Role of dimension-eight operators in an EFT for the 2HDM

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The Standard Model and SMEFT

- The Standard Model (SM) is the renormalizable field theory of $SU(3) \times SU(2) \times U(1)$ gauge theory describing electroweak and strong interactions among the known fields
- Has three generations of quarks and leptons
- Has one Higgs doublet
- Has no right-handed neutrinos
- The Standard Model Effective Field Theory (SMEFT) includes non-renormalizable operators with the same field content

	spin	$SU(3)$	$SU(2)$	$U(1)$
H	0	**1**	**2**	1
ℓ_L	$\frac{1}{2}$	**1**	**2**	-1
e_R	$\frac{1}{2}$	**1**	**1**	-2
q_L	$\frac{1}{2}$	**3**	**2**	$\frac{1}{3}$
u_R	$\frac{1}{2}$	**3**	**1**	$\frac{4}{3}$
d_R	$\frac{1}{2}$	**3**	**1**	$-\frac{2}{3}$

What SMEFT looks like (at dimension 6)

- So-called "Warsaw basis", Grzadkowski et al, J. High Energ. Phys. 2010, 85 (2010).

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p d_r \varphi)$
$Q_{\widetilde{W}}$	$\varepsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu}G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{l}_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}^A_{\mu\nu}G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{l}_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu}W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{\varphi}\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{e}_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}^I_{\mu\nu}W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{\varphi}\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{q}_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{\varphi}\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi)(\bar{q}_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{u}_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu}B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\varphi)(\bar{d}_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}^I_{\mu\nu}B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde{\varphi}^\dagger D_\mu\varphi)(\bar{u}_p\gamma^\mu d_r)$

Table 2: Dimension-six operators other than the four-fermion ones.

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$				

Table 3: Four-fermion operators.

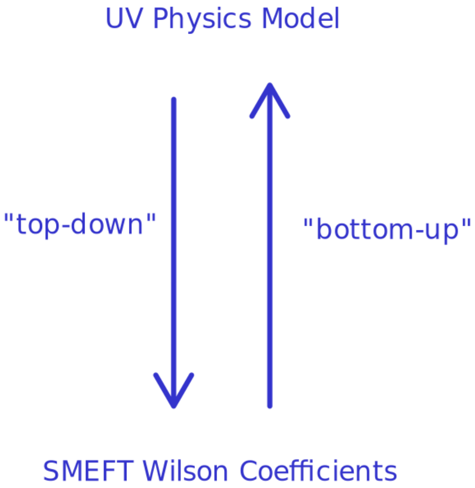
- 59 B-conserving operators not including flavor
- 2499 (!) B-conserving operators with flavor structure

Dimension 8 operators

- Murphy, JHEP 10 (2020) 174, and Li et al., Phys.Rev.D 104 (2021) 1, 015026, wrote down a complete basis of dimension 8 SMEFT operators
- There are **44807** operators when including flavor structure
  - Beyond feasible to include all dimension 8 operators in any bottom-up analysis
- Tiny sample of some dim 8 operators (one of many tables on many pages):

$\mathbf{8 : XH^4D^2}$	
$Q^{(1)}_{WH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H) W^I_{\mu\nu}$
$Q^{(2)}_{WH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger \tau^I D^\nu H) \widetilde{W}^I_{\mu\nu}$
$Q^{(3)}_{WH^4D^2}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H) W^K_{\mu\nu}$
$Q^{(4)}_{WH^4D^2}$	$\epsilon^{IJK}(H^\dagger \tau^I H)(D^\mu H^\dagger \tau^J D^\nu H) \widetilde{W}^K_{\mu\nu}$
$Q^{(1)}_{BH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H) B_{\mu\nu}$
$Q^{(2)}_{BH^4D^2}$	$(H^\dagger H)(D^\mu H^\dagger D^\nu H) \widetilde{B}_{\mu\nu}$

Top-down vs bottom-up approaches



- Bottom-up approach starts with arbitrary Wilson coefficients, tries to get to UV model
    - E.g. experiments fits to Wilson coefficients, then attempts to explain what model any deviations could come from
- Top-down approach starts with UV model, then matches onto SMEFT to get Wilson coefficients in SMEFT
    - This is the sort of analysis I will talk about with the 2HDM

Two Higgs doublet model

- Two Higgs doublet models (2HDMs) are extremely popular scalar sector extensions
- Doublets don't mess up electroweak precision
- Most of the literature focuses on the case of (softly broken) Z_2 symmetry, to remove tree-level flavor-changing neutral currents, and with no CP violation in the scalar sector
- There are multiple different "types" which have different Yukawa relations

Two Higgs doublet model Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm kin} &= (D_\mu H_1)^\dagger (D^\mu H_1) + (D_\mu H_2)^\dagger (D^\mu H_2) \\
V &= Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + \left(Y_3 H_1^\dagger H_2 + \text{h.c.} \right) \\
&+ \frac{Z_1}{2} \left(H_1^\dagger H_1 \right)^2 + \frac{Z_2}{2} \left(H_2^\dagger H_2 \right)^2 \\
&+ Z_3 \left(H_1^\dagger H_1 \right) \left(H_2^\dagger H_2 \right) + Z_4 \left(H_1^\dagger H_2 \right) \left(H_2^\dagger H_1 \right) \\
&+ \left\{ \frac{Z_5}{2} \left(H_1^\dagger H_2 \right)^2 + Z_6 \left(H_1^\dagger H_1 \right) \left(H_1^\dagger H_2 \right) \right. \\
&+ \left. Z_7 \left(H_2^\dagger H_2 \right) \left(H_1^\dagger H_2 \right) + \text{h.c.} \right\} \\
\mathcal{L}_Y &= -\lambda_u^{(1)} \bar{u}_R \tilde{H}_1^\dagger q_L - \lambda_u^{(2)} \bar{u}_R \tilde{H}_2^\dagger q_L - \lambda_d^{(1)} \bar{d}_R H_1^\dagger q_L - \lambda_d^{(2)} \bar{d}_R H_2^\dagger q_L \\
&- \lambda_l^{(1)} \bar{e}_R H_1^\dagger l_L - \lambda_l^{(2)} \bar{e}_R H_2^\dagger l_L + \text{h.c.}
\end{aligned} \tag{1}
$$

Particle content of the 2HDM

- By performing a field redefinition such that only H_1 gets a vev (the so-called Higgs basis) the doublets break down as follows:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}\left(v + \sin(\beta-\alpha)\, h_{125} + \cos(\beta-\alpha)\, H_0 + iG_0\right) \end{pmatrix}$$

$$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}\left(\cos(\beta-\alpha)\, h_{125} - \sin(\beta-\alpha)\, H_0 + iA\right) \end{pmatrix}$$

- h_{125} is the 125 GeV light scalar state, H_0, A, H^+ are the heavy scalar states, G_0, G^+ are the Goldstones
- The mixing $\beta - \alpha$ changes the couplings of h_{125} to other Standard Model particles

Types of 2HDMs

- The Yukawas in the Higgs basis can be written as:

$$\lambda_f^{(1)} = \frac{\sqrt{2}}{v} m_f, \qquad \lambda_f^{(2)} = \frac{\eta_f}{\tan\beta} \lambda_f^{(1)}$$

- For the different types of 2HDM, η_f takes the following values:

	Type-I	Type-II	Type-L	Type-F
η_u	1	1	1	1
η_d	1	$-\tan^2\beta$	1	$-\tan^2\beta$
η_l	1	$-\tan^2\beta$	$-\tan^2\beta$	1

- $\tan\beta$ is the ratio of vevs from the Z_2 symmetric basis

Matching the 2HDM to dimension 8 at tree level

- $F_{n,m}$ denotes terms suppressed by $1/\Lambda^{(n-4)}$ of operator dimension m

$$
\begin{aligned}
F_{6,2} &= |Y_3|^2 (H_1^\dagger H_1), \\
F_{6,4} &= Y_3 \lambda_u^{(2)*} H_1^\dagger \widehat{q}_L u_R + Y_3 \lambda_d^{(2)} \bar{d}_R H_1^\dagger q_L + Y_3 Z_6^* (H_1^\dagger H_1)^2 + \text{h.c.}, \\
F_{6,6} &= (H_1^\dagger H_1) \left[|Z_6|^2 (H_1^\dagger H_1)^2 + \left\{ Z_6 \lambda_u^{(2)*} H_1^\dagger \widehat{q}_L u_R + Z_6 \lambda_d^{(2)} \bar{d}_R H_1^\dagger q_L + \text{h.c.} \right\} \right] + 4\text{F}
\end{aligned}
$$

$$
\begin{aligned}
F_{8,4} &= |Y_3|^2 (D_\mu H_1)^\dagger (D^\mu H_1) - (H_1^\dagger H_1)^2 \left[|Y_3|^2 Z_{34} + \frac{1}{2}(Y_3)^2 Z_5^* + \frac{1}{2}(Y_3^*)^2 Z_5 \right], \qquad (22a) \\
F_{8,6} &= \{ Y_3 Z_6^* + Y_3^* Z_6 \} (H_1^\dagger H_1)(D_\mu H_1)^\dagger (D^\mu H_1) + \{ Y_3 Z_6^* (D_\mu H_1)^\dagger H_1 + \text{h.c.} \} \partial^\mu (H_1^\dagger H_1) \\
&\quad + \left\{ Y_3^* \lambda_u^{(2)} \left(D_\mu (\widehat{q}_L u_R) \right)^\dagger (D^\mu H_1) + Y_3^* \lambda_d^{(2)*} \left(D_\mu (\bar{d}_R q_L) \right)^\dagger (D^\mu H_1) + \text{h.c.} \right\} \\
&\quad - (H_1^\dagger H_1)^3 \left[Y_3 Z_{34} Z_6^* + Y_3 Z_5^* Z_6 + \text{h.c.} \right] \\
&\quad - (H_1^\dagger H_1) \left[H_1^\dagger \widehat{q}_L u_R \left(Y_3 Z_{34} \lambda_u^{(2)*} + Y_3^* Z_5 \lambda_u^{(2)*} \right) + \bar{d}_R H_1^\dagger q_L \left(Y_3 Z_{34} \lambda_d^{(2)} + Y_3^* Z_5 \lambda_d^{(2)} \right) + \text{h.c.} \right] \qquad (22b) \\
F_{8,8} &= |Z_6|^2 (H_1^\dagger H_1)^2 (D_\mu H_1)^\dagger (D^\mu H_1) + 2|Z_6|^2 (H_1^\dagger H_1) \partial_\mu (H_1^\dagger H_1) \partial^\mu (H_1^\dagger H_1) \\
&\quad - (H_1^\dagger H_1)^4 \left[Z_{34} |Z_6|^2 + \frac{1}{2} Z_5^* Z_6^2 + \frac{1}{2} Z_5 (Z_6^*)^2 \right] \\
&\quad - (H_1^\dagger H_1)^2 \left[H_1^\dagger \widehat{q}_L u_R \left(Z_{34} Z_6 \lambda_u^{(2)*} + Z_5 Z_6^* \lambda_u^{(2)*} \right) + \bar{d}_R H_1^\dagger q_L \left(Z_{34} Z_6 \lambda_d^{(2)} + Z_5 Z_6^* \lambda_d^{(2)} \right) + \text{h.c.} \right] \\
&\quad + \left\{ \left[Z_6^* \lambda_u^{(2)} \left(D_\mu (\widehat{q}_L u_R) \right)^\dagger + Z_6^* \lambda_d^{(2)*} \left(D_\mu (\bar{d}_R q_L) \right)^\dagger \right] \left[\partial^\mu (H_1^\dagger H_1) H_1 + (H_1^\dagger H_1)(D^\mu H_1) \right] + \text{h.c.} \right\} \\
&\quad + 4\text{F}, \qquad (22c)
\end{aligned}
$$

- We ignore the 4-fermion operators

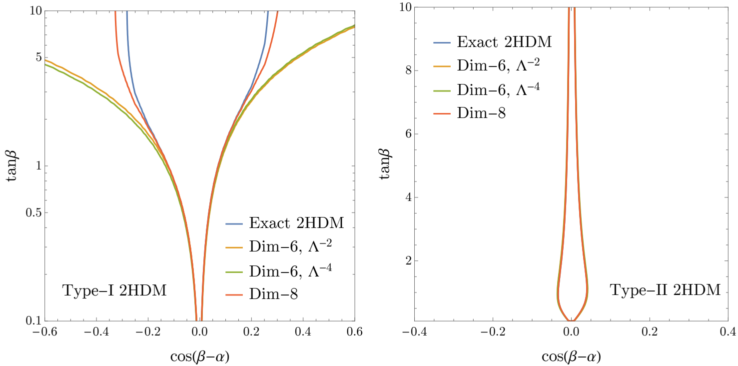
Physical parameters in the 2HDM and power counting

- Practically all 2HDM limit plots are in terms of $\tan\beta$ and $\cos(\beta-\alpha)$; we really want to change to these from the Lagrangian parameters after we do the matching
- We will also take the decoupling limit:

$$m_A^2 \sim m_{H_0}^2 \sim m_{H^\pm}^2 \sim Y_2 \equiv \Lambda^2 \gg v^2, \qquad m_h^2 \simeq v^2$$

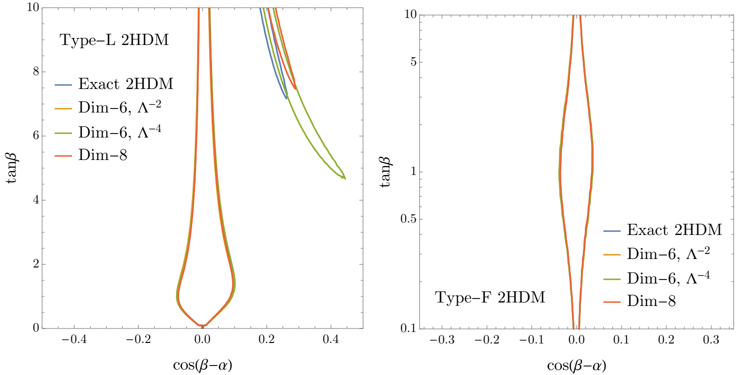
- Decoupling requires $\cos(\beta-\alpha) \sim v^2/\Lambda^2$
- Keeping a consistent power counting during the conversion is key: we matched up to $\mathcal{O}(\Lambda^{-4})$, so we should only keep expressions in terms of physical parameters up to $\mathcal{O}(\Lambda^{-4})$

Are the dimension 8 terms relevant?



- Limits for exact 2HDM, dimension 6 expansion, dimension 6 expansion including squared terms, and dimension 8 expansion
- Type-I is not reproduced well until dimension 8!
- Type-II is already well-constrained even with just dimension 6 matching, in contrast

Dimension 8 effects for the other types



- Type-L and Type-F are well-described by dimension 6 except for the second, disconnected region in type-L, where the EFT contribution to lepton Yukawa couplings dominates over the SM contribution

Why does type-I need dimension 8?

- In the Type-I model, all the Yukawas of the heavy doublet were suppressed by $\tan\beta$, and the high $\tan\beta$ region is where dimension 8 is important
- The 2HDM also changes the couplings of the 125 GeV Higgs to W and Z bosons; where is that in the matching?
- That comes only from the following Wilson coefficient at dimension 8

$$\frac{C^{(1)}_{\mathcal{H}^6}}{\Lambda^4} = -\cos(\beta-\alpha)^2\,(\sqrt{2}G_F)^2$$

- This Wilson coefficient corresponds to the dimension 8 operator $(H^\dagger H)^2 D_\mu H^\dagger D^\mu H^\dagger$
- So, for all types, the 2HDM doesn't change hWW and hZZ couplings at dimension 6, which are important for constraining the Type-I model

Is this generic?

- There is a similar dimension 6 operator $H^\dagger H D_\mu H^\dagger D^\mu H^\dagger$
  - It matches onto $(H^\dagger H)\Box(H^\dagger H)$ and other dimension 6 operators in the Warsaw basis using field redefinitions
- Both this operator and the dimension 8 operator $(H^\dagger H)^2 D_\mu H^\dagger D^\mu H^\dagger$ have similar effects
  - hWW and hZZ couplings
  - Momentum-dependent hhh couplings
- The 2HDM happens to be a model that doesn't generate this dimension 6 operator
- Other models *can* generate the dimension 6 operator at tree-level, like scalar or vector triplets or singlets

Conclusions

- A top-down analysis of the 2HDM shows that including dimension 8 operators can be necessary, since the hWW and hZZ coupling changes are missing at dimension 6
- Even so, going to dimension 8 is opening Pandora's box of 44807 additional Wilson coefficients
- Some general SMEFT takeaways:
  - SMEFT has more subtleties than one might think
  - You can't be sure that dimension 6 matching is good enough for a model without checking
  - Determining the UV model from measurements of Wilson coefficients requires accurately attributing effects to the correct operators

Thank you!