Metaplectic Flavor symmetries from magnetized tori

(talk based on Almumin, Yahya, Mu-Chun Chen, Víctor Knapp-Pérez, Saúl Ramos-Sánchez, Michael Ratz, and Shreya Shukla. "Metaplectic flavor symmetries from magnetized tori." Journal of High Energy Physics 2021, no. 5 (2021): 1-41.)

Víctor Knapp Pérez (UCI) **(in collaboration with Yahya Almumin, Mu-Chun Chen, Saul Ramos-Sanchez, Michael Ratz, Shreya Shukla)**

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Why modular physics for flavor physics?

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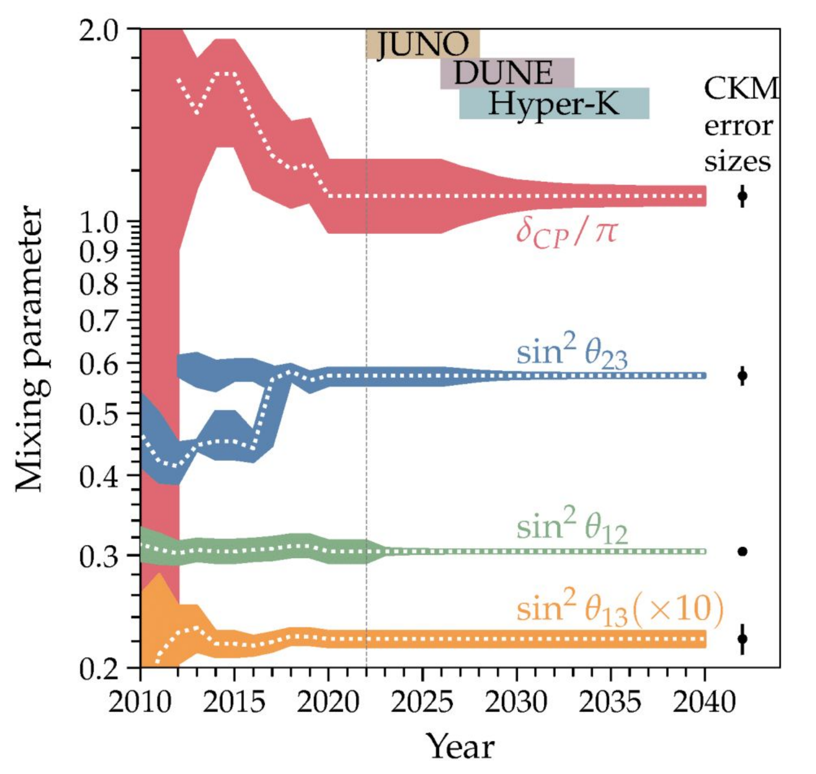

Future experiments to increase precision in neutrino flavor parameter measurements.

Our theoretical uncertainties shouldn't be bigger than the experimental error bars

Song, Li, Argüelles, Bustamente (2021)

Why modular physics for flavor physics?

Model (A4):

Feruglio (2017)

	E_1^c	E_2^c	E_3^c	N^c	L	H_d	H_u
$SU(2)_L \times U(1)_Y$	$(1,+1)$	$(1,+1)$	$(1,+1)$	$(1,0)$	$(2,-1/2)$	$(2,-1/2)$	$(2,+1/2)$
$\Gamma_3 \equiv A_4$	1	$1''$	$1'$	3	3	1	1

$$w_e = \alpha\, E_1^c H_d (L\, Y)_1 + \beta\, E_2^c H_d (L\, Y)_{1'} + \gamma\, E_3^c H_d (L\, Y)_{1''}$$

$$w_\nu = g(N^c H_u L\, Y)_1 + \Lambda (N^c N^c Y)_1$$

Why modular physics for flavor physics?

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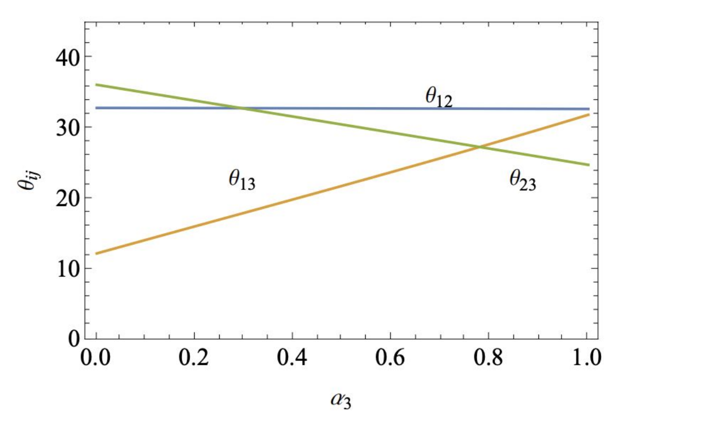
Chen, Ramos–Sánchez, Ratz (2019)



Non-diagonal terms to Kähler potential

$$\Delta K \ = \ \alpha_1 \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(1)}} (Y\,L)_{\mathbf{3}^{(1)}} + \alpha_2 \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(2)}} (Y\,L)_{\mathbf{3}^{(2)}} + \alpha_3 \left[\left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(1)}} (Y\,L)_{\mathbf{3}^{(2)}} + \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(2)}} (Y\,L)_{\mathbf{3}^{(1)}}\right] + \ldots\ .$$

Why modular physics for flavor physics?

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How do we control these Kähler terms? Where do these modular forms come from?

Non-diagonal terms to Kähler potential

Chen, Ramos–Sánchez, Ratz (2019)

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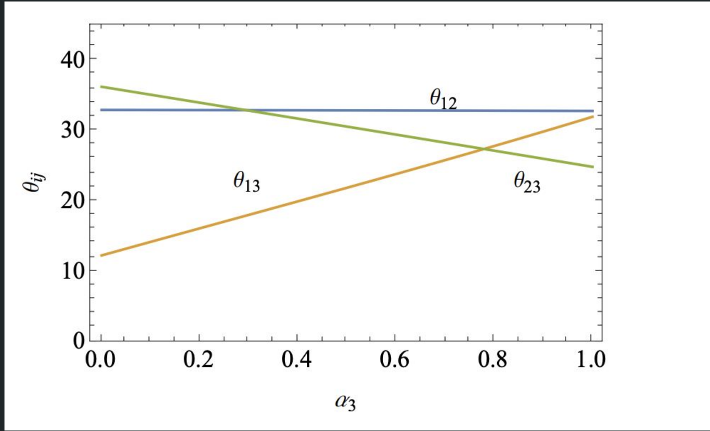

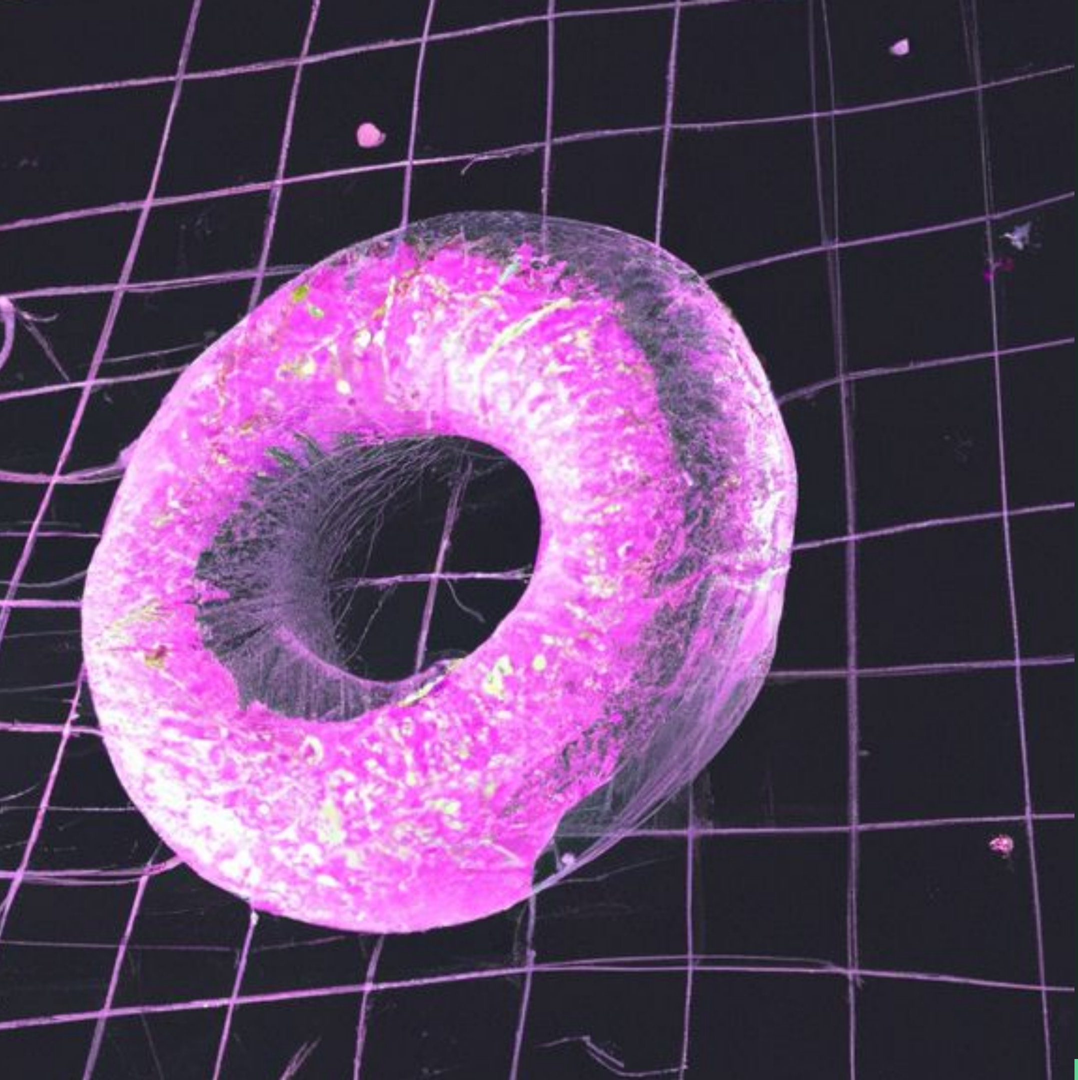

Flavor physics from magnetized tori (Recipe)

Flavor physics from magnetized tori

Recipe:

- Choose a compact space

Flavor physics from magnetized tori

Recipe:

- Choose a compact space
- Derive Yukawa couplings

Flavor physics from magnetized tori

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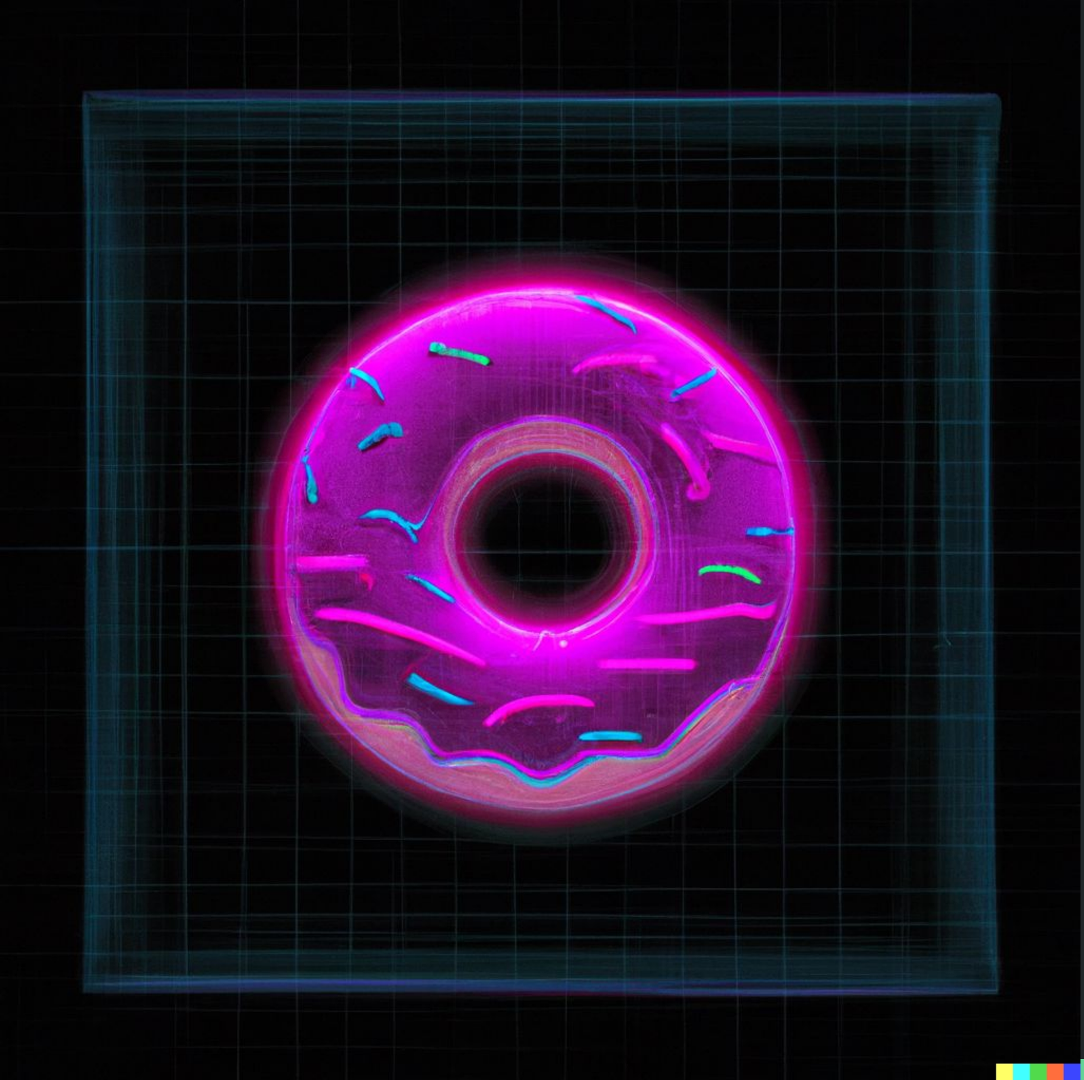
- Choose a compact space
- Derive Yukawa couplings
- Obtain modular symmetry representation matrices

Flavor physics from magnetized tori

Recipe:

- Choose a compact space
- Derive Yukawa couplings
- Obtain modular symmetry representation matrices
- Obtain modular group



Compact space

Flavor physics from magnetized tori

6D = Minkowski field + Torus wavefunction

Cremades, Ibáñez and Marchesano. (2004)

$$\underbrace{\Omega^{j,M}}_{6D} = \underbrace{\phi^{j,M}(x^\mu)}_{4D} \otimes \underbrace{\psi^{j,M}(z,\tau)}_{2D}$$

Gauge Group: U(N)

Flavor physics from magnetized tori

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Gauge Group: U(N)

Add magnetic field

$$F_{z\bar{z}} = \frac{\pi \mathrm{i}}{\mathrm{Im}\tau} \begin{pmatrix} \frac{m_a}{N_a} \mathbb{1}_{N_a \times N_a} & 0 & 0 \\ 0 & \frac{m_b}{N_b} \mathbb{1}_{N_b \times N_b} & 0 \\ 0 & 0 & \frac{m_c}{N_c} \mathbb{1}_{N_c \times N_c} \end{pmatrix}$$

Flavor physics from magnetized tori

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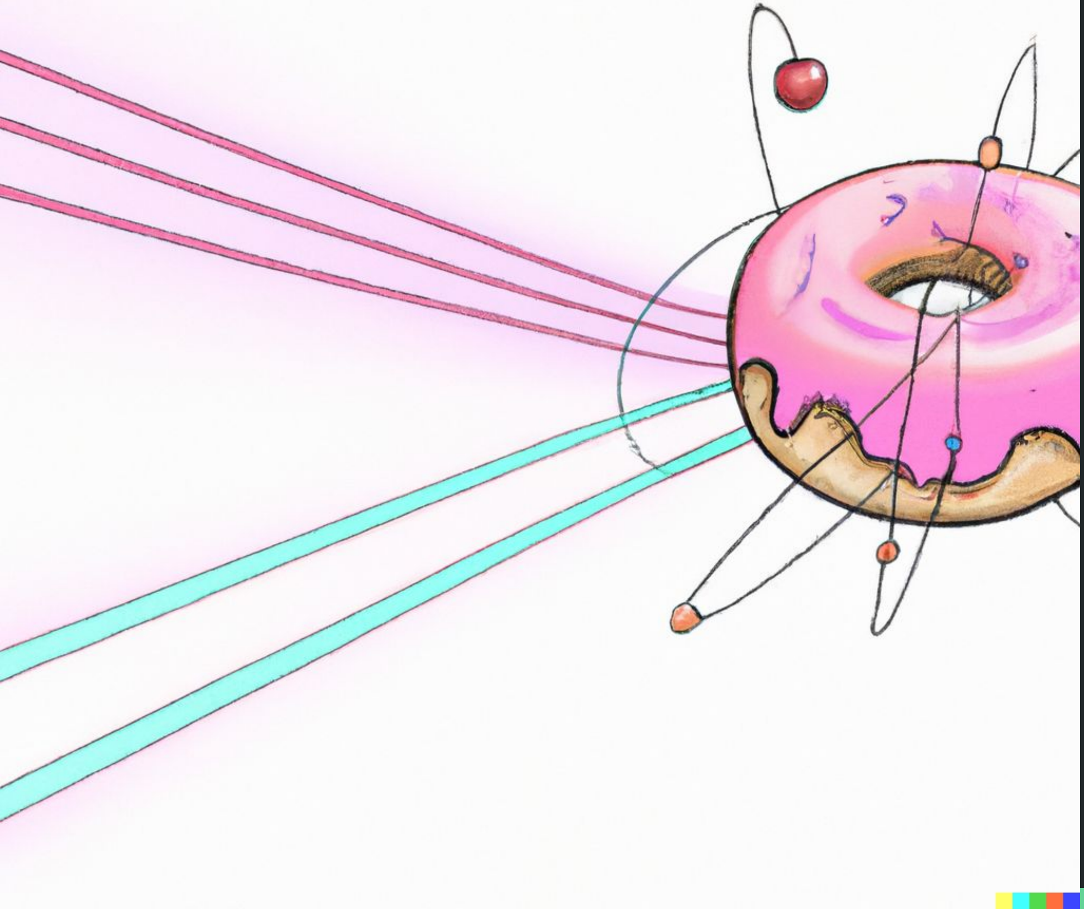
$$U(N) \to U(N_a) \times U(N_b) \times U(N_c)$$



Yukawa couplings from magnetized tori

Flavor physics from magnetized tori

We obtain 3 types of chiral fields with

Cremades, Ibáñez and Marchesano. (2004)

$$M_1,\ M_2,\ M_3 \text{ with } M_1 + M_2 + M_3 = 0$$

number of copies. These numbers depend on the magnetic fluxes.

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The superpotential is

$$\omega \supset Y_{ijk} \phi^{i,M_1} \phi^{j,M_2} \phi^{k,M_3}$$

Yukawa couplings from magnetized tori

The wavefunctions are obtained by solving the Dirac equation in the torus:

Cremades, Ibáñez and Marchesano. (2004)

$$\psi^{j,M}(z,\tau,\zeta) = \mathcal{N}\, \mathrm{e}^{\pi\, \mathrm{i}\, M\,(z+\zeta)\, \frac{\mathrm{Im}(z+\zeta)}{\mathrm{Im}\,\tau}}\, \vartheta\begin{bmatrix} \frac{j}{M} \\ 0 \end{bmatrix}\big(M\,(z+\zeta), M\,\tau\big)\ .$$

z is the torus coordinate

τ is the half-period ratio

ζ is Wilson line

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$$\vartheta\begin{bmatrix}\alpha \\ \beta\end{bmatrix}(z,\tau) = \sum_{\ell=-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\pi\,(\alpha+\ell)^2\,\tau}\, \mathrm{e}^{2\pi\mathrm{i}\,(\alpha+\ell)\,(z+\beta)}$$

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Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

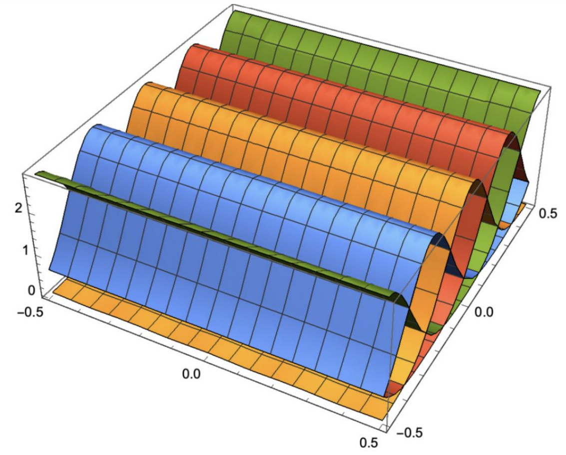
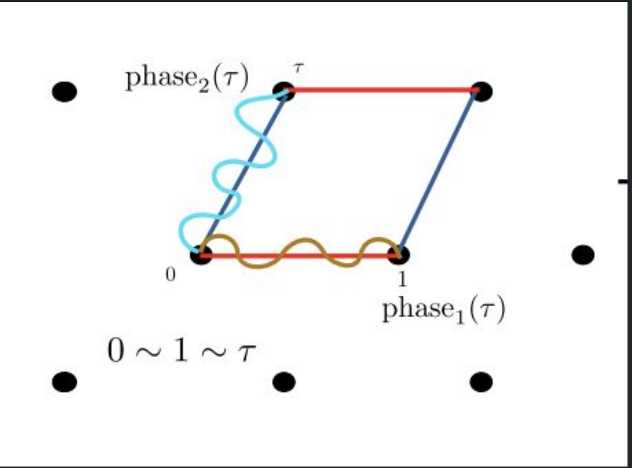


Figure 2.1: Squares of the absolute values of the wave functions on a quadratic torus for $M = 4$.

Yukawa couplings from magnetized tori

The wavefunctions need to satisfy boundary conditions



Yukawa couplings from magnetized tori

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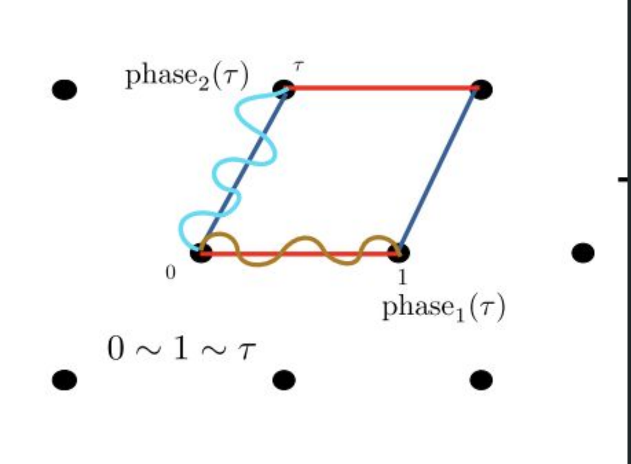

Do modular transformed wavefunctions follow correct boundary conditions?

$$\psi(z,\tau,\underbrace{0}_{\text{Assume } \zeta=0}) \xrightarrow{S} \psi\left(-\frac{z}{\tau},-\frac{1}{\tau},0\right) \quad \text{and} \quad \psi(z,\tau,0) \xrightarrow{T} \psi(z,\tau+1,0)$$

Yukawa couplings from magnetized tori

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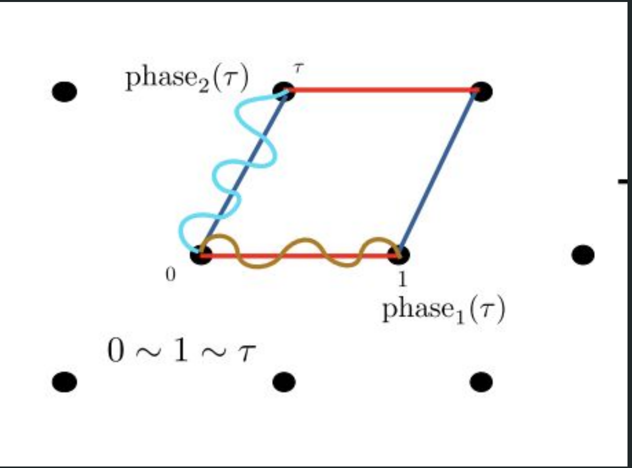

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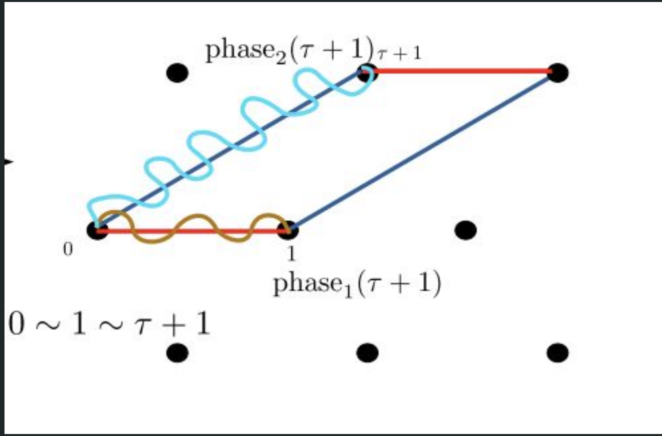
Magnetized tori wavefunctions were thoroughly studied and it was stated that T transformation for odd M didn't follow boundary conditions

Ohki, Uemura, Watanabe (2020);
Kikuchi, Kobayashi, Takada, Tatsuishi, and Uchida (2020)

Yukawa couplings from magnetized tori

However, odd M are fine!



This was also studied through Scherk-Schwarz phases: Shota, Kobayashi, and Uchida. (2021)

Yukawa couplings from magnetized tori

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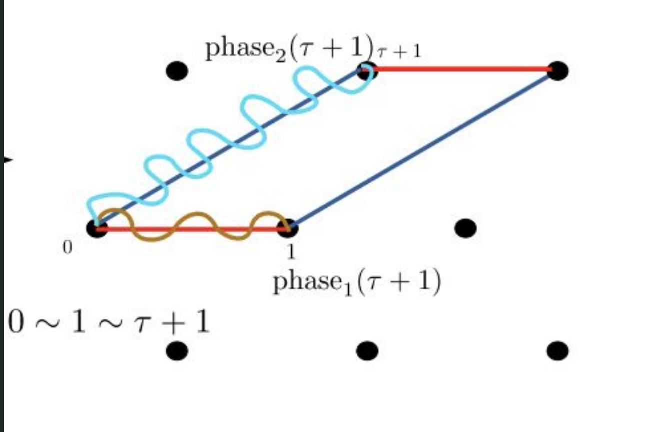

The transformation rule is

$$\psi^{j,M}(z,\tau,0) \xrightarrow{T} \underbrace{e^{i\pi M \frac{\mathrm{Im} z}{\mathrm{Im}\tau}}}_{\text{ugly phase}} \underbrace{\rho(T)}_{\text{rep. matrix}} \psi^{j,M}(z - \underbrace{\frac{1}{2}}_{\text{translation}}, \tau, 0),$$

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Yukawa couplings from magnetized tori

From the normalization constant

$$\mathcal{A} = (2\pi R)^2 \mathrm{Im}\tau$$

$$\mathcal{N} = \left(\frac{2M\ \mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4}$$

Cremades, Ibáñez and Marchesano. (2004)

Yukawa couplings from magnetized tori

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This implies that

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Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

object	$\psi^{j,M}$	$\phi^{j,M}$	$\Omega^{j,M}$	Y_{ijk}	$\mathscr{W}$
modular weight k	$1/2$	$-1/2$	0	$1/2$	-1

Table 4.1: Modular weights of the $\mathbb{T}^2$ wave functions $\psi^{j,M}$, 4D fields $\phi^{j,M}$, 6D fields $\Omega^{j,M}$, Yukawa couplings Y_{ijk}, and superpotential $\mathscr{W}$.

Yukawa couplings from magnetized tori

Using this wavefunctions, the overlap integral was calculated to be

Cremades, Ibáñez and Marchesano. (2004)

$$Y_{ijk}(\widetilde{\zeta},\tau) = \mathcal{N}_{abc}\, \mathrm{e}^{\frac{H(\widetilde{\zeta},\tau)}{2}} \sum_{m\in\mathbb{Z}_{\mathcal{I}_{bc}}} \delta_{k,i+j+\mathcal{I}_{ab}\, m}\, \vartheta\begin{bmatrix} \frac{\mathcal{I}_{ca} i - \mathcal{I}_{ab} j + \mathcal{I}_{ab}\mathcal{I}_{ca} m}{-\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}} \\ 0 \end{bmatrix} (\widetilde{\zeta}, \tau\, |\mathcal{I}_{ab}\mathcal{I}_{bc}\mathcal{I}_{ca}|)\ ,$$

where

$$\widetilde{\zeta} := -\mathcal{I}_{ab}\,\mathcal{I}_{ca}\,(\zeta_{ca} - \zeta_{ab})$$

$$\mathcal{N}_{abc} = g\,\sigma_{abc}\left(\frac{2\,\mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4} \left|\frac{\mathcal{I}_{ab}\mathcal{I}_{ca}}{\mathcal{I}_{bc}}\right|^{1/4}$$

$$\frac{H(\widetilde{\zeta},\tau)}{2} := \frac{\pi \mathrm{i}}{\mathrm{Im}\,\tau}(\mathcal{I}_{ab}\,\zeta_{ab}\ \mathrm{Im}\,\zeta_{ab} + \mathcal{I}_{bc}\,\zeta_{bc}\ \mathrm{Im}\,\zeta_{bc} + \mathcal{I}_{ca}\,\zeta_{ca}\ \mathrm{Im}\,\zeta_{ca})$$

Yukawa couplings from magnetized tori

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Sum over several theta-functions

where

$$\tilde{\zeta} := -\mathcal{I}_{ab}\,\mathcal{I}_{ca}\,(\zeta_{ca}-\zeta_{ab})$$

$$\mathcal{N}_{abc} = g\,\sigma_{abc}\left(\frac{2\,\mathrm{Im}\,\tau}{\mathcal{A}^2}\right)^{1/4}\left|\frac{\mathcal{I}_{ab}\mathcal{I}_{ca}}{\mathcal{I}_{bc}}\right|^{1/4}$$

$$\frac{H(\tilde{\zeta},\tau)}{2} := \frac{\pi \mathrm{i}}{\mathrm{Im}\,\tau}\left(\mathcal{I}_{ab}\,\zeta_{ab}\;\mathrm{Im}\,\zeta_{ab} + \mathcal{I}_{bc}\,\zeta_{bc}\;\mathrm{Im}\,\zeta_{bc} + \mathcal{I}_{ca}\,\zeta_{ca}\;\mathrm{Im}\,\zeta_{ca}\right)$$

Yukawa couplings from magnetized tori

However, this expression was not simplified for $d \neq 1$

$$d := \gcd\left(|\mathcal{I}_{ab}|, |\mathcal{I}_{ca}|, |\mathcal{I}_{bc}|\right)$$

The simplified equation is

Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

$$Y_{ijk}(\widetilde{\zeta}, \tau) = \mathcal{N}_{abc}\, \mathrm{e}^{\frac{H(\widetilde{\zeta},\tau)}{2}}\, \Delta^{(d)}_{i+j,k}\, \vartheta\begin{bmatrix} \frac{\mathcal{I}'_{ca}\, i - \mathcal{I}'_{ab}\, j + \mathcal{I}'_{ca}\, \left(\mathcal{I}'_{ab}\right)^{\phi(|\mathcal{I}'_{bc}|)}\,(k-i-j)}{\lambda} \\ 0 \end{bmatrix}\left(\frac{\widetilde{\zeta}}{d}, \lambda\,\tau\right)$$

$$\lambda := \operatorname{lcm}\left(|\mathcal{I}_{ab}|, |\mathcal{I}_{ca}|, |\mathcal{I}_{bc}|\right)$$

Yukawa couplings from magnetized tori

This expression has a nice geometrical interpretation. Basically,

$$Y_{\widehat{\alpha}} \propto (\operatorname{Im}\tau)^{-1/4}\,\vartheta\begin{bmatrix}\widehat{\alpha}/\lambda\\0\end{bmatrix}(0,\lambda\tau) = (\operatorname{Im}\tau)^{-1/4}\sum_{\ell=-\infty}^{\infty} \mathrm{e}^{-\pi\,\lambda(\operatorname{Im}\tau-\mathrm{i}\,\operatorname{Re}\tau)(\widehat{\alpha}/\lambda+\ell)^2}\ ,$$

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The exponential suppression can be thought as the overlap of two gaussians

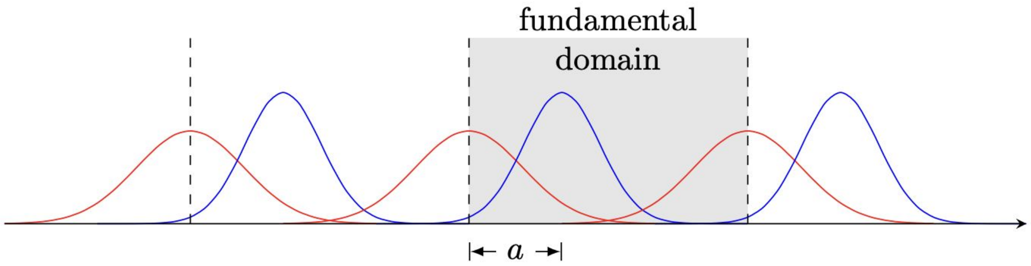

Figure 4.1: Overlap of two Gaussians on a torus. The overlap of a given, say red, curve is not just the overlap with one blue curve but with infinitely many of them, thus leading to an expression of the form (4.67).

Modular transformations of Yukawa couplings

Modular transformations of Yukawa couplings

Under S and T modular transformations

$$\tau \xrightarrow{S} -\frac{1}{\tau} \quad \text{and} \quad \tau \xrightarrow{T} \tau + 1 \; .$$

Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

Modular transformations of Yukawa couplings

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Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

$$\tau \xmapsto{S} -\frac{1}{\tau} \quad \text{and} \quad \tau \xmapsto{T} \tau + 1 \;.$$

the Yukawa couplings transform as

$$\mathcal{Y}_{\widehat{\alpha}}(\tau) \xmapsto{\widetilde{\gamma}} \mathcal{Y}_{\widehat{\alpha}}(\widetilde{\gamma}\,\tau) = \pm(c\,\tau + d)^{1/2}\,\rho_{\boldsymbol{\lambda}}(\widetilde{\gamma})_{\widehat{\alpha}\widehat{\beta}}\,\mathcal{Y}_{\widehat{\beta}}(\tau)$$

$$\rho_{\boldsymbol{\lambda}}(\widetilde{S})_{\widehat{\alpha}\widehat{\beta}} = -\frac{\mathrm{e}^{\mathrm{i}\pi/4}}{\sqrt{\lambda}}\,\exp\left(\frac{2\pi\mathrm{i}\,\widehat{\alpha}\,\widehat{\beta}}{\lambda}\right) ,$$

$$\rho_{\boldsymbol{\lambda}}(\widetilde{T})_{\widehat{\alpha}\widehat{\beta}} = \exp\left(\frac{\mathrm{i}\pi\,\widehat{\alpha}^2}{\lambda}\right)\delta_{\widehat{\alpha}\widehat{\beta}} \;.$$

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Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

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$$\rho_{\boldsymbol{\lambda}}(\widetilde{S})_{\widehat{\alpha}\widehat{\beta}} = -\frac{e^{i\pi/4}}{\sqrt{\lambda}} \exp\left(\frac{2\pi i\, \widehat{\alpha}\, \widehat{\beta}}{\lambda}\right) ,$$

$$\rho_{\boldsymbol{\lambda}}(\widetilde{T})_{\widehat{\alpha}\widehat{\beta}} = \exp\left(\frac{i\pi\, \widehat{\alpha}^2}{\lambda}\right) \delta_{\widehat{\alpha}\widehat{\beta}} \, .$$

Metaplectic modular symmetries!!!

Modular transformations of Yukawa couplings

We need to consider the metaplectic group

$$\widetilde{\Gamma} = \left\{ \widetilde{\gamma} = (\gamma, \varphi(\gamma, \tau)) \mid \gamma \in \Gamma,\ \varphi(\gamma, \tau) = \pm(c\tau + d)^{1/2} \right\}$$

Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

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with the multiplication rule

$$(\gamma_1,\, \varphi(\gamma_1,\tau))(\gamma_2,\, \varphi(\gamma_2,\tau)) = (\gamma_1\gamma_2,\, \varphi(\gamma_1,\gamma_2\,\tau)\varphi(\gamma_2,\tau))$$

Modular transformations of Yukawa couplings

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$$(\gamma_1,\, \varphi(\gamma_1,\tau))(\gamma_2,\, \varphi(\gamma_2,\tau)) = (\gamma_1\gamma_2,\, \varphi(\gamma_1,\gamma_2\,\tau)\varphi(\gamma_2,\tau))$$

and the generators

$$\widetilde{S} = (S, -\sqrt{-\tau}) \quad \text{and} \quad \widetilde{T} = (T, +1)$$

Almumin,Chen, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

Modular transformations of Yukawa couplings

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We need to consider the metaplectic group

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$$\widetilde{S}^8 = (\widetilde{S}\,\widetilde{T})^3 = \mathbb{1} \quad \text{and} \quad \widetilde{S}^2\widetilde{T} = \widetilde{T}\,\widetilde{S}^2$$

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# of generations	Modular group
$M_1 = 1$ $M_2 = 1$ $M_3 = -2$	$\tilde{\Gamma}_{2\cdot 2} = \tilde{\Gamma}_4 \cong [96, 67]$
$M_1 = 3$ $M_2 = 3$ $M_3 = -6$	$\tilde{\Gamma}_{6\cdot 2} = \tilde{\Gamma}_{12} \cong [2304, ?]$
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Analyzed models

Modular transformations of Yukawa couplings

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Conjecture: Magnetized tori exhibit a finite modular symmetry of the form

$$\tilde{\Gamma}_{2\lambda} \text{ with } \lambda = \text{l.c.m}(M_1, M_2, M_3)$$

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Thank you!

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