An SZ-Like Effect on Stochastic Gravitational Wave Backgrounds

Particles, Strings, and Cosmology Conference

Marcell Howard

University of Pittsburgh

Co-authors: Morgane Konig, Tatsuya Daniel

arxiv: 2307.xxxxx

Co-authors

Outline

- Background and motivation
- Assumptions and derivation
- Results
- Conclusions

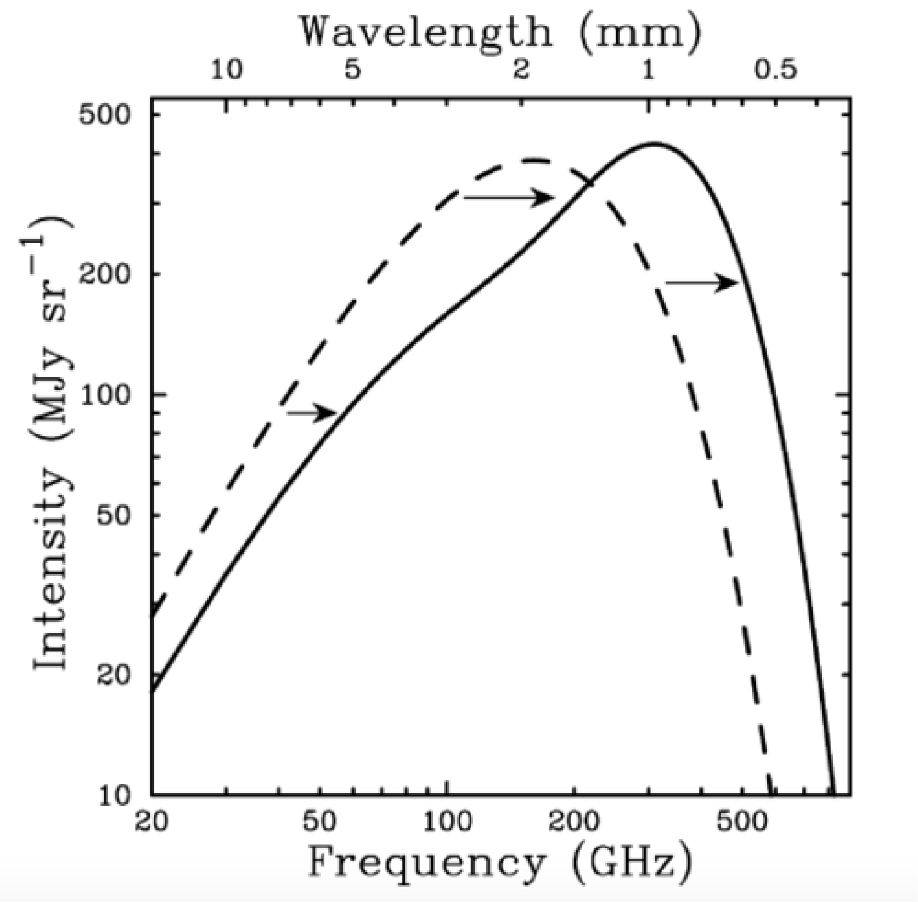
Stochastic Gravitational Wave Backgrounds

- Gravitational wave signals from unresolvable sources
- Astrophysical and Cosmological
- Many different potential sources for both:
  - Inflation, phase transitions, and many more

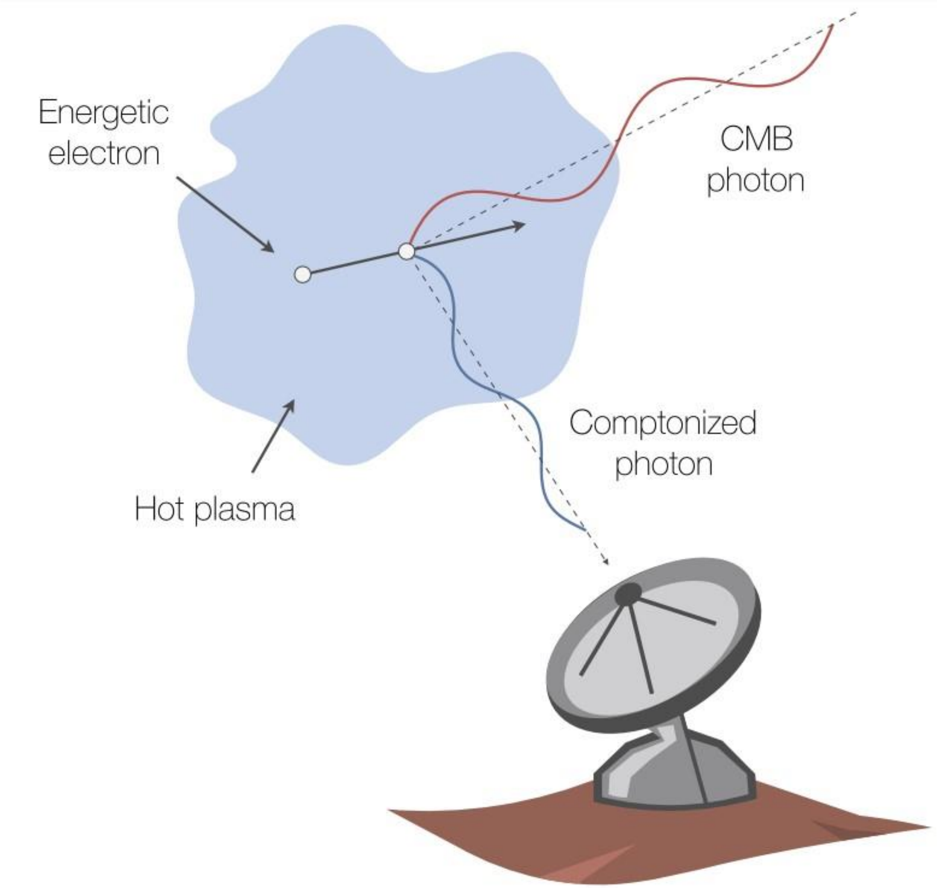
Why Primordial Gravitational Waves Are Interesting

- Gravitons decouple early
- Large optical depth
- (Some) inflation models can be constrained by LISA

The Sunyaev-Zeldovich Effect



Carlstrom et al. 2002



Mroczkowski et al. 2019

Order of Magnitude/Back of Envelope Estimate

- Very tiny effect!

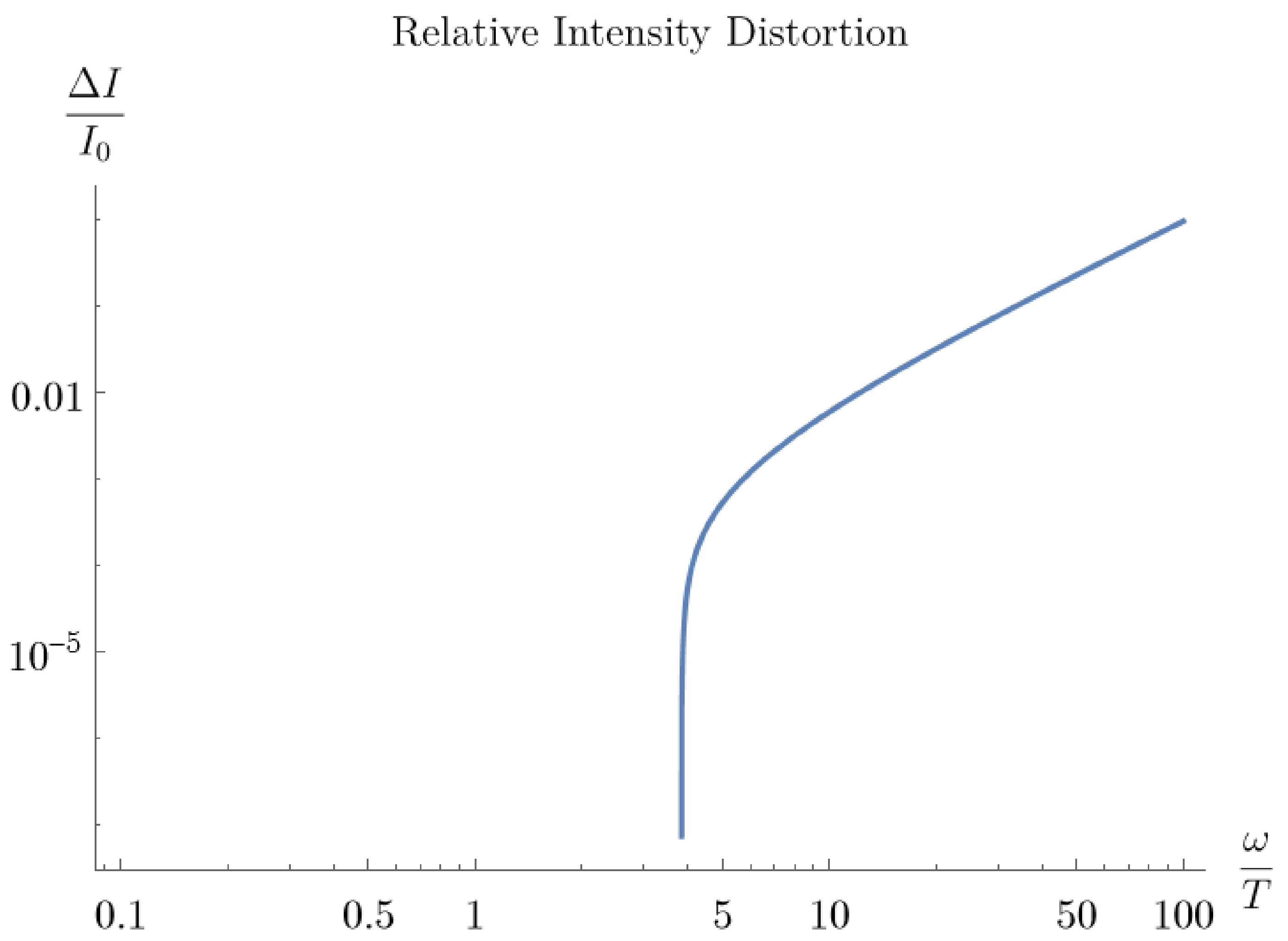
$$\Delta n \sim \tau \frac{T}{m}, \qquad \tau \sim n_\chi \sigma_s \ell$$

- For $m = 10\ \mathrm{TeV}, T = 2.7\mathrm{K},\ z = 10^7,\ \tau \sim 10^{-53},$

$$\Delta n \sim 10^{-70}$$

- We are more interested in the formalism here

Kompaneets Equation



$$\frac{\partial n}{\partial y} = \frac{1}{x_e^2}\frac{\partial}{\partial x_e}x_e^4\left(\frac{\partial n}{\partial x_e} + n + n^2\right) \longrightarrow \frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}x^4\frac{\partial n}{\partial x}, \qquad \frac{\partial n}{\partial x} \gg n, n^2,$$

$$\Delta n = xy\frac{e^x}{(e^x-1)^2}(x\coth(x/2) - 4), \qquad y = \int n_e \sigma_T \, \mathrm{d}\ell \, \frac{T_e}{m_e} = \tau_e \frac{T_e}{m_e}$$

BSM Candidate and Graviton Assumptions

- Minimal coupling to gravity
- Thermal equilibrium at temperature, T
- Non-relativistic $|\mathbf{p}|, T \ll m$
- Soft gravitons $\omega \ll m, \quad \omega \sim T$
- Small energy shift $\frac{\omega' - \omega}{T} = x' - x \equiv \Delta \ll 1$

Boltzmann-Master Equation

$$\frac{\partial n(\mathbf{k},t)}{\partial t} - H\omega\frac{\partial n}{\partial \omega} = a\int \mathrm{d}^3\mathbf{p}\int \mathrm{d}^3\mathbf{p}'\int \mathrm{d}^3\mathbf{k}'\left[\mathcal{N}(\mathbf{p}')n(\mathbf{k}',t)w(p',k'\to p,k)(1+n(\mathbf{k},t))\right.$$
$$\left. - \mathcal{N}(\mathbf{p})n(\mathbf{k},t)w(p,k\to p',k')(1+n(\mathbf{k}',t))\right],$$

$$w(p,k\to p',k')\,\mathrm{d}^3\mathbf{p}'\,\mathrm{d}^3\mathbf{k}' = \frac{\mathrm{d}^3\mathbf{p}'\,\mathrm{d}^3\mathbf{k}'}{\omega E\omega' E'}W(p,k\to p',k') = \left(1-\frac{\mathbf{p}}{m}\cdot\hat{\mathbf{n}}\right)\frac{\mathrm{d}\sigma_s(\mathbf{p},\mathbf{k},\mathbf{p}',\mathbf{k}')}{\mathrm{d}\Omega}\,\mathrm{d}\Omega\,,$$

$$\mathcal{N}(\mathbf{p})\,\mathrm{d}^3\mathbf{p} = \mathcal{N}_{\mathrm{eq}}(|\mathbf{p}|)\,\mathrm{d}^3\mathbf{p} = n_\chi(2\pi mT)^{-3/2}\exp\left(-\frac{\mathbf{p}^2}{2mT}\right)\mathrm{d}^3\mathbf{p}\,,\quad n_\chi = \frac{\rho_\chi}{m},$$

$$E' = E - T\Delta,\qquad \mathcal{N}(\mathbf{p}') = \mathcal{N}(\mathbf{p})\left(1+\Delta+\frac{\Delta^2}{2}\right)$$

$$\Delta(\omega,\mathbf{p},\hat{\mathbf{n}}',\hat{\mathbf{n}}) = \frac{\omega}{T}\frac{\mathbf{p}\cdot(\hat{\mathbf{n}}'-\hat{\mathbf{n}}) - \omega(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')}{(E-\mathbf{p}\cdot\hat{\mathbf{n}}'+\omega(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}'))}.$$

Gravitational Kompaneets Equation

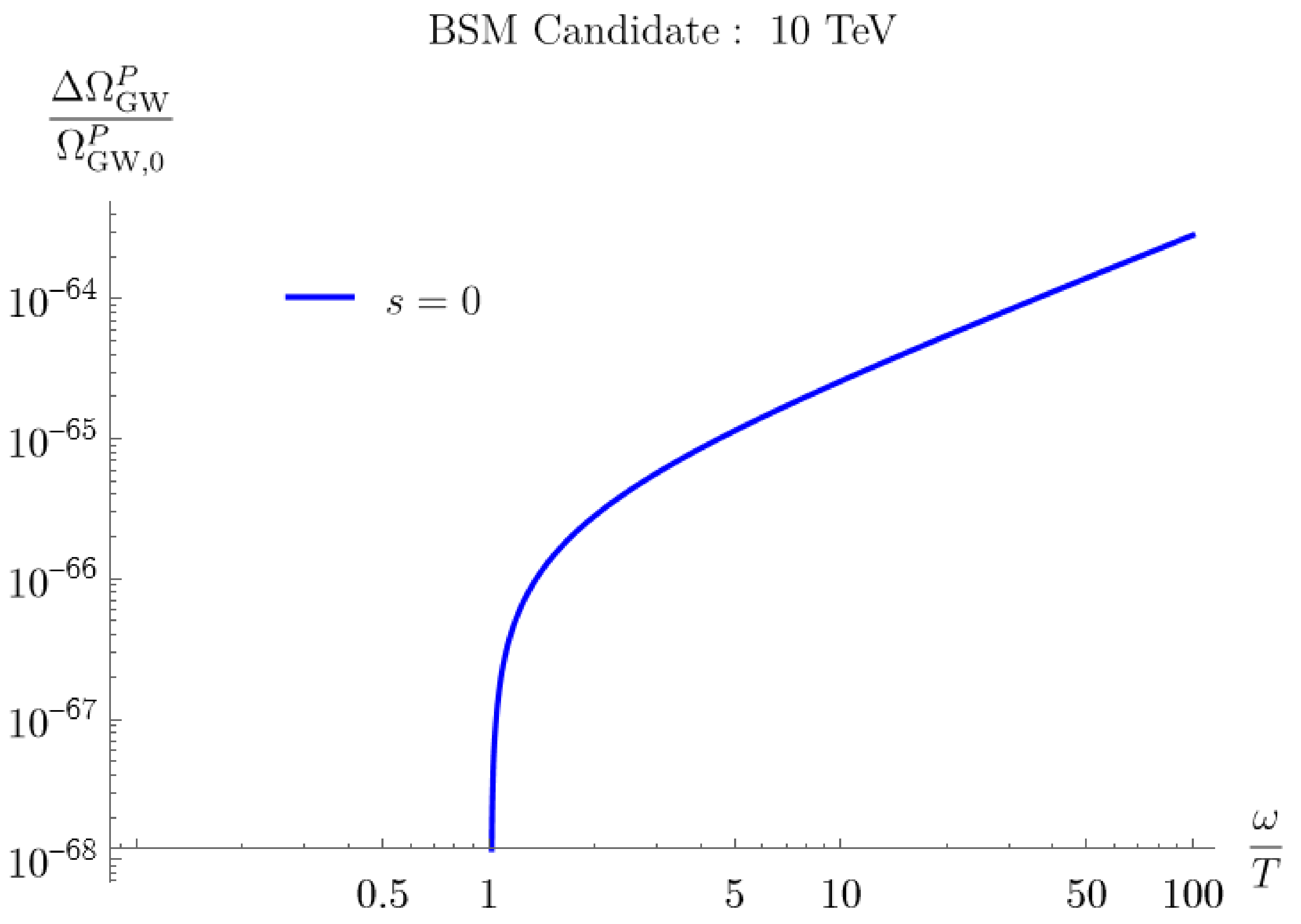
$$\frac{\partial n}{\partial t} = \frac{J_2(x;s)}{2}\frac{\partial}{\partial x}\left[\frac{\partial n}{\partial x} + n(1+n)\right] + \left(J_1(x;s) + \frac{J_2(x;s)}{2}\right)\left[\frac{\partial n}{\partial x} + n(1+n)\right],$$

$$J_\ell(x,\lambda;s) = 2\pi\int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \sin\theta\,\mathrm{d}\theta \int \mathrm{d}^3\mathbf{p}\left(1 - \frac{\mathbf{p}}{m}\cdot\hat{\mathbf{n}}\right)\frac{\mathrm{d}\sigma_s(\mathbf{p},x,\theta)}{\mathrm{d}\theta}\mathcal{N}_{\mathrm{eq}}(\mathbf{p})\Delta^\ell(\mathbf{p},x,\theta)$$

$$\Delta n(x,z) = y\frac{16}{\pi}\left(\frac{H}{M_{\mathrm{Pl}}}\right)^2\left[1 - \frac{2(\tilde{J}_1(x,\lambda;s) + \tilde{J}_2(x,\lambda;s))}{x} + \frac{6\tilde{J}_2(x,\lambda;s)}{2x^2}\right]x^{-2},$$

$$y = \tau_s(z_O, z_{em})\frac{T}{m}, \qquad \tau_s(z_O, z_{em}) = 16\pi\int_{z_{em}}^{z_O} n_\chi(z)\sigma_s(m,\omega,z)a(z)\frac{\mathrm{d}\eta}{\mathrm{d}z}\,\mathrm{d}z\,,$$

(Preliminary) Results



Future Work

- BSM Thermal Equilibrium
  - Add spin-1 (finished this morning!)
  - Higher spin
- Non-Thermal Candidates
  - Primordial Black Holes

Summary and Conclusions

An analogous SZ signal is present in the cosmic gravitational wave background

Particles of various masses, spins can leave distinguishable imprints

SZ signal scales with mass but is redshift dependent

Thank You

Marcell Howard
mah455@pitt.edu

References

- J.E. Carlstrom, G.P. Holder and E.D. Reese, Cosmology with the Sunyaev-Zel'dovich effect, Annual Review of Astronomy and Astrophysics 40 (2002) 643.
- T Mrockowski et al., Astrophysics with the Spatially and Spectrally Resolved Sunyaev-Zeldovich Effects: A Millimetre/Submillimetre Probe of the Warm and Hot Universe, Space Science Reviews 215 (2018) 1.

Backup Slide: Minimal Gravitational Couplings

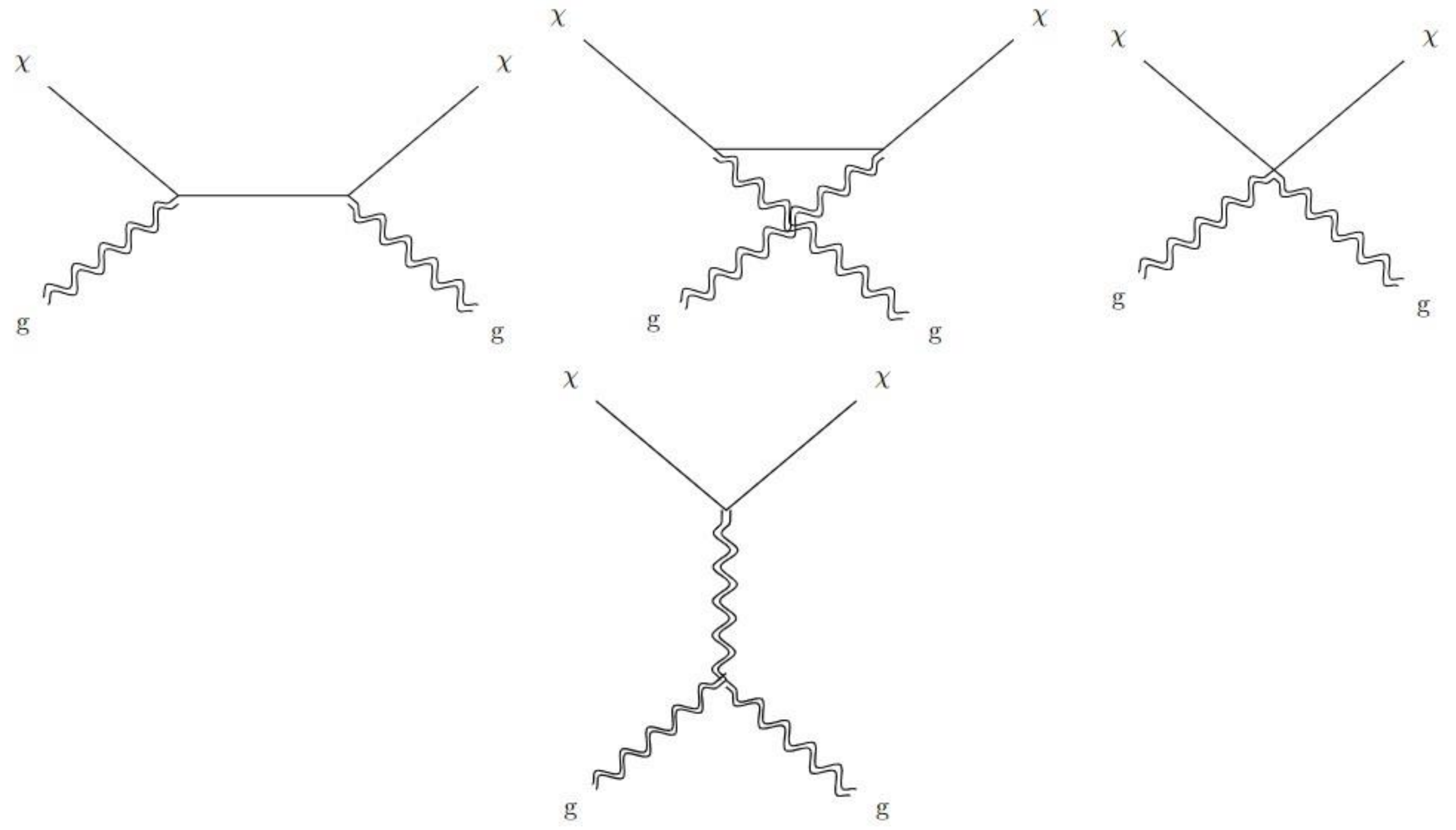
$$S_{s=0} = -\frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\left(g^{\mu\nu}\nabla_\mu\chi\nabla_\nu\chi + m^2\chi^2\right),$$

$$S_{s=\frac{1}{2}} = \int \mathrm{d}^4x\,\sqrt{-g}\,\overline{\chi}\left(i\not{D} - m\right)\chi,$$

$$S_{s=1} = \int \mathrm{d}^4x\,\sqrt{-g}\left[-\frac{1}{4}g^{\mu\nu}g^{\lambda\rho}\chi_{\mu\lambda}\chi_{\nu\rho} - \frac{1}{2}m^2 g^{\mu\nu}\chi_\mu\chi_\nu\right],$$

$$\not{D} = \gamma^\mu\left(\partial_\mu + \frac{1}{2}\omega_{\mu ab}\gamma^{ab}\right), \qquad \partial_\mu \equiv e^a_\mu\partial_a, \qquad \omega^{ab}_\mu = -e^{b\nu}\nabla_\mu e^a_\nu, \qquad \gamma^{ab} = -\frac{i}{4}\left[\gamma^a, \gamma^b\right]$$

Feynman Diagrams



Backup Slide: Gravitational Compton Amplitude and Cross Section

$$\frac{\mathrm{d}\sigma_s(\mathbf{p},\hat{\mathbf{n}},\hat{\mathbf{n}}')}{\mathrm{d}\Omega} = \frac{\sigma_s(\lambda)}{(4\pi)^2}\frac{1}{(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2}\left(\frac{\omega'(\mathbf{p},\hat{\mathbf{n}},\hat{\mathbf{n}}')}{\omega}\right)^2 \left|\overline{\mathcal{M}_s}\right|^2$$

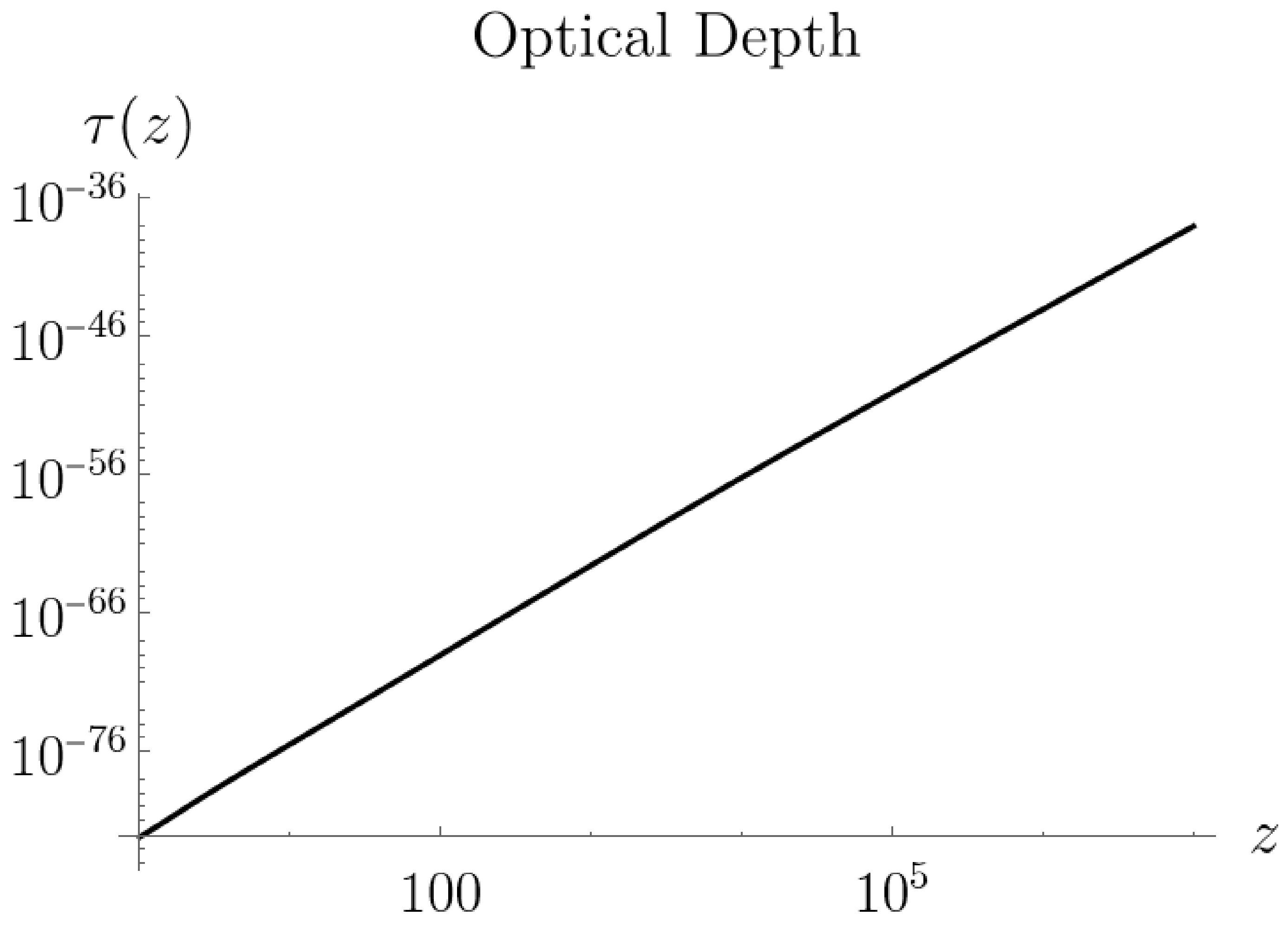
$$|\mathcal{M}_{s=0}|^2 = \frac{\kappa^4}{8}\left[\frac{m^8(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2}{(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2} + \frac{[2(E-\mathbf{p}\cdot\hat{\mathbf{n}})(E-\mathbf{p}\cdot\hat{\mathbf{n}}') - m^2(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')]^4}{(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2}\right]$$

$$\left|\mathcal{M}_{s=\frac{1}{2}}\right|^2 = \frac{\kappa^4}{4}\left[\frac{m^6(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2(m^2+\omega\omega'(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}'))}{(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2} + \frac{[2(E-\mathbf{p}\cdot\hat{\mathbf{n}})(E-\mathbf{p}\cdot\hat{\mathbf{n}}') - m^2(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')]^3}{(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2}\times\right.$$
$$\left.\times\left(\frac{\omega'}{\omega}(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2 + \frac{\omega}{\omega'}(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2 - m^2(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')\right)\right]$$

$$|\mathcal{M}_{s=1}|^2 = \frac{\kappa^4}{2}\left[\frac{m^8(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2 + 4m^4(\omega\omega')^2(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')}{(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2} + \frac{2m^6(m^2+\omega\omega'(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}'))(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2}{(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2}\right.$$
$$\left. + \frac{[2(E-\mathbf{p}\cdot\hat{\mathbf{n}})(E-\mathbf{p}\cdot\hat{\mathbf{n}}') - m^2(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')]^2}{(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')^2(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2}\left(\frac{\omega'}{\omega}(E-\mathbf{p}\cdot\hat{\mathbf{n}}')^2 + \frac{\omega}{\omega'}(E-\mathbf{p}\cdot\hat{\mathbf{n}})^2 - m^2(1-\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}')\right)^2\right],$$

Backup Slide: Optical Depth

$$\tau_s(z_O, z_{em}) = 16\pi \int_{\eta_{em}(z_{em})}^{\eta_O(z_O)} \mathrm{d}\eta\, n_\chi(\eta)\sigma(m,\eta)a(\eta)$$



Backup Slide: Gravitational Compton Cross Section Cont.

$$\sigma_{s=0}(m,\lambda) = 2\pi(Gm)^2 \int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \mathrm{d}\theta \left(\frac{\omega'(\theta)}{\omega}\right)^2 \left[\cot^4\left(\frac{\theta}{2}\right)\cos^4\left(\frac{\theta}{2}\right) + \sin^4\left(\frac{\theta}{2}\right)\right]$$

$$\begin{aligned}\sigma_{s=1/2}(m,\lambda) = 2\pi(Gm)^2 \int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \mathrm{d}\theta \left(\frac{\omega'(\theta)}{\omega}\right)^3 &\left[\cot^4\left(\frac{\theta}{2}\right)\cos^4\left(\frac{\theta}{2}\right) + \sin^4\left(\frac{\theta}{2}\right)\right. \\ &\left. + \frac{2\omega}{m}\left(\cot^2\left(\frac{\theta}{2}\right)\cos^6\left(\frac{\theta}{2}\right) + \sin^6\left(\frac{\theta}{2}\right)\right) + 2\left(\frac{\omega}{m}\right)^2\left(\cos^6\left(\frac{\theta}{2}\right) + \sin^6\left(\frac{\theta}{2}\right)\right)\right]\end{aligned}$$

$$\begin{aligned}\sigma_{s=1}(m,\lambda) = 2\pi(Gm)^2 \int_{\theta_{\min}(\lambda)}^{\theta_{\max}} \mathrm{d}\theta \left(\frac{\omega'(\theta)}{\omega}\right)^4 &\left[\left(\cot^4\left(\frac{\theta}{2}\right)\cos^4\left(\frac{\theta}{2}\right) + \sin^4\left(\frac{\theta}{2}\right)\right)\left(1 + \frac{2\omega\sin^2\left(\frac{\theta}{2}\right)}{m}\right)^2\right. \\ &\left. + \frac{16}{3}\frac{\omega^2}{m^2}\left(\cos^6\left(\frac{\theta}{2}\right) + \sin^6\left(\frac{\theta}{2}\right)\right)\left(1 + \frac{2\omega\sin^2\left(\frac{\theta}{2}\right)}{m}\right) + \frac{16}{3}\frac{\omega^4}{m^4}\sin^2\left(\frac{\theta}{2}\right)\left(\cos^4\left(\frac{\theta}{2}\right) + \sin^4\left(\frac{\theta}{2}\right)\right)\right],\end{aligned}$$

Infrared Divergence of Low Energy Coulomb Scattering

- The cross section has an infrared divergence due to Coulomb scattering:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \frac{1}{\theta^4}, \quad \text{for} \quad \theta \to 0$$

- Introduce a cutoff at Geometric Optics limit:

$$\theta_{\text{min}} = \frac{\lambda_C}{b_{\text{max}}}, \qquad b_{\text{max}} = (2\sqrt{3}\lambda^2 r_s)^{\frac{1}{3}}, \qquad r_s = 2GM$$