Precise Estimate of Decay of Charged Fermion in Electroweak-Charged Dark Matter Model

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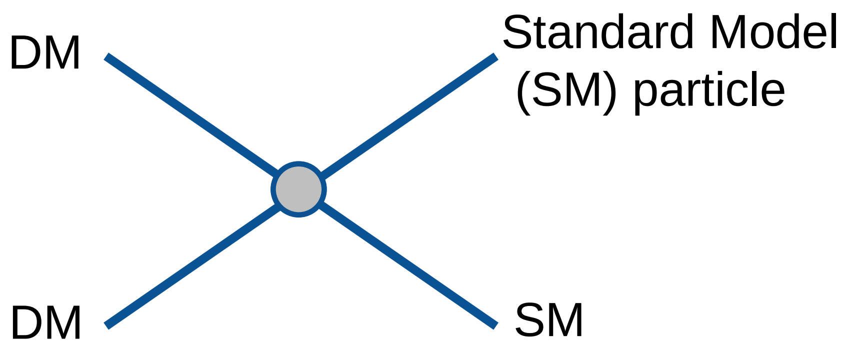
Based on arXiv:2210.16035 and work in progress with

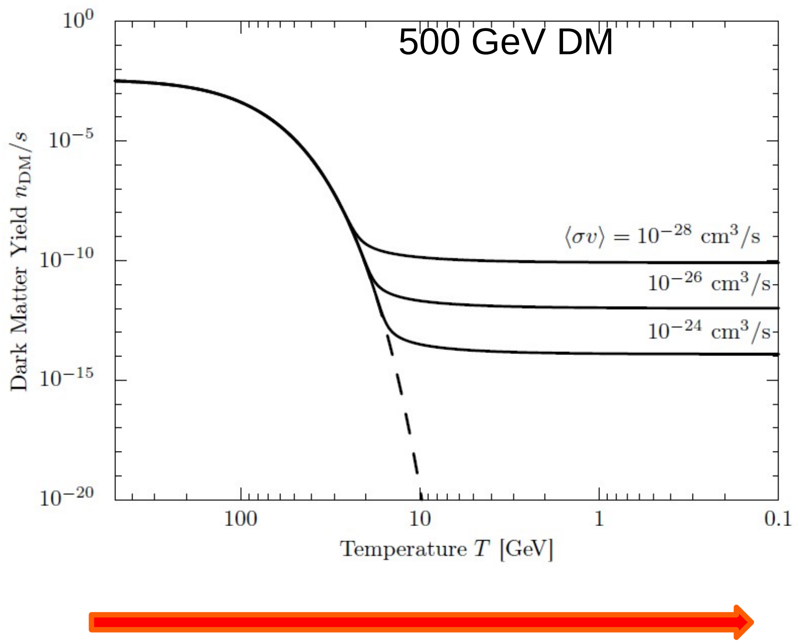
Masahiro Ibe, Masataka Mishima, and Yuhei Nakayama

WIMP Dark Matter

Weakly Interacting Massive Particle

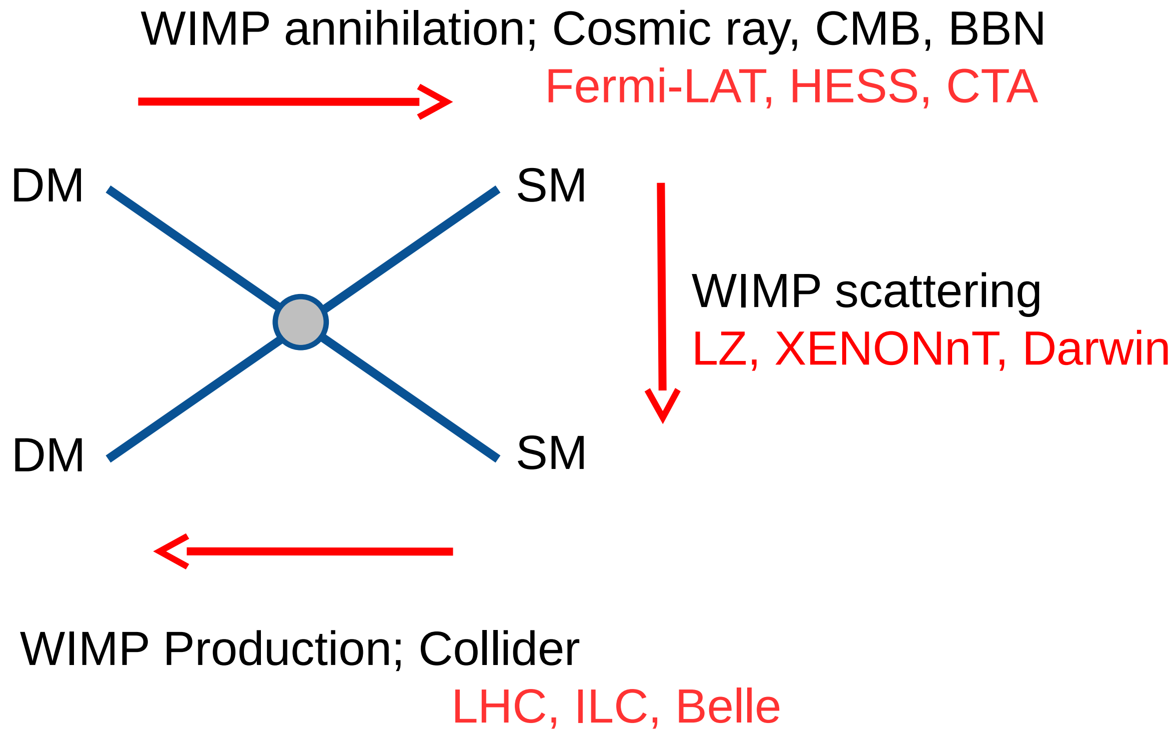


DM abundance



Time

WIMP Detection



Minimal WIMP Model

Add one particle and parameter. UV-complete (renormalizable theory).

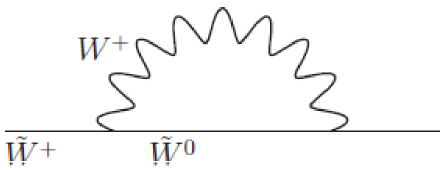
Gauge Portal dark matter

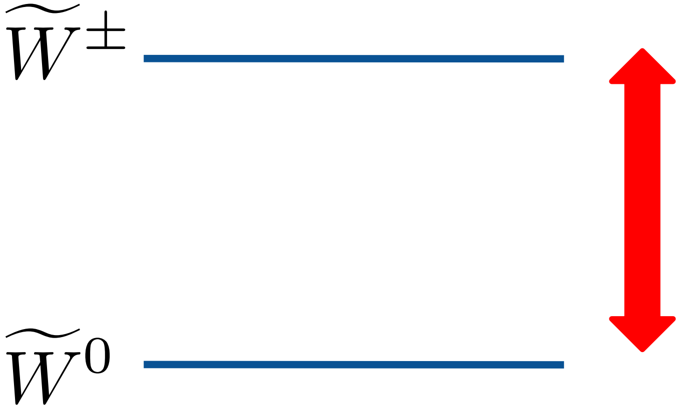
- DM charged weak interaction.
- Minimal choice of electroweak charge is triplet.
- Wino dark matter in SUSY model.

Wino

- Majorana fermion $\widetilde{W}$
- Hypercharge Y=0
- $SU(2)_L$ triplet $\begin{pmatrix} \widetilde{W}^+ \\ \widetilde{W}^0 \\ \widetilde{W}^- \end{pmatrix}$

Wino Spectrum





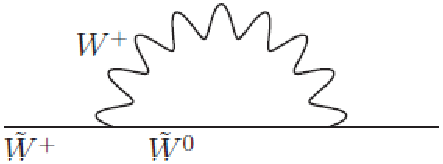
Radiative correction

$$\Delta m \simeq 165\ \mathrm{MeV}$$

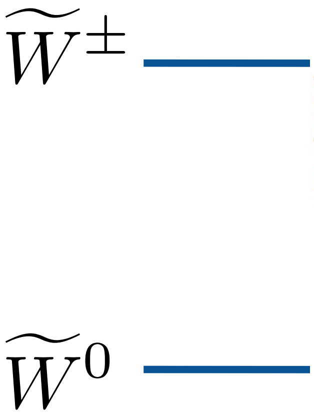
$$\mathcal{O}\left(\frac{\alpha}{4\pi} m_Z\right)$$

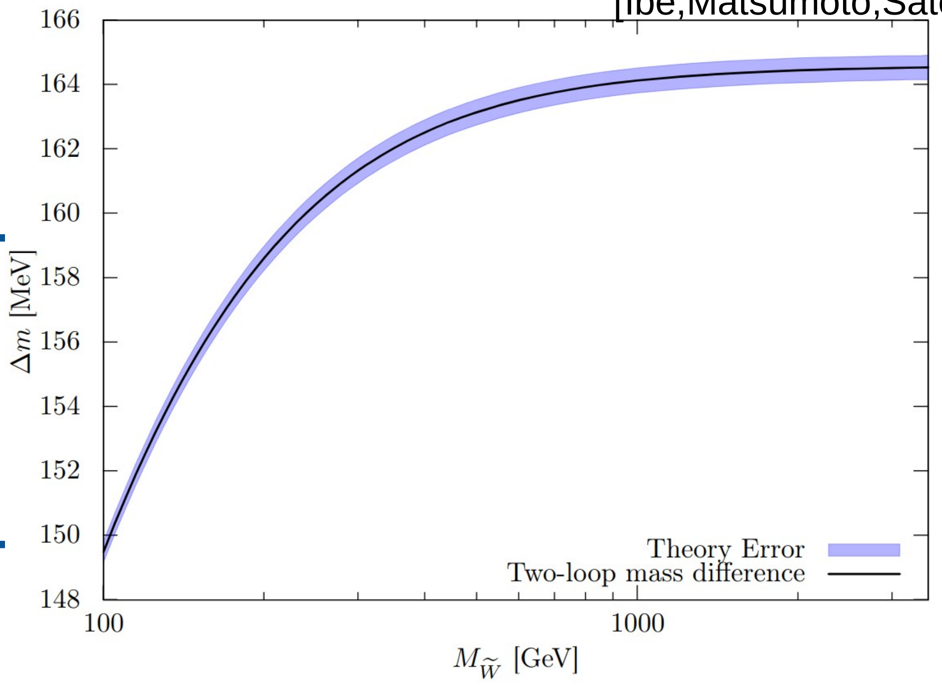
$$c\tau(\widetilde{W}^{\pm} \to \widetilde{W}^{0}\pi^{\pm}) \simeq 7\ \mathrm{cm}\left(\frac{\Delta m}{165\ \mathrm{MeV}}\right)^{-3}$$

Wino Spectrum



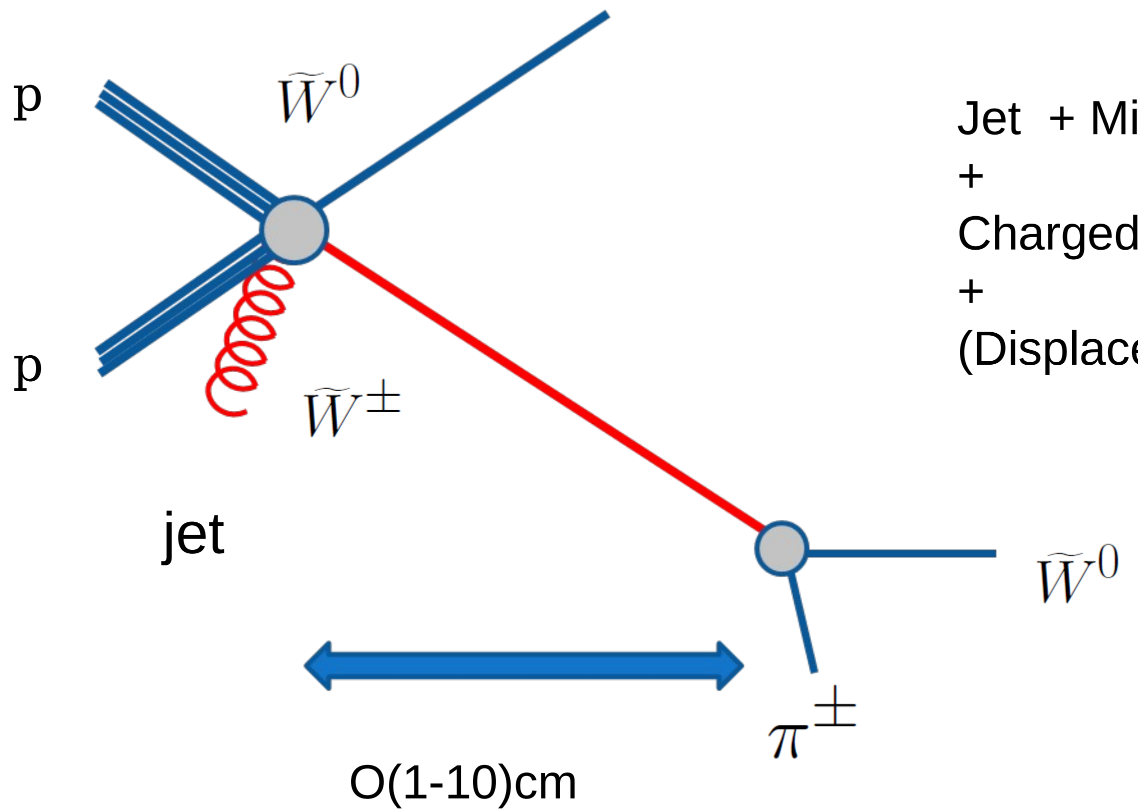
[Ibe,Matsumoto,Sato 12]





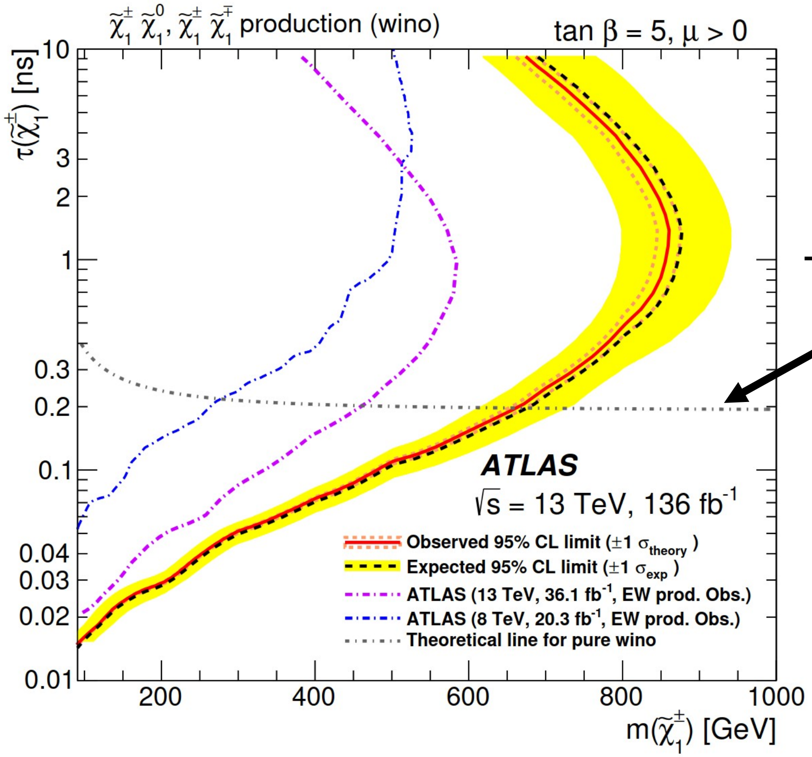
$$c\tau(\widetilde{W}^{\pm} \to \widetilde{W}^{0}\pi^{\pm}) \simeq 7 \text{ cm} \left(\frac{\Delta m}{165 \text{ MeV}}\right)^{-3}$$

Direct LHC Signals



Jet + Missing energy
+
Charged track
+
(Displaced)Soft pion

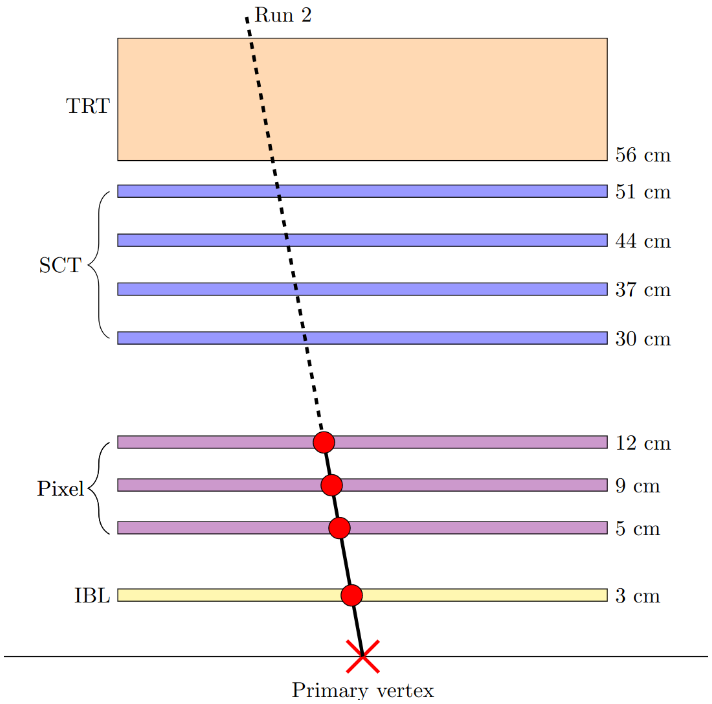
LHC Search



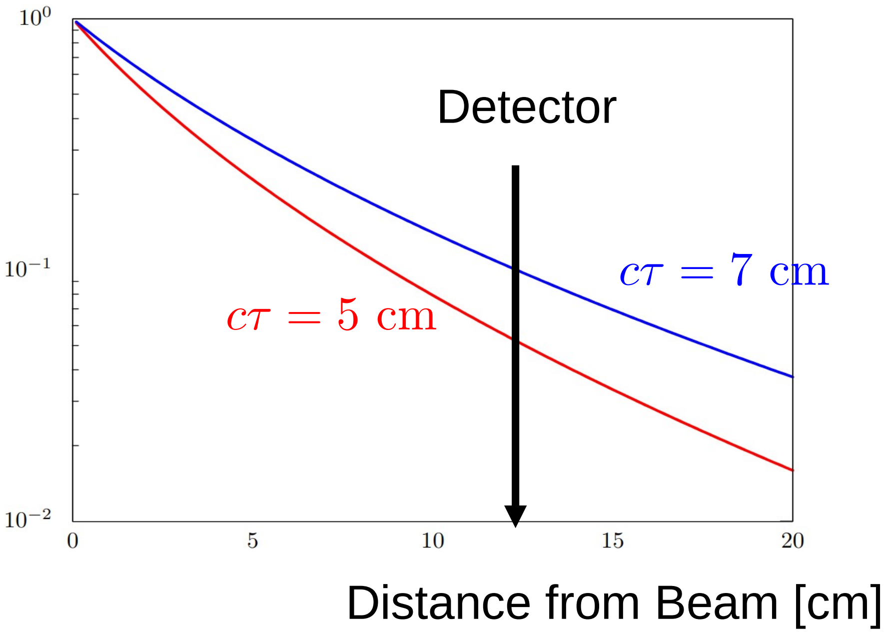
Theory prediction

Direct LHC Signals

ATLAS detector



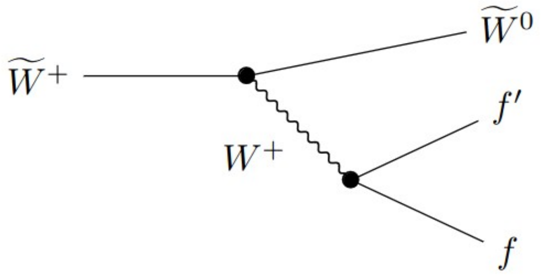
Survival Probability



10% error of lifetime → 50% error of signal.

Charged Wino Decay

Charged Wino Decay



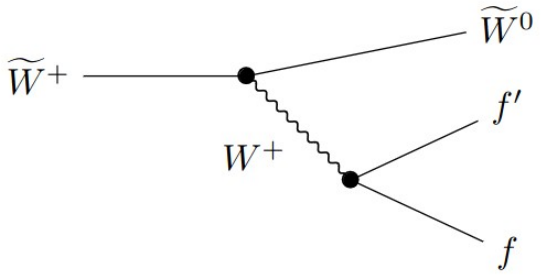
Wino and SM fermion effective interaction

$$G_F(\bar{f}'_L \gamma^\mu f_L)(\bar{\psi}_\pm \gamma_\mu \psi_0)$$

$$\Rightarrow \quad \Gamma = \frac{2(G_F)^2 \Delta m^5}{15\pi^3}$$

$$\longrightarrow \quad c\tau \simeq 1 \text{ m} \left(\frac{\Delta m}{160 \text{ MeV}}\right)^{-5}$$

Charged Wino Decay



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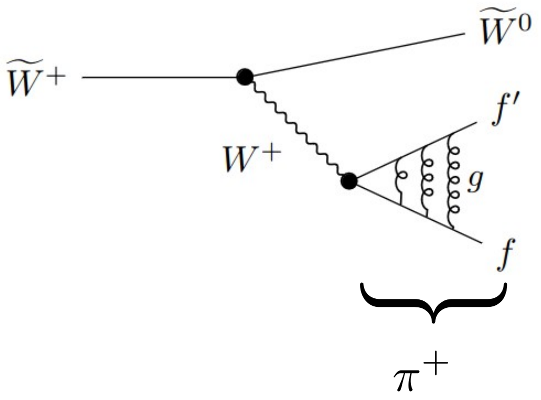
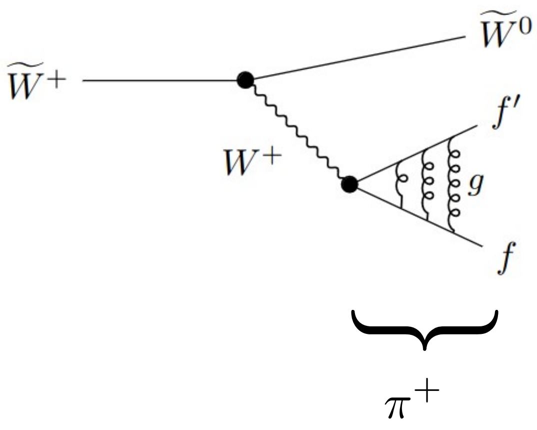


Due to small mass difference,
QCD effect is strong.

Decay into pion



Quark current to pion

$$\bar{d}\gamma^\mu\gamma^5 u \to F_\pi p_\pi^\mu$$

Coupling Wino and pion

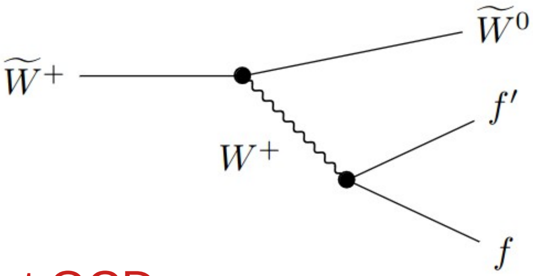
$$2\sqrt{2}F_\pi G_F(\partial_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0)$$

$$i\mathcal{M}_{\rm tree} = 2\sqrt{2}F_\pi G_F \Delta m\, \bar{u}_\pm(q)u_0(p)$$

$$\Gamma = \frac{4}{\pi}F_\pi^2(G_F)^2\Delta m^3\left(1-\frac{m_\pi^2}{\Delta m^2}\right)^{1/2}$$

$$\longrightarrow \quad c\tau \simeq 5\ \text{cm}\left(\frac{\Delta m}{160\ \text{MeV}}\right)^{-3}$$

Charged Wino Decay



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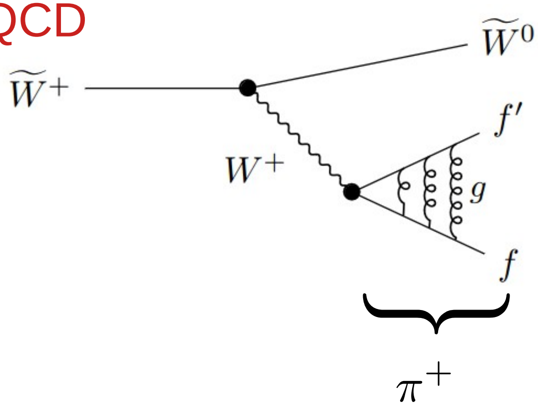
Without QCD

With QCD



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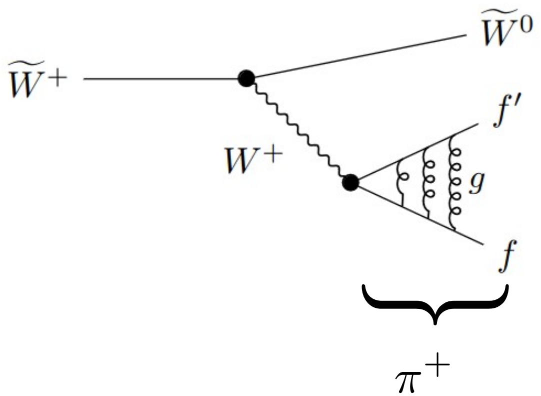
QCD correction is very strong. Electroweak correction?

Charged Wino Decay



$$m_{\widetilde{W}} \gg m_W \gg \Lambda_{\rm QCD} \sim \Delta m$$

EW correction includes multi-scale physics.

Large logarithm? e.g., $\frac{\alpha}{4\pi} \log\left(\frac{m_{\widetilde{W}}}{\Lambda_{\rm QCD}}\right)$?

Which energy scale parameter? $\alpha(m_{\widetilde{W}})$? $\alpha(m_W)$? $\alpha(\Lambda_{\rm QCD})$?

EW next-to-leading order calculation.

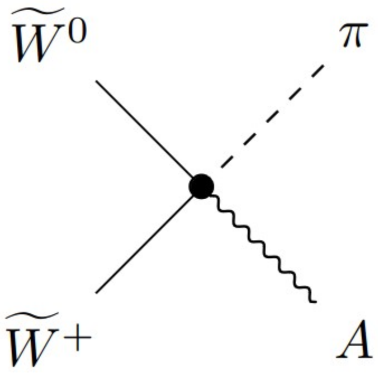
Inclusion of QED

Coupling Wino and pion

covariant derivative

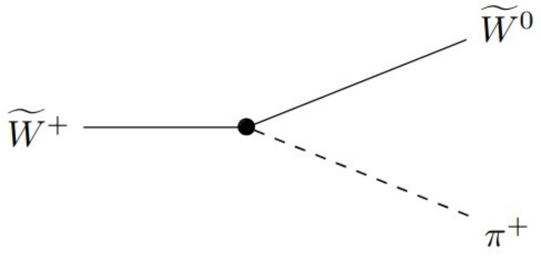
$$2\sqrt{2}F_\pi G_F(\partial_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0) \Rightarrow 2\sqrt{2}F_\pi G_F(D_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0)$$

$$D_\mu = \partial_\mu + ieA_\mu$$



$$2\sqrt{2}eF_\pi G_F$$

Photon Loop Example

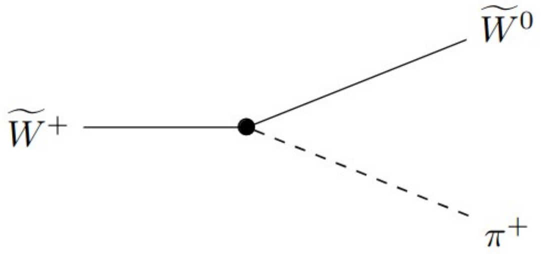


$$\mathcal{M}_{\rm tree} \simeq F_\pi G_F m_{\widetilde{W}} \Delta m$$

c.f., $\Delta m \sim \frac{\alpha}{4\pi} m_Z$

Photon Loop Example

$$2\sqrt{2}F_\pi G_F(\partial_\mu \pi^-)\times(\bar{\psi}_\pm \gamma^\mu \psi_0)$$



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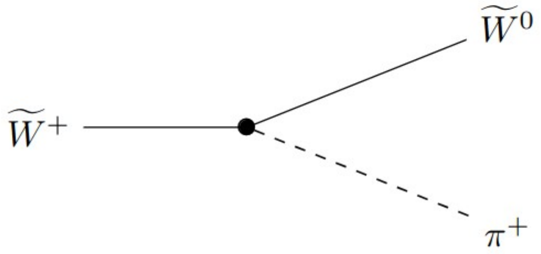

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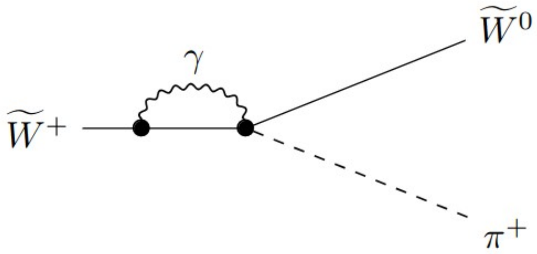


$$\mathcal{M}_{\rm loop} \simeq F_\pi G_F m_{\widetilde{W}}^2 \times \frac{\alpha}{4\pi}\left(\frac{1}{\bar{\epsilon}} - 2\log\frac{m_{\widetilde{W}}}{\mu} + \frac{4}{3}\right)$$

Loop is much larger than tree?

Break of shift-symmetry

Coupling Wino and pion

$$2\sqrt{2}F_\pi G_F(\partial_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0) \quad \Rightarrow \quad 2\sqrt{2}F_\pi G_F(D_\mu \pi^-) \times (\bar{\psi}_\pm \gamma^\mu \psi_0)$$

$$D_\mu = \partial_\mu + ieA_\mu$$

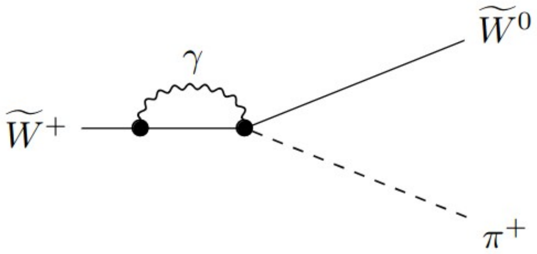
With QED included, pion shift symmetry is completely broken.

⇓

Suppression of Wino decay is no longer guaranteed.

$$m_{\widetilde{W}} F_\pi G_F(\pi^-) \times (\bar{\psi}_\pm \psi_0)$$

To get observable effect



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UV divergent

We need specify counter-terms relevant for Wino decay.

Matching with electroweak theory and chiral perturbation (ChPT)

Matching procedure

Strategy is similar to precision of pion decay calculation in SM.

[Descotes-Genon & Moussallam 2005]

Electroweak $\qquad \mathcal{L}^{\rm EW} = \mathcal{L}^{\rm EW}_{\rm (Wino)} + \mathcal{L}^{\rm EW}_{\rm (SM)} + \mathcal{L}^{\rm EW}_{\rm CT}$

↓ Matching of decay into free quarks

Four-Fermi $\qquad \mathcal{L}^{\rm FF} = \mathcal{L}^{\rm FF}_{\rm (Wino)} + \mathcal{L}^{\rm FF}_{\rm (SM)} + \mathcal{L}^{\rm FF}_{\rm CT}$

↓ Matching of current correlators

ChPT $\qquad \mathcal{L}^{\rm ChPT+QED} = \mathcal{L}^{\rm QED}_{({\rm Wino}+\pi)} + \mathcal{L}^{\rm ChPT}_{({\rm Wino}+\pi)} + \mathcal{L}^{\rm ChPT+QED}_{\rm CT}$

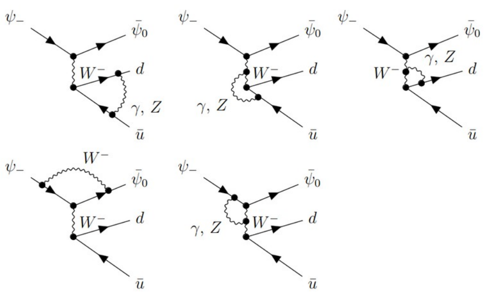
Compute both Wino decay and pion decay with EW/QCD corrections.

Relating

$$\Gamma_{\rm loop}(\pi^+ \to \mu^+ \nu(\gamma)) \quad \Longleftrightarrow \quad \Gamma_{\rm loop}(\widetilde{W}^+ \to \widetilde{W}^0 \pi^+(\gamma))$$

Computing...

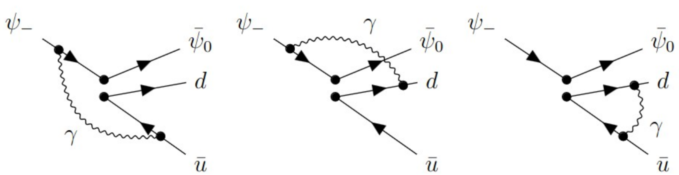
$$\frac{\mathcal{M}^{\rm WF,Box,Vertex(EW)}}{\mathcal{M}^{\rm quark}_{\rm tree}} = \frac{\alpha}{4\pi}\left[\frac{3}{2}\log\frac{M_Z^2}{m_\gamma^2} - \left(\frac{1}{s_W^2}-\frac{4}{s_W^4}\right)\log c_W + \frac{3}{s_W^2}\right] - \frac{\alpha(4+6c_W+c_W^2)}{4(1+c_W)s_W^2}\frac{M_W}{m_\chi} + \mathcal{O}(M_W^2/m_\chi^2)\,.$$



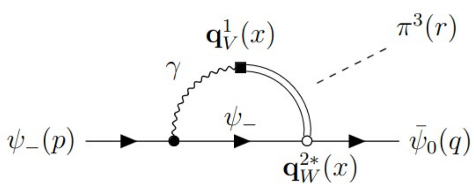
$$\hat{\mathcal{M}}^{\rm quark}_{\rm tree}\times\frac{\alpha}{8\pi}\left[-Q_\chi^2\left(\frac{1}{\bar\epsilon_{\rm FF}}+\log\frac{\mu_{\rm FF}^2}{m_\chi^2}+2\log\frac{m_\gamma^2}{m_\chi^2}+4\right)-(Q_d^2+Q_{\bar u}^2)\left(\frac{1}{\bar\epsilon_{\rm FF}}+\log\frac{\mu_{\rm FF}^2}{m_\gamma^2}-\frac{1}{2}\right)\right]$$

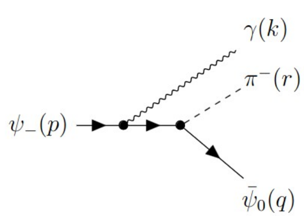
$$\begin{aligned}F_{VW}^{(\rm loop,FF)} = \frac{\alpha}{16\pi\sqrt{2}}F_0G_F\Bigg[&\frac{5}{\bar\epsilon_{\rm ChPT}}+5\log\frac{\mu_{\rm FF}^2}{m_\chi^2}+\log\frac{m_\chi^2}{m_\gamma^2}+\log\frac{m_\chi^2}{M_V^2}\\ &-\frac{5\pi^2}{3}-4\log\frac{m_\gamma^2}{M_V^2}-\log^2\frac{m_\gamma^2}{4\Delta m^2}\\ &+\frac{6M_A^2-9M_V^2}{M_A^2-M_V^2}+3\frac{M_V^4}{(M_A^2-M_V^2)^2}\log\frac{M_A^2}{M_V^2}\\ &-\frac{4}{\Delta m}\frac{\pi M_A M_V}{M_A+M_V}\Bigg]\times\bar u_0(q)\not{r}\,u_-(p)\,.\end{aligned}$$

$$\begin{aligned}\mathcal{L}_K = e^2F_0^2\Big\{&\frac{1}{2}K_1\,\langle(\mathcal{Q}_L)^2+(\mathcal{Q}_R)^2\rangle\langle u_\mu u^\mu\rangle + K_2\,\langle\mathcal{Q}_L\mathcal{Q}_R\rangle\langle u_\mu u^\mu\rangle\\ &-K_3\,[\langle\mathcal{Q}_Lu_\mu\rangle\langle\mathcal{Q}_Lu^\mu\rangle+\langle\mathcal{Q}_Ru_\mu\rangle\langle\mathcal{Q}_Ru^\mu\rangle]+K_4\,\langle\mathcal{Q}_Lu_\mu\rangle\,\langle\mathcal{Q}_Ru^\mu\rangle\\ &+K_5\,\langle[(\mathcal{Q}_L)^2+(\mathcal{Q}_R)^2]u_\mu u^\mu\rangle+K_6\,\langle(\mathcal{Q}_L\mathcal{Q}_R+\mathcal{Q}_R\mathcal{Q}_L)u_\mu u^\mu\rangle\\ &+\frac{1}{2}K_7\,\langle(\mathcal{Q}_L)^2+(\mathcal{Q}_R)^2\rangle\,\langle\chi_+^{(\rm sp)}\rangle+K_8\,\langle\mathcal{Q}_L\mathcal{Q}_R\rangle\,\langle\chi_+^{(\rm sp)}\rangle\\ &+K_9\,\langle[(\mathcal{Q}_L)^2+(\mathcal{Q}_R)^2]\chi_+^{(\rm sp)}\rangle+K_{10}\,\langle(\mathcal{Q}_L\mathcal{Q}_R+\mathcal{Q}_R\mathcal{Q}_L)\chi_+^{(\rm sp)}\rangle\\ &-K_{11}\,\langle(\mathcal{Q}_L\mathcal{Q}_R-\mathcal{Q}_R\mathcal{Q}_L)\chi_-^{(\rm sp)}\rangle\\ &-iK_{12}\,\langle[(\mathcal{Q}_{L\mu}\mathcal{Q}_L-\mathcal{Q}_L\mathcal{Q}_{L\mu})-(\mathcal{Q}_{R\mu}\mathcal{Q}_R-\mathcal{Q}_R\mathcal{Q}_{R\mu})]u^\mu\rangle\\ &+K_{13}\,\langle\mathcal{Q}_{L\mu}\mathcal{Q}_R^\mu\rangle+K_{14}\,\langle(\mathcal{Q}_{L\mu}\mathcal{Q}_L^\mu)+(\mathcal{Q}_{R\mu}\mathcal{Q}_R^\mu)\rangle\Big\}\,.\end{aligned}$$



$$\begin{aligned}\mathcal{L}_Y = e^2\Big\{&\sqrt{2}F_0^2G_F\Big[Y_1\bar\psi_-\gamma_\mu\psi_0\langle u^\mu\{\mathcal{Q}_R,\mathcal{Q}_W\}\rangle+Y_1\psi_-\gamma_\mu\psi_0\langle u^\mu\{\mathcal{Q}_L,\mathcal{Q}_W\}\rangle\\ &+Y_2\bar\psi_-\gamma_\mu\psi_0\langle u^\mu[\mathcal{Q}_R,\mathcal{Q}_W]\rangle+\hat Y_2\bar\psi_-\gamma_\mu\psi_0\langle u^\mu[\mathcal{Q}_L,\mathcal{Q}_W]\rangle\\ &+Y_3m_\chi\bar\psi_-\psi_0\langle\mathcal{Q}_R\mathcal{Q}_W\rangle\\ &+iY_4\bar\psi_-\gamma_\mu\psi_0\langle\mathcal{Q}_L^\mu\mathcal{Q}_W\rangle+iY_5\bar\psi_-\gamma_\mu\psi_0\langle\mathcal{Q}_R^\mu\mathcal{Q}_W\rangle+h.c.\Big]\\ &+\hat Y_6\bar\psi_-(i\not\partial-e\not A)\psi_-+\hat Y_7m_\chi\bar\psi_-\psi_-\Big\}\,.\end{aligned}$$





$$\begin{aligned}\frac{\delta\Gamma_\chi}{\Gamma_\chi} = &-\frac{\alpha M_AM_V}{\Delta m(M_A+M_V)}+\frac{\alpha}{16\pi}g_\chi\left(\frac{M_V}{M_A},\frac{\Delta m}{M_A}\right)+\frac{\alpha}{\pi}f_\chi\left(\frac{m_\pi}{\Delta m}\right)\\ &+e^2(2f^r_{\chi\chi}(\mu_{\rm FF})+f^r_{\chi d}(\mu_{\rm FF})-f^r_{\chi\bar u}(\mu_{\rm FF})+2f^r_{d\bar u}(\mu_{\rm FF}))\\ &+\frac{8}{3}e^2(K_1^r(\mu_{\rm ChPT})+K_2)+\frac{20}{9}e^2(K_5^r(\mu_{\rm ChPT})+K_6)\\ &+\frac{3\alpha}{8\pi}\log\frac{\mu_{\rm ChPT}^2}{M_V^2}+\frac{3\alpha}{4\pi}\log\frac{\mu_{\rm FF}^2}{\mu_{\rm ChPT}^2}+\frac{\alpha}{4\pi}\log\left(\frac{\Delta m^2M_V^4}{m_\pi^6}\right)+\frac{2\alpha}{\pi}\log 2\end{aligned}$$

Final Result

$$\Gamma^{\text{loop}}_{\widetilde{W}^\pm} = \Gamma^{\text{tree}}_{\widetilde{W}^\pm} \left\{ 1 + \frac{\alpha}{4\pi} \left[c_{-2} \left(\frac{m_{\widetilde{W}}}{\Delta m} \right)^2 + c_{-1} \left(\frac{m_{\widetilde{W}}}{\Delta m} \right) + c_{\log} \log \left(\frac{m_{\widetilde{W}}}{\Delta m} \right) + c_0 + \cdots \right] \right\}$$

Final Result

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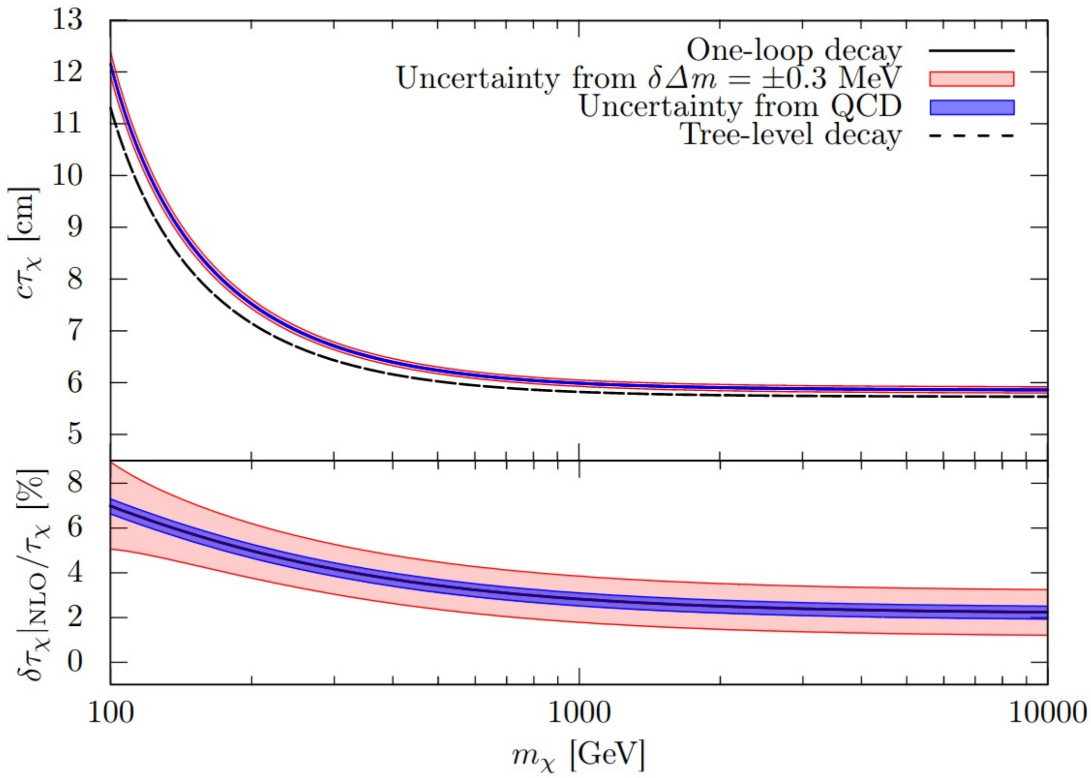
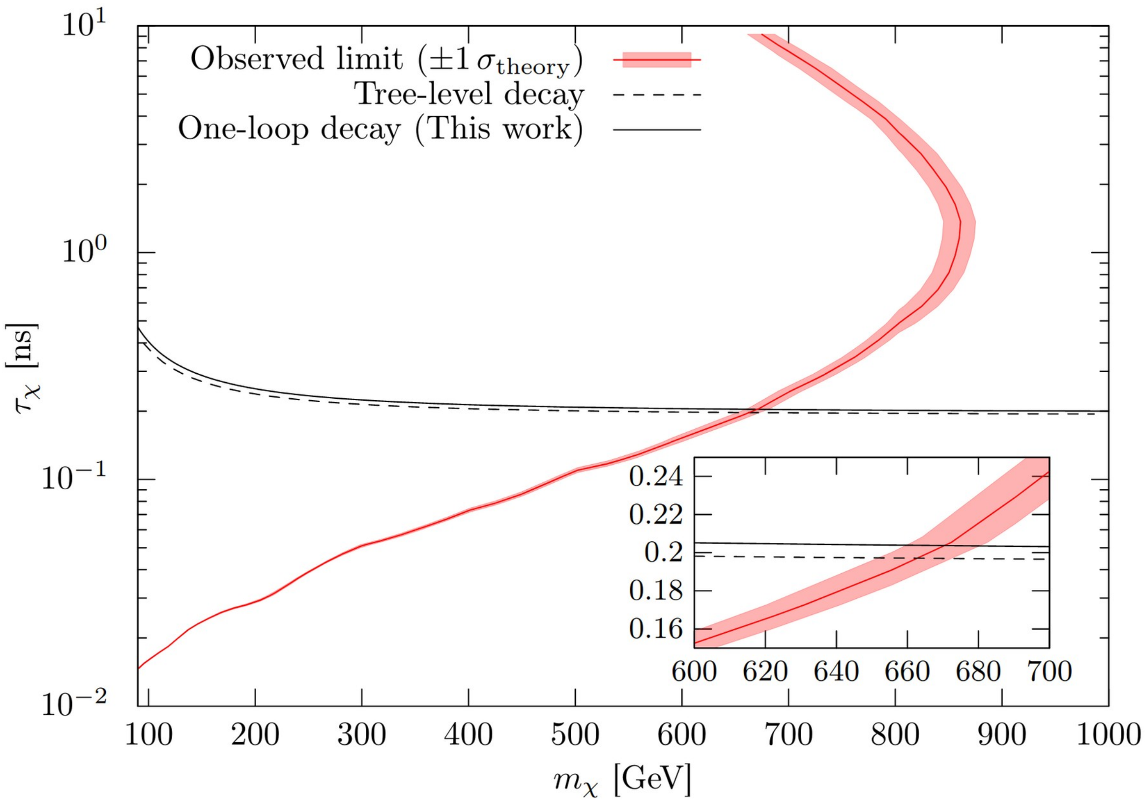
No Wino mass enhancement effect is found!

Final Result



Main theory errors from unknown piece of three-loop mass difference $\Delta_{3-\mathrm{loop}} m$

Impact on LHC Search

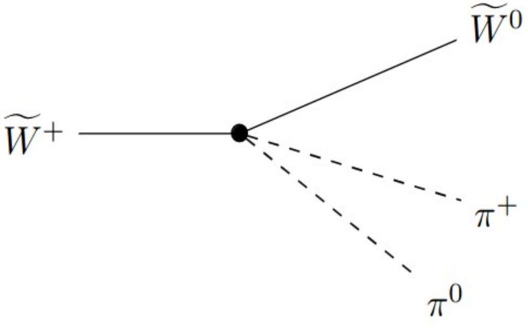


10 GeV shift

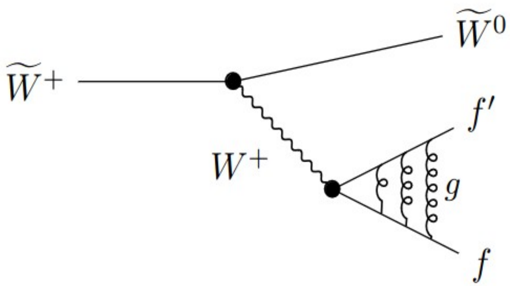
General Mass Difference

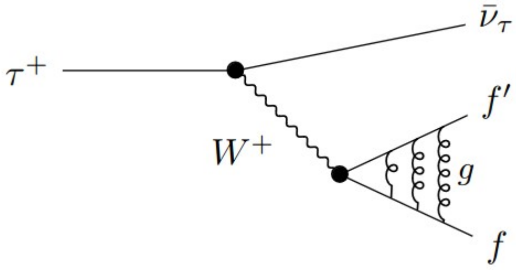
	Δm
Pure Wino DM	~160 MeV
Higgsino-like DM	~300 MeV – 2 GeV from gaugino mixing
5-plet DM	~160 MeV and ~500 MeV

For larger mass difference, lepton and multi-meson decay modes are dominant.



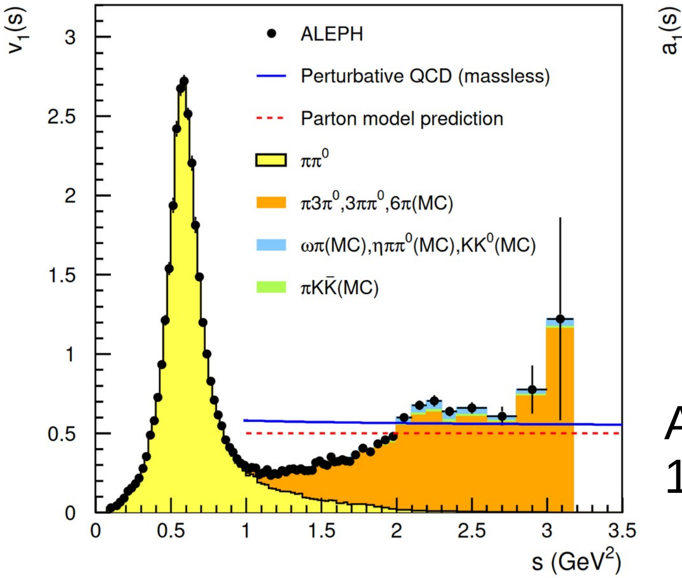
Tau Decay





Tau decay has similar structure.
Hadron data for tau decay is available for BSM particle decay.

[Chen, Drees & Gunion, 96]

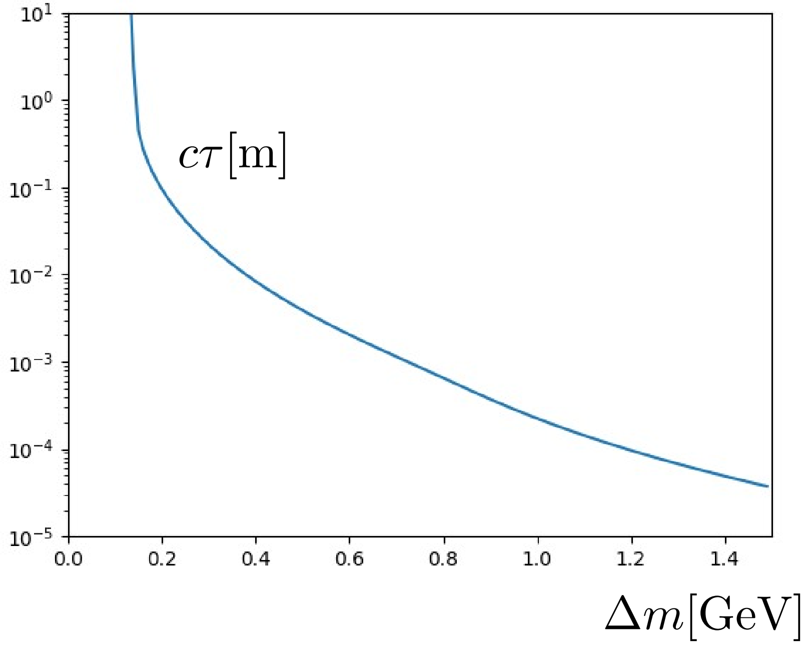


Aleph data
1312.1501

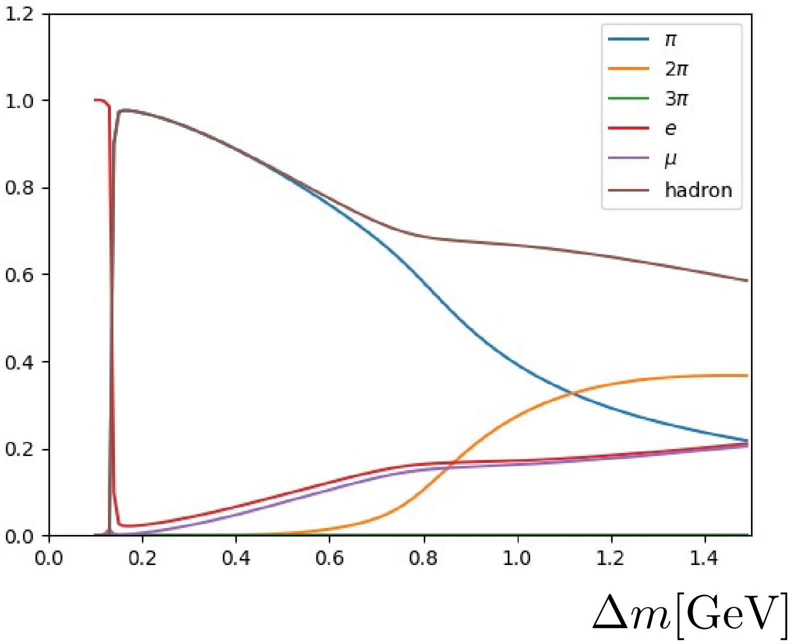
Charged Higgsino Decay

With EW correction and latest tau decay data

Decay length



Branching fraction



Summary

- Precise estimation of EW charged fermion is crucial for LHC search.
- All the large enhancements from heavy DM are completely canceled.
  - Non-relativistic version of Appelquist-Carazzone's decoupling theorem.
- Minor effect on the LHC searches.
- Application for more generic case, charge and mass difference.