GRAVITATIONAL PARTICLE PRODUCTION WITH NON-MINIMAL COUPLING

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Based on: arXiv:2206.08940, arXiv:2303.07359 w/ M. A.G. Garcia and M. Pierre

PASCOS 2023

JUNE 29TH, 2023

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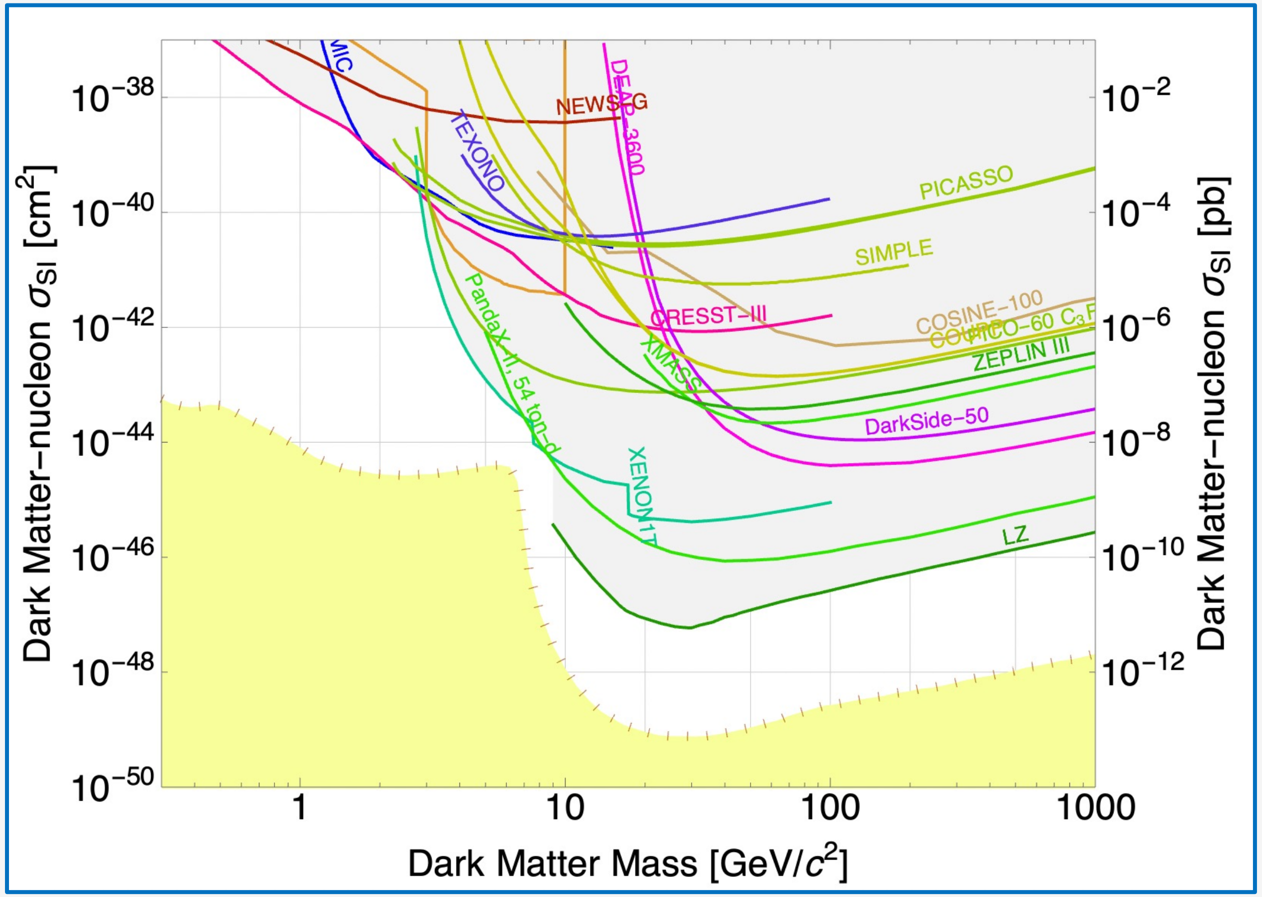
Motivation

WIMPS have not been detected yet!



SuperCDMS Dark Matter Limit Plotter

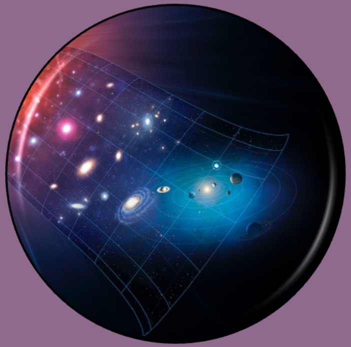
Consider feebly-interacting massive particles (FIMPs) instead:

- They are never in thermal equilibrium.
- Production occurs via freeze-in mechanism.
- Avoids direct and indirect detection.
- Depends on the initial conditions of inflation and the reheating mechanism.

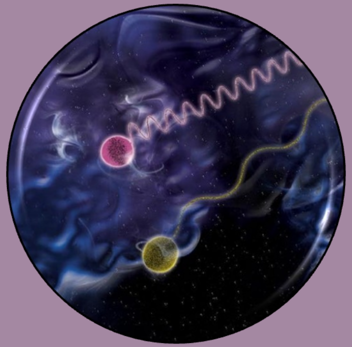
In such models, the coupling between the visible and dark sectors is assumed to be sufficiently small to ensure out-of-equilibrium production.

One can consider purely gravitational production of dark matter!

Overview



Particle creation in expanding universes.

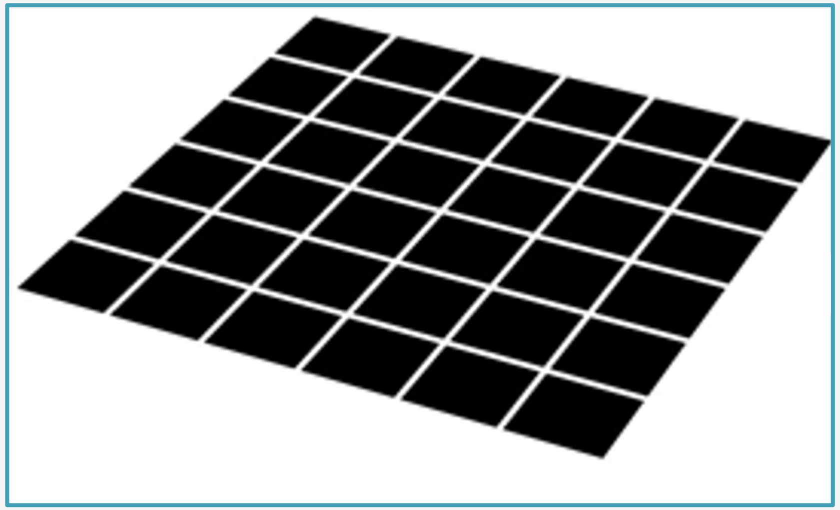


Gravitationally produced scalar dark matter.

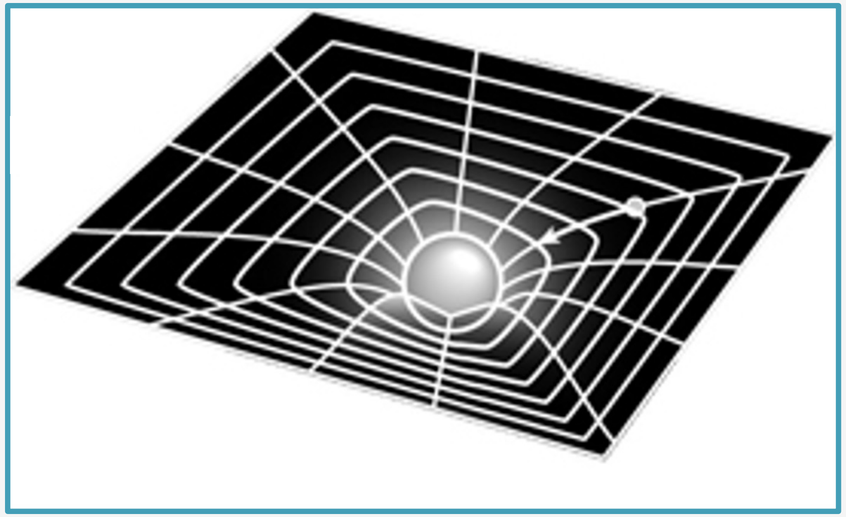
Dark matter creation during inflation and reheating.

Quantization in Curved Spacetime

Our starting point: QFT in curved spacetime.



Flat spacetime



Curved spacetime

- Massive scalar field action in curved spacetime:

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial_\mu\phi)^2 - V(\phi) + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 - \frac{1}{2}(M_P^2 - \xi\chi^2)R\right]$$

$g \equiv \det(g_{\mu\nu})$ is the metric determinant

$M_P = 1/\sqrt{8\pi G_N} \simeq 2.435 \times 10^{18}$ GeV

R is the Ricci curvature

Inflationary Potential

DM Mass Term

DM Coupling to the Curvature

Non-Perturbative Production of Dark Matter

We introduce a scalar dark matter field, χ,

$$V_\chi = \frac{1}{2}m^2\chi^2$$

Dark matter action

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi\chi^2 R\right]$$

Non-Minimal Coupling

It leads to : $$(\partial_\eta^2 - \nabla^2 + a^2 m_{\text{eff}}^2)X = 0, \qquad m_{\text{eff}}^2 = m_\chi^2 + \frac{1}{6}(1-6\xi)R$$

↓

Introduce a rescaled field : $$X \equiv a\chi$$

V. Mukhanov and S. Winitzki "Quantum Effects in gravity"

Conformal time

$$dt/d\tau = a$$

$$' \equiv d/d\tau$$

The momentum modes of the field X

$$X(\tau,\mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}}\left[X_k(\tau)\hat{a}_\mathbf{k} + X_k^*(\tau)\hat{a}_{-\mathbf{k}}^\dagger\right]$$

where $\mathbf{k}$ is the comoving momentum and $\hat{a}_\mathbf{k}^\dagger$ and $\hat{a}_\mathbf{k}$ are the creation and annihilation operators

Non-Perturbative Production of Dark Matter

We impose the Wronskian condition

$$X_k X_k^{*\prime} - X_k^* X_k' \;=\; i$$

To solve the mode equations, we choose the positive frequency of the Bunch-Davies vacuum:

$$X_k(\tau_0) \;=\; \frac{1}{\sqrt{2\omega_k}}\,, \qquad X_k'(\tau_0) \;=\; -\frac{i\omega_k}{\sqrt{2\omega_k}}$$

T.S Bunch and P.C. W. Davies, 1977

These initial conditions correspond to the zero-particle initial state.

We find the mode equations:

$$X_k^2 + \omega_k^2 X_k \;=\; 0$$

Angular Frequency

$$\omega_k^2 = k^2 + a^2 m_{\text{eff}}^2$$

$$m_{\text{eff}}^2 = m_\chi^2 + \frac{1}{6}(1-6\xi)R$$

de Sitter value

$$R \simeq -12H^2$$

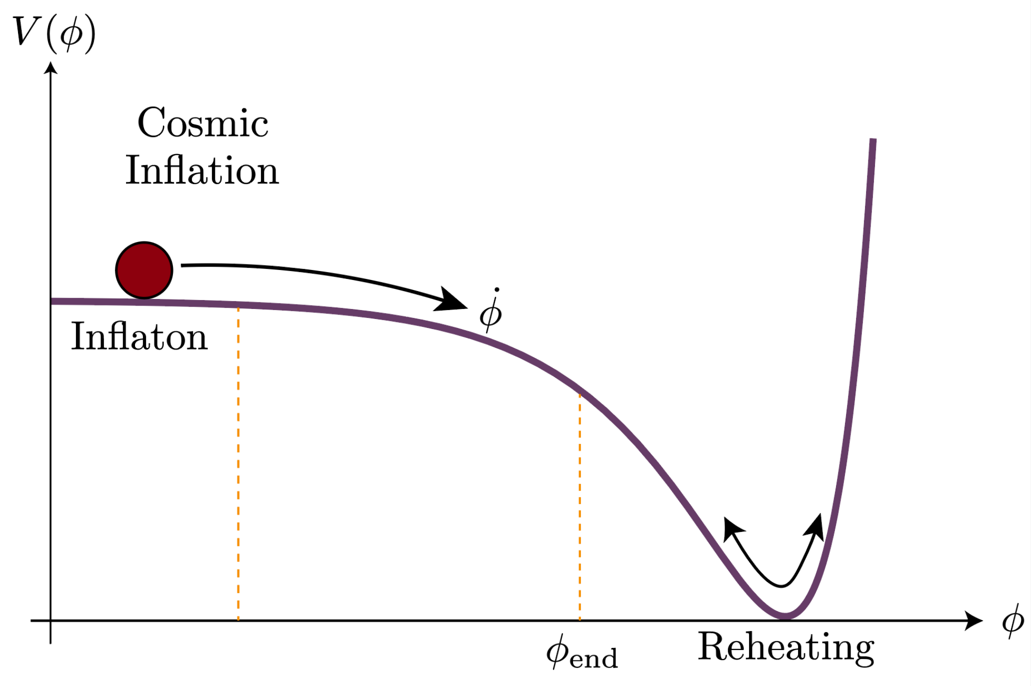
Minimal Coupling

$$m_{\text{eff}}^2 = m_\chi^2 - 2H^2$$

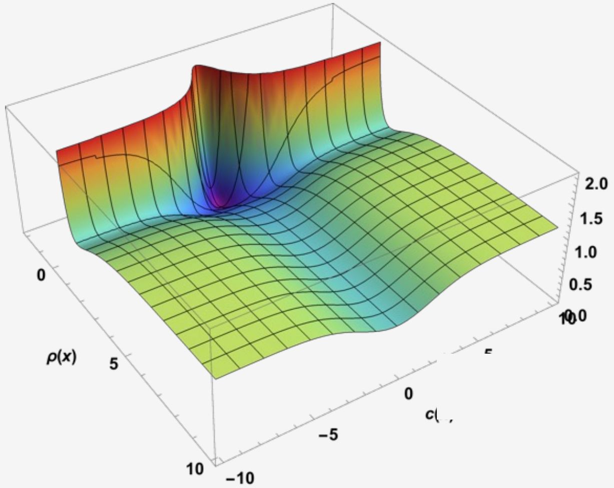
Conformal Coupling

$$m_{\text{eff}}^2 = m_\chi^2$$

Dark Matter Production During Inflation



End of inflation, when $V_{\rm end} = \dot{\phi}^2_{\rm end}$ and $H^2_{\rm end} M_P^2 = V_{\rm end}/2$



$$\omega_k^2 \equiv k^2 + a^2 \left(m_\chi^2 + \frac{1}{6}(1-6\xi)R \right)$$

Since the scale factor a grows, the mode frequency increases
This leads to the growth of n_k.

n_χ – comoving number density

$$n_\chi \left(\frac{a}{a_{\rm end}} \right)^3 = \int_{k_0}^{\infty} \frac{dk}{k} \mathcal{N}_k$$

$a_{\rm end}$ – scale factor at the end of inflation

$\mathcal{N}_k$ – comoving number density spectrum

$$\mathcal{N}_k = \frac{k^3}{2\pi^2} f_\chi(k,t)$$

$f_\chi(k,t)$ – DM phase space distribution

Phase Space Distribution

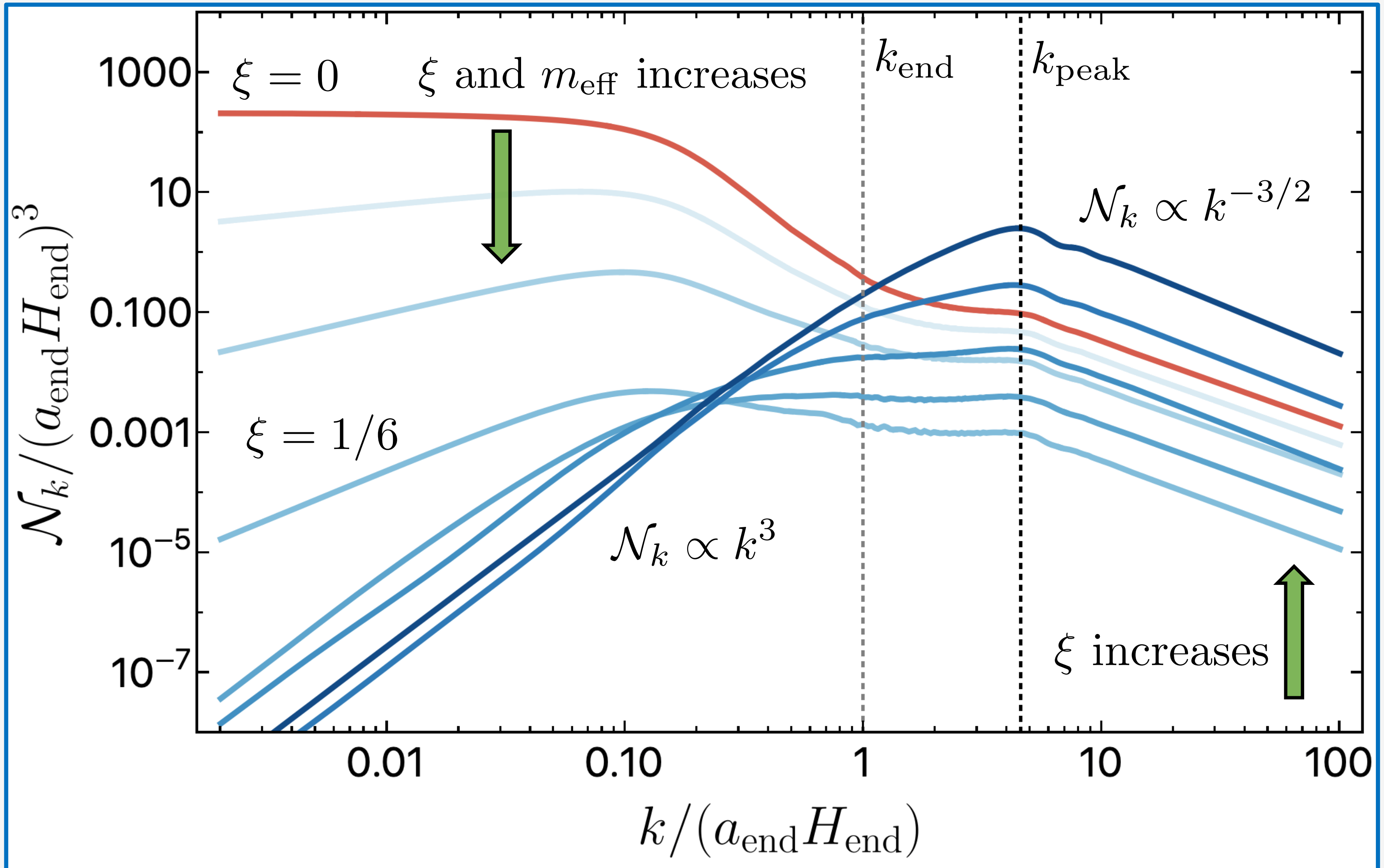
Long-wavelength (IR) regime:

$$f_\chi \propto k^{-2\nu}$$

$$\mathcal{N}_k \propto k^{3-2\nu}$$

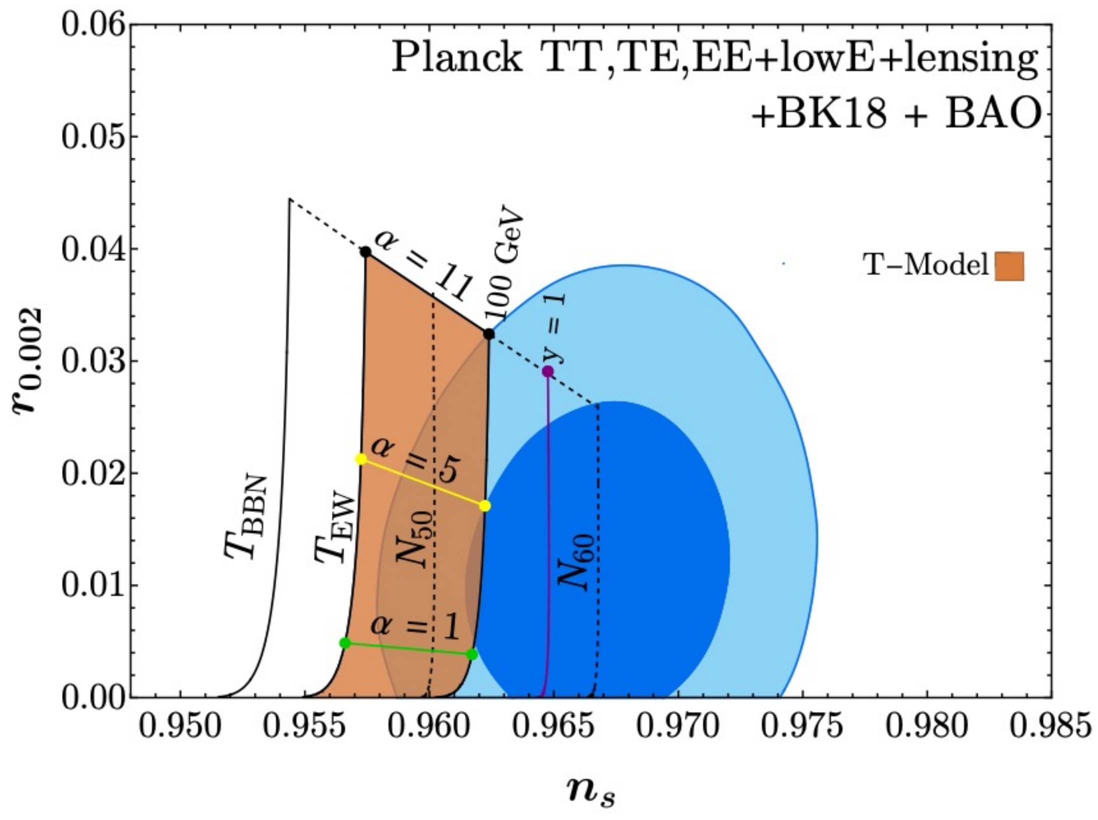
$$\nu = \sqrt{9/4 - 12\xi - m_\chi^2/H^2}$$

$$\nu \in \mathbb{R}$$



T-Model Constraints

T-model of Inflation



BICEP/Keck constraints on T-models of inflation. The panel compares the 68% and 95% C. L. constraints in the (n_s , r) plane with the model predictions for different number of e-folds N_{50} and N_{60} .

$$V(\phi) = \lambda M_P^4 \left[\sqrt{6}\tanh\left(\frac{\phi}{\sqrt{6}M_P}\right)\right]^2$$

$\lambda \simeq 2 \times 10^{-11}$ $\quad H_{\rm end} \simeq 6 \times 10^{12}\,\text{GeV}$

Friedmann-Boltzmann equations:

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi$$

$$\dot{\rho}_r + 4H\rho_r = \Gamma_\phi \rho_\phi$$

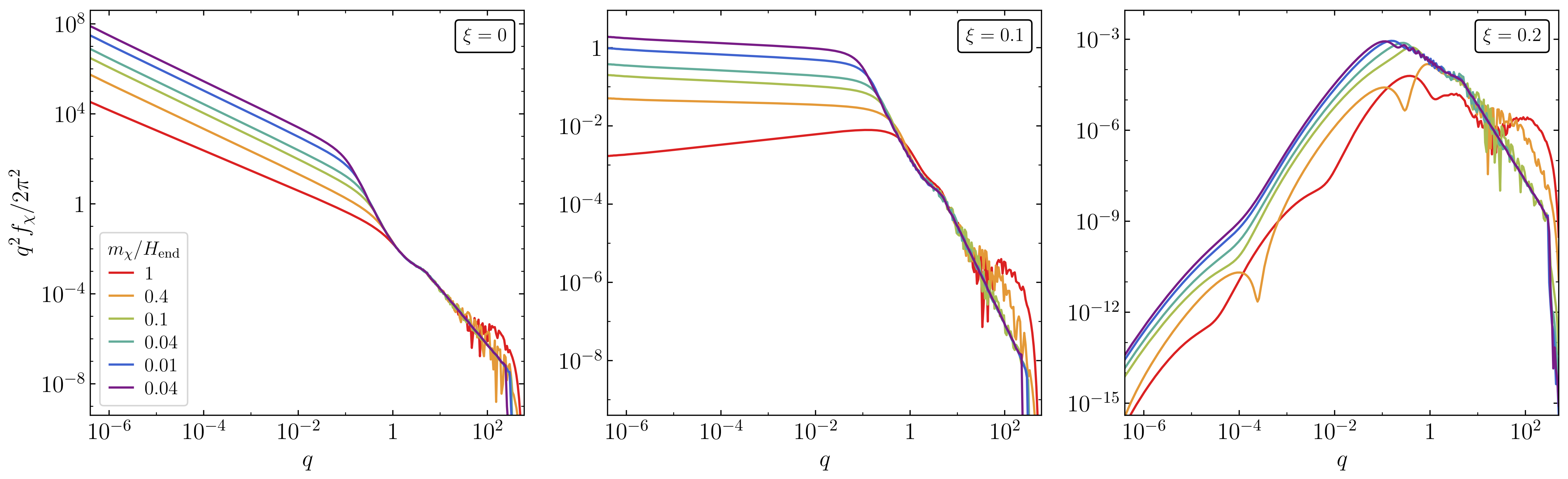
$$\rho_\phi + \rho_r = 3M_P^2 H^2$$

J. Ellis, M. Garcia, D. Nanopoulos, K. Olive, S. Verner, 2021

Overview

- $\Omega_\chi h^2$ — Relic abundance constraints.
- $T_{\rm reh}$ — Reheating Temperature
- $\mathrm{Ly}-\alpha$ — Lyman-α
- $\mathcal{P}_{\mathcal{S}}(k)$ — Isocurvature Constraints

Dark Matter Production During Inflation and Reheating



Re-scaled dimensionless comoving momentum

$$q \equiv \frac{K}{H_{\rm end}}\left(\frac{a}{a_{\rm end}}\right)$$

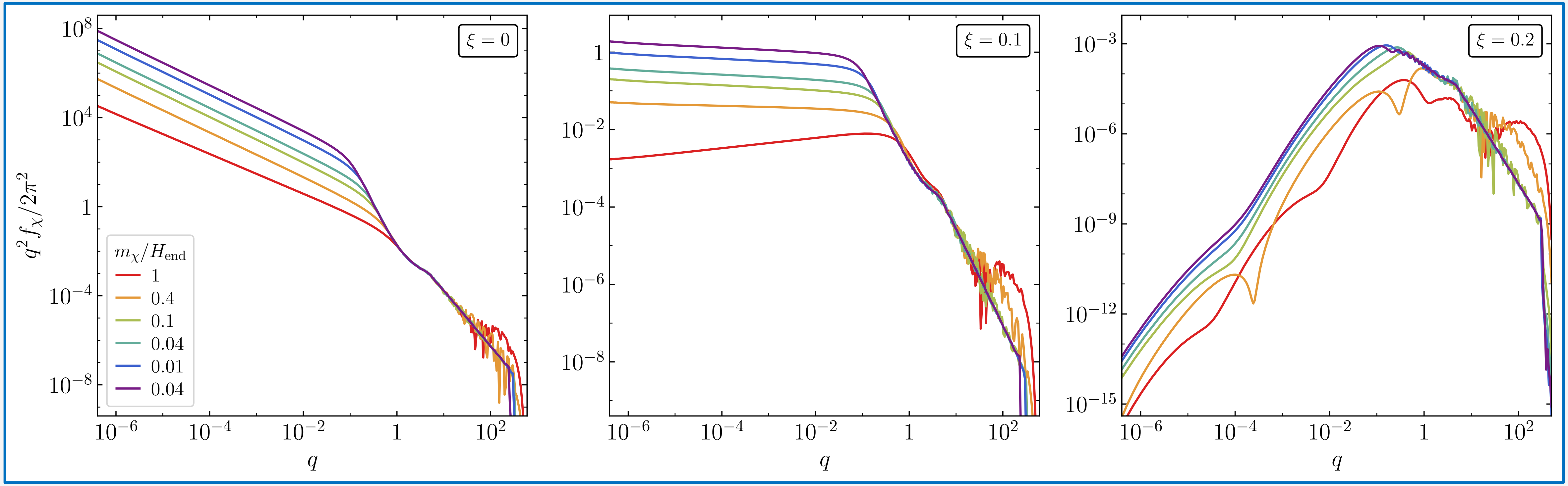
K is the physical momentum
m_ϕ is the inflaton mass

n_χ – comoving number density

$$n_\chi \left(\frac{a}{a_{\rm end}}\right)^3 = \frac{m_\phi^3}{2\pi^2}\int dq\, q^2 f_\chi(q,t)$$

Particle occupation number: $f_\chi(k,t) = \frac{1}{2\omega_k}\left|\omega_k X_k - i X_k'\right|^2$

Dark Matter Production During Inflation and Reheating



Long-wavelength (IR) regime:

$$f_\chi \propto q^{-2\nu}$$

$$\nu = \sqrt{9/4 - 12\xi - m_\chi^2/H^2}$$

$$\nu \in \mathbb{R}$$

n_χ – comoving number density

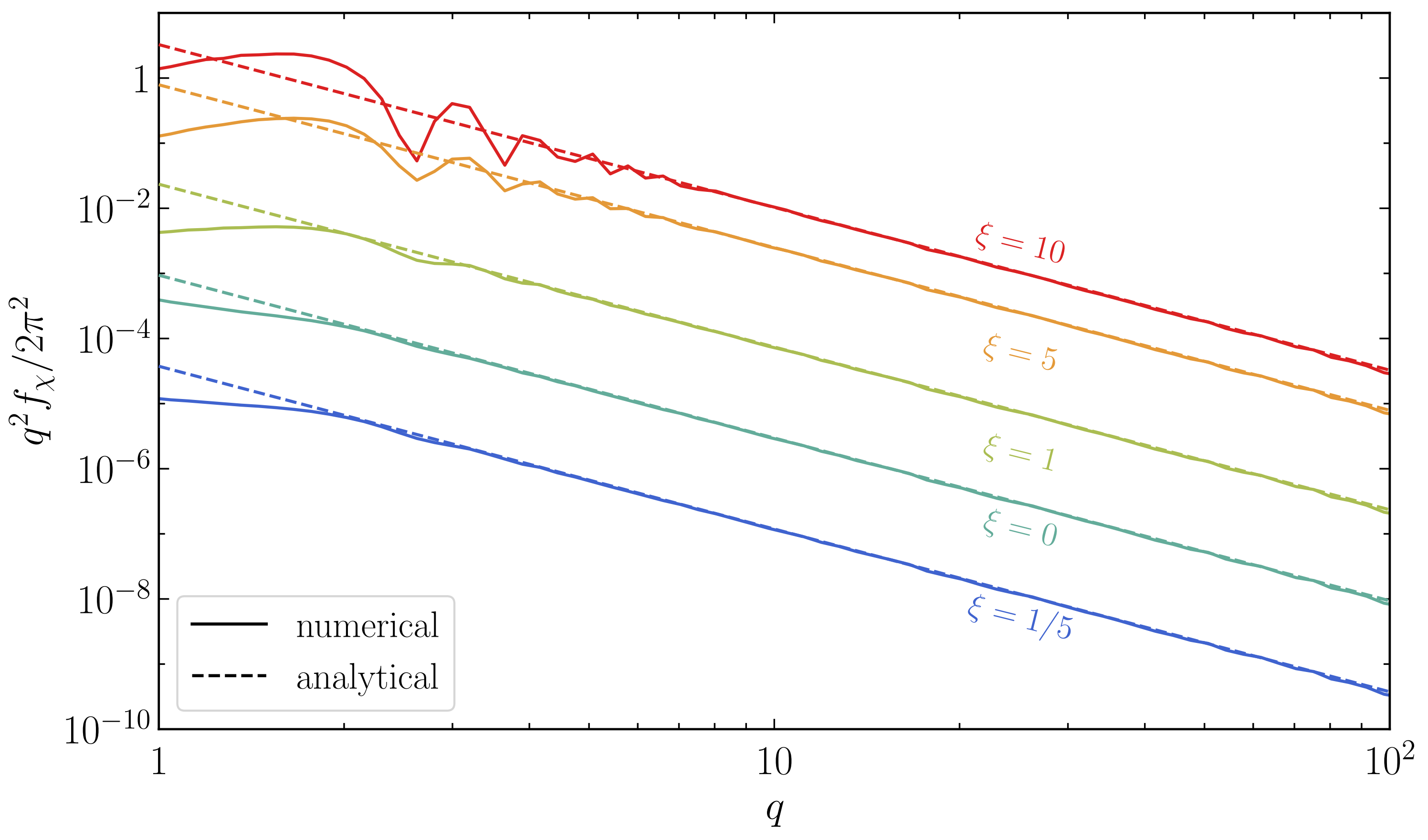
$$n_\chi \left(\frac{a}{a_{\rm end}}\right)^3 = \frac{m_\phi^3}{2\pi^2} \int dq\, q^2 f_\chi(q,t)$$

$$q \ll 1$$

$$\xi \lesssim 0.3$$

IR-dominated regime

Short-wavelength (UV) regime:



$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{K}|\frac{\partial f_\chi}{\partial |\boldsymbol{K}|} = \frac{\pi |\mathcal{M}|^2}{2m_\phi^2}\delta(|\boldsymbol{K}| - m_\phi)$$

Inflaton condensate amplitude:

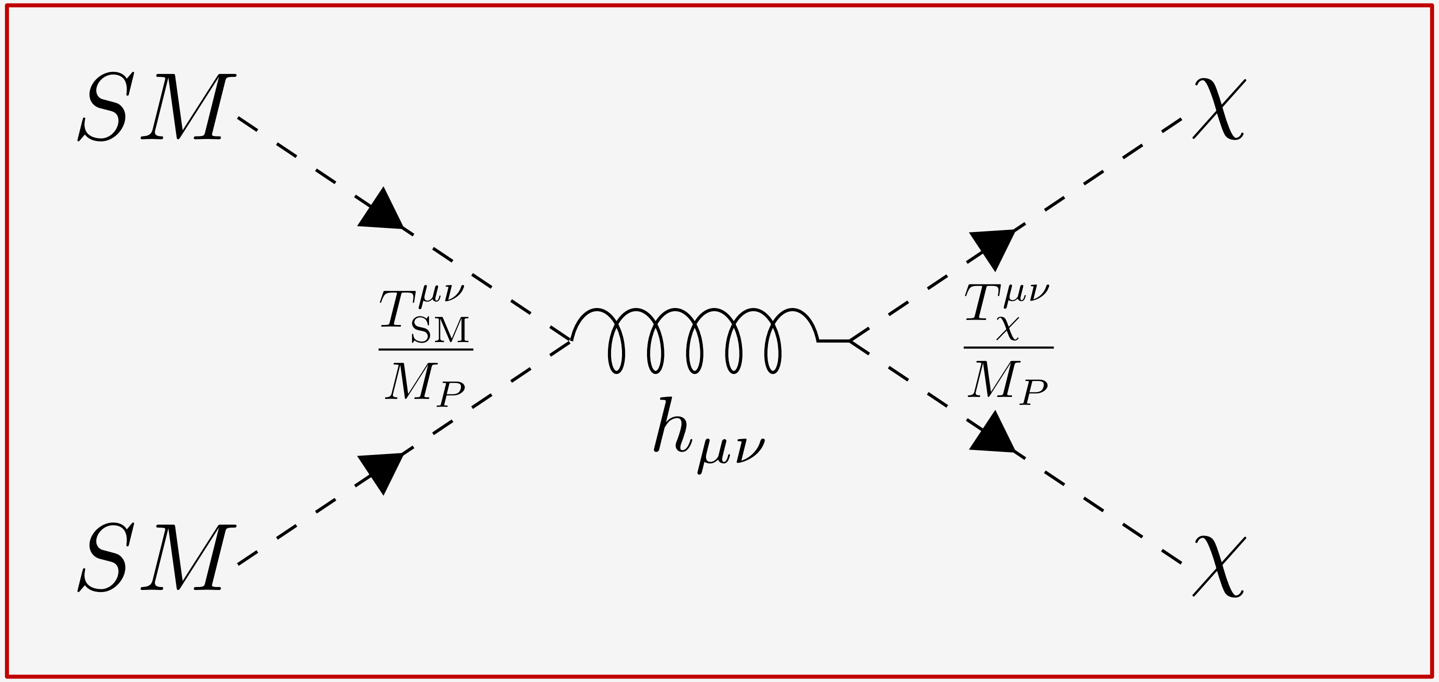
$$|\mathcal{M}|^2 = \frac{1}{8}\frac{\rho_\phi^2}{m_\phi^4}\lambda^2 (1-6\xi)^2$$

Particle occupation number:

$$f_\chi = \frac{\pi |\mathcal{M}(\hat{t})|^2}{2m_\phi^3 H(\hat{t})}\theta(t-\hat{t})\theta(\hat{t}-t_{\rm end})$$

$$\frac{a(t)}{a(\hat{t})} = \frac{m_\phi}{|\boldsymbol{K}|}$$

Thermal Production of Scalar Dark Matter



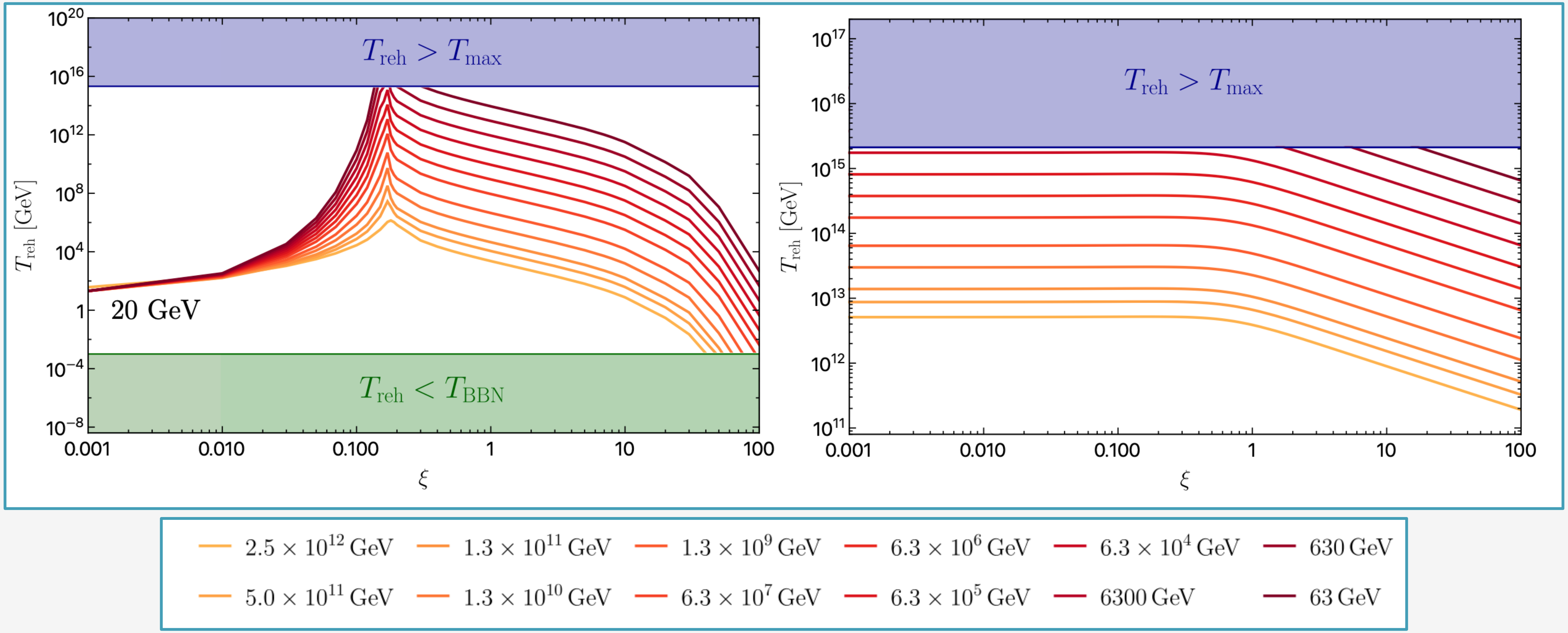
$$\sqrt{-g}\mathcal{L}_{\rm int} = -\frac{h_{\mu\nu}}{M_P}\left(T^{\mu\nu}_{\rm SM} + T^{\mu\nu}_{\chi}\right) \longrightarrow \Omega_{\rm DM}h^2 \simeq 0.12$$

$h_{\mu\nu}$ – canonically-normalized metric perturbation

$$\frac{dn_\chi}{dt} + 3Hn_\chi = R_\chi \xrightarrow{\text{Production rate}} R_\chi = \frac{\pi^3(2560\xi(3\xi-1)+3997)}{41472000}\frac{T^8}{M_P^4}$$

Dark Matter Abundance Constraints

Production from the Inflaton — Thermal Production

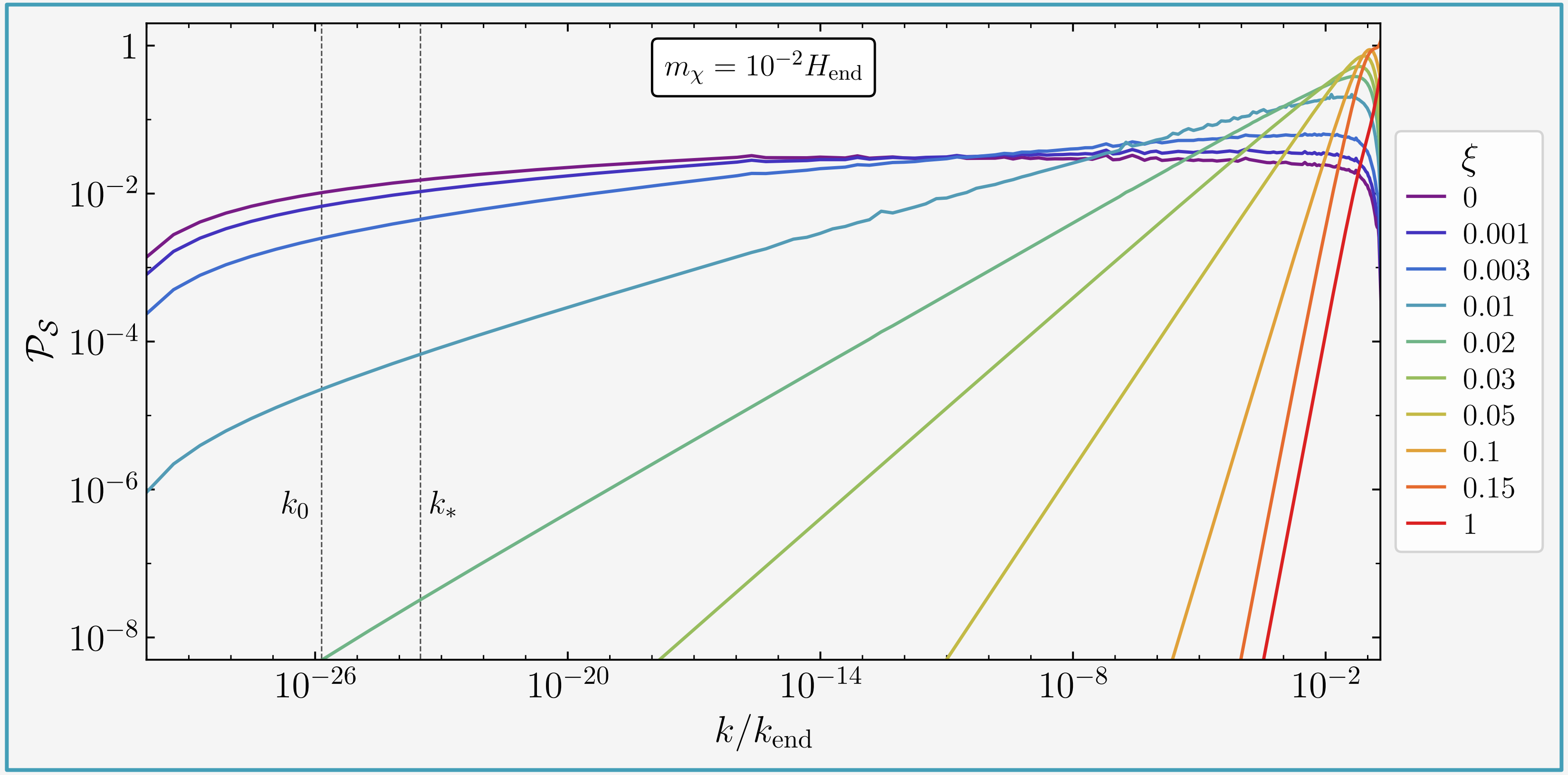


$$\rho_r(t_{\rm reh}) = \frac{\pi^2 g_{\rm reh} T_{\rm reh}^4}{30} \equiv \frac{12}{25}(\Gamma_\phi M_P)^2$$

$$T_{\rm max} = \left(90 H_{\rm end}^2 M_P^2/\pi^2 g_{\rm reh}\right)^{1/4} \simeq 2 \times 10^{15}\,{\rm GeV}$$

$$T_{\rm BBN} \sim 1\,{\rm MeV}$$

Isocurvature Constraints



DM isocurvature power spectrum for different non-minimal couplings, with each coupling represented by a different color. The vertical lines indicate the present horizon scale and the Planck pivot scale.

$$\mathcal{P}_S(k) = \frac{k^3}{2\pi^2 \rho_\chi^2} \int d^3\mathbf{x} \, \langle \delta\rho_\chi(\mathbf{x}) \delta\rho_\chi(0) \rangle e^{-i\mathbf{k}\cdot\mathbf{x}}$$

where ρ_χ and $\delta\rho_\chi$ denote the DM energy density and its fluctuation

$$\mathcal{P}_S(k_*) \lesssim 8.3 \times 10^{-11}$$

from *Planck*

$$m_\chi \gtrsim 0.54\, H_*$$

$$\xi \gtrsim 0.03$$

Structure Formation Constraints

For out-of-equilibrium DM production mechanisms, the Lyman-α bound is dependent on the details of the production and decoupling of the dark particles

$$m_\chi^{\text{Ly-}\alpha} = m_{\text{WDM}}^{\text{Ly-}\alpha}\left(\frac{T_\star}{T_{\text{WDM},0}}\right)\sqrt{\frac{\langle q^2\rangle}{\langle q^2\rangle_{\text{WDM}}}}$$

$T_{\text{WDM},0}$ is the WDM temperature

$T_\star$ is the characteristic energy scale of the produced DM,

$$T_\star = m_\phi(a_{\text{end}}/a_0)$$

$$\langle q^2\rangle \equiv \frac{\int dq\, q^4 f_\chi(q)}{\int dq\, q^2 f_\chi(q)} \simeq 2.43\sqrt{\frac{a_{\text{reh}}}{a_{\text{end}}}}$$

$$\langle q^2\rangle_{\text{WDM}} \simeq 12.93$$

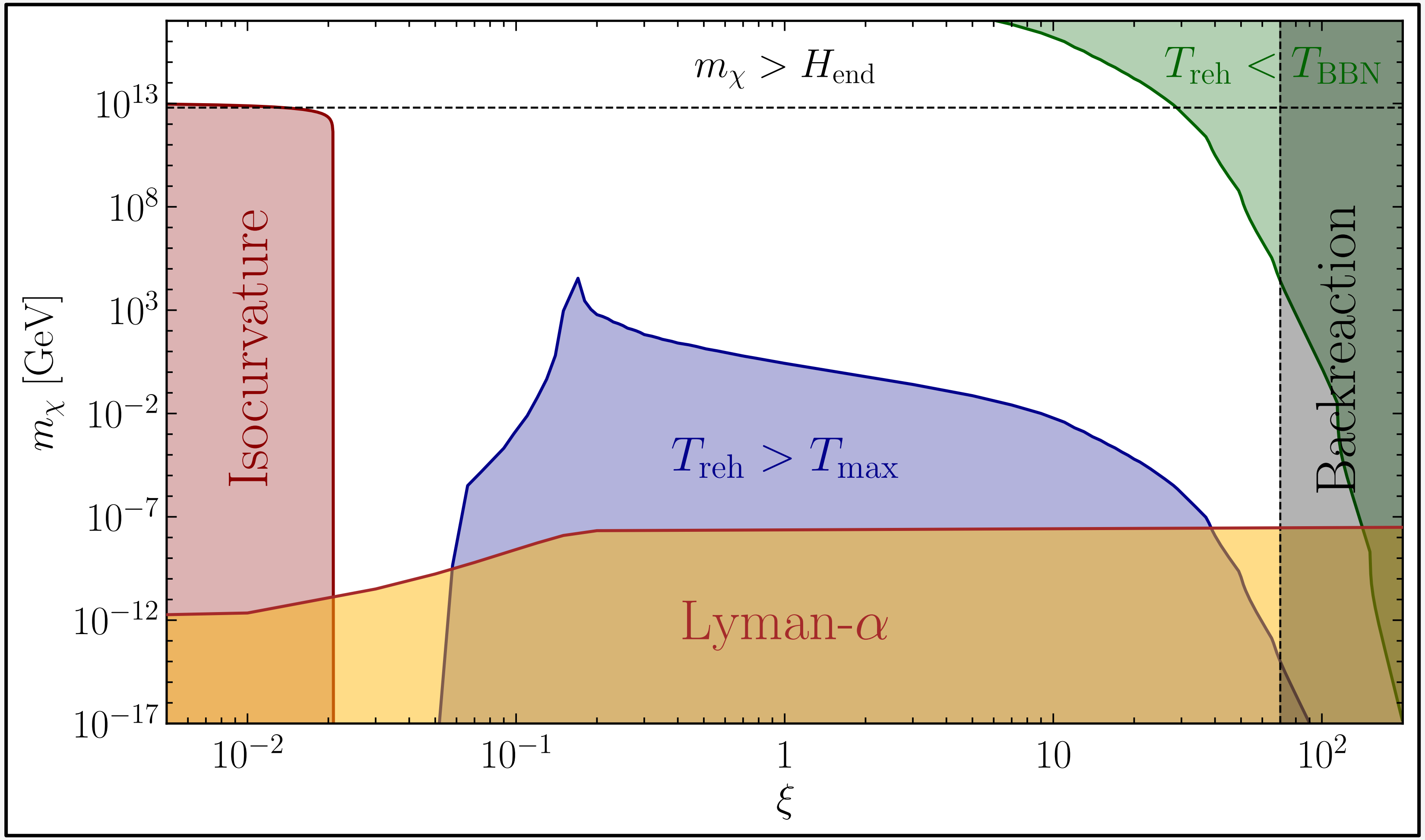
$$\xi > 1/4$$

$$m_\chi > 32.4\,\text{eV}$$

$$\xi \ll 1$$

$$m_\chi > 3\times 10^{-4}\ \text{eV}$$

Combined Constraints



Parameter space of the dark matter mass as a function of the non-minimal coupling, ξ. The white region displays the space compatible with the observed dark matter abundance. The red region is ruled out by isocurvature constraints, the blue region is ruled out by the maximum possible reheating temperature, the green region is excluded because of the BBN temperature, and finally, the yellow region displays the Lyman-α constraint.

Conclusions

- In the absence of a non-minimal coupling, the isocurvature power spectrum is nearly scale-invariant and must satisfy the constraint:

$$m_\chi \gtrsim 0.54\, H_*$$

- In the limit where the bare dark matter mass is much smaller than the Hubble scale, the isocurvature constraint is satisfied when:

$$\xi \gtrsim 0.03$$

- The Lyman-α constraint excludes masses lighter than :

$$\xi > 1/4$$

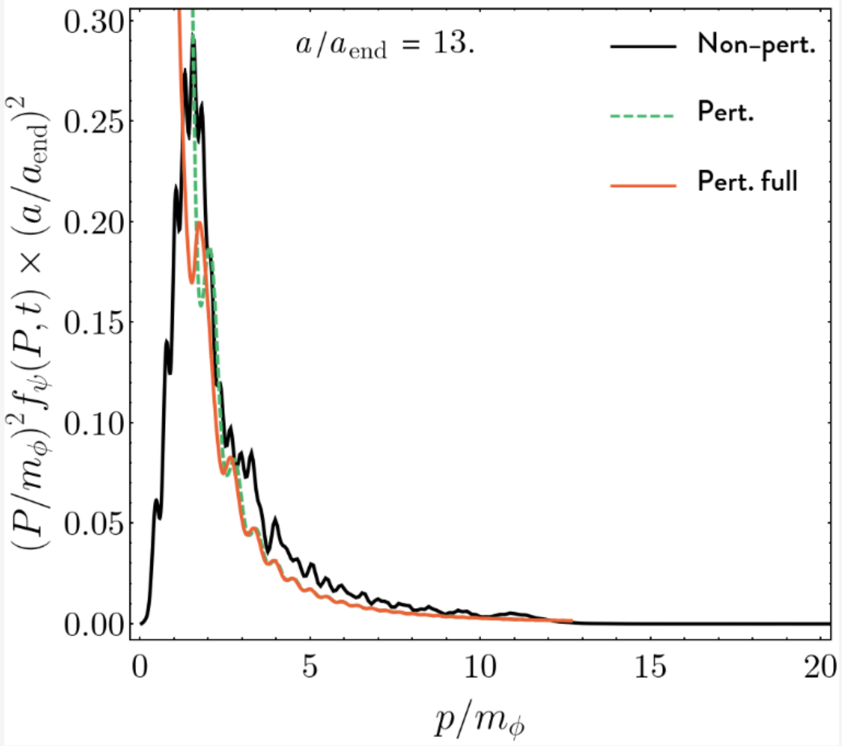
$$m_\chi > 32.4\,\mathrm{eV}$$

$$\xi \ll 1$$

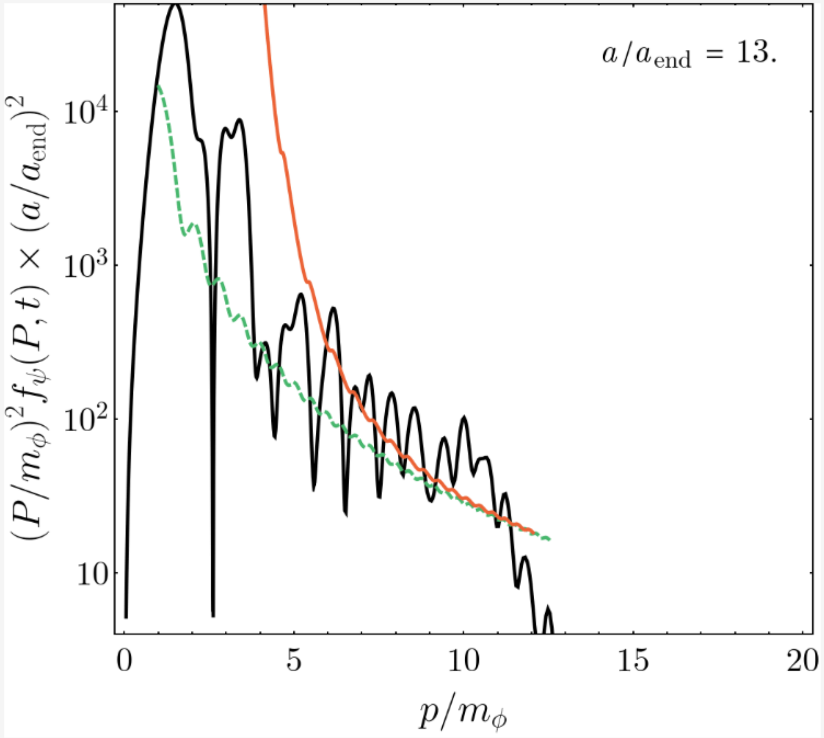
$$m_\chi > 3 \times 10^{-4}\ \mathrm{eV}$$

- CMB-S4 and LiteBIRD experiments could detect the B-modes or the isocurvature modes in the CMB narrowing down the gravitational particle production models.

Thank you!



$a/a_{\rm end}$ = 13.
Non-pert.
Pert.
Pert. full
$(P/m_\phi)^2 f_\psi(P,t) \times (a/a_{\rm end})^2$
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
5
10
15
20
p/m_ϕ



$a/a_{\rm end}$ = 13.
$(P/m_\phi)^2 f_\psi(P,t) \times (a/a_{\rm end})^2$
10^4
10^3
10^2
10
0
5
10
15
20
p/m_ϕ