Dark Photons and Magnetic Charge

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28 June 2023

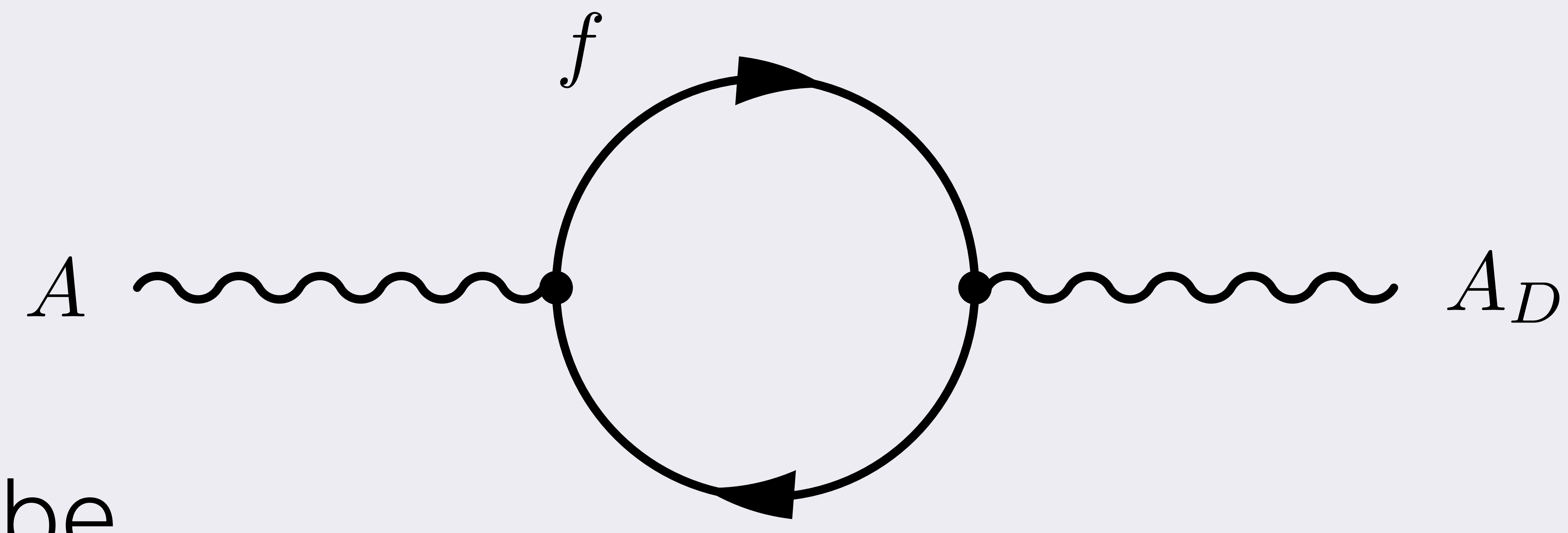
With John Terning

Dark Photons

Dark matter motivates dark sectors

Perhaps rich and complicated, but composed of simple parts

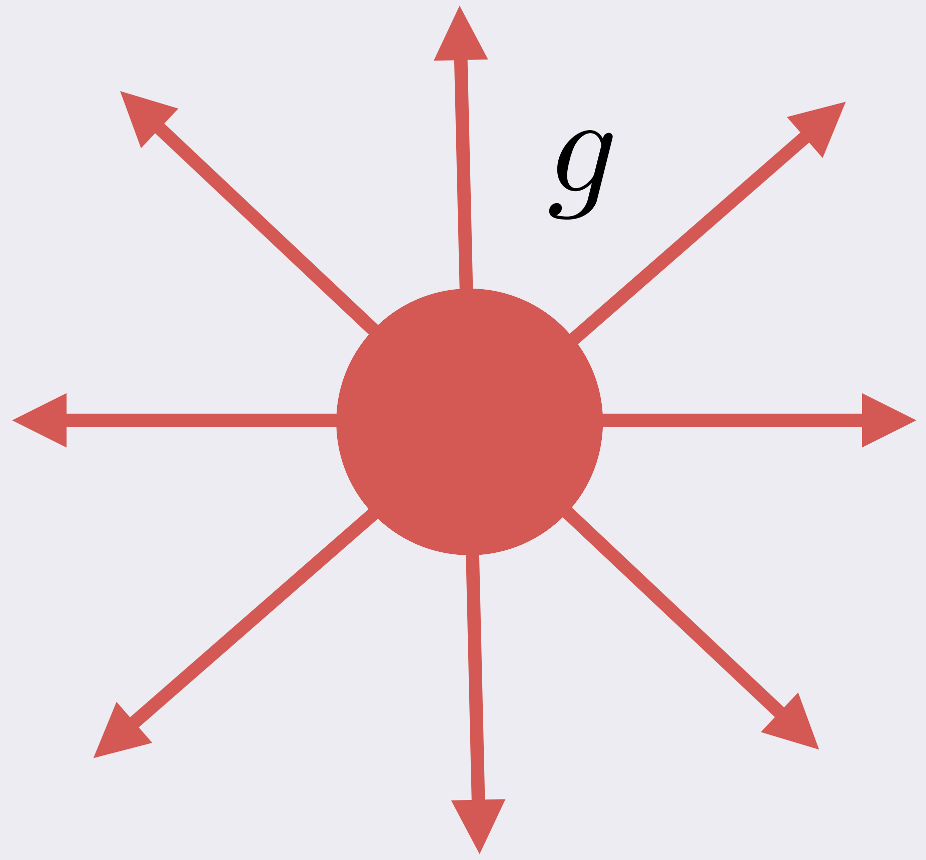
Visible and dark U(1) gauge bosons can be mixed together by particles charged under both



Typically discussed for electrically charged states

What if magnetically charged states are involved?

Magnetic Monopoles



Interesting particles with a variety of motivations

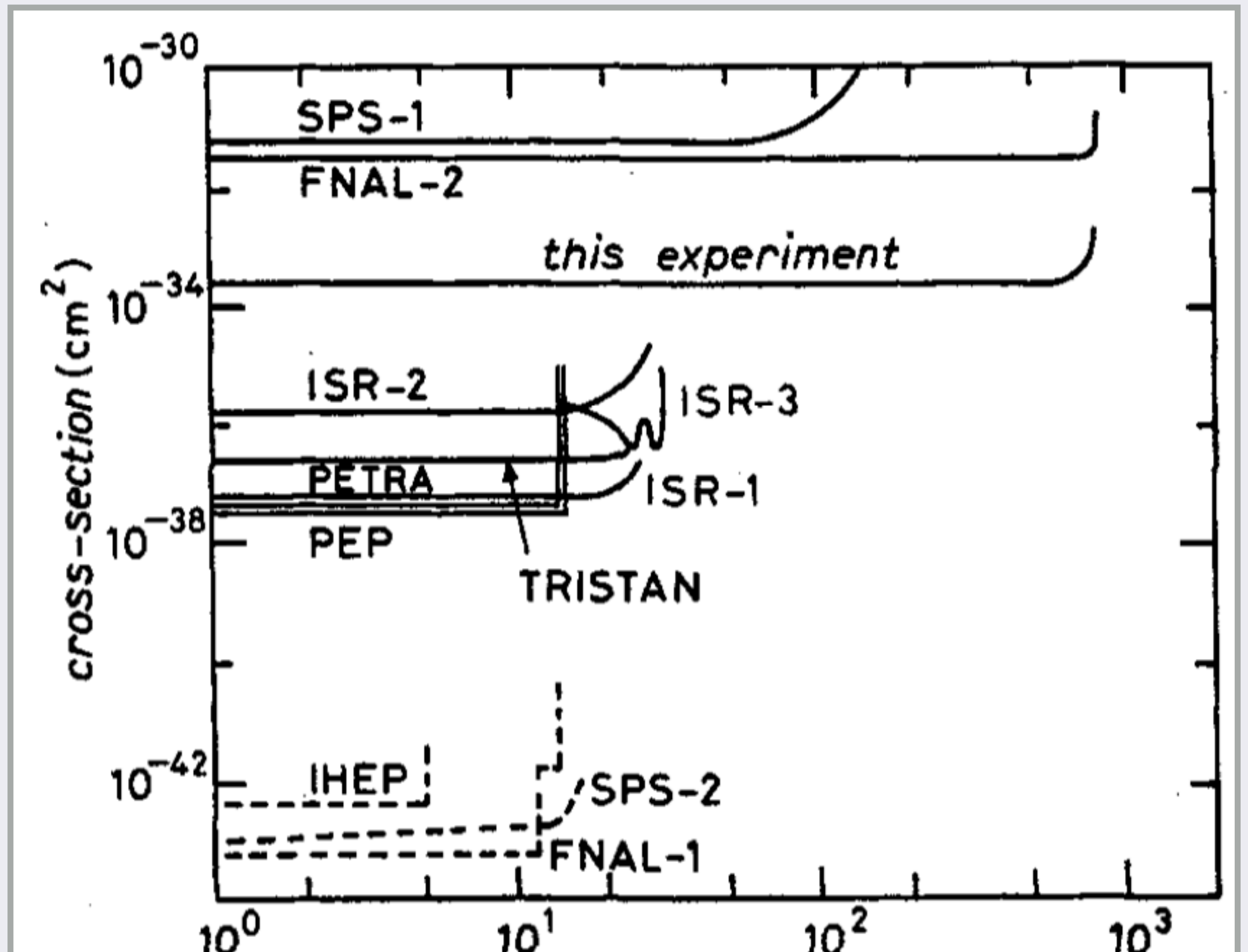
1) Makes Maxwell's equations more "symmetric"

$$\partial_\nu F^{\mu\nu} = J^\mu \qquad \partial_\nu {}^*F^{\mu\nu} = K^\mu \qquad\qquad {}^*F^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

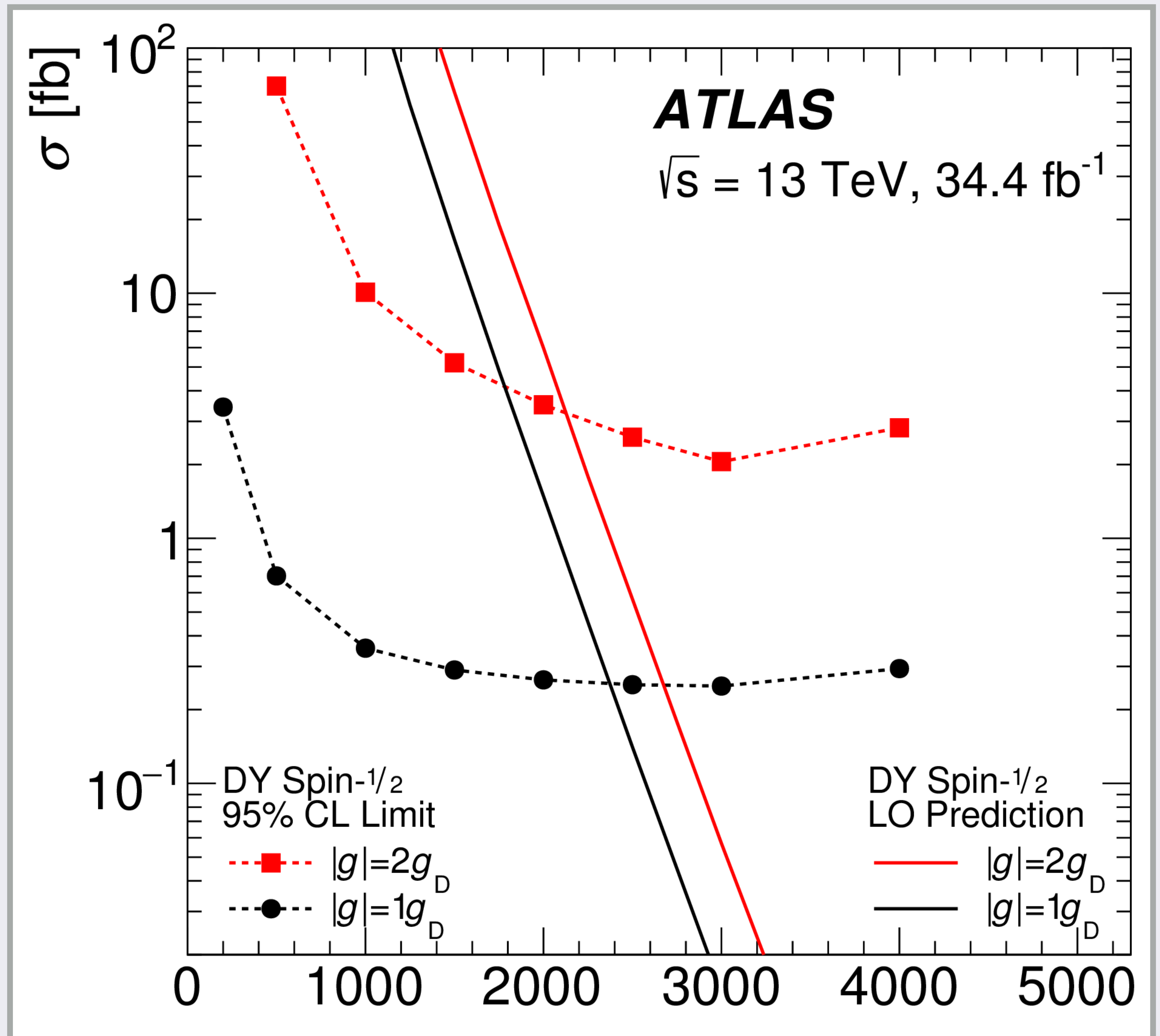
2) Provides an explanation of electric charge quantization

3) Generic prediction of the grand unified theories

Sought experimentally for decades



1990 Tevatron



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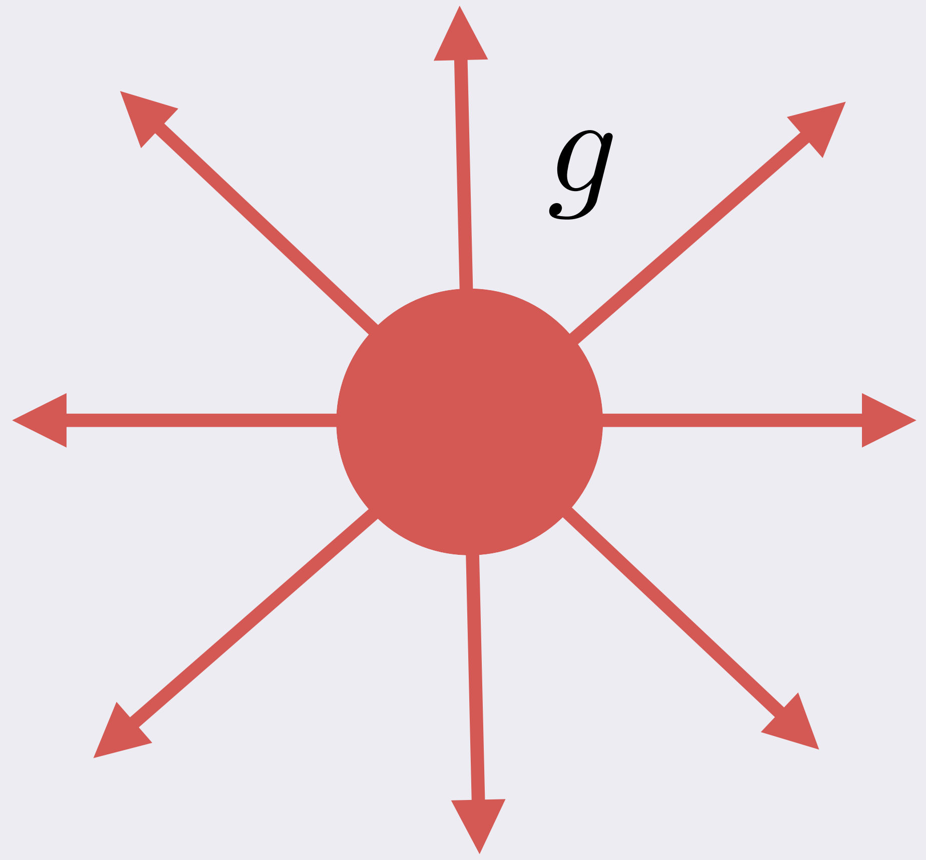
Dirac Monopoles



Monopole are something like $\boldsymbol{B} = \frac{g}{r^2}\hat{\boldsymbol{r}}$

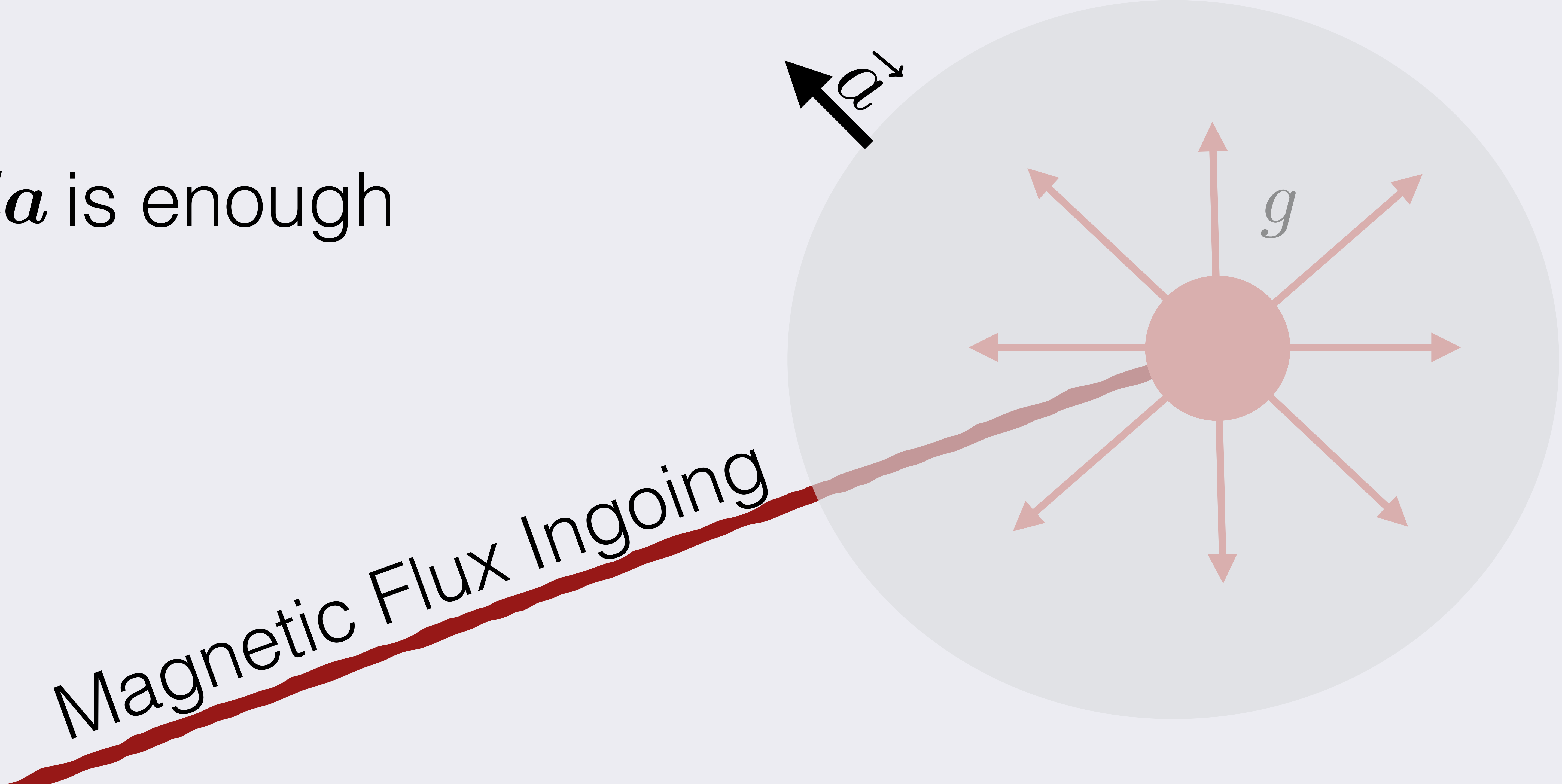
Implies that $\nabla \cdot \boldsymbol{B} \neq 0$, which is a bit worrying

We know how to do quantum mechanics in terms of a vector potential

$$\boldsymbol{B} = \nabla \times \boldsymbol{A} \;\Rightarrow\; \nabla \cdot \boldsymbol{B} = 0$$

Dirac's idea: requiring $\oint \boldsymbol{B} \cdot d\boldsymbol{a}$ is enough

All the flux out of the pole
is piped in through a "string"



Dirac Monopoles

Change of string location is a gauge transformation

$$\vec{A}' = \vec{A} + g\nabla\Omega_C$$

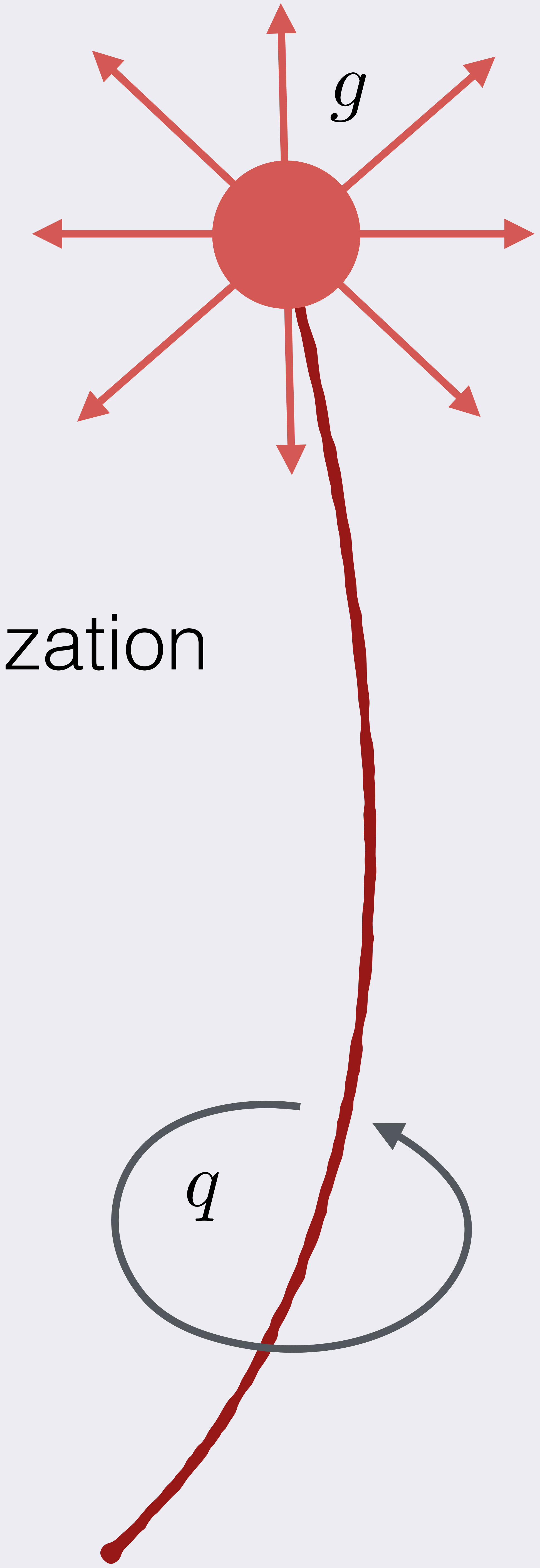
not physical

Aharonov-Bohm phase around string leads to charge quantization

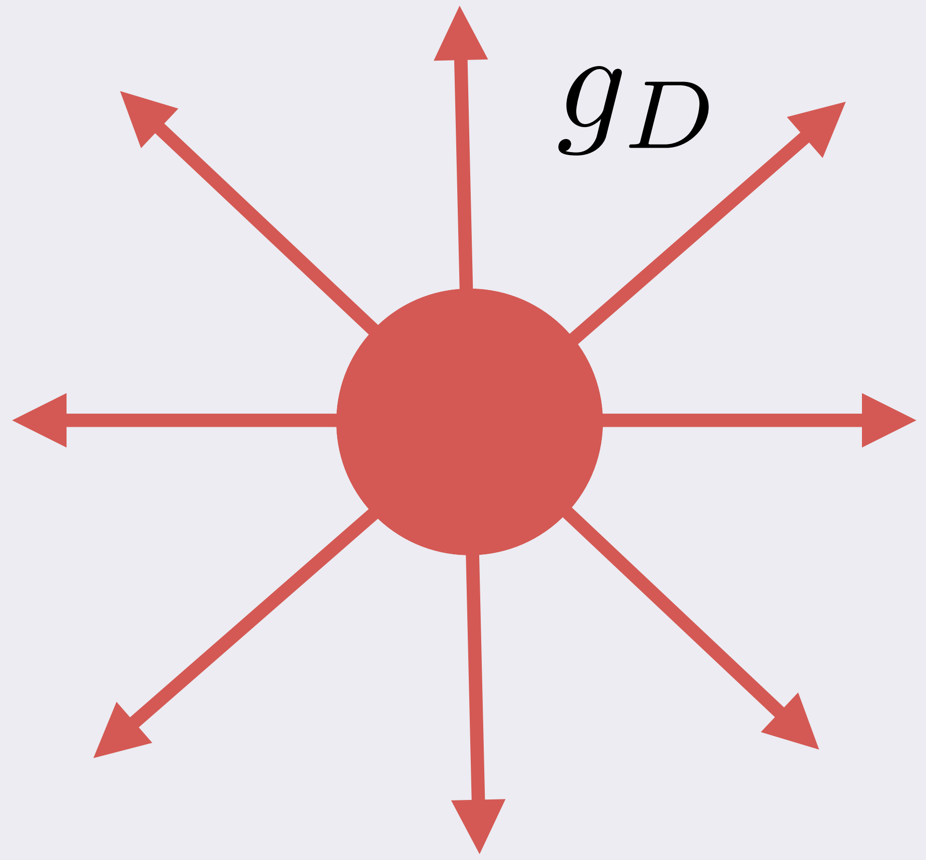
$$qg = \frac{N}{2}$$

Same result from quantized angular momentum of the field between electric and magnetic charges

$$\vec{L} = qg\hat{r}$$



Dark Monopoles



Currently, no robust evidence of monopoles in our sector

They may be hiding in the dark

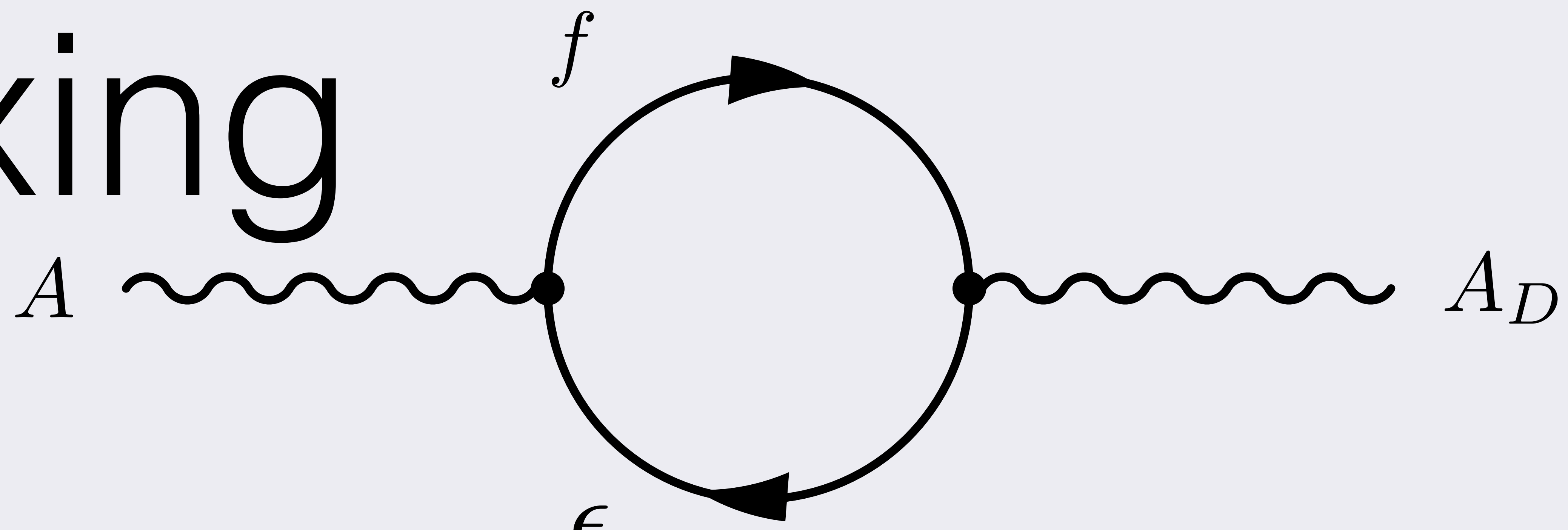
Kinetic mixing between the dark photon and our photon can reveal them

Interesting for at least two reasons

1) A less studied dark sector state with some novel phenomenology

2) A theoretical laboratory for understanding the interactions between electric and magnetic particles in a perturbative framework

Kinetic Mixing



From

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + A_\mu J^\mu - \frac{1}{4}F_{D\mu\nu}F_D^{\mu\nu} + A_{D\mu}J_D^\mu + \frac{\epsilon}{2}F_{\mu\nu}F_D^{\mu\nu}$$

we have

$$\partial_\nu F^{\mu\nu} - \epsilon\partial_\nu F_D^{\mu\nu} = J^\mu \qquad \partial_\nu {}^*F^{\mu\nu} = 0$$

$$\partial_\nu F_D^{\mu\nu} - \epsilon\partial_\nu F^{\mu\nu} = J_D^\mu \qquad \partial_\nu {}^*F_D^{\mu\nu} = 0$$

Can unmix with

$$A_\mu \to (\cos\phi + \epsilon\sin\phi)\, A_\mu + (-\sin\phi + \epsilon\cos\phi)\, A_{D\phi}$$

$$A_{D\mu} \to A_\mu \sin\phi + A_{D\mu}\cos\phi$$

Kinetic Mixing & Magnetic

After unmixing

$$\partial_\nu F^{\mu\nu} = J^\mu \left(\cos\phi + \epsilon \sin\phi\right) + J_D^\mu \sin\phi \qquad \partial_\nu {}^*F^{\mu\nu} = 0$$

$$\partial_\nu F_D^{\mu\nu} = J_D^\mu \cos\phi + \left(-\sin\phi + \epsilon\cos\phi\right) J^\mu \qquad \partial_\nu {}^*F_D^{\mu\nu} = 0$$

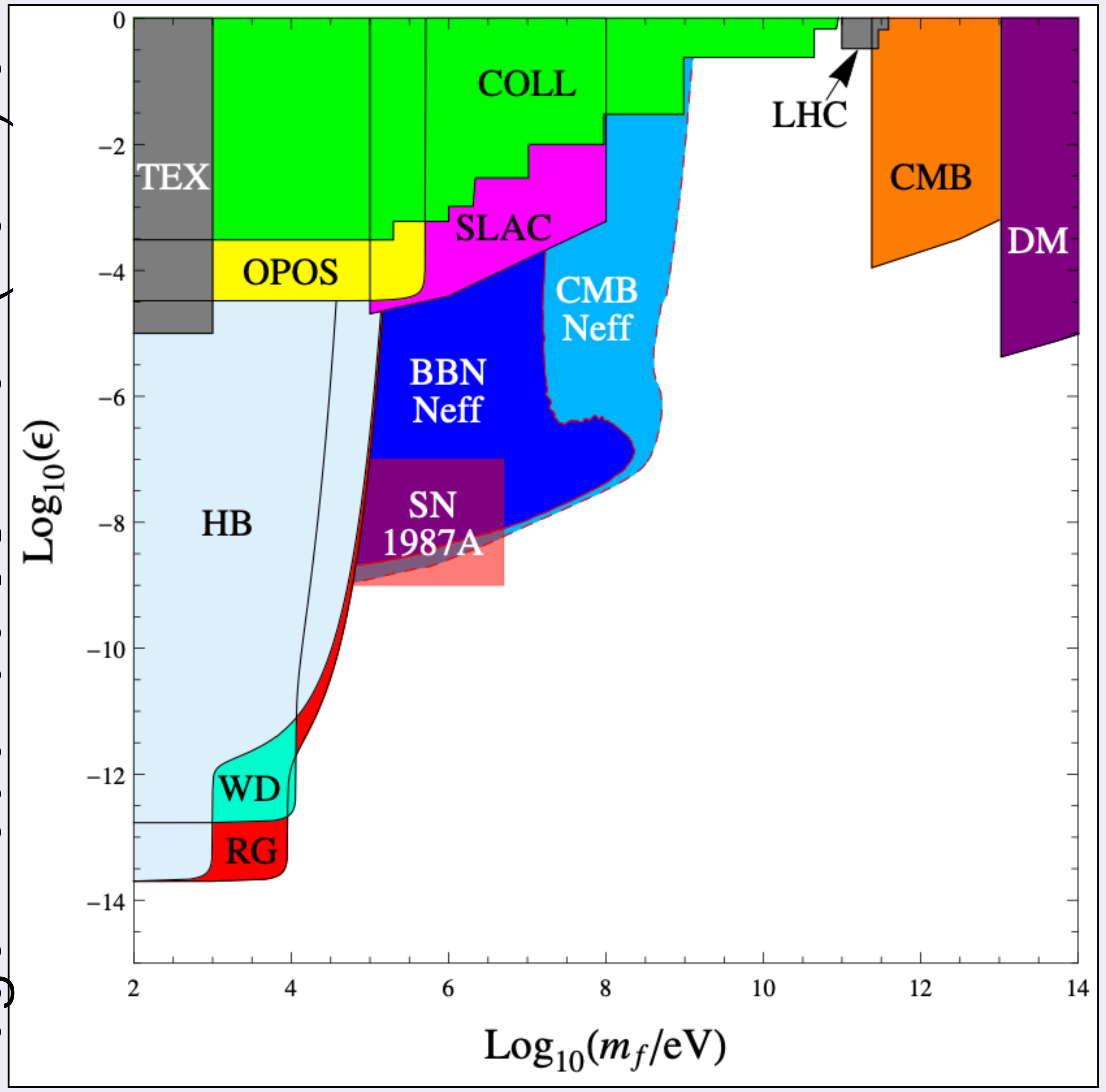
Angle shows ambiguity of having two massless U(1)s

Physical quantities are independent of ϕ

For $\tan\phi = \epsilon$ the dark photon does not couple to visible matter, dark sector matter has a small coupling to the visible photon

Bound are often set in this "millicharged" matter basis

Vogel Redondo *JCAP* 02 (2014) 029



Kinetic Mixing

If the dark U(1) is broken by a photon mass

$$m_D^2 A_{D\mu} A_D^\mu \to m_D^2 \left(A_\mu A^\mu \sin^2\phi + 2 A_D^\mu A_\mu \cos\phi \sin\phi + A_D^\mu A_{D\mu} \cos^2\phi \right)$$

only $\phi = 0$ keeps the visible photon massless

No millicharge under visible photon $\partial_\nu F^{\mu\nu} = J^\mu$

Visible matter has a small coupling to the dark photon

$$\partial_\nu F_D^{\mu\nu} + m_D^2 A_{D\mu} A_D^\mu = J_D^\mu + \epsilon J^\mu$$

Kinetic Mixing & Magnetic

Include magnetic sources (and a dark photon mass)

$$\partial_\nu F^{\mu\nu} - \epsilon \partial_\nu F_D^{\mu\nu} = J^\mu \qquad \partial_\nu {}^*F^{\mu\nu} = K^\mu$$

$$\partial_\nu F_D^{\mu\nu} + m_D^2 A_{D\mu} A_D^\mu - \epsilon \partial_\nu F^{\mu\nu} = J_D^\mu \qquad \partial_\nu {}^*F_D^{\mu\nu} = K_D^\mu$$

Leads to

$$\partial_\nu F^{\mu\nu} = J^\mu \qquad \partial_\nu {}^*F^{\mu\nu} = K^\mu - \epsilon K_D^\mu$$

$$\partial_\nu F_D^{\mu\nu} + m_D^2 A_{D\mu} A_D^\mu = J_D^\mu + \epsilon J^\mu \qquad \partial_\nu {}^*F_D^{\mu\nu} = K_D^\mu$$

Visible electric matter gets a small coupling to the dark photon

Dark magnetic matter gets a small coupling to the visible photon

Charge Quantization

Before mixing we had charge quantization in each sector

$$qg = \frac{N}{2} \qquad q_D g_D = \frac{N_D}{2}$$

After mixing the particles are charged under both $U(1)$s, the Aharonov-Bohm phase depends on

$$q(g - \epsilon g_D) + (q_D + \epsilon q) g_D = \frac{N + N_D}{2}$$

Single charge quantization while neither $q(g - \epsilon g_D)$ nor $(q_D + \epsilon q) g_D$ are half-integer

Physical Strings

When the dark $U(1)$ is broken by the mass term $\frac{m_D^2}{2} A_{D\mu} A_D^\mu$

The dark magnetic charges confine

Monopole-antimonopole pairs are connected by Nielsen-Olesen flux tubes which behave like strings with tension $\sim \mathcal{O}(m_D^2)$

Observables depend on this **physical** flux tube (not like Dirac string)

Small Magnetic Charge

Below the photon mass, one contribution to AB phase:

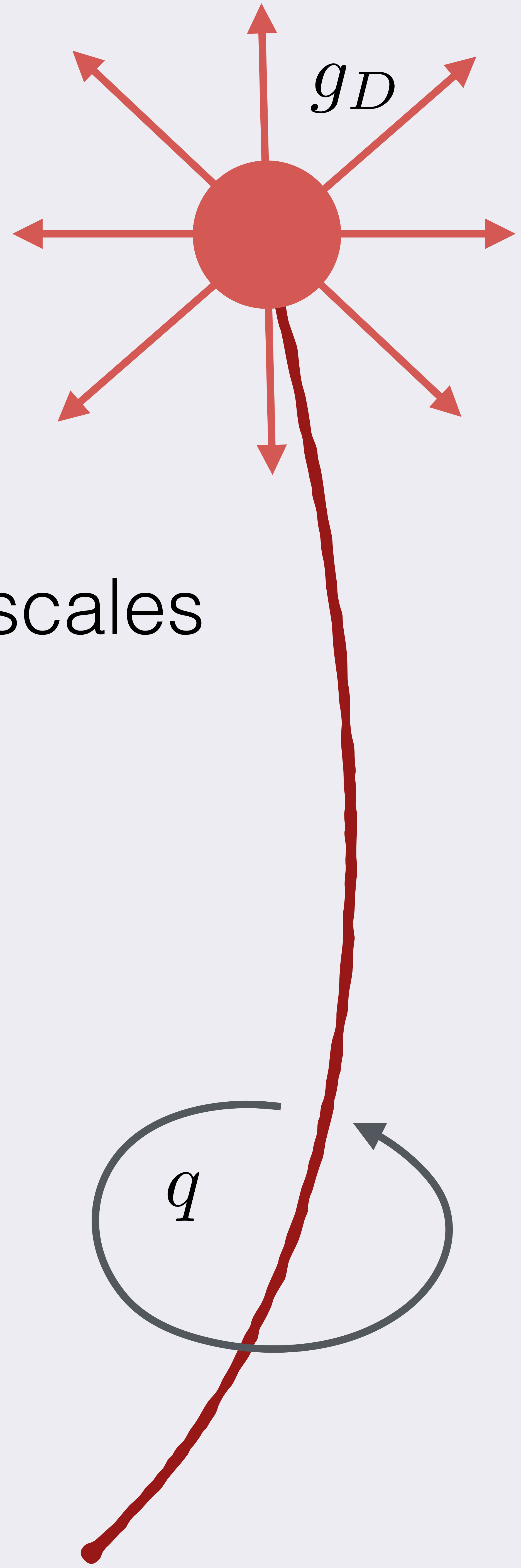
$$\Phi_{\mathrm{AB}} = 4\pi\varepsilon q g_D$$

Physical phase shows charge quantization “violated” at low scales

Flux string connecting the dark monopoles is **physical**

If such monopoles make up some fraction of the dark matter, can search using AB phase shifts

Terning CV *JHEP* 12 (2019) 152



Phenomenology

Little phenomenology of perturbative magnetic charge has been done (see Hook & Huang *Phys Rev* D 96 (2017) 5, 055010 regarding magnetars)

Lagrangian formulation aids systematic study

But such formulations are…painful

Dirac (1948) developed a theory with **non-local** coupling between the photon and magnetic charges

Zwanziger (1968) developed a **local** theory…but uses **two potentials** and a **constant vector**

Zwanziger's Lagrangian

$$\mathcal{L} = -\frac{1}{2}\left(F^{\mu\nu}\partial_\mu A_\nu + {}^*F^{\mu\nu}\partial_\mu B_\nu\right) - eA_\mu J^\mu - bB_\mu K^\mu$$

Local Electric

Local Magnetic

Produces the usual Maxwell equations with

$$F_{\mu\nu} = \frac{n^\alpha}{n^2}\left(n_\mu F^A_{\alpha\nu} - n_\nu F^A_{\alpha\mu} - \varepsilon_{\mu\nu\alpha}{}^\beta n^\gamma F^B_{\gamma\beta}\right)$$

$${}^*F_{\mu\nu} = \frac{n^\alpha}{n^2}\left(n_\mu F^B_{\alpha\nu} - n_\nu F^B_{\alpha\mu} + \varepsilon_{\mu\nu\alpha}{}^\beta n^\gamma F^A_{\gamma\beta}\right)$$

where $F^X_{\mu\nu} \equiv \partial_\mu X_\nu - \partial_\nu X_\mu$

Zwanziger's Lagrangian

Or

$$\mathcal{L} = -\frac{n^\alpha}{2n^2}\left[n^\mu g^{\beta\nu}\left(F^A_{\alpha\beta}F^A_{\mu\nu} + F^B_{\alpha\beta}F^B_{\mu\nu}\right) - \frac{n_\mu}{2}\varepsilon^{\mu\nu\gamma\delta}\left(F^B_{\alpha\nu}F^A_{\gamma\delta} - F^A_{\alpha\nu}F^B_{\gamma\delta}\right)\right]$$

$$- eJ_\mu A^\mu - \frac{4\pi}{e}K_\mu B^\mu$$

$$\Delta^{AB}_{\mu\nu}(k) = -\frac{\epsilon_{\mu\nu\alpha\beta}\, n^\alpha k^\beta}{n \cdot k}\,\frac{i}{k^2 + i\epsilon}$$

Cons:
Two potentials to describe one photon

Dependance on constant vector n^μ

Pros:
Local, leads to familiar types of calculation of kinetic mixing
(Terning CV *JHEP* 12 (2018) 123)

SL(2,Z) duality structure is manifest
(Csáki, Terning, Shirman *Phys Rev* D 81 (2010) 125028)

What About n^μ?

Plays a technical role of reducing propagating degrees of freedom

Essential in form of electric-magnetic propagator

$$\Delta^{AB}_{\mu\nu}(k) = -\frac{\epsilon_{\mu\nu\alpha\beta}\, n^\alpha k^\beta}{n \cdot k}\frac{i}{k^2 + i\epsilon}$$

Weinberg found similar form without a Lagrangian (*Phys Rev* 138 (1965) B988)

Can be associated with direction of Dirac string

Shown to vanish from all orders soft corrections (Terning CV *JHEP* 03 (2019) 177)

The trouble arises in diagrams in which a photon is exchanged between a charge and monopole. Since the charge current $J_\mu(x)$ is coupled to $A^\mu(x)$ and the monopole current $M_\nu(y)$ is coupled to $B^\nu(y)$, the photon propagator will be

$$-i\Delta_{AB}{}^{\mu\nu}(q)=\int d^4x\, e^{-iq\cdot(x-y)}\langle T\{A^\mu(x),B^\nu(y)\}\rangle_0. \qquad (8.1)$$

This can be easily calculated using (3.22) and (3.23) and the results of Appendix A we find

$$\Delta_{AB}{}^{\mu\nu}(q)=\frac{\Xi^{\mu\nu}(q)(q^0/|\mathbf{q}|)}{q^2-i\epsilon} \qquad (8.2)$$

$$\Xi^{\mu\nu}(q)=i\sum\nolimits_\pm(\pm)e_\pm{}^\mu(\mathbf{q})e_\pm{}^\nu(\mathbf{q})^*$$

$$=\epsilon^{\mu\nu\lambda\rho}q_\lambda n_\rho/|\mathbf{q}|\,. \qquad (8.3)$$

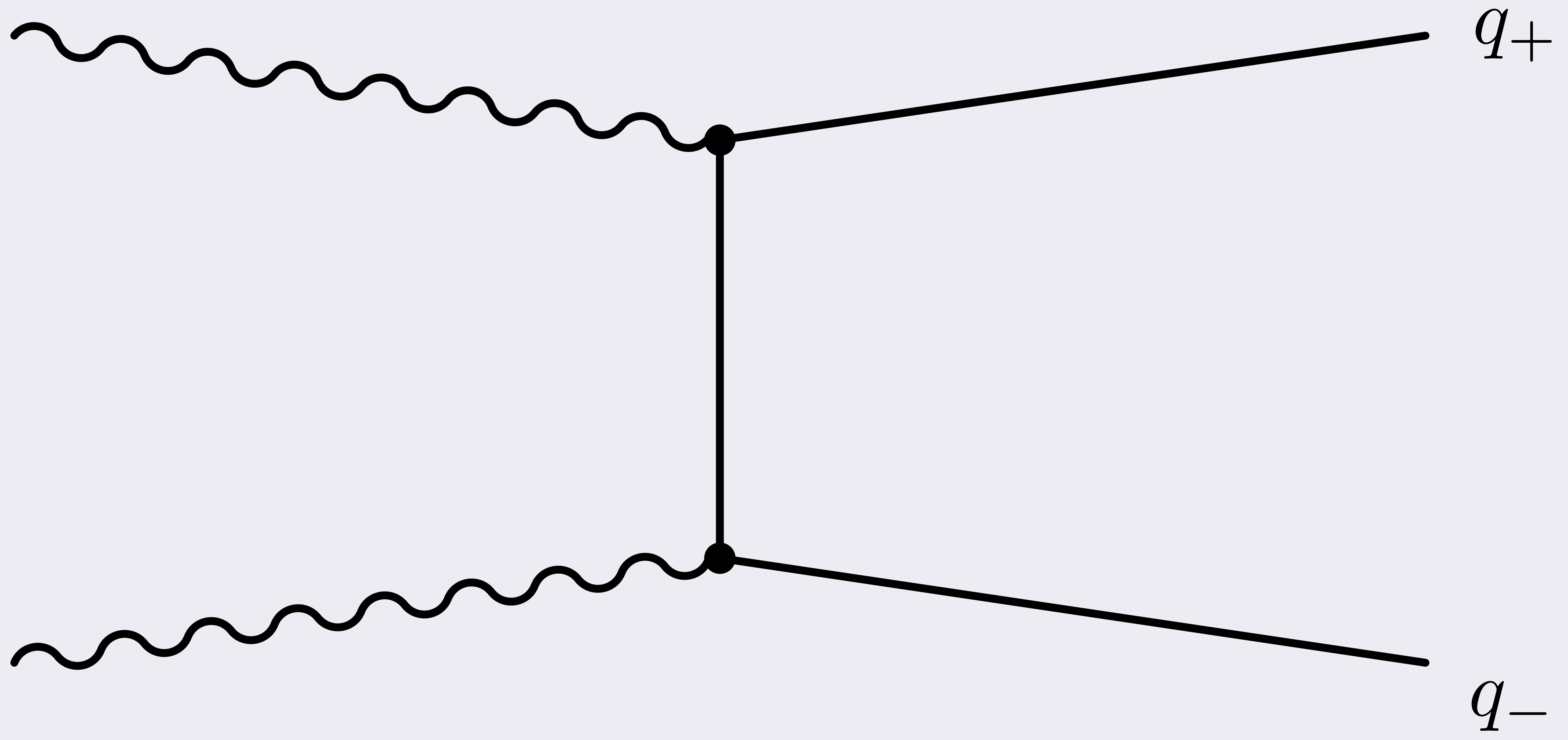
Spurious Pole?

What about the $n \cdot k$ in the denominator of the mixed propagator?

$$\Delta^{AB}_{\mu\nu}(k) = -\frac{\epsilon_{\mu\nu\alpha\beta}\, n^{\alpha} k^{\beta}}{n \cdot k} \frac{i}{k^2 + i\epsilon}$$

Shown to cancel in physical amplitudes when $n^{\mu} \propto q^{\mu}_{+} - q^{\mu}_{-}$
(Terning CV *JHEP* 12 (2020) 153)

That is, when the vector is taken along
the physical flux string
between the bound monopoles



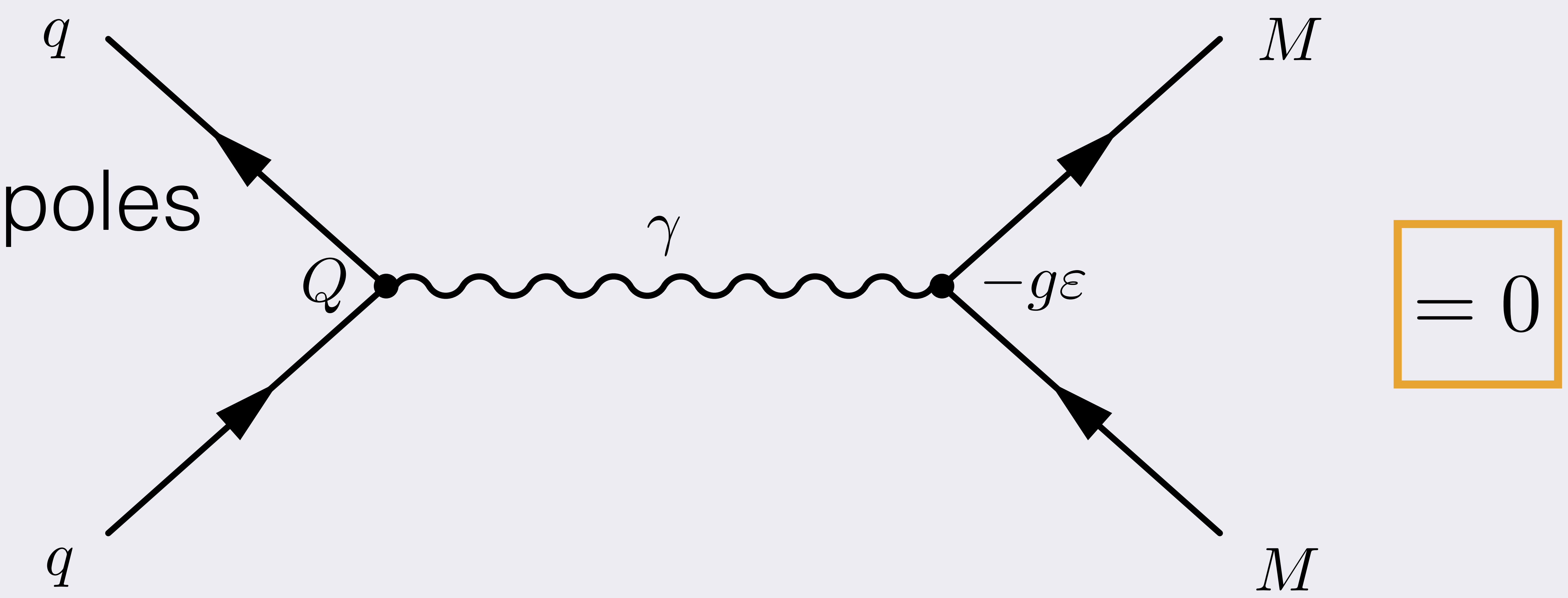
Beginning Pheno

We also discovered:

Spherically symmetric magnetic bound states have no “charge radius” to electric probes

Need a magnetic dipole moment to have nonzero interactions with electric particles

Single photon production of monopoles by electric annihilation vanishes



Photon fusion is nonzero

Lots more to do!

Summary

The simple extension of a dark U(1) is **not** fully explored

Abelian gauge theory is rich and deep

Magnetic charges in the dark sector can lead to novel phenomenology

Perturbative magnetic charges (through kinetic mixing) allow new understanding of electric-magnetic interactions

Currently working on loops and the the running of electric and magnetic couplings

Extra Credit

Electric-Magnetic Scattering

Real bound state constituents are not separated by a fixed distance

The ground state is spherically symmetric, how do we address physical bound state?

Consider non-relativistic, elastic scattering

Bound State Wavefunction

$$\mathcal{M} = \int d^3y\, e^{-i\vec{k}\cdot\vec{y}} \Delta(y) \int d^3x'\, e^{-i\vec{k}\cdot\vec{x}'} |\psi(x')|^2$$

$$= \int d^3y\, e^{-i\vec{k}\cdot\vec{y}} \Delta(y)\, F(k)$$

Form Factor

We write this in terms of single particle charge densities in CM frame

$$\mathcal{M} = \int d^3y\, e^{-i\vec{k}\cdot\vec{y}} \int d^3x'\, e^{-i\vec{k}\cdot\vec{x}'} \Delta(y) \left[\rho_p(x') + \rho_{\overline{p}}(x') \right]$$

Electric-Magnetic Scattering

$$\mathcal{M} = \int d^3y\, e^{-i\vec{k}\cdot\vec{y}} \int d^3x' e^{-i\vec{k}\cdot\vec{x}'} \Delta(y)\, [\, \rho_p(x') + \rho_{\overline{p}}(x')]$$

For mixed propagator define

$$\Delta(k) = \widetilde{\Delta}(k) \cdot \frac{\vec{n} \times \vec{k}}{\vec{n} \cdot \vec{k}}$$

where n^μ points along the flux string

$$\vec{n} \propto \vec{x}'$$

The amplitude is

$$\mathcal{M} = \widetilde{\Delta}(k) \int d^3x e^{-i\vec{k}\cdot\vec{x}} \frac{\vec{x} \times \vec{k}}{\vec{x} \cdot \vec{k}} [\, \rho_p(x) + \rho_{\overline{p}}(x)]$$

Electric-Magnetic Scattering

$$\Delta(k)F_{p\overline{p}}(k) = \widetilde{\Delta}(k)\int d^3x e^{-i\vec{k}\cdot\vec{x}}\frac{\vec{x}\times\vec{k}}{\vec{x}\cdot\vec{k}}\left[\rho_p(x)+\rho_{\overline{p}}(x)\right]$$

Parity implies spatial charge distributions are equal, but differ in sign

Amplitude is zero: no explicit cancellation of the pole

This follows from the spherical symmetry of the bound state

There is no nonzero expectation value for n^μ

Even for different mass constituents this implies spherically symmetric ground states have no “charge radius” to electric probes

External Field

If the bound state is an external magnetic field the dipole moment is

$$gL\langle\hat{n}\rangle = \int d^3x\,\vec{x}\,[\,\rho_p(x) + \rho_{\overline{p}}(x)]$$

This orientation tells us the direction we must choose for n^μ

$$F_{p\overline{p}}(k) = \int d^3x e^{-i\vec{k}\cdot\vec{x}}\,\frac{\vec{x}' \times \vec{k}}{\vec{x}' \cdot \vec{k}}\,[\,\rho_p(x) + \rho_{\overline{p}}(x)]$$

Parity implies

$$\int d^3x'\,\rho_p(x') = -\int d^3x'\,\rho_{\overline{p}}(x')$$

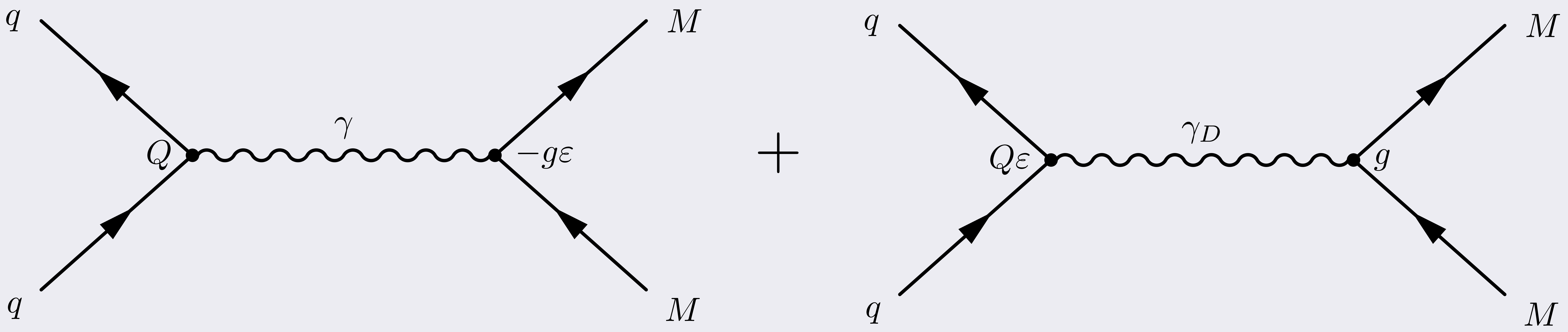
Odd Function

Pole cancels in expansion of the exponential, leading term agrees with static dipole result

$$F_{p\overline{p}}(k) \approx -i\int d^3x\,\vec{k} \times \vec{x}\,[\,\rho_p(x) + \rho_{\overline{p}}(x)] = igL\langle\hat{n}\rangle \times \vec{k}$$

Direct Production

Again there are two diagrams that contribute



The cross sections goes like

$$\frac{1}{k^4} \to \frac{m_D^4}{k^4\left(k^2 - m_D^2\right)^2}$$

Must consider bound state production

Discrete Symmetries Again

Bound states are best characterized by discrete symmetries

$$P = (-1)^{L+1} \qquad C = (-1)^{L+S} \qquad \text{Electric Fermion Bound State}$$

$$P = (-1)^{L} \qquad C = (-1)^{L} \qquad \text{Electric Scalar Bound State}$$

Electric P is Magnetic CP and vice versa

$$P = (-1)^{S+1} \qquad C = (-1)^{L+S} \qquad \text{Magnetic Fermion Bound State}$$

$$P = +1 \qquad C = (-1)^{L} \qquad \text{Magnetic Scalar Bound State}$$

What does this mean for single photon production?

Bound State Production

Consider producing a bound state of charged scalars

The lowest lying states are

${}^{d}L_J$	Scalar J^{PC}	
	electric	magnetic
${}^1S_0$	0^{++}	0^{++}
${}^1P_1$	1^{--}	1^{+-}
${}^1D_2$	2^{++}	2^{++}

Can be produced by a single photon

The spin-1 states have opposite CP

Single photon production forbidden by discrete symmetries, no explicit cancellation of the pole

Pointed out for scalar monopole production by Ignatiev and Joshi hep-ph/9710553

Bound State Production

Consider producing a bound state of charged fermions

Now there are allowed transitions between electric and magnetic

But, the states that couple to a single photon have no overlap

Not Single Photon → $^1P_1$

^dL_J	Fermionic J^{PC} electric	Fermionic J^{PC} magnetic
$^1S_0$	0^{-+}	0^{-+}
$^3S_1$	1^{--}	1^{+-}
$^1P_1$	1^{+-}	1^{--}
$^3P_0$	0^{++}	0^{++}
$^3P_1$	1^{++}	1^{++}
$^3P_2$	2^{++}	2^{++}
$^3D_1$	1^{--}	1^{+-}

We find the amplitude goes like

$$\mathcal{M} = \frac{n \cdot k}{n \cdot k} \times 0$$

The pole cancels, even thought the amplitude vanishes!