PASCOS 2023

Excited Q-balls

Y. Almumin, J. Heeck, A. Rajaraman, and C. B. Verhaaren, “Excited Q-balls,” Eur. Phys. J. C 82 no. 9, (2022) 801, [2112.00657]

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University of California-Irvine

Date: 6/28/2023

Outline

- What are Q-balls & why are they interesting.
- Constructing Q-ball solutions.
- “Excited Q-balls” paper discussion.

What are Q-balls & why are they interesting?

Q-balls

Stable **solitons** which are composed of complex scalars that carry a $\mathscr{U}(1)$ **non-topological charge.**

Defining a **special potential** is key to get these stable non-topological solitons.

(S. R. Coleman, Q Balls, Nucl. Phys. B262 (1985) 263. [Erratum: Nucl. Phys. B269, 744 (1986)])

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Q-balls & their excited states are interesting

- Solutions to the some field theories.

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- Dark matter candidates.

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- Some supersymmetric theories predict Q-balls.

  - **A. Kusenko, "Solitons in the supersymmetric extensions of the standard model," Phys. Lett. B 405 (1997) 108, [hep-ph/9704273]**
  - **K. Enqvist and J. McDonald, "Q balls and baryogenesis in the MSSM," Phys. Lett. B 425 (1998) 309–321, [hep-ph/9711514]**

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Constructing Q-ball solutions

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$$\mathscr{L} = |\partial_\mu \phi|^2 + U(|\phi|)$$

Coleman conditions

- Symmetric under unbroken $\mathscr{U}(1)$

- Potential is zero at the vacuum

- $\dfrac{U(|\phi|)}{|\phi|^2}$ has a minimum at $\dfrac{\phi_0}{\sqrt{2}}$

Coleman (1986)

Constructing Q-ball solutions

$$\mathscr{L} = |\partial_\mu \phi|^2 + U(|\phi|)$$

Coleman conditions

- Symmetric under unbroken $\mathscr{U}(1)$

$$\phi(x) = \frac{\phi_0}{\sqrt{2}} f(r) e^{i\omega t}$$

$$\omega_0 < \omega < m_\phi$$

- Potential is zero at the vacuum

- $\dfrac{U(|\phi|)}{|\phi|^2}$ has a minimum at $\dfrac{\phi_0}{\sqrt{2}}$

$$\frac{U(f)}{\phi_0^2} = \frac{1}{2}(m_\phi^2 - \omega_0^2) f^2 (1 - f^2)^2 + \frac{\omega_0^2}{2} f^2$$

Heeck, Rajaraman, Riley, & Verhaarn (2021)

Q-ball equation of motion

$$\mathscr{L} = |\partial_\mu \phi|^2 + U(|\phi|)$$

Dimensionless Quantities

$$\rho := r\sqrt{m_\phi - \omega_0}$$

$$\kappa^2 := \frac{\omega^2 - \omega_0^2}{m_\phi^2 - \omega_0^2}, \quad \kappa \in (0,1)$$

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Effective potential

$$V(f) = \frac{1}{m_\phi^2 - \omega_0^2}\left(\frac{\omega^2}{2} f^2 - \frac{U(f)}{\phi_0^2}\right) = \frac{1}{2}\left[f^2\kappa^2 - f^2(1-f^2)^2\right]$$

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Q-ball equation of motion

$$f''(\rho) + \frac{2}{\rho} f'(\rho) + \frac{dV}{df} = 0$$

↔

Particle rolling in a potential with friction

$$x''(t) + \frac{2}{t} x'(t) + \frac{dV}{dx} = 0$$

Heeck, Rajaraman, Riley, & Verhaarn (2021)

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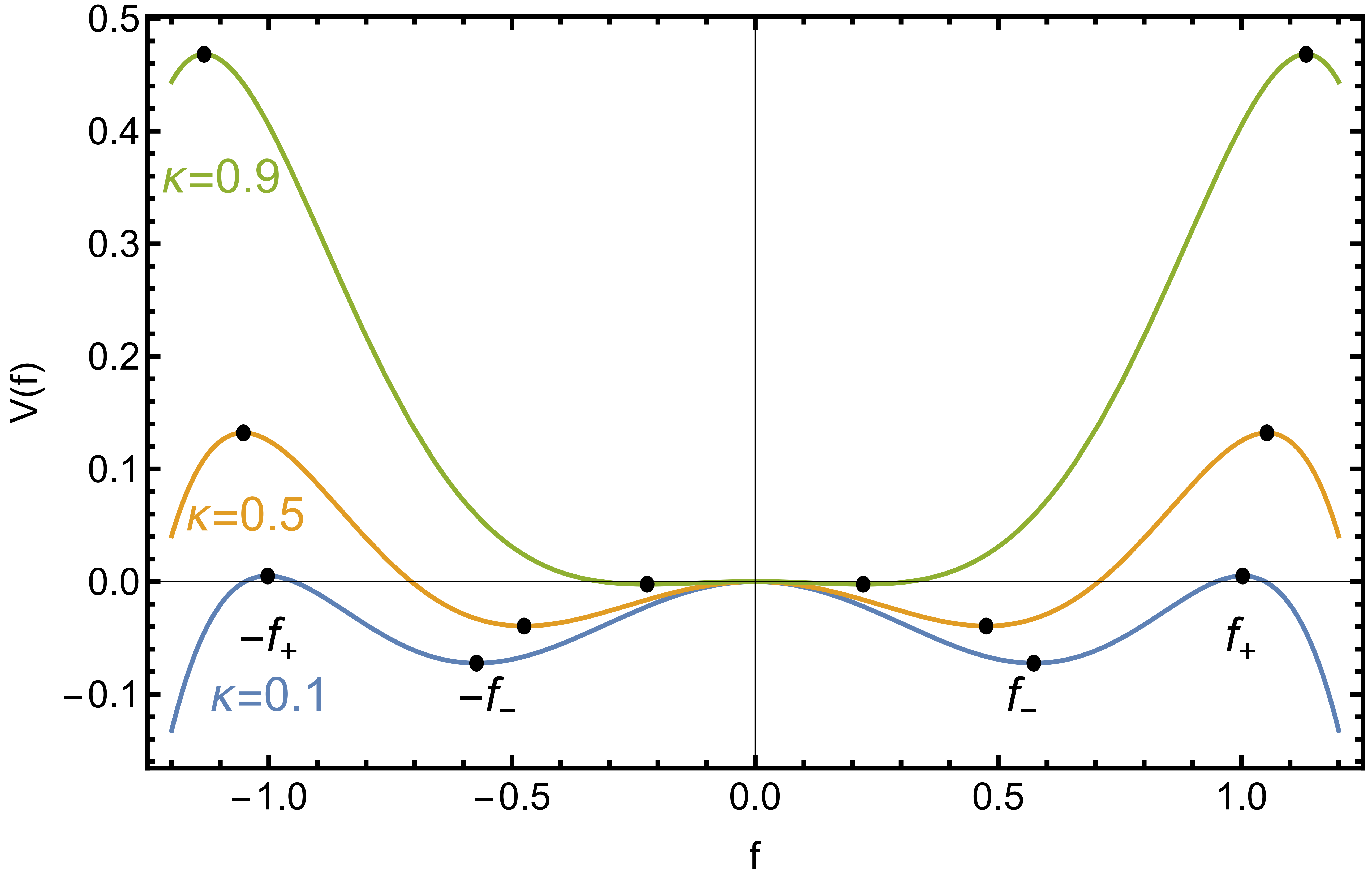

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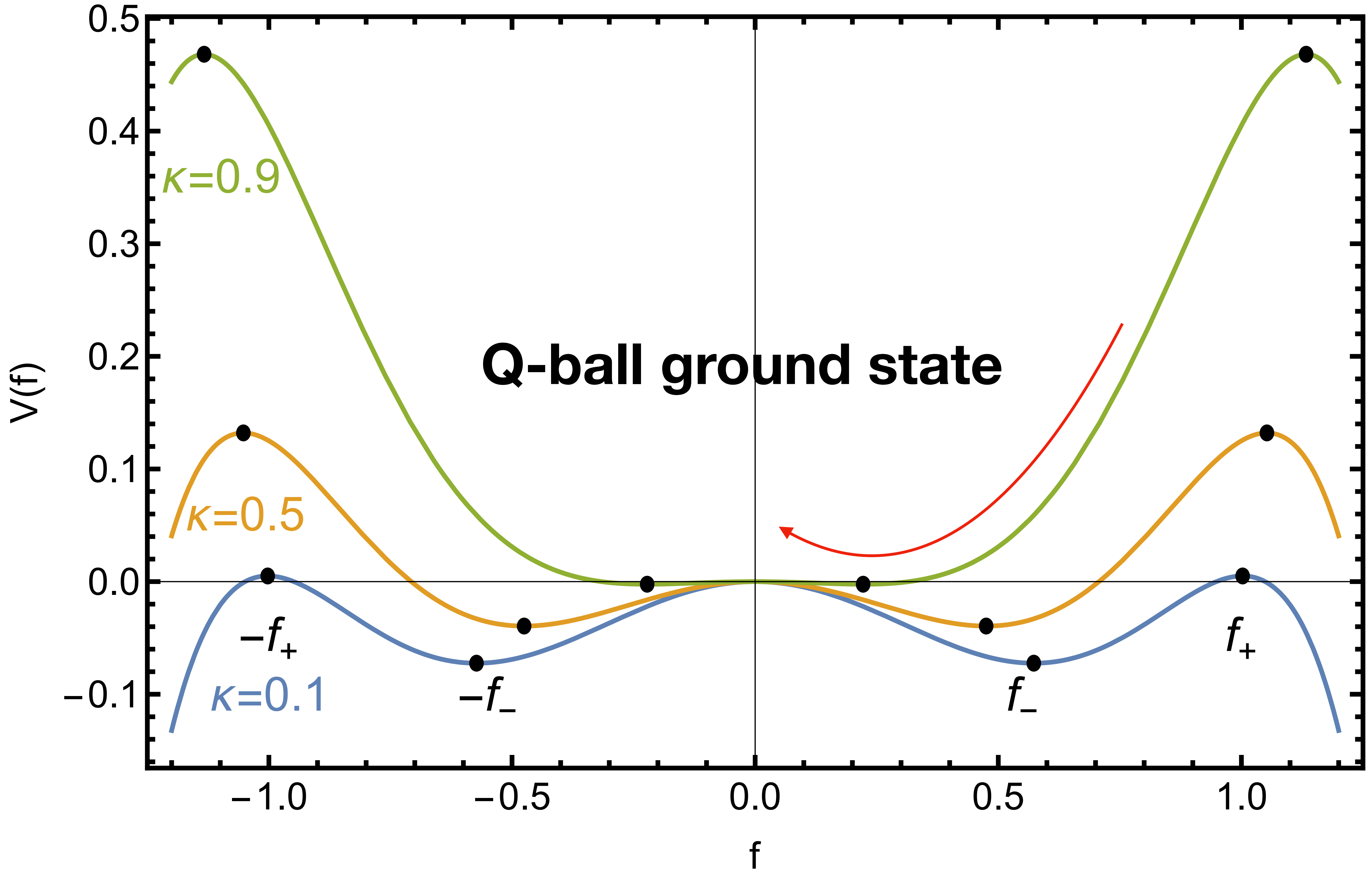

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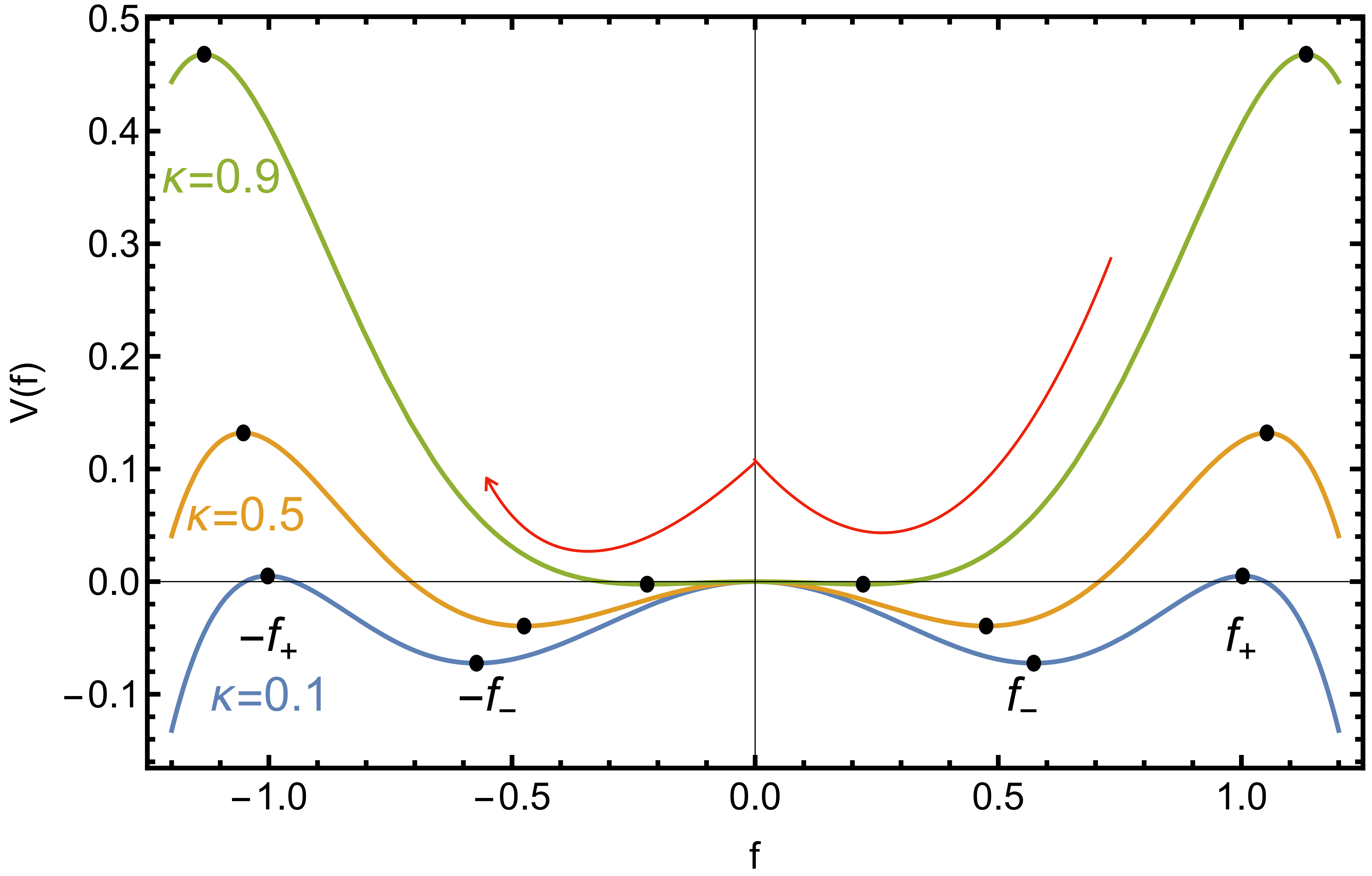

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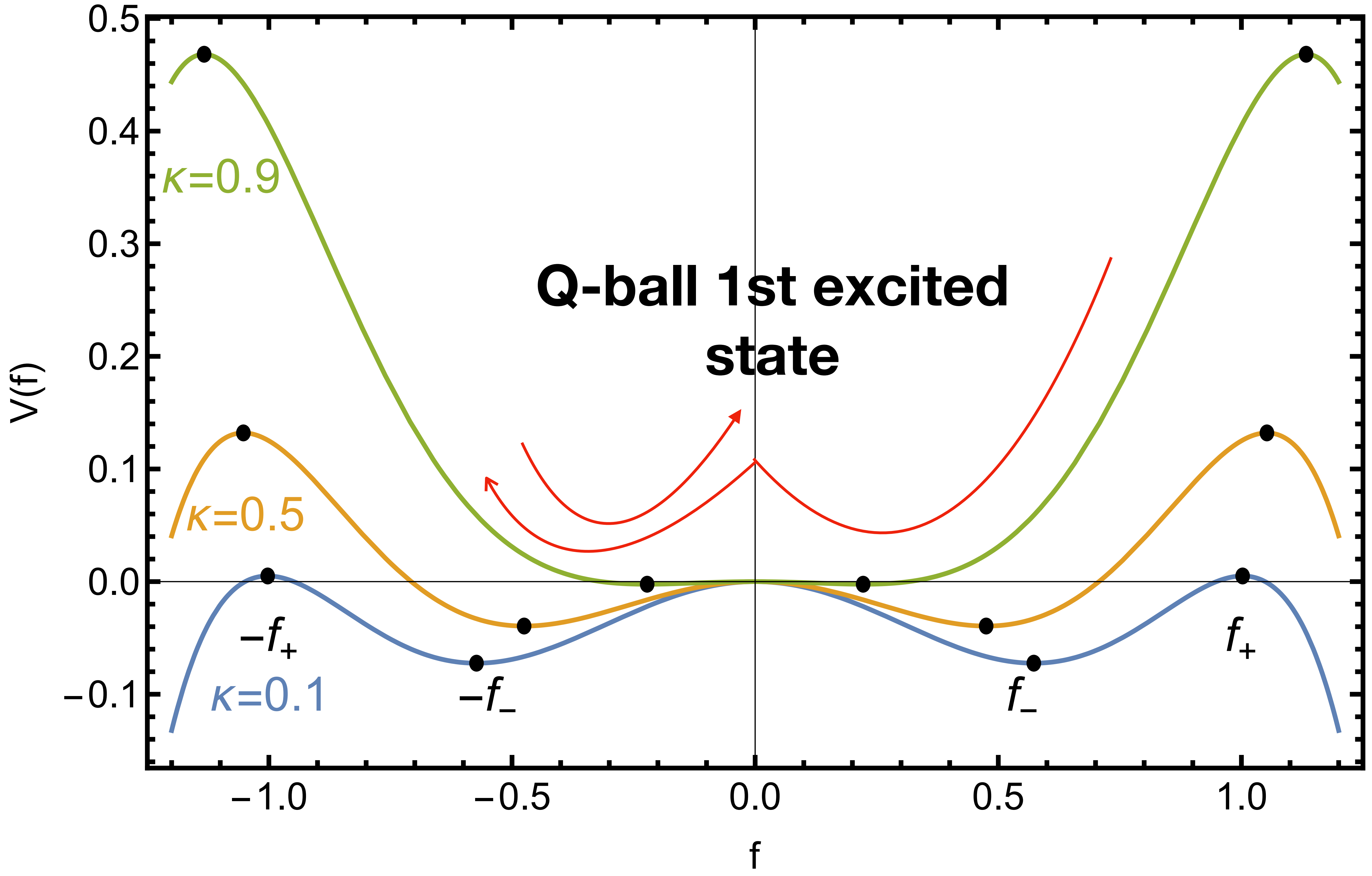

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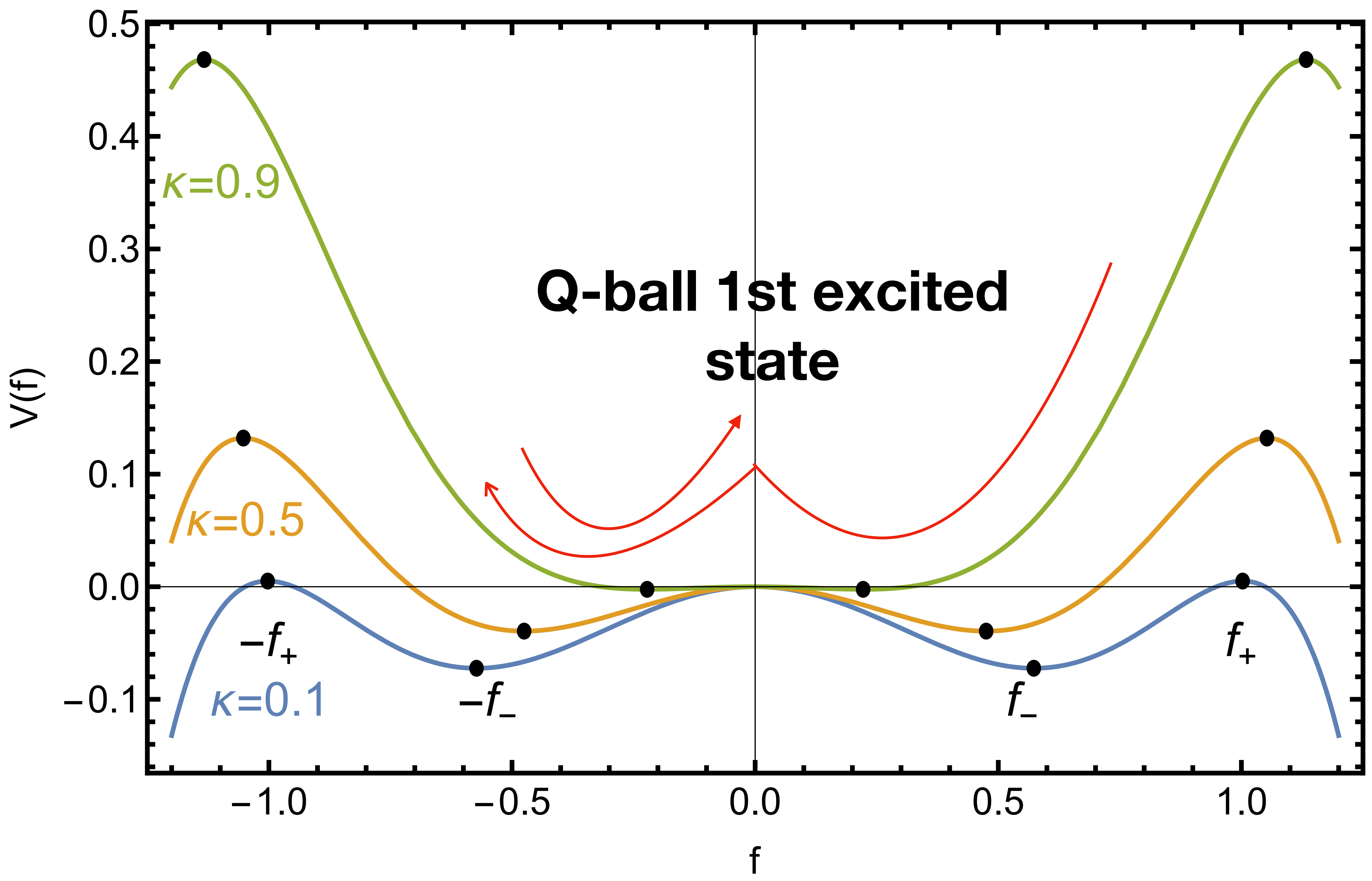
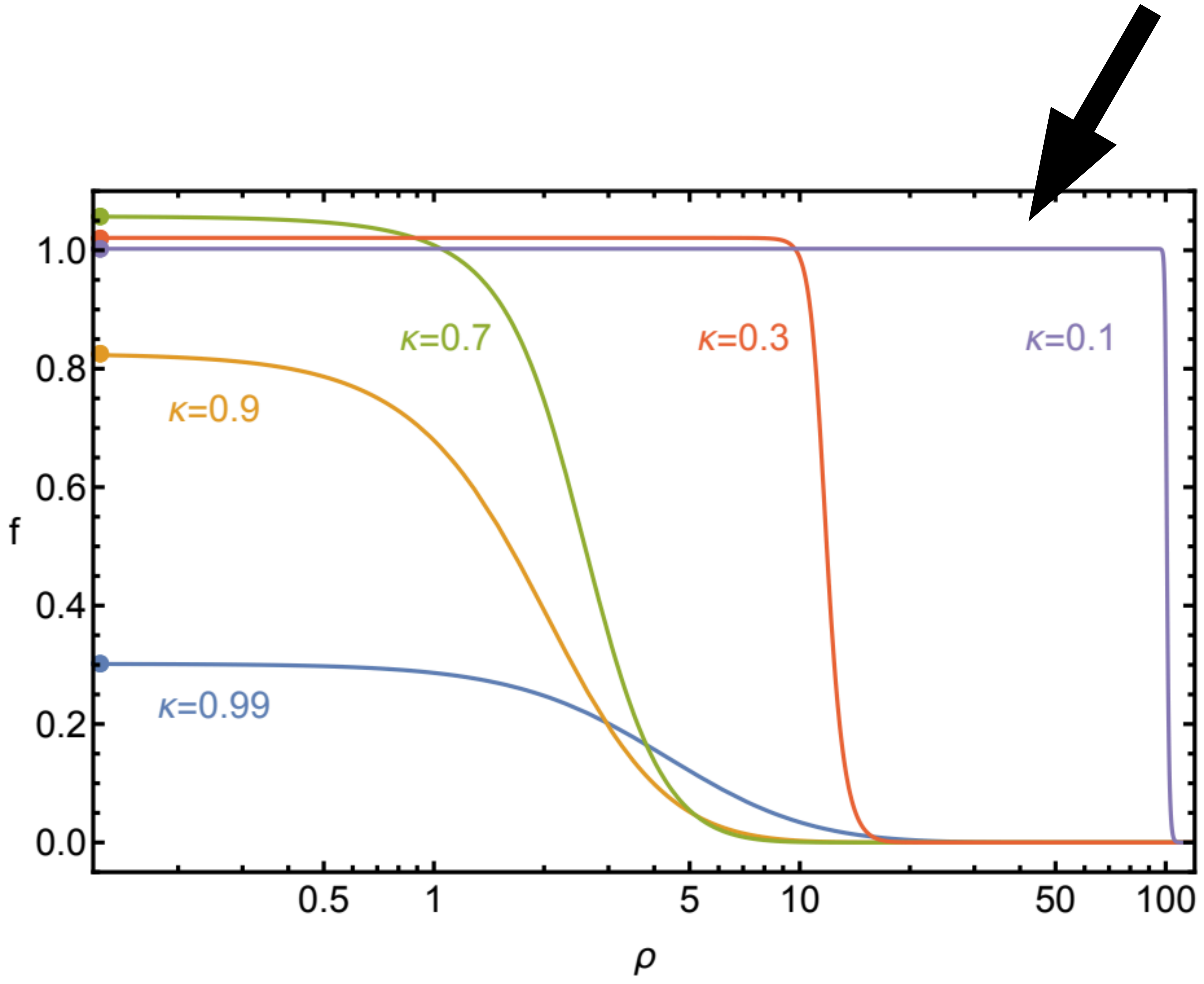
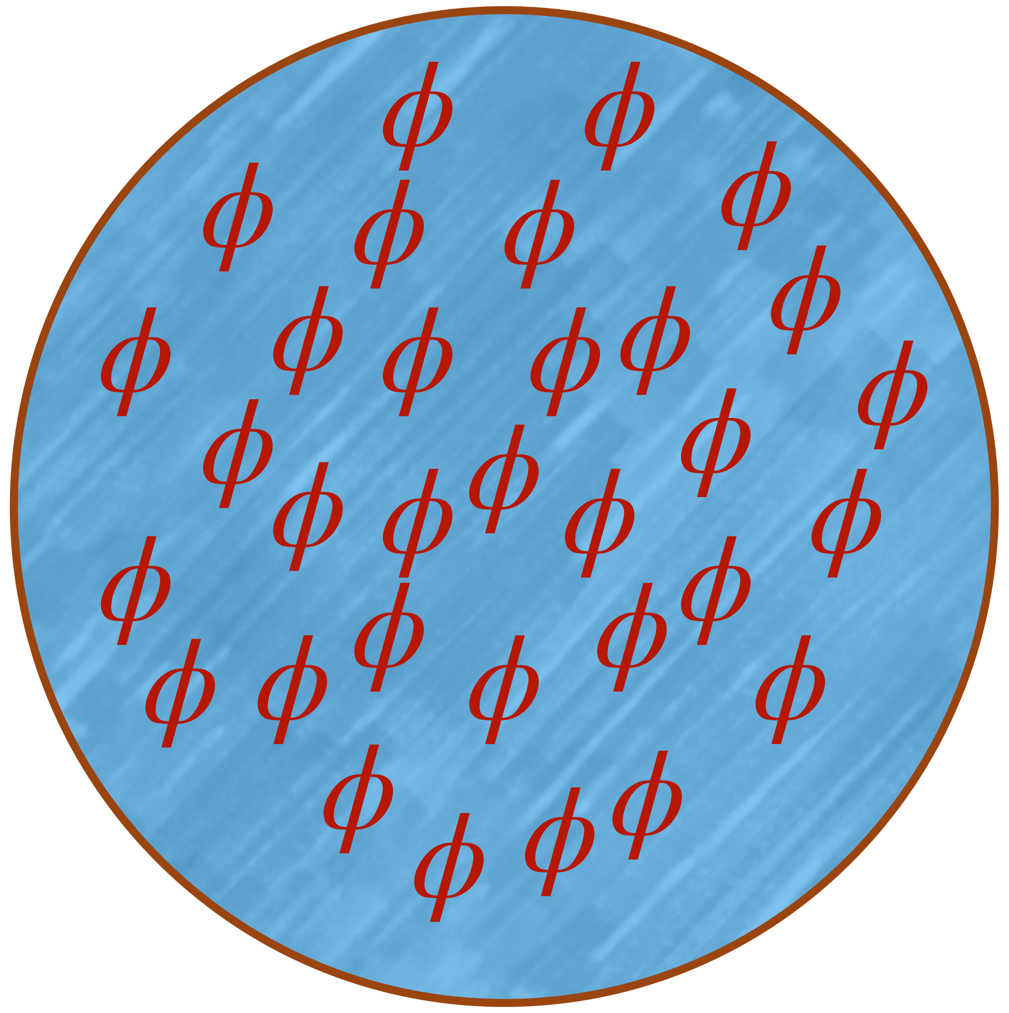


Radius approximation & Q-ball transition function





$$R* = \frac{1}{\kappa^2}$$

Thin-wall limit

$$f(\rho) = \begin{cases} 1, & \rho < R* \\ 0, & \rho > R* \end{cases}$$

Heeck, Rajaraman, Riley, & Verhaarn (2021)

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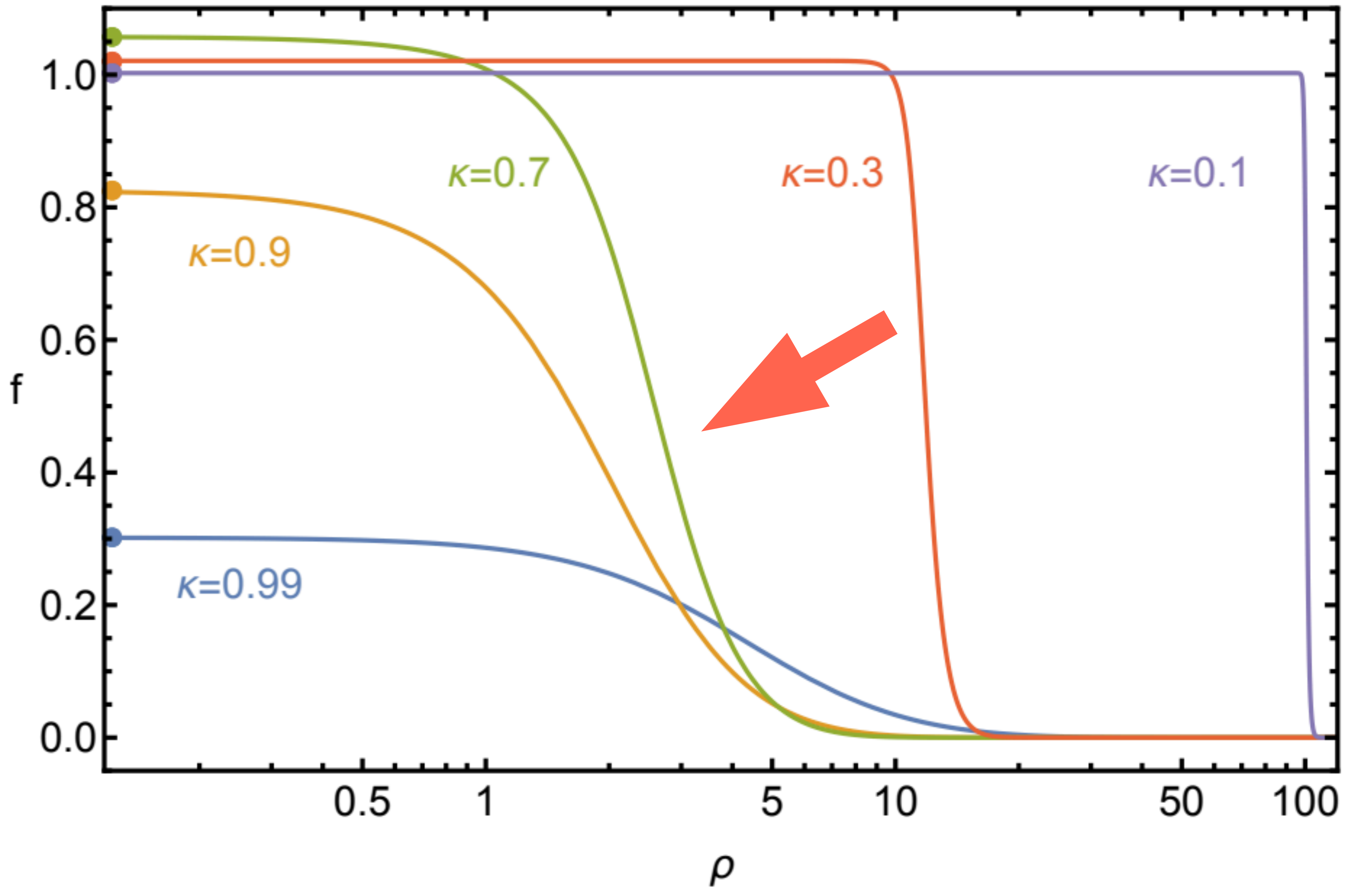
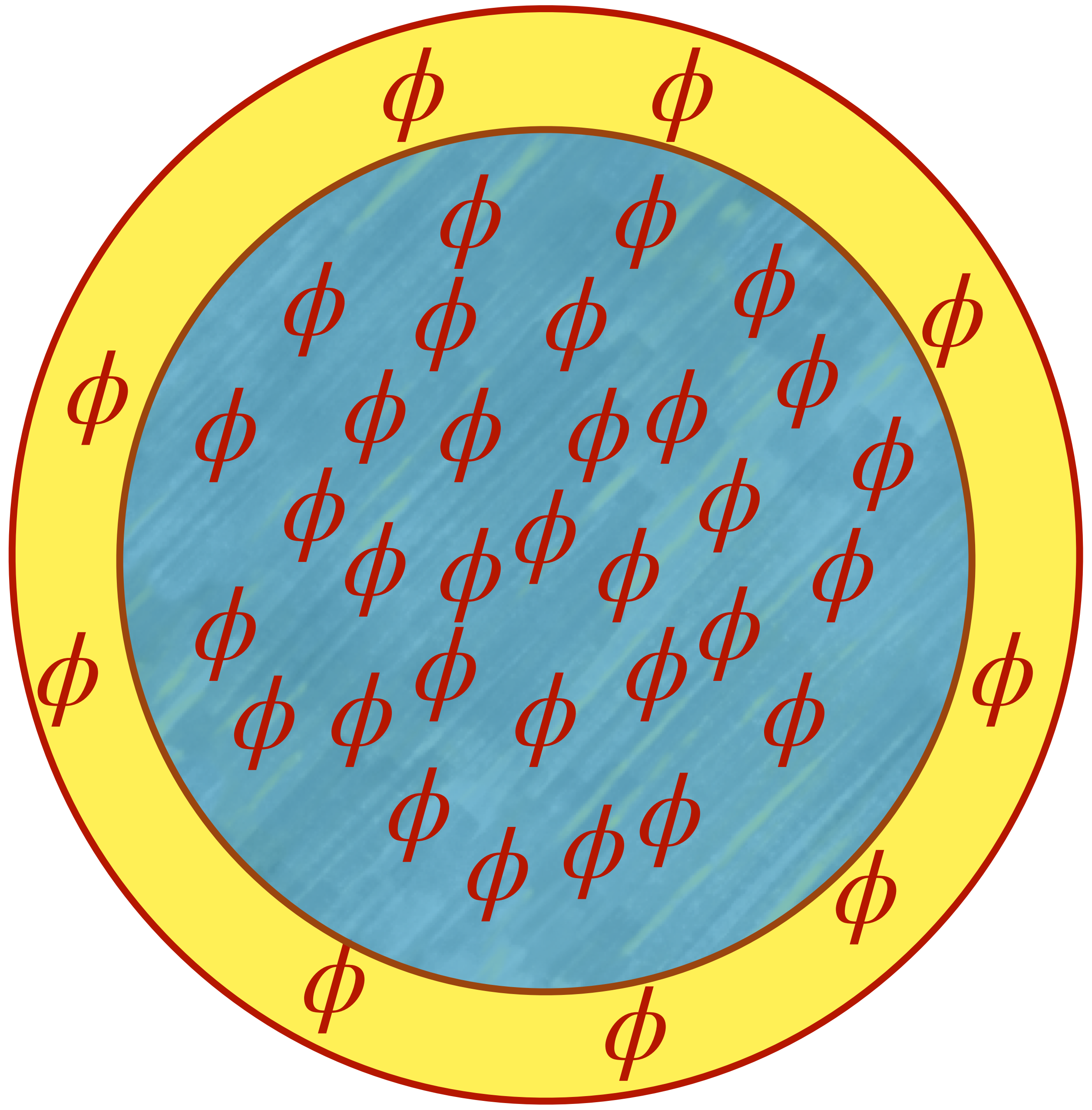




$$R* = \frac{1}{\kappa^2} + \frac{1}{4} - \frac{5\kappa^2}{16} + \mathcal{O}(\kappa^4)$$

Transition function

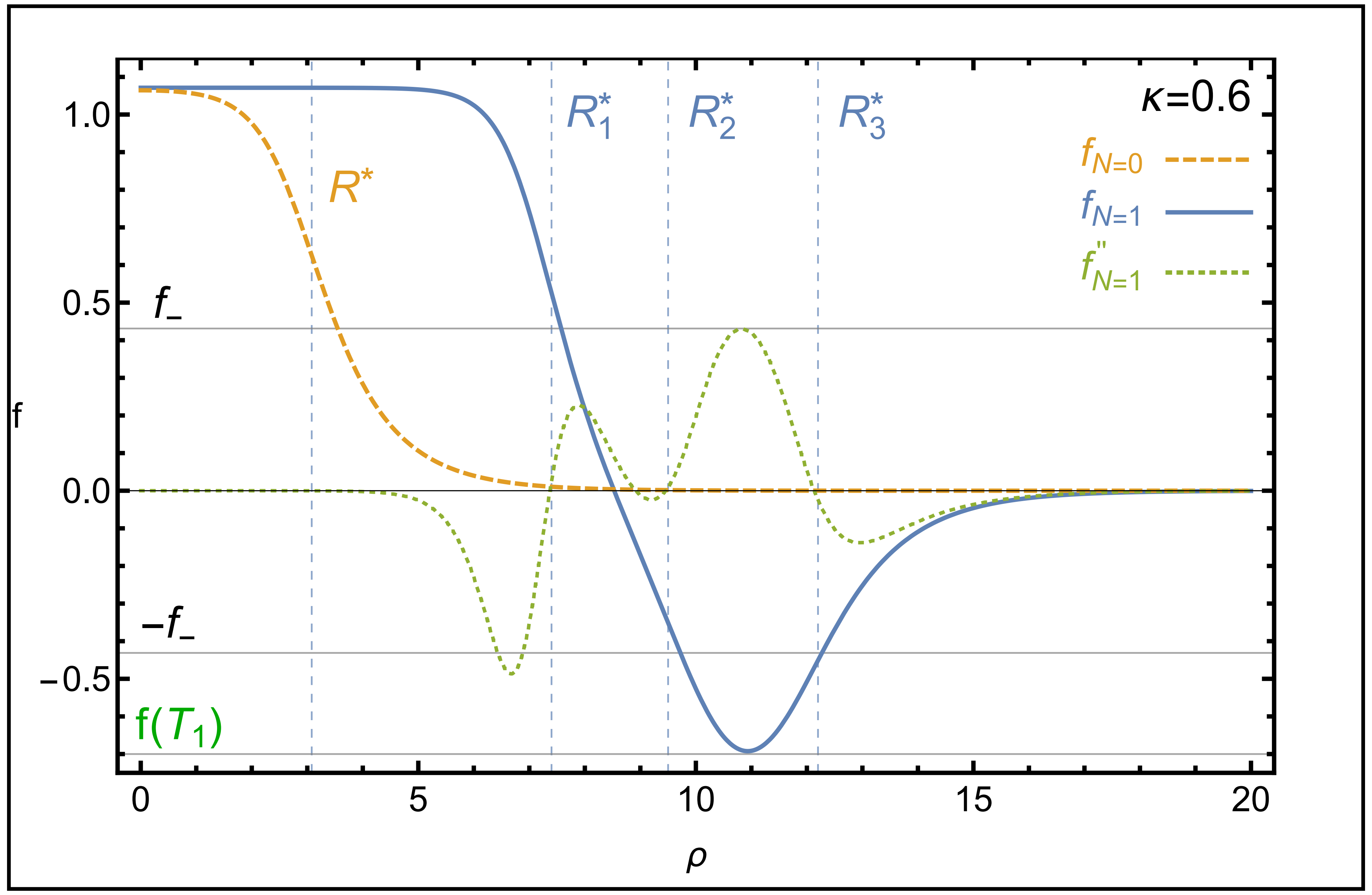
$$f(\rho)_T = [1 + 2e^{2(\rho - R*)}]^{-1/2}$$

Heeck, Rajaraman, Riley, & Verhaarn (2021)

Excited Q-balls discussion

Excited Q-balls effective potential & profile

Q-ball profiles



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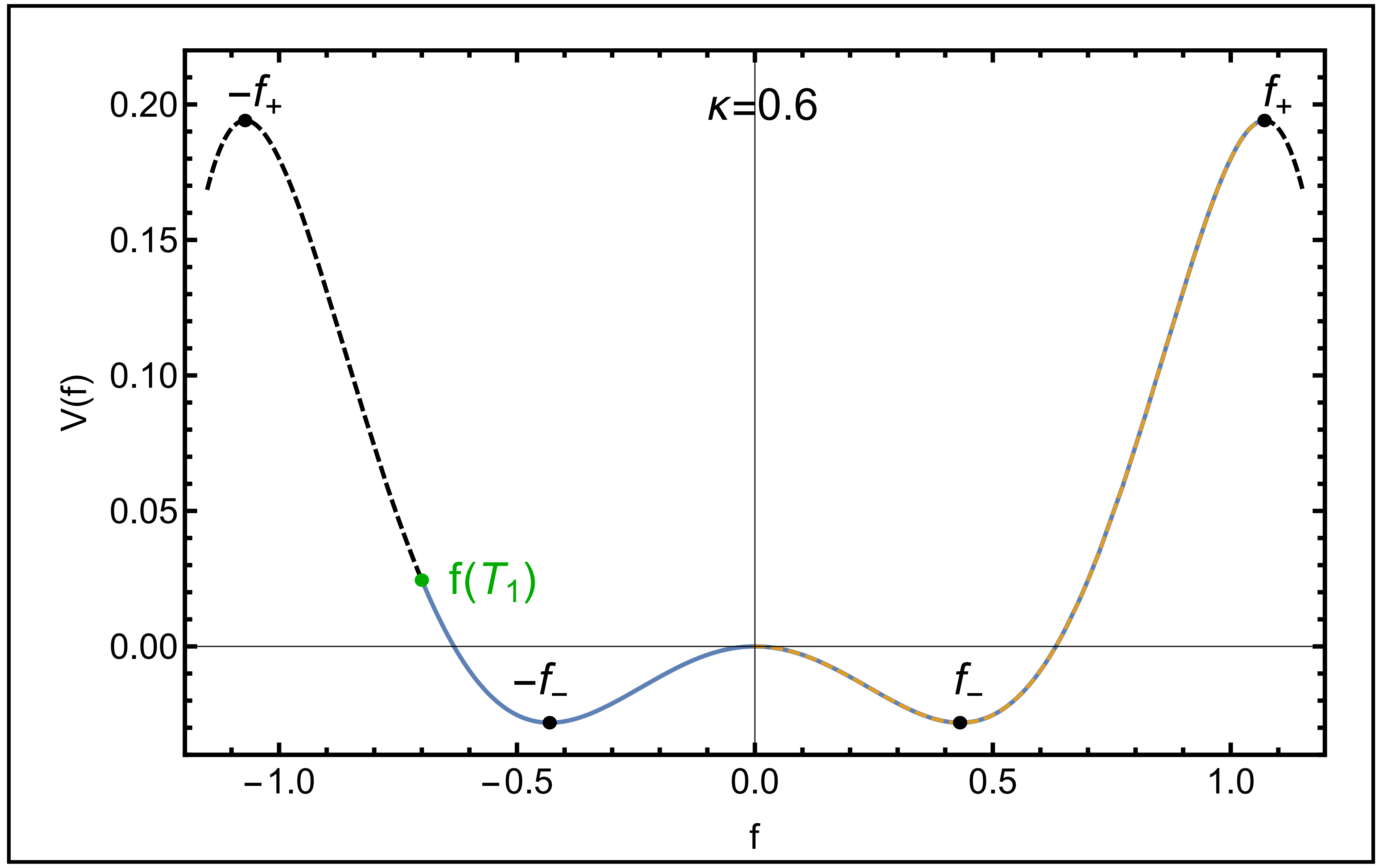

Almumin, Heeck, Rajaraman, & Verhaaren (2022)

Excited Q-balls effective potential & profile

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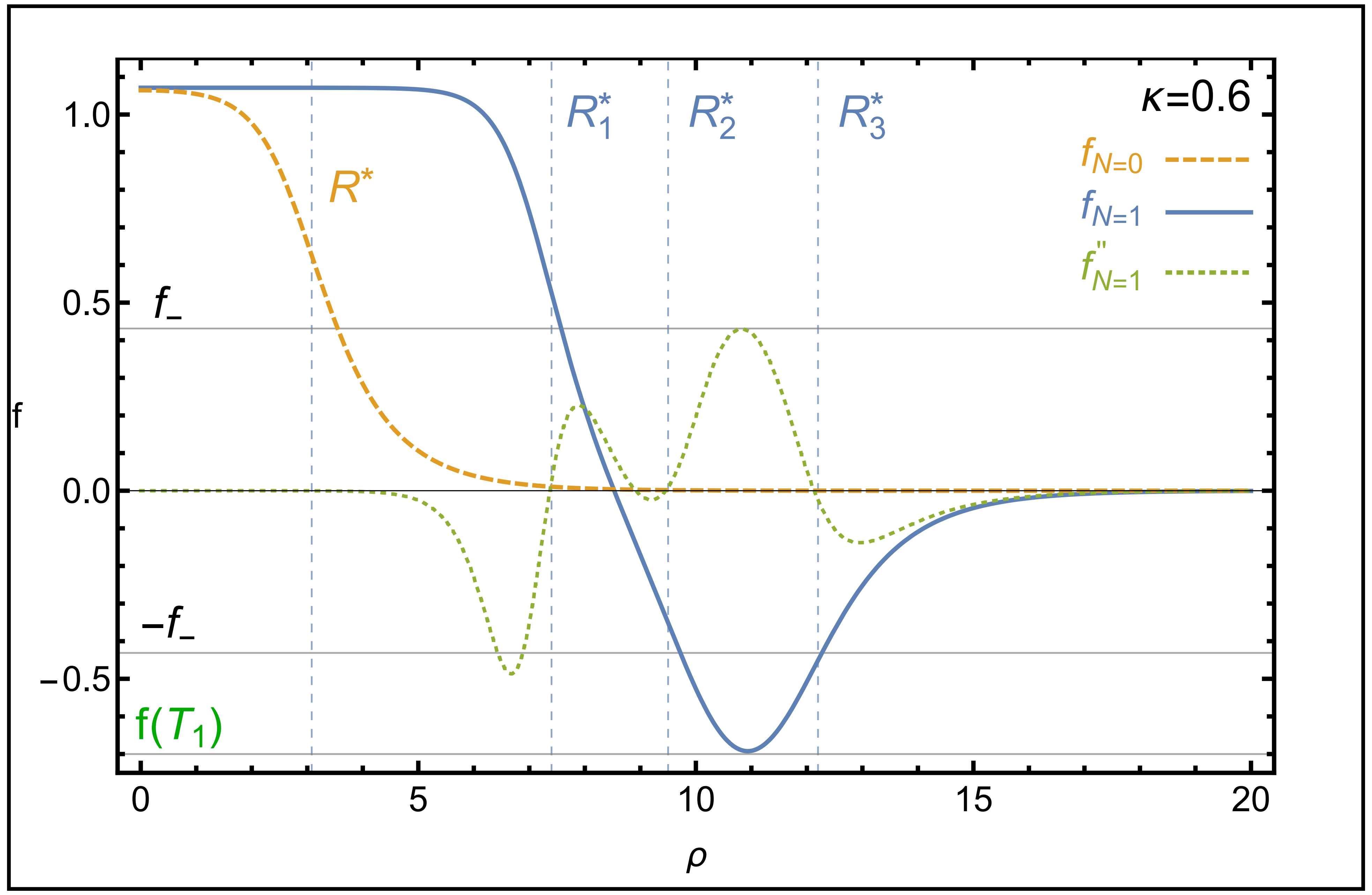

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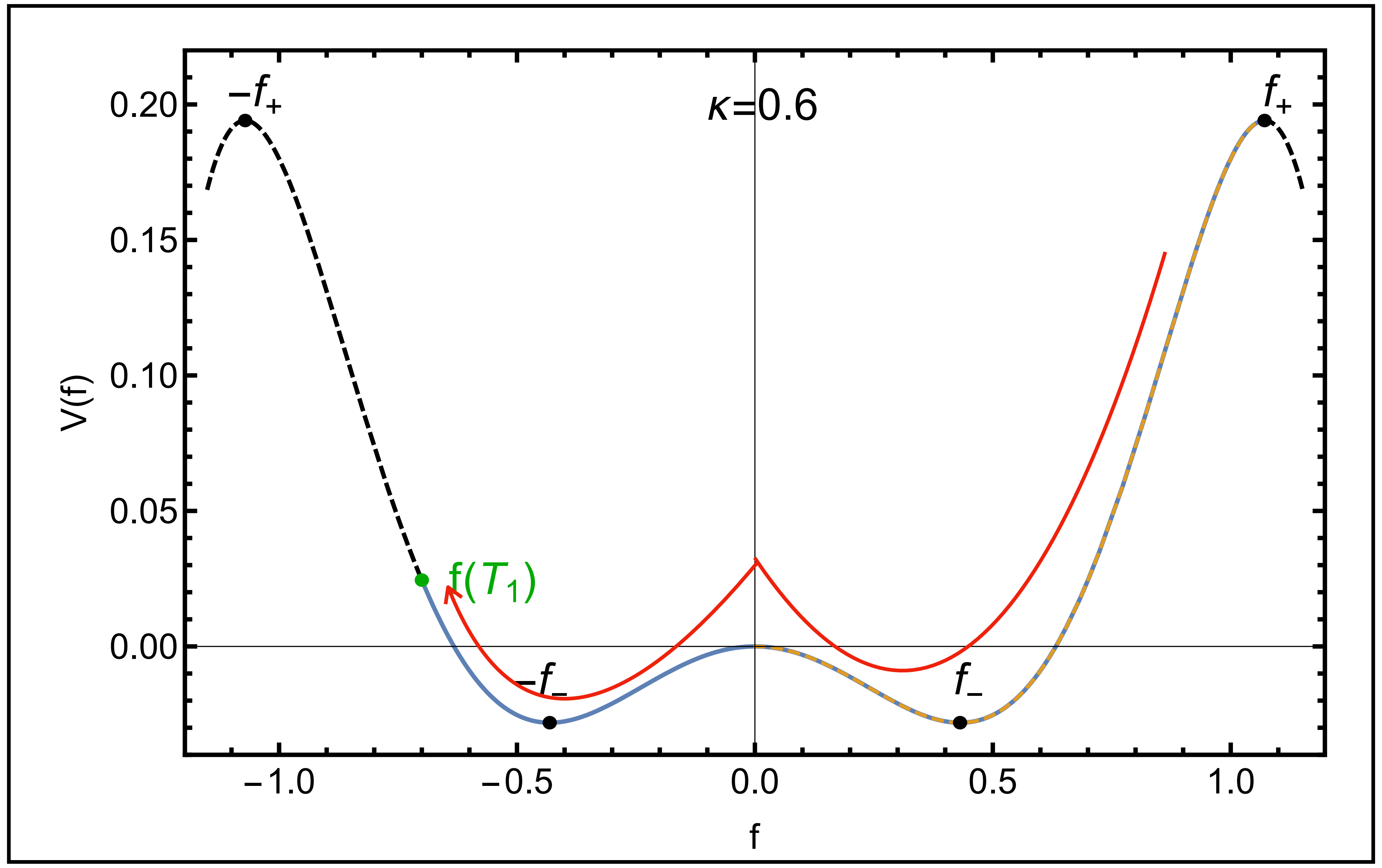

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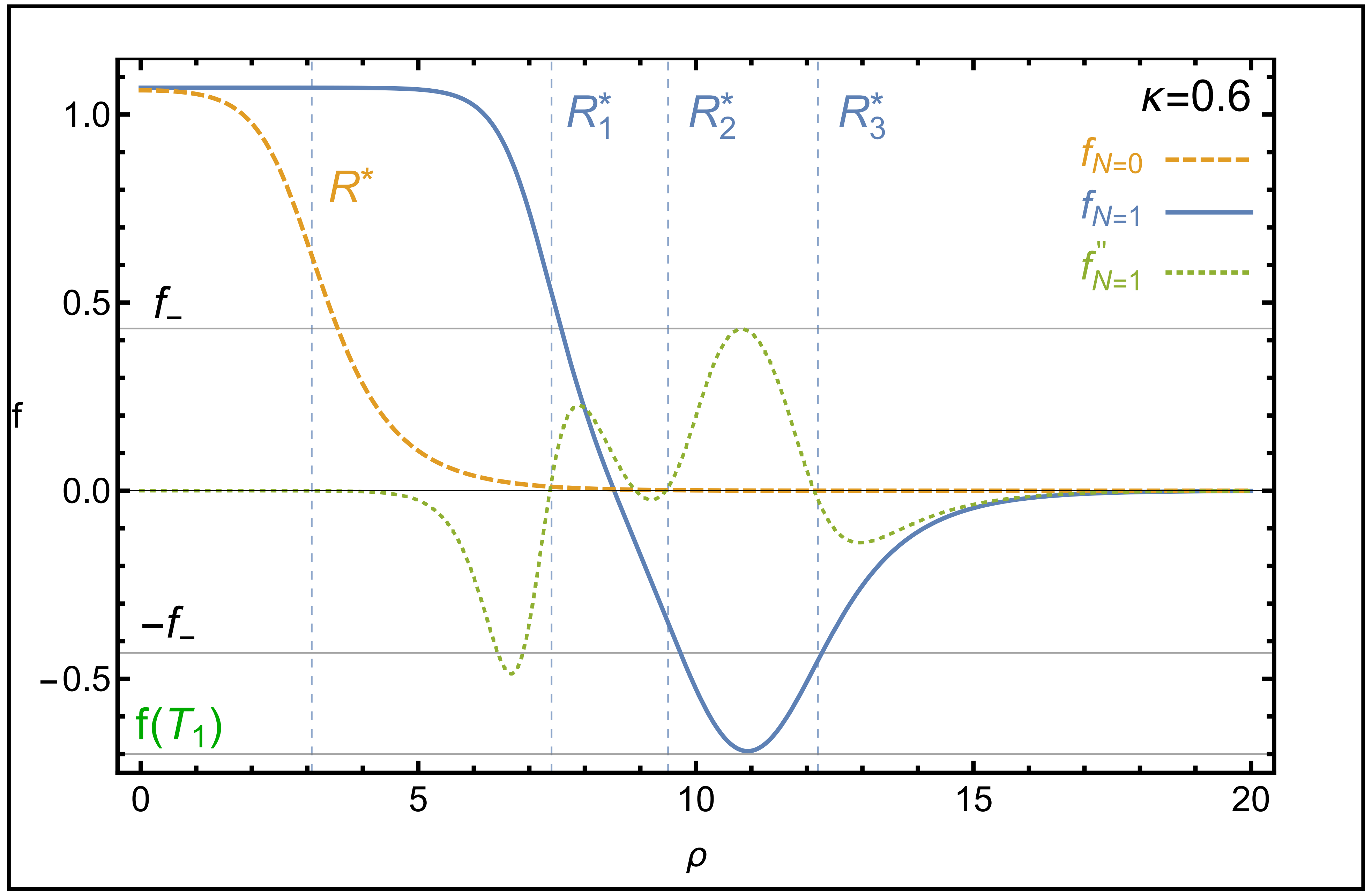

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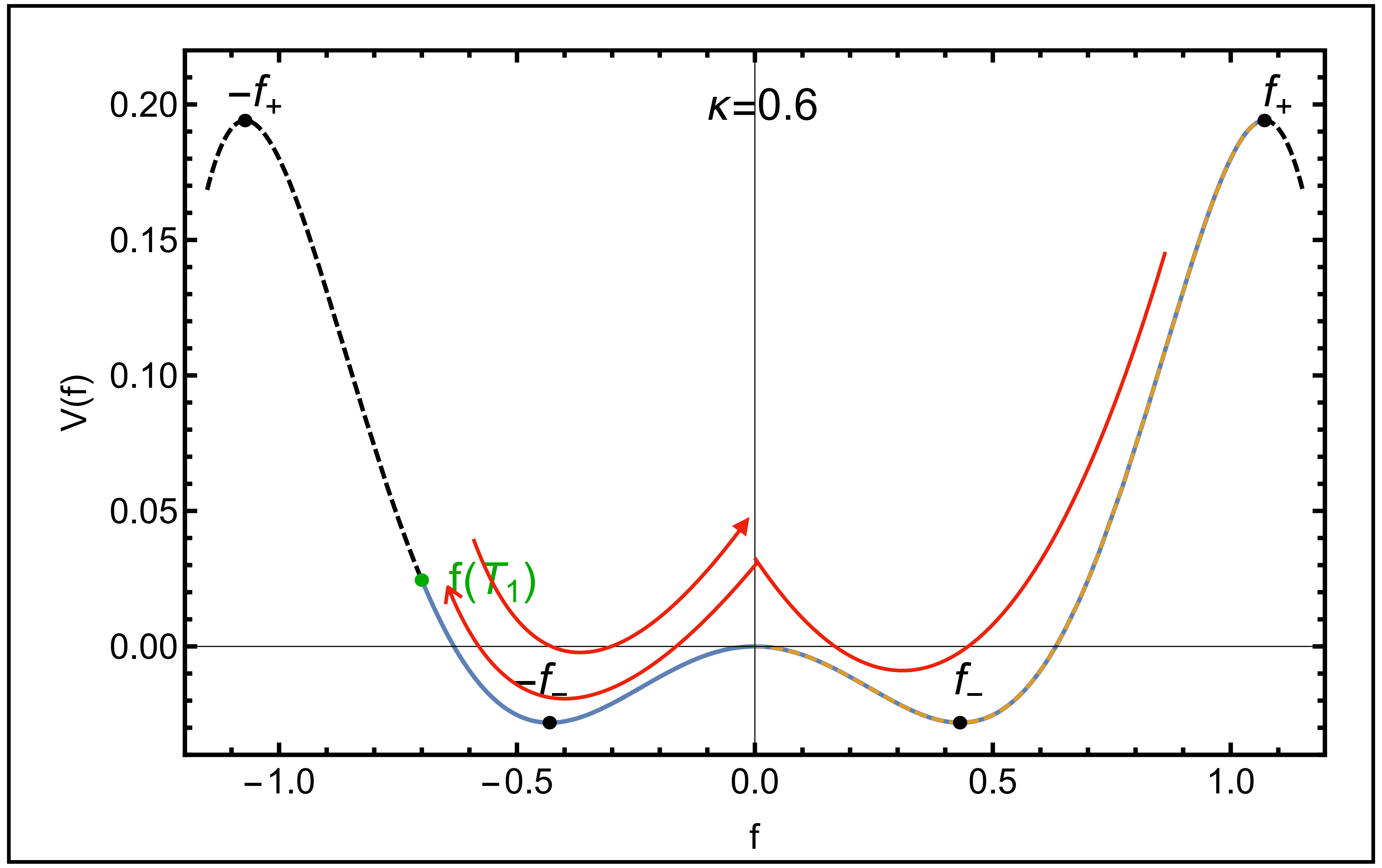

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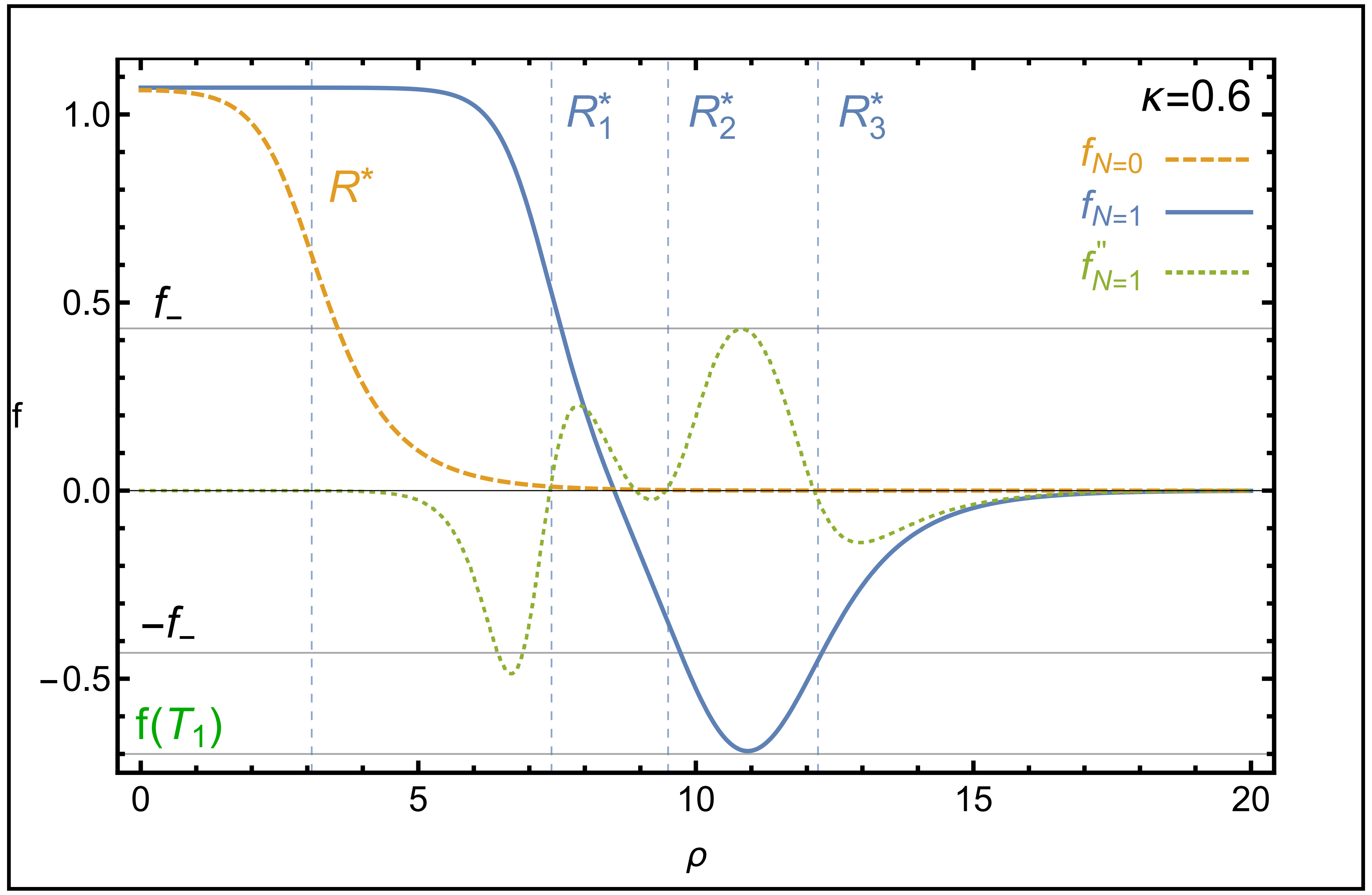
Excited Q-balls radii approximation

Q-ball profiles



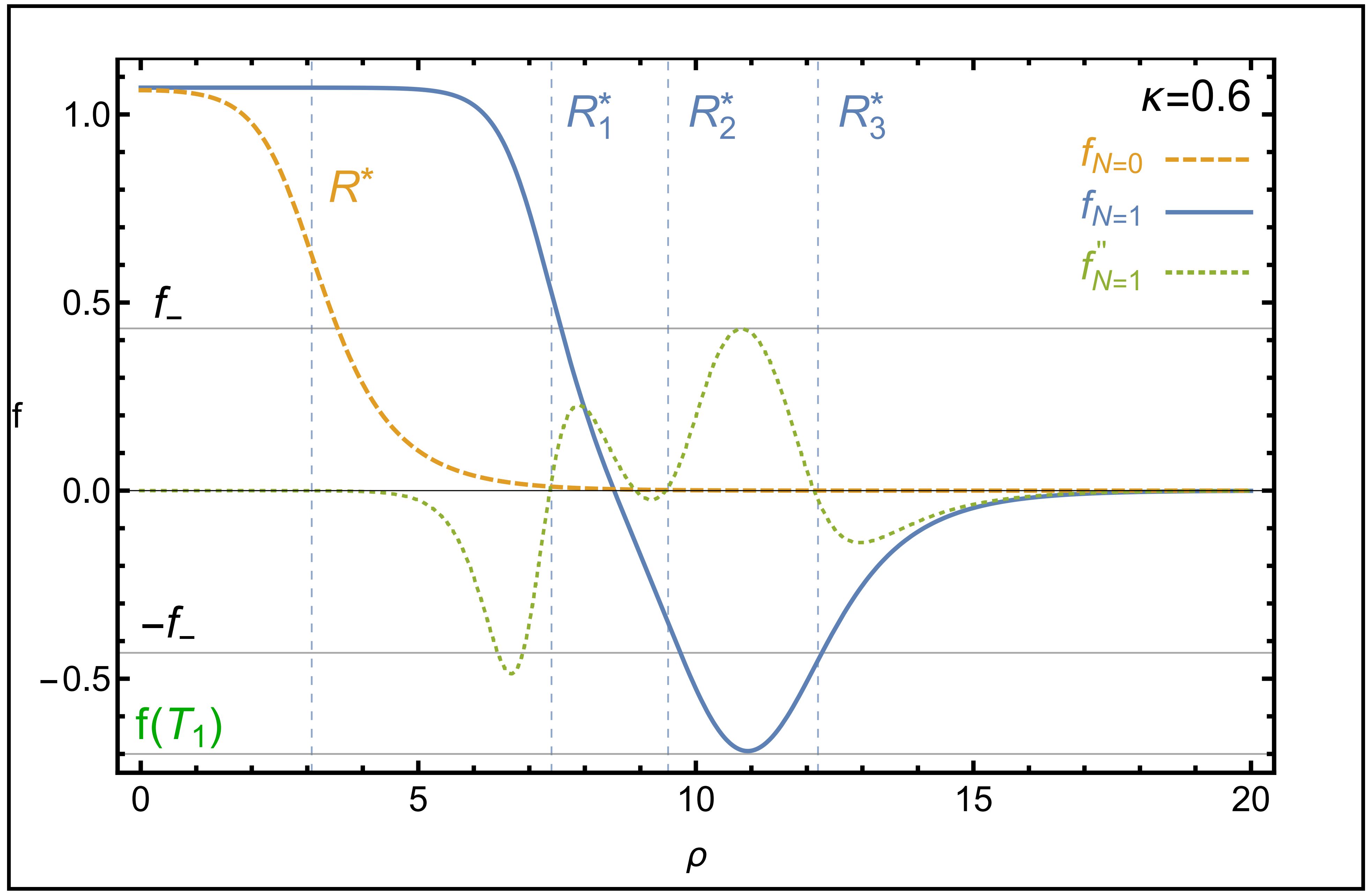
Radii approximation terms of κ

$$R^*_{N,n} = \frac{2N+1}{\kappa^2} + \begin{cases} \frac{c_{N,n-1}}{\sqrt{1-\kappa^2}} - \left(\frac{3}{2}n - 1\right)\ln\kappa, & \text{even } n, \\ \frac{c_{N,n}}{\sqrt{1-\kappa^2}} - \left(\frac{3}{2}n - \frac{3}{2}\right)\ln\kappa, & \text{odd } n, \end{cases}$$

Almumin, Heeck, Rajaraman, & Verhaaren (2022)

Excited Q-balls radii approximation

Q-ball profiles



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Leading order

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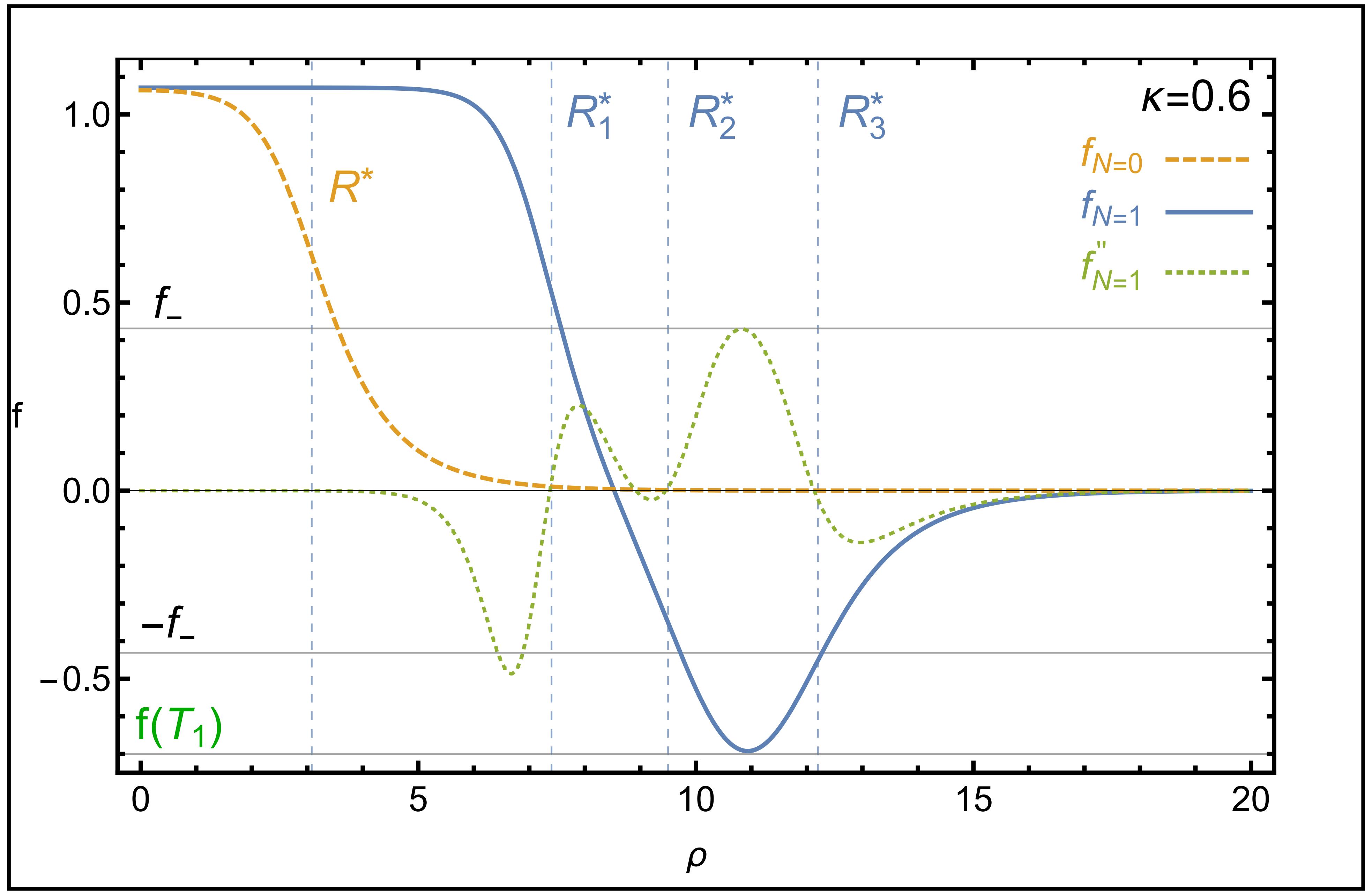
Excited Q-balls radii approximation

Q-ball profiles



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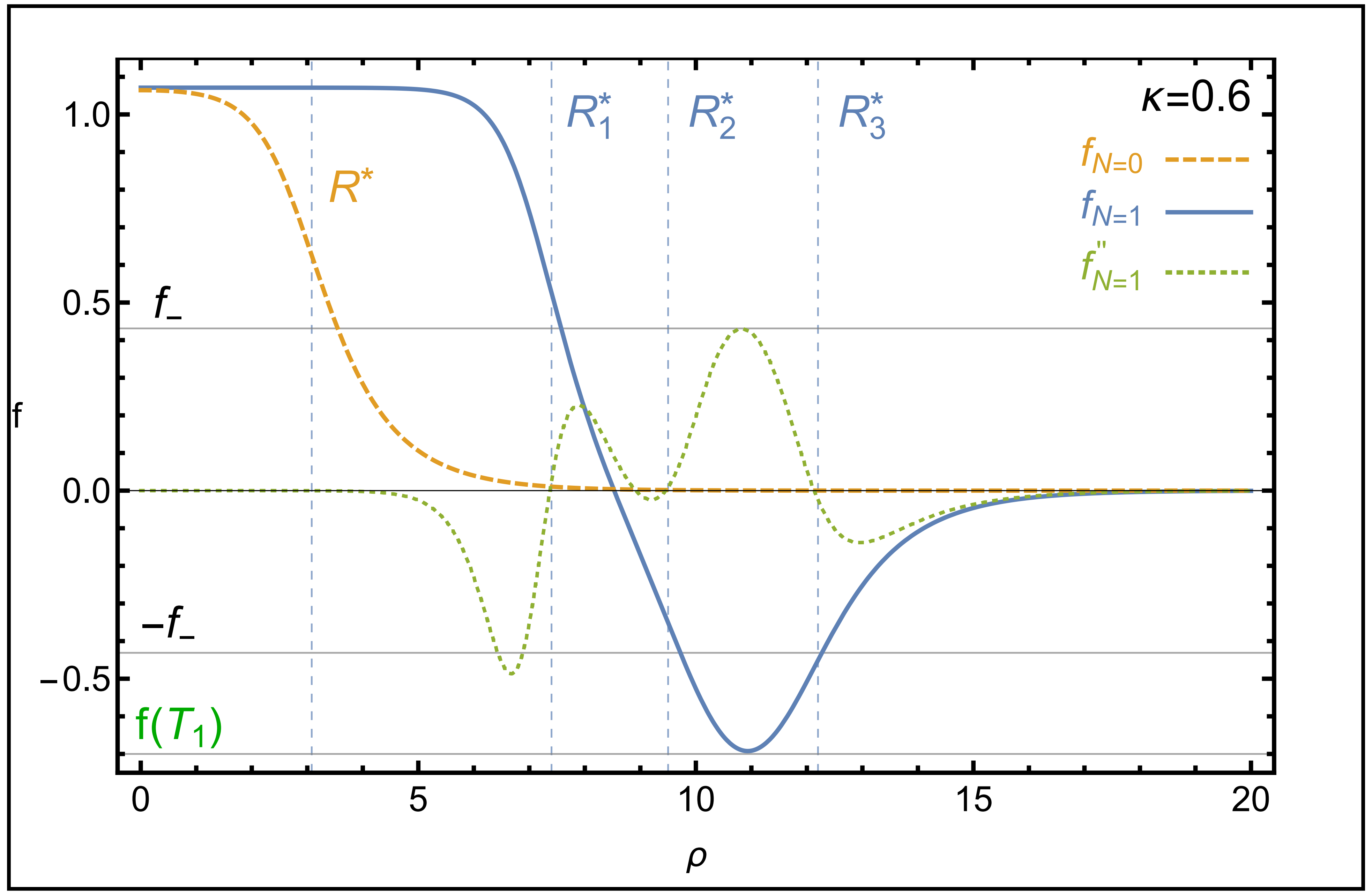
Leading order

Small κ

Almumin, Heeck, Rajaraman, & Verhaaren (2022)

Excited Q-balls radii approximation

Q-ball profiles



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Leading order — Thick-wall limit — Small κ

Almumin, Heeck, Rajaraman, & Verhaaren (2022)

Transition function approximation

Transitions function ansatz

$$f(\rho, R^*_{N,n})_T = [1 + 2e^{2(\rho - R^*_{N,n})}]^{-1/2}$$

$$f_N = [f_T(\rho, R^*_{N,1}) - f_T(-\rho, -R^*_{N,2})] \ldots [f_T(\rho, R^*_{N,2N-1}) - f_T(-\rho, -R^*_{N,2N})] \;\; f_T(\rho, R^*_{N,2N+1})$$

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Example: transitions function of 1st excited state

$$f_1 = [f_T(\rho, R*_{1,1}) - f_T(-\rho, -R*_{1,2})]\ \ f_T(\rho, R*_{1,3})$$

Almumin, Heeck, Rajaraman, & Verhaaren (2022)

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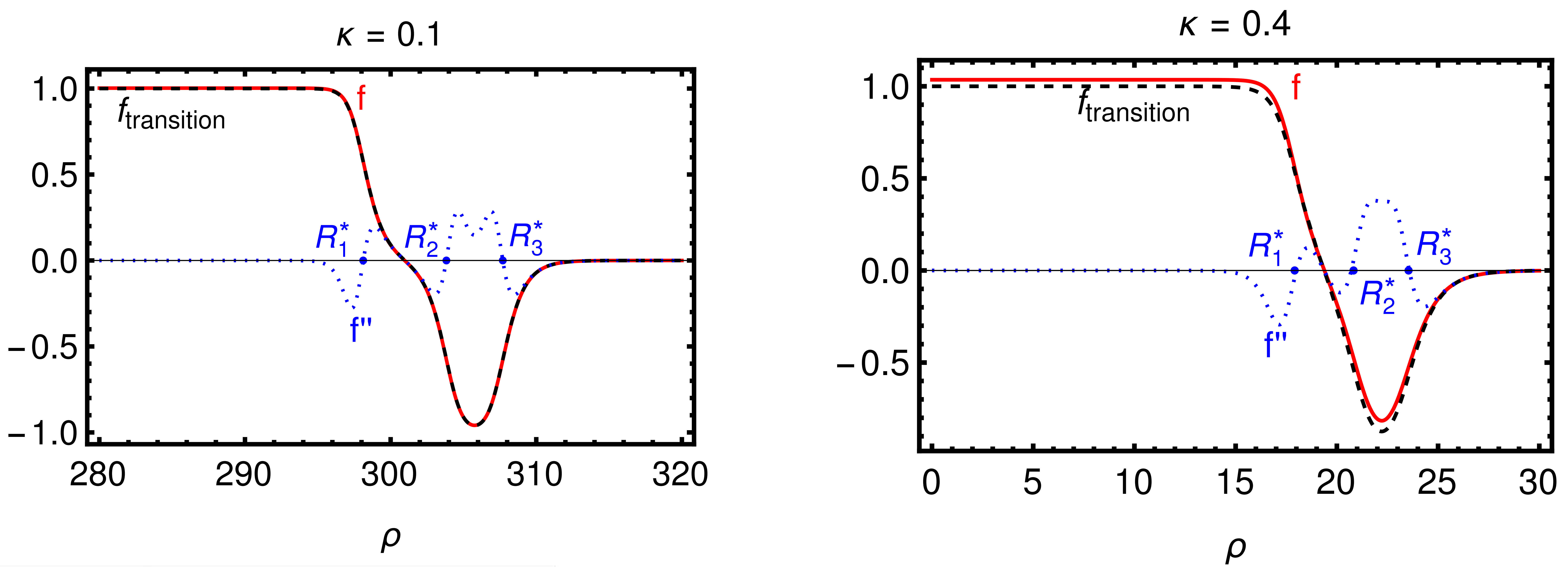

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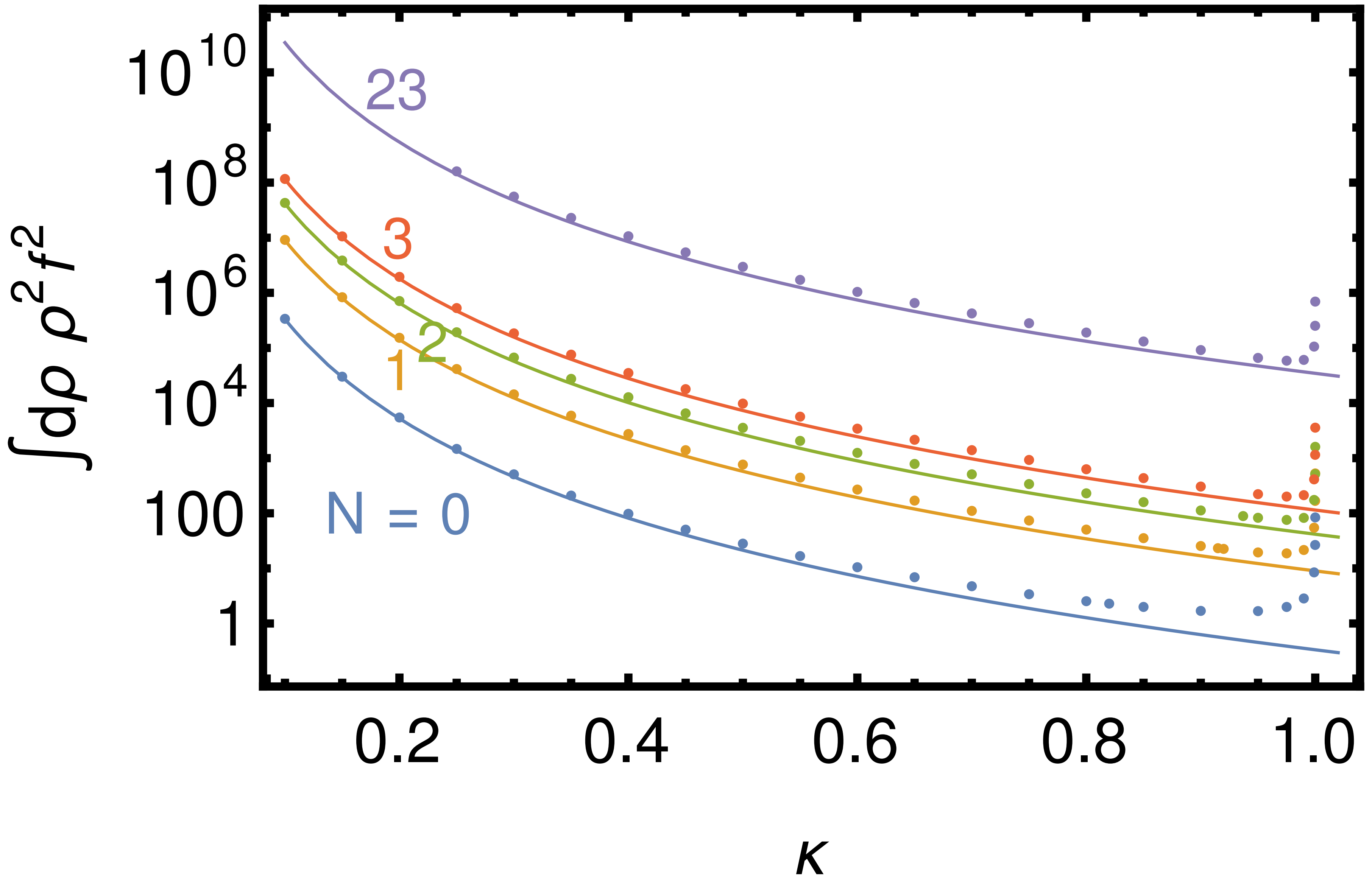
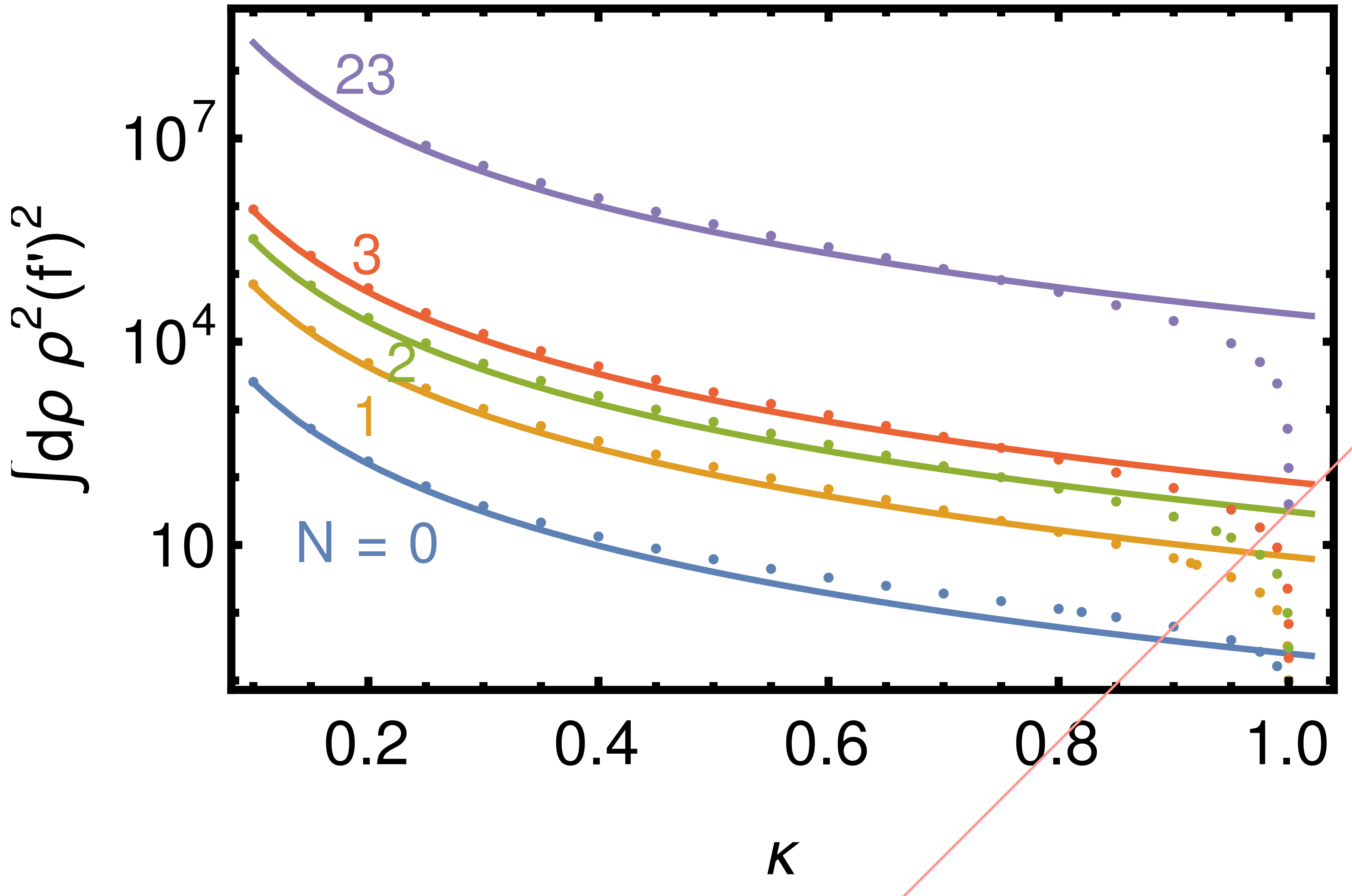
Charge & energy of excited Q-balls





$$Q = \frac{4\pi\omega\phi_0}{(m_\phi^2 - \omega_0^2)^{3/2}} \int d\rho \rho^2 f^2$$

$$E = \omega Q + \frac{4\pi\phi_0}{3\sqrt{m_\phi^2 - \omega_0^2}} \int d\rho \rho^2 f'^2$$

Energy & charge approximation using the leading order of the radii

$$\int \mathrm{d}\rho\, \rho^2 f^2 \simeq \frac{(2N+1)^3}{3\kappa^6},$$

$$\int \mathrm{d}\rho\, \rho^2 f'^2 \simeq \frac{(2N+1)^3}{4\kappa^4},$$

Almumin, Heeck, Rajaraman, & Verhaaren (2022)

Summary of the paper

- We found analytical approximation of excited Q-balls radii in terms of κ.

- We found a transition function of excited Q-balls, which we use to produce excited Q-ball profiles.

- We produced the charge and energy of excited Q-balls using the radii analytical approximation.

Almumin, Heeck, Rajaraman, & Verhaaren (2022)

References

- (S. R. Coleman, Q Balls, Nucl. Phys. B262 (1985) 263. [Erratum: Nucl. Phys. B269, 744 (1986)])
- J. Heeck, A. Rajaraman, R. Riley, and C. B. Verhaaren, “Understanding Q-Balls Beyond the Thin-Wall Limit,” Phys. Rev. D 103 (2021) 045008, [2009.08462]
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- A. Kusenko, V. Kuzmin, M. E. Shaposhnikov, and P. G. Tinyakov, “Experimental signatures of supersymmetric dark matter Q balls,” Phys. Rev. Lett. 80 (1998) 3185–3188, [hep-ph/9712212]
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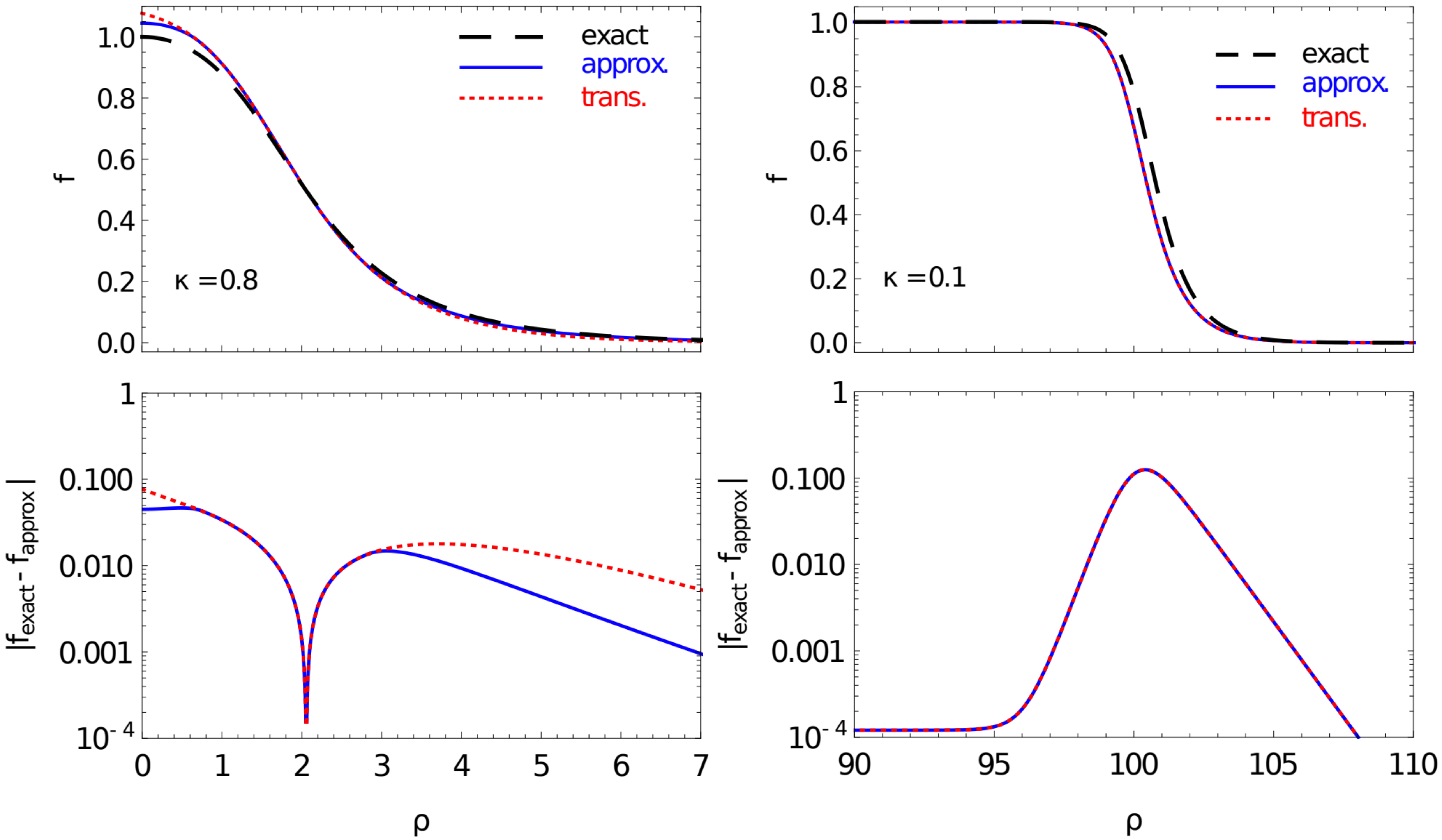
Thank you

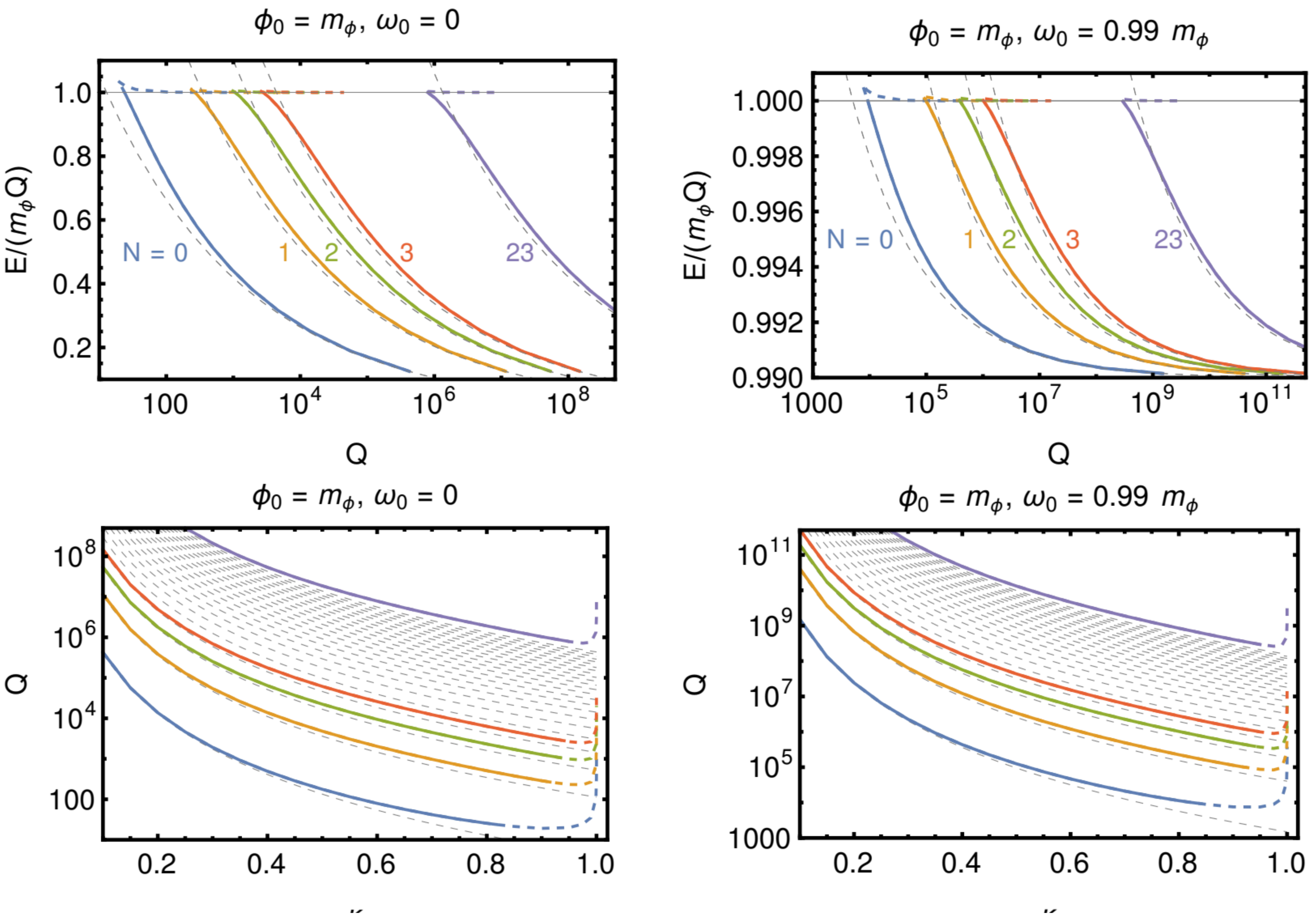
Back up



exact
approx.
trans.
f
κ =0.8
κ =0.1
|f_exact - f_approx|
ρ

N	$g(0)$	$c_{N,1}$	$c_{N,3}$	$c_{N,5}$	$c_{N,7}$	$c_{N,9}$
0	2.168693539	0.345758	–	–	–	–
1	7.051791599	0.106101	1.70188	–	–	–
2	14.565602713	0.0513571	0.793518	2.93172	–	–
3	24.6803496815	0.0303081	0.464925	1.64239	4.15909	–
4	37.38615404998	0.0200077	0.306252	1.0686	2.58726	5.39492

TABLE I. Initial value $g(0)$ for use in shooting-method solutions to Eq. (55) with $\varepsilon = 0$ as well as coefficient values for Eq. (56).



$\phi_0 = m_\phi,\ \omega_0 = 0$
$\phi_0 = m_\phi,\ \omega_0 = 0.99\ m_\phi$
E/(m_ϕQ)
N = 0
1
2
3
23
Q
$\phi_0 = m_\phi,\ \omega_0 = 0$
$\phi_0 = m_\phi,\ \omega_0 = 0.99\ m_\phi$
Q