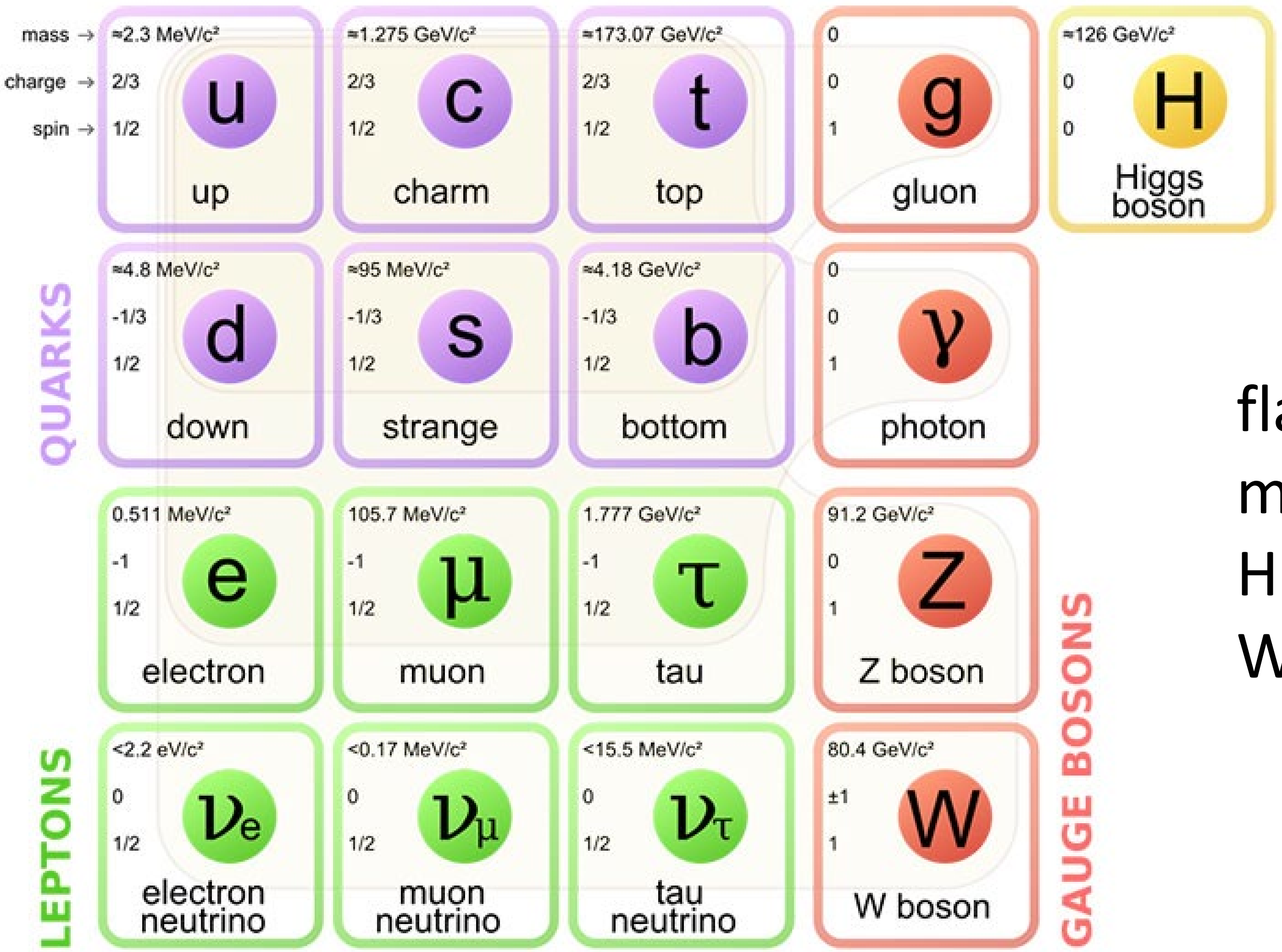
Standard Model



flavor structure,
mixing patterns,
Higgs mass 125 GeV,…
Why?

Goal

- Belief: SM parameters are free
- ChatGPT:

  ……

  In summary, the Standard Model parameters are free in the sense that their values cannot be determined by the theory alone, and experimental measurements play a crucial role in determining their values.
- To explain them, introduce new physics, but…

- Will show that at least some of the SM parameters are not free, but constrained dynamically for internal consistency

QCD sum rules for resonance masses

- Two-current correlator

$$J_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/\sqrt{2}$$

$$\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T[J_\mu(x)J_\nu(0)]|0\rangle$$

$$= (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)$$ ← vacuum polarization function

- Sum rules

unknown spectral density

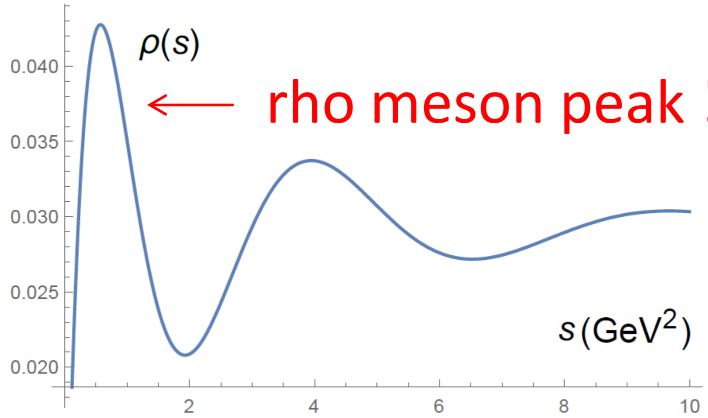
operator-product-expansion input

$$\frac{1}{\pi}\int_0^R ds\frac{\mathrm{Im}\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_0^R ds\frac{\mathrm{Im}\Pi^{\mathrm{pert}}(s)}{s-q^2} + \frac{1}{12\pi}\frac{\langle\alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q\bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa\alpha_s\langle\bar{q}q\rangle^2}{(q^2)^3}$$

insert $|n><n|$ any real state n, created or annihilated by J, contributes

solve sum rule directly as an inverse problem

➡ rho resonance emerges



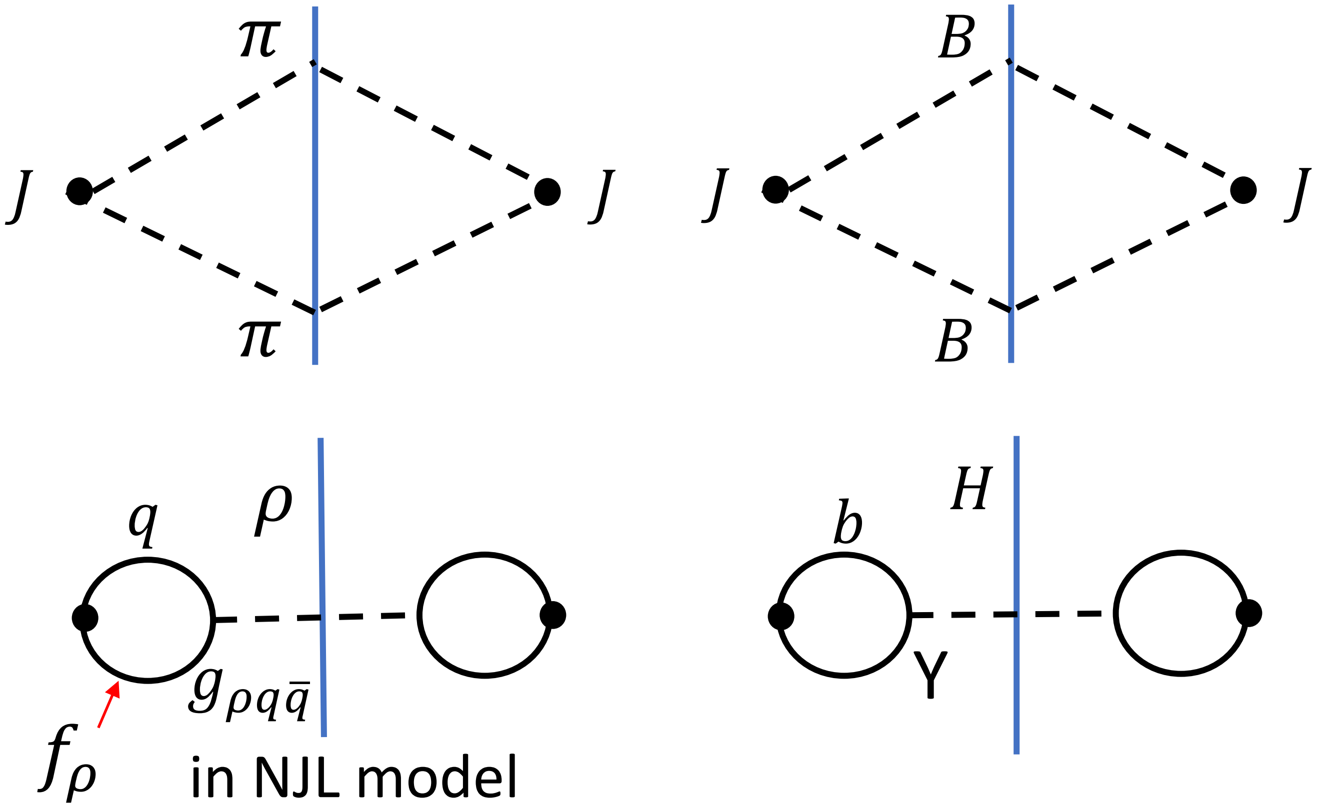
2109.04956 Li

Speculation

- Consider correlator of two b quark scalar currents
- Higgs can be created or annihilated by this current, $H \to b\bar{b}$
- Higgs contributes to spectral function $\mathrm{Im}\Pi(s)$
- Could this correlator reveal Higgs property by solving for spectral function directly?
- If yes, Higgs mass is not free parameter, but determined dynamically, like rho meson mass
- Fine-tuning problem of SM may not be serious…
- c quark currents work too, but many scalars decay into $c\bar{c}$

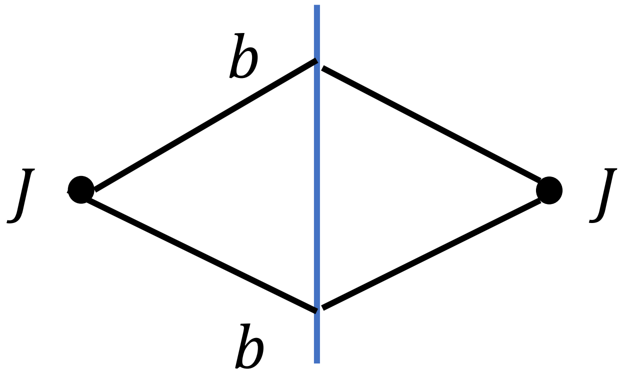
Comparison to rho meson

"hadron" side: Im from real physical particles



Polleri et al, 1997

"quark" side: Im from real b quarks plus power corrections theoretically calculated



these two ways give same result at large q^2
--- sum rules

Dispersion relation

Gorishinii, Kataev, Larin 1984

- Perturbative input

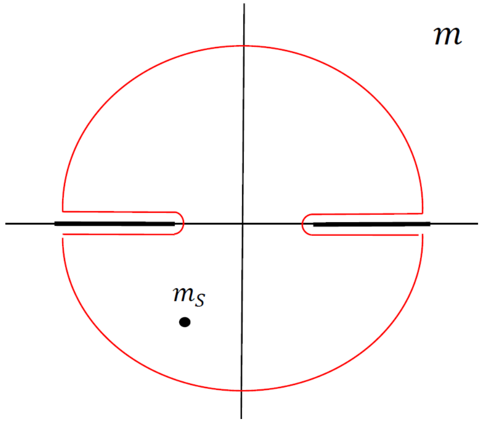
$$\mathrm{Im}\Pi^{\mathrm{p}}(m_S) \propto \frac{(m_S^2 - 4m_b^2)^{3/2}}{m_S}\left[1 + \frac{\alpha_s(\mu)}{\pi} C_F \left(\frac{17}{4} + \frac{3}{2}\ln\frac{\mu^2}{m_S^2}\right)\right]$$

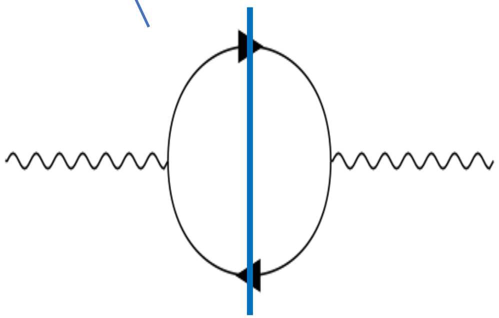
- Starting with correlator, derive

suppress low m singularity

$$\int_{4m_B^2}^{R^2} \frac{m\mathrm{Im}\Pi(m)}{m_S^2 - m^2} dm^2 = \int_{4m_b^2}^{R^2} \frac{m\mathrm{Im}\Pi^{\mathrm{p}}(m)}{m_S^2 - m^2} dm^2$$

analyticity



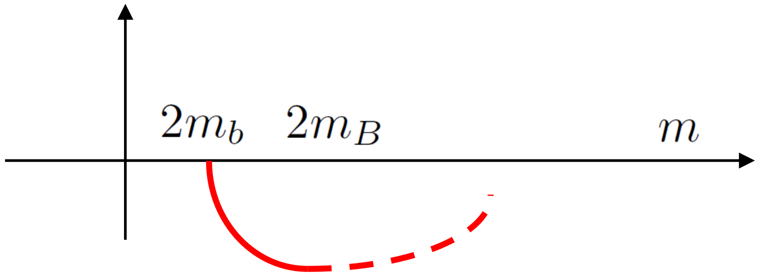


- Move RHS to LHS,

$\Delta\Pi(m)$ is fixed to $-\mathrm{Im}\Pi^{\mathrm{p}}(m)$ in the interval $(2m_b, 2m_B)$

$$\int_{4m_b^2}^{\infty} \frac{\Delta\rho(m)}{(m_S^2 - m^2)} dm^2 = 0,$$

$$\Delta\rho(m) \equiv m\Delta\Pi(m), \quad \Delta\Pi(m) = \mathrm{Im}\Pi(m) - \mathrm{Im}\Pi^{\mathrm{p}}(m)$$



- mb=mB, trivial solution $\mathrm{Im}\Pi(m) = \mathrm{Im}\Pi^{\mathrm{p}}(m)$
- Power corrections (mB-mb)/mb crucial

Polynomial expansion

arbitrary scale ↓

- Introduce dimensionless variables, $m_S^2 - 4m_b^2 = u\Lambda$, $m^2 - 4m_b^2 = v\Lambda$

$$\int_0^\infty dv \frac{\Delta\rho(v)}{u-v} = 0$$

$\Delta\rho(v) \to 0$ at large v, because $\mathrm{Im}\Pi(m) \to \mathrm{Im}\Pi^{\mathrm{p}}(m)$

power series in $1/u$ using $1/(u-v) = \sum_{i=1}^\infty v^{i-1}/u^i$

- Start with case of N vanishing coefficients, N large

contained in $L_0^{(\alpha)}(v), L_1^{(\alpha)}(v), \cdots, L_{N-1}^{(\alpha)}(v)$

$$\int_0^\infty dv v^{i-1} \Delta\rho(v) = 0, \quad i = 1, 2, 3 \cdots, N$$

- Imply expansion in generalized Laguerre polynomials because of orthogonality

weight

$$\Delta\rho(v) = \sum_{j=N}^{N'} a_j v^\alpha e^{-v} L_j^{(\alpha)}(v), \quad N' > N \qquad \int_0^\infty y^\alpha e^{-y} L_m^{(\alpha)}(y) L_n^{(\alpha)}(y) dy = \frac{\Gamma(n+\alpha+1)}{n!}\delta_{mn}$$

↑ fixed by initial condition in principle, needs not be infinite

Solution

- Large j approximation, subject to correction of $1/\sqrt{j}$

$$L_j^{(\alpha)}(v) \approx j^{\alpha/2} v^{-\alpha/2} e^{v/2} J_\alpha(2\sqrt{jv})$$

- Solution in variable m

arbitrary degree and scale appear in ratio

$$\Delta\rho(m) \approx \sum_{j=N}^{N'} a_j \sqrt{\frac{j(m^2-4m_b^2)}{\Lambda}}^{\alpha} e^{-(m^2-4m_b^2)/(2\Lambda)} J_\alpha\left(2\sqrt{\frac{j(m^2-4m_b^2)}{\Lambda}}\right)$$

- Scaling variable $\omega \equiv \sqrt{N/\Lambda}$, large N limit $N'/\Lambda = \omega^2 + (N'-N)/N \approx \omega^2$

$$J_\alpha(2\sqrt{j(m^2-4m_b^2)/\Lambda}) \approx J_\alpha(2\omega\sqrt{m^2-4m_b^2}) \qquad e^{-(m^2-4m_b^2)/(2\Lambda)} = e^{-\omega^2(m^2-4m_b^2)/(2N)}$$

≈ 1

$$\Delta\rho(m) \approx y\left(\omega\sqrt{m^2-4m_b^2}\right)^{\alpha} J_\alpha\left(2\omega\sqrt{m^2-4m_b^2}\right)$$

$\sum_{j=N}^{N'} a_j$

solution in terms of single Bessel function

3 unknowns

Scale invariance

Xiong, Wei, Yu 2022

- Solution to this type of integral (Fredholm) equation, if existing, is unique, given boundary condition.
- It must be insensitive to arbitrary Λ , i.e., to ω from variable change
- To realize this insensitivity, consider

minimal to maximize stability window

$$\Delta\rho(m_S) = \Delta\rho(m_S)|_{\omega=\bar{\omega}} + \frac{d\Delta\rho(m_S)}{d\omega}\Big|_{\omega=\bar{\omega}}(\omega-\bar{\omega}) + \frac{1}{2}\frac{d^2\Delta\rho(m_S)}{d\omega^2}\Big|_{\omega=\bar{\omega}}(\omega-\bar{\omega})^2 + \cdots$$

fit to initial condition to determine $\bar{\omega}$, α y

$d\Delta\rho(m_S)/d\omega|_{\omega=\bar{\omega}} = 0$ discrete roots! stability window exists

- Single root of m_S is allowed ➡ Higgs mass ?
- Both N and Λ can be arbitrarily large, large N approximation justified

Initial condition

- Compare solution with perturbative input in low end $m_S \to 2m_b$

$$-m_S \mathrm{Im}\Pi^{\mathrm{P}}(m_S) \propto (m_S^2 - 4m_b^2)^{3/2}$$

simple power of $m^2 - 4m_b^2$

explain the modified integrand $m^2\Pi(m)$

$$\alpha = 3/2$$

- Boundary value at high end $m_S = 2m_B$

$$y = -2m_B \mathrm{Im}\Pi^{\mathrm{P}}(2m_B)\left[\left(2\omega\sqrt{m_B^2 - m_b^2}\right)^{3/2} J_{3/2}\left(4\omega\sqrt{m_B^2 - m_b^2}\right)\right]^{-1}$$

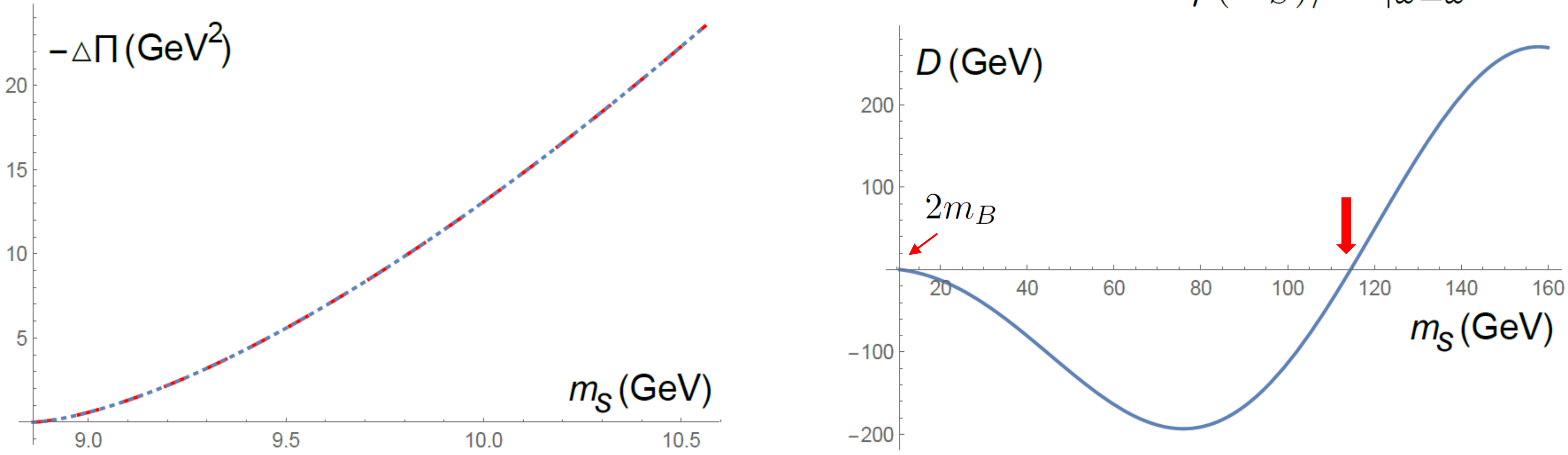
- Best fit to initial condition for $m_b = 4.43$ GeV, $m_B = 5.28$ GeV

$$\bar{\omega} = 0.0254\ \mathrm{GeV}^{-1}$$

Higgs mass

- Excellent match to initial condition

$$d\Delta\rho(m_S)/d\omega|_{\omega=\bar{\omega}} = 0$$



- First root of vanishing derivative with minimal second derivative

$m_S = 114$ GeV 9% deviation from data $m_H = (125.25 \pm 0.17)$ GeV

- Renormalization scale $\mu = m_S/2$ $(\mu = 2m_S)$ gives 126 (112) GeV

Z mass

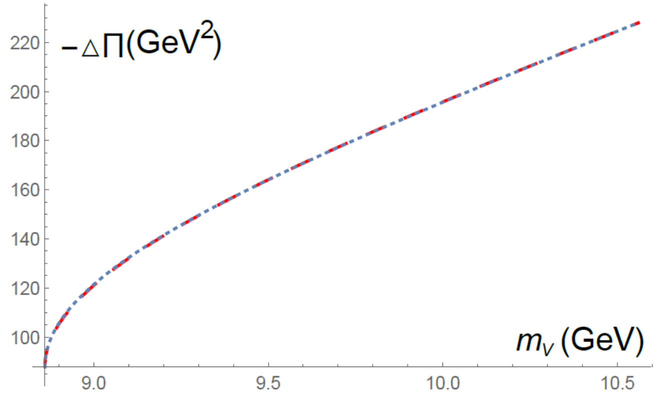
constant couplings

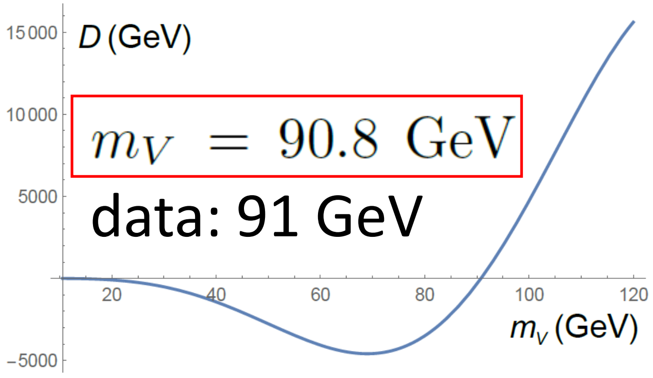
- Z decays into b pair via vertex $\gamma_\mu(v_b + a_b\gamma_5)$
- Vector, axial-vector couplings independent in mathematical viewpoint
- Consider correlator of vector current $J_\mu = \bar{b}\gamma_\mu b$
- Perturbative input

Schwinger 1973

$$\mathrm{Im}\Pi^{\mathrm{p}}(m_V) \propto m_V^2\beta(m_V)[3-\beta^2(m_V)]\left\{1+\frac{4\alpha_s(m_V)}{3}\left[\frac{\pi}{2\beta(m_V)}-\frac{3+\beta(m_V)}{4}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\right]\right\}$$

- Same b quark, B meson masses, $\bar{\omega} = 0.0249\ \mathrm{GeV}^{-1}$





if fixing W mass $m_W = 80.377$ GeV

consider m_V-dependent coupling

$$v_b(m_V) = -1 + \frac{4}{3}\left(1-\frac{m_W^2}{m_V^2}\right)$$

$m_Z = 38$ GeV wrong mass

Conclusion

- Dispersion relations physical observables must obey impose stringent constraints on dynamics at various scales
- Appearance of scaling variable crucial for constructing physical solution with stability
- Particles must take specific values (not arbitrary) for existence of physical solution
- Strong interaction provides necessary power corrections
- Particle masses (including top mass) over large hierarchy, 0.1-100 GeV, and fermion mixing angles understood
- Fine-tuning problem of SM may not be serious

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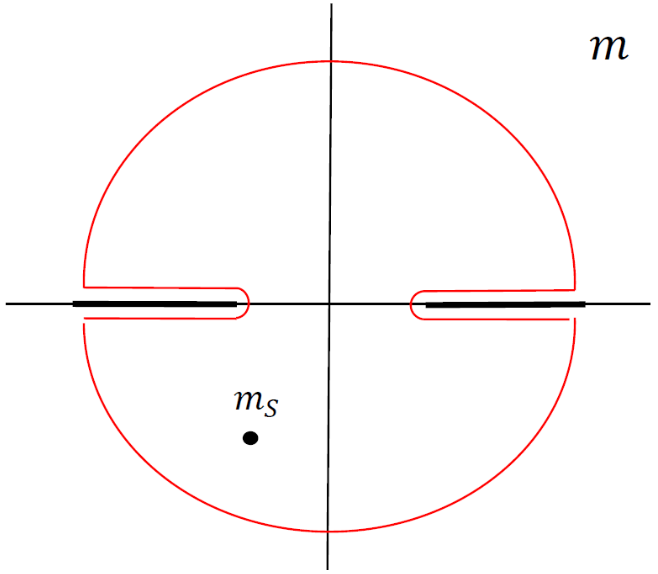
Back-up slides

Framework

- Two-current correlator

scalar current $J = \bar{b}b$

$m_S = \sqrt{q^2}$



$$\Pi(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T[J(x)J(0)]|0\rangle$$

- Contour integration

physical correlator

perturbative correlator

suppress low m singularity

$$\frac{1}{2\pi i}\oint \frac{m^2\Pi(m)}{m-m_S}dm = m_S^2\Pi^{\mathrm{p}}(m_S) = \frac{1}{2\pi i}\oint \frac{m^2\Pi^{\mathrm{p}}(m)}{m-m_S}dm$$

due to analyticity

- Big circle contributions cancel

$$\int_{2m_B}^{R}\frac{m^2\mathrm{Im}\Pi(m)}{m-m_S}dm - \int_{-R}^{-2m_B}\frac{m^2\mathrm{Im}\Pi(m)}{m-m_S}dm$$

$$= \int_{2m_b}^{R}\frac{m^2\mathrm{Im}\Pi^{\mathrm{p}}(m)}{m-m_S}dm - \int_{-R}^{-2m_b}\frac{m^2\mathrm{Im}\Pi^{\mathrm{p}}(m)}{m-m_S}dm.$$

left branch cut

W mass

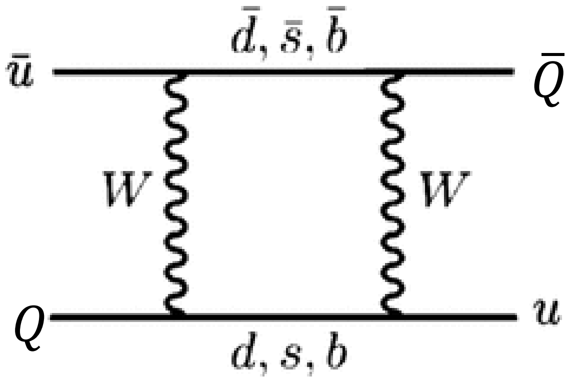
- Z mass in agreement with data can be obtained, only when vector coupling is constant, i.e., when Z and W masses are proportionate
- Conform to Higgs mechanism for electroweak symmetry breaking
- Given the three couplings of SU(3), SU(2), U(1)
- Once Z mass is determined by dispersive relation, W mass is known

$$M_W = M_Z \cos\theta_W \qquad g_2 \sin\theta_W = g_1 \cos\theta_W$$

- Together with predicted Higgs mass, parameters in Higgs potential are also determined

$$V(\varphi) = \mu^2 \varphi^\dagger \varphi + \lambda(\varphi^\dagger \varphi)^2$$

Top mass

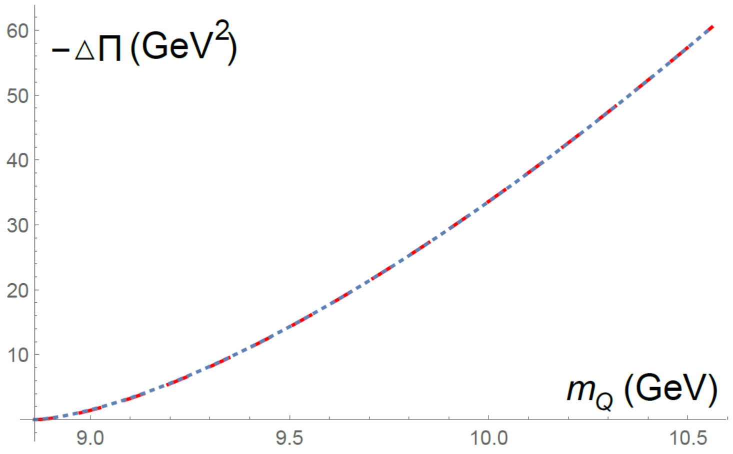


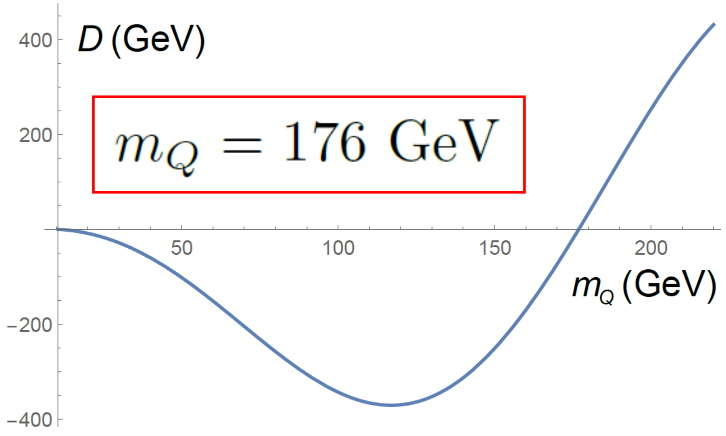
- Inspired by our previous study of neutral meson mixing
- Consider $Q\bar{u}$-$\bar{Q}u$ mixing through box diagrams
- Only $b\bar{b}$ channel (same threshold), (V-A)(V-A) operator
- Perturbative input, solution same as for Higgs case

Wilson coefficient

Cheng 1982; Buras et al 1984

$$\mathrm{Im}\Pi^{\mathrm{P}}(m_Q) \propto C_2(m_Q)\frac{\sqrt{m_Q^2-4m_b^2}}{m_Q(m_W^2-m_b^2)^2}\left[3\left(1+\frac{m_b^4}{4m_W^4}\right)m_W^2(m_Q^2-4m_b^2)-4m_b^2(m_Q^2-2m_b^2)\right]$$





2% deviation from data

$m_t = (172.69 \pm 0.30)$ GeV

explained by choosing

$\mu = 0.98 m_Q$