Stasis, Stasis, Stasis

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Work with Lucien Heurtier, Fei Huang, Doojin Kim, Tim M.P. Tait, and Brooks Thomas

- arXiv:2111.04753
- arXiv:2212.01369

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- arXiv:2307.nnnnn

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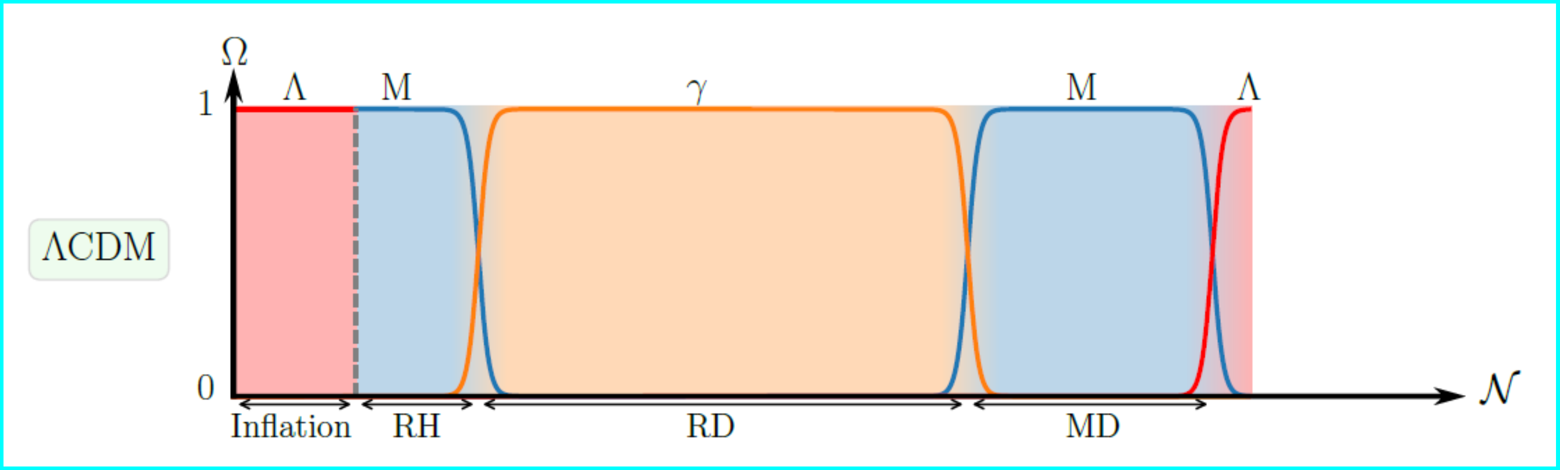
I'm going to start this talk by giving away the punchline.

We are all used to the standard ΛCDM timeline...



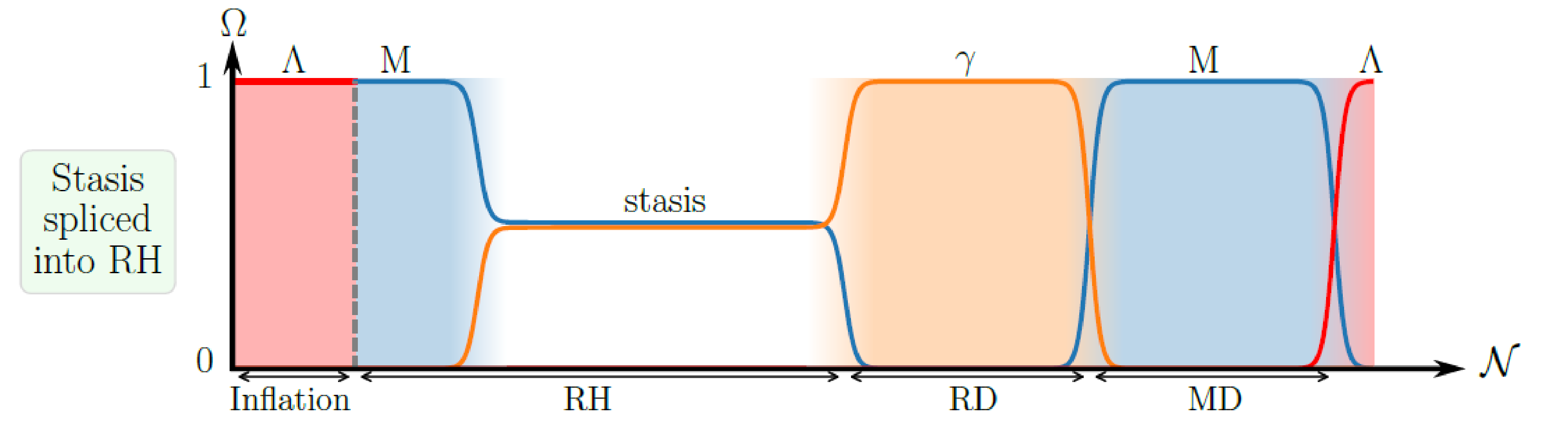
My main message: If you believe that physics at higher energies is governed by some form of BSM physics ---
(*pick your favorite theory: extra spacetime dimensions, strongly coupled sectors, string theory, etc.*),
---- then this picture is wrong. Not just "modified" or generalized in some corrective way, but *with a big piece missing.*

> Instead, a whole new type of epoch must open up in the early universe...
>
> … an epoch of ***cosmological stasis.***
>
> Such an epoch has been missed until now in most discussions of ΛCDM physics and early-universe cosmology, but it's there and must be dealt with.

Stasis

A cosmological epoch during which the abundances of different energy components (matter, radiation, vacuum energy, etc.) remain constant despite cosmological expansion.

For example,



The universe continues to expand, but the abundances stay fixed.
Time passes as measured in *e*-folds, but not as measured by abundances.

Already your alarm bells should be ringing.

Energy densities for each component scale differently. Cannot keep their ratios constant.

Ah, but we can!
Stasis is a fundamental epoch which need not be dominated by any one component over many *e*-folds.

Even if you could arrange this somehow, it must be freakishly fine-tuned.

No, it's not fine-tuned at all!

Even if it can be done without fine-tuning, why should the universe happen to fall into such a stasis state?

It's an attractor!
It cannot be avoided.

Even if it's an attractor for one model of BSM physics, what happens for a different model of BSM physics?

It's fairly general. The attractor behavior survives across a wide swath of BSM models, and for a wide range of parameters within each model.

Stasis sounds like an eternal thing. How can it form the basis of a cosmological epoch?

Stasis comes with its own mechanisms for starting at one time and ending later on, with a potentially large number of e-folds in between. No problem either entering stasis or exiting from it.

So if high-energy physics is governed by a BSM theory such as extra dimensions, strongly coupled sectors, or string theory, are you saying that the early universe will *necessarily* experience a stasis epoch?

Well, yes. There are, of course, various important caveats and provisos. However, the critical point is that such stasis epochs are a rather generic feature of such BSM cosmologies, and one would need to understand why they might *not* arise in certain circumstances if one doesn't take them into account.

You sound excited about this.

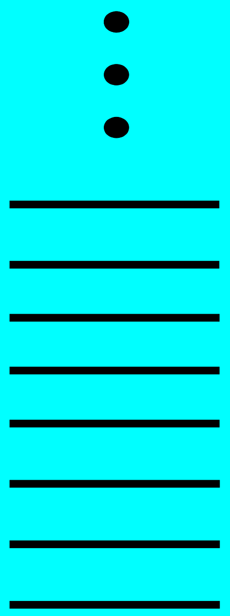
You bet.

The basic idea

So what's so special about BSM physics?

A wide variety of scenarios for BSM physics predict towers of massive, unstable states --- *e.g.*,

- Theories with extra compactified spacetime dimensions (KK towers)
- Scenarios with confining dark/ hidden-sector gauge groups (towers of bound-state resonances)
- String theory (infinite towers of KK/winding states, string resonances –- especially for bulk fields such as string moduli, axions, etc.)
- Scenarios which lead to the production of primordial black holes with an extended mass spectrum (towers of massive PBHs)



In general, such states share certain general properties

- Towers of states are potentially infinite (or bounded by a relevant cutoff) --- generally stretch across many orders of magnitude in mass.
- Such states are generally unstable and decay.

Moreover, two features tend to govern these decays

- Heavy states at top of tower tend to have largest decay widths and decay first, then lighter ones. Decays thus proceed "down the tower".
- For any state, the dominant decay mode is to the lightest states available. Such decay products are therefore produced with huge amounts of kinetic energy (relativistic), and are effectively radiation.

So what is the effect of such infinite towers of states on early-universe cosmology?

These decays establish a sequential process working its way down the tower which continually converts matter into radiation.

This may seem rather trivial, but there is actually a competing effect which pushes the other way: ***cosmological expansion*****!**

- radiation scales as a^{-4} (a = FRW scale factor)
- matter scales as a^{-3}

Thus, *even if nothing else happens*, cosmological expansion causes the *relative* fractional energy densities ("abundances") of matter and radiation to change

- abundance of radiation Ω_γ *drops*
- abundance of matter Ω_M *rises*

(Total remains fixed at 1 for a radiation/matter universe.)

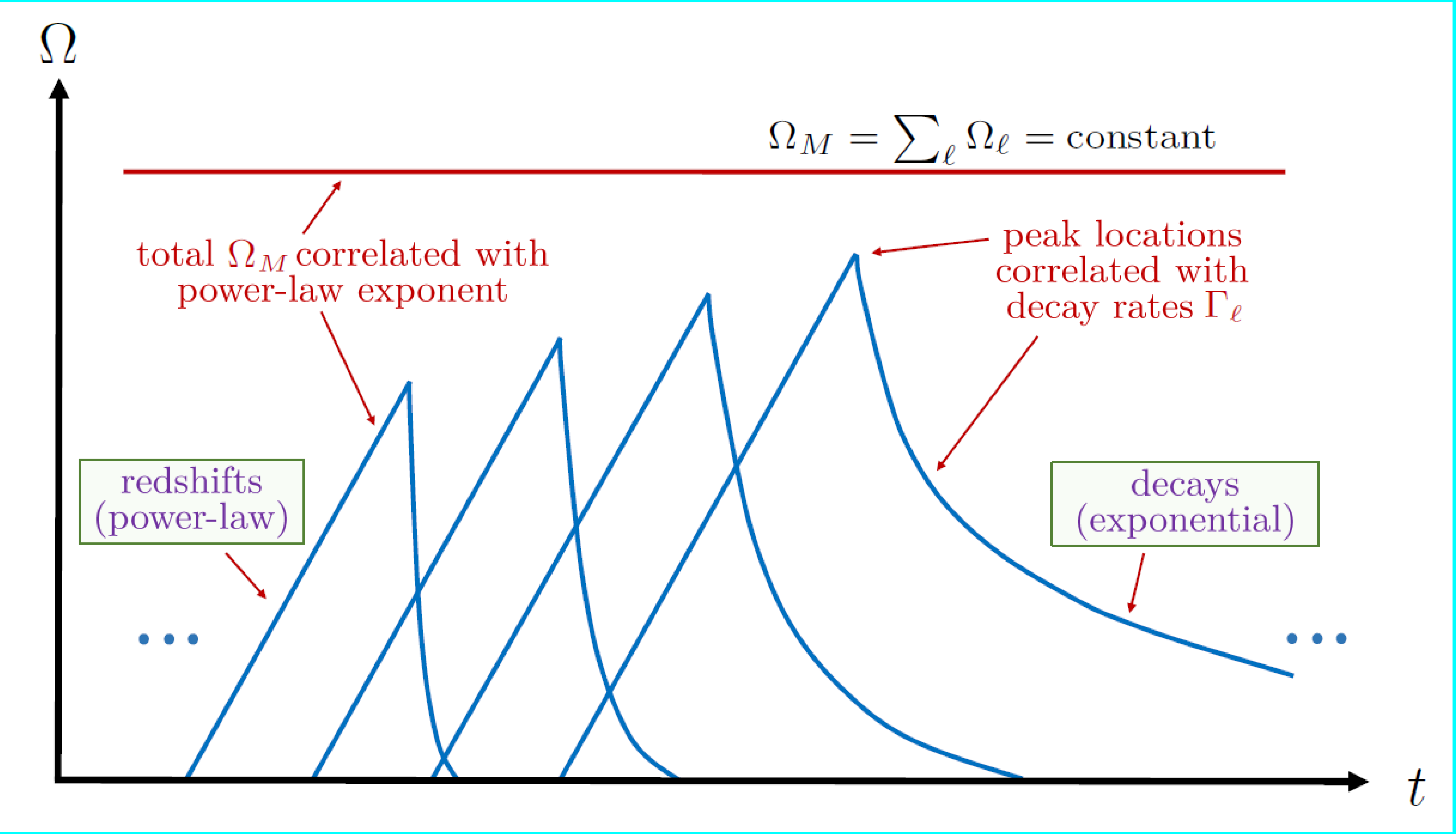
Indeed, this is how a radiation-dominated universe becomes matter-dominated universe *simply as the result of cosmic expansion*.

We thus see that

- *decays along tower*: convert $\Omega_M \rightarrow \Omega_\gamma$
- *cosmic expansion*: converts $\Omega_\gamma \rightarrow \Omega_M$

Can these two effects cancel?

This would be a way of keeping the matter and radiation abundances fixed --- at least through the time interval (which may stretch across many *e*-folds) during which the decays are proceeding sequentially down the tower.



Seems like too much to ask for!

But….

… they CAN balance

… they DO balance

… *even if* they don't start out by balancing,
the balanced solution is an ***attractor***
and the system will quickly come into balance all by itself!

Especially remarkable because particle decay and cosmological expansion are very different things!

To understand this how this can happen,
let us analyze this system mathematically….

Very simple ingredients from Cosmology 101!

$$\Omega_i \equiv \frac{8\pi G}{3H^2}\rho_i$$

abundances Ω_i in terms of energy densities ρ_i

H = Hubble parameter (falls with time)

$$\Rightarrow \quad \frac{d\Omega_i}{dt} = \frac{8\pi G}{3}\left(\frac{1}{H^2}\frac{d\rho_i}{dt} - 2\frac{\rho_i}{H^3}\frac{dH}{dt}\right)$$

Friedmann "acceleration" equation

$$\begin{aligned}\frac{dH}{dt} &= -H^2 - \frac{4\pi G}{3}\left(\sum_i \rho_i + 3\sum_i p_i\right)\\ &= -\tfrac{1}{2}H^2\left(2 + \Omega_M + 2\Omega_\gamma\right)\\ &= -\tfrac{1}{2}H^2\left(4 - \Omega_M\right) .\end{aligned}$$

$$\Rightarrow \quad \frac{d\Omega_i}{dt} = \frac{8\pi G}{3H^2}\frac{d\rho_i}{dt} + H\Omega_i\left(4 - \Omega_M\right)$$

Now insert "EOMs" for ρ_i:

$$\frac{d\rho_\ell}{dt} = -3H\rho_\ell - \Gamma_\ell \rho_\ell$$

$$\frac{d\rho_\gamma}{dt} = -4H\rho_\gamma + \sum_\ell \Gamma_\ell \rho_\ell$$

cosmological expansion

decay sources and sinks

$$\frac{d\Omega_M}{dt} = -\sum_\ell \Gamma_\ell \Omega_\ell + H\left(\Omega_M - \Omega_M^2\right)$$

General dynamical evolution of Ω_M

Setting $d\Omega_M/dt=0$ then yields

$$\sum_\ell \Gamma_\ell \Omega_\ell = H(\Omega_M - \Omega_M^2) .$$

A minimum condition for stasis!

But for actual stasis, need Ω_M to stay fixed *over an extended period*!

- $d^n\Omega_M/dt^n = 0$ for *all* $n > 1$
- Instead, demand that $d\Omega_M/dt = 0$ for all t!

During stasis

$$H(t) = \left(\frac{2}{4-\overline{\Omega}_M}\right)\frac{1}{t} \quad \Longleftrightarrow \quad \overline{\kappa} = \frac{6}{4-\overline{\Omega}_M}$$

$H(t) = \kappa/(3t)$

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \frac{\overline{\kappa}}{3}\,\overline{\Omega}_M(1-\overline{\Omega}_M)\,\frac{1}{t} = \left(2-\overline{\kappa}\right)\overline{\Omega}_M\,\frac{1}{t}\,.$$

For stasis, this relation must be true *as a function of t*.

where each individual $\Omega_\ell(t)$ is given by

$$\Omega_\ell(t) = \Omega_\ell^{(0)}\, h(t^{(0)},t)\, e^{-\Gamma_\ell(t-t^{(0)})}$$

with

$$h(t^{(0)},t) = h(t^{(0)},t_*)\, h(t_*,t) = h(t^{(0)},t_*)\left(\frac{t}{t_*}\right)^{2-6/(4-\overline{\Omega}_M)}$$

So what does BSM physics tell us about the Ω_ℓ and Γ_ℓ across the tower?

Let us parametrize these quantities in a general way in order to encapsulate a wide variety of different BSM scenarios…

mass spectrum

$$m_\ell \;=\; m_0 + (\Delta m)\ell^\delta$$

e.g., $(m_0, \Delta m, \delta)$ =

- $(m, (2mR^2)^{-1}, 2)$ for KK on circle, $mR \gg 1$
- $(m, R^{-1}, 1)$ for KK on circle, $mR \ll 1$
- $\delta = ½$ for string/ strong-coupling resonances $(\alpha' M_\ell^2 = \ell)$

decay widths

$$\Gamma_\ell = \Gamma_0 \left(\frac{m_\ell}{m_0}\right)^\gamma$$

Exponent γ determined by dominant ϕ_ℓ decay mode, e.g., if ϕ_ℓ decays to photons via dimension-d contact operator $c_\ell \phi_\ell F/\Lambda^{d-4}$, then $\boldsymbol{\gamma = 2d-7}$.

abundances at production

$$\Omega_\ell^{(0)} = \Omega_0^{(0)} \left(\frac{m_\ell}{m_0}\right)^\alpha$$

Exponent α determined by production mechanism. Typically $\alpha<0$ for misalignment production, but either sign for thermal freeze-out.

So for what scaling exponents (α,γ,δ) can we achieve stasis?

Given our constraint equation, we need to evaluate the sum $\sum \Gamma_\ell \, \Omega_\ell \, (t)$!

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \frac{\overline{\kappa}}{3} \, \overline{\Omega}_M (1 - \overline{\Omega}_M) \, \frac{1}{t} = \left(2 - \overline{\kappa}\right) \overline{\Omega}_M \, \frac{1}{t} \, .$$

Convenient to take continuum limit $\Delta m \to 0, \; N \to \infty$

Sum becomes integral

$$\sum_\ell \to \int d\tau \, n_\tau(\tau)$$

where

$$n_\tau(t) \equiv \left| \frac{d\ell}{d\tau} \right|_{\tau = t} = \frac{1}{\gamma\delta} \left(\frac{m_0}{\Delta m} \right)^{1/\delta} \Gamma_0^{-1/\gamma\delta} \, t^{-1-1/\gamma\delta}$$

density of states

Easy to integrate. Consider situation far from "edge" effects – *i.e.*, $t \gg t^{(0)}$.

Constraints to impose for stasis:

1. Realize proper 1/t scaling
2. Avoid potential *logarithmic time-dependence* in abundances
3. Match *prefactors* as well.

→

We find that all constraints are subsumed into a single relation:

$$\frac{1}{\gamma} \left(\alpha + \frac{1}{\delta} \right) = 2 - \overline{\kappa} \, ,$$

$$\frac{1}{\gamma}\left(\alpha + \frac{1}{\delta}\right) \;=\; 2 - \overline{\kappa}\;,$$

So what do we learn?

This is not a constraint on (α,γ,δ) so much as
a *prediction for* κ during stasis!

Thus, so long as

$$\gamma > 0\;, \qquad 0 \;<\; \alpha + \frac{1}{\delta} \;\leq\; \frac{\gamma}{2}$$

so that
$3/2 < \kappa < 2$

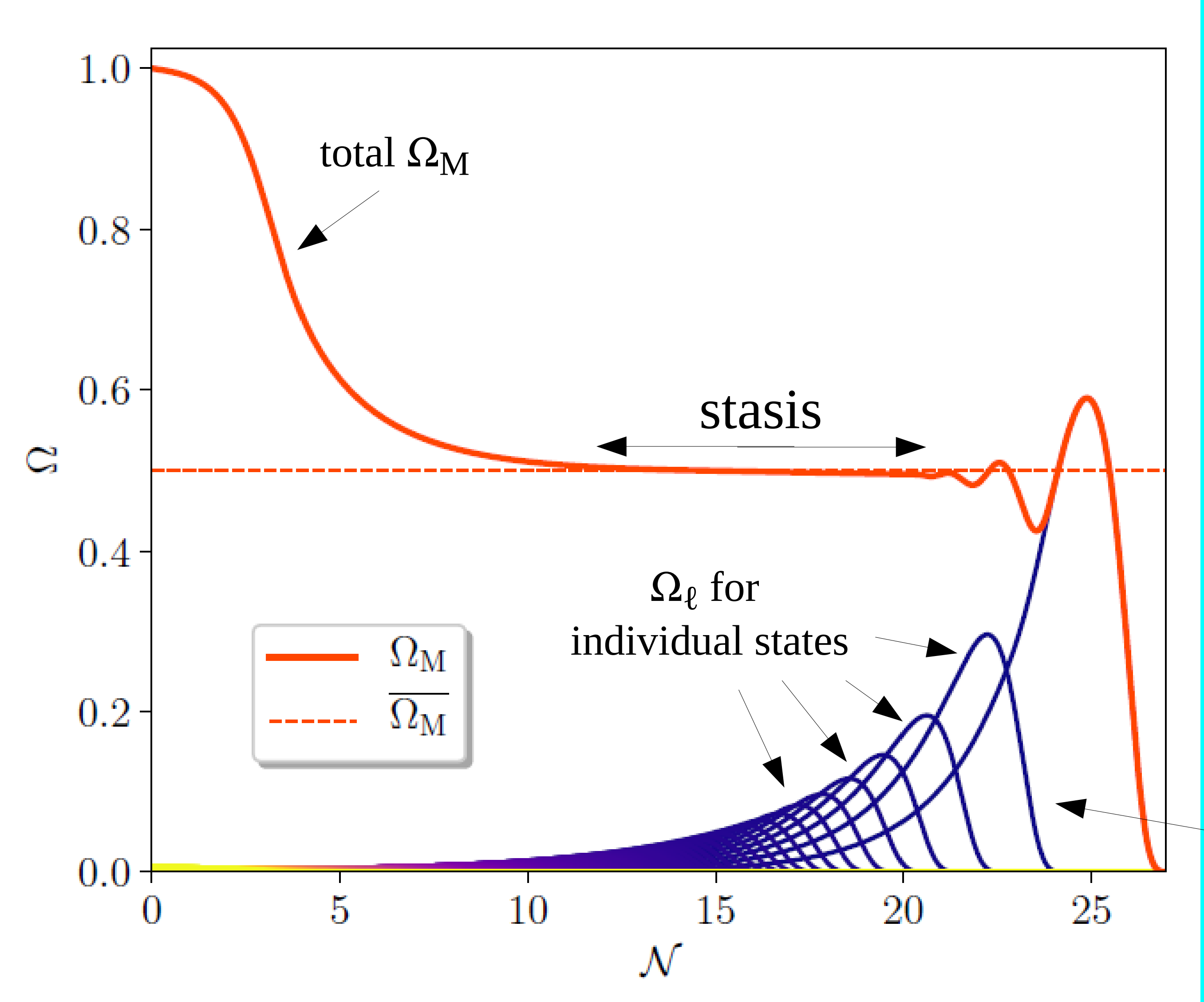
we will ***always*** have stasis (!),
and indeed the corresponding stasis abundance will be given by

$$\overline{\Omega}_M \;=\; \frac{2\gamma\delta - 4(1+\alpha\delta)}{2\gamma\delta - (1+\alpha\delta)}\;.$$

For example, if

$$\frac{1+\alpha\delta}{\gamma\delta} = \frac{2}{7}.$$

An extended stasis epoch with *matter-radiation equality*!



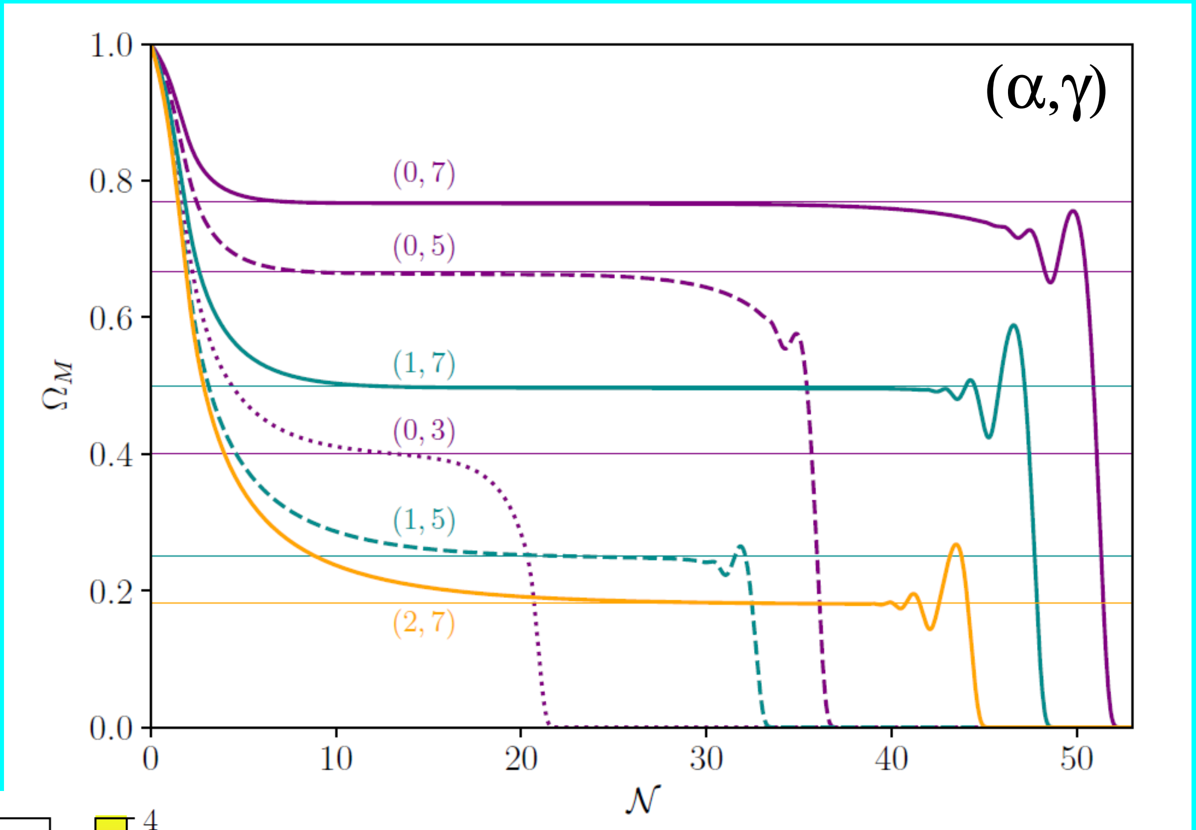
$(\alpha,\gamma,\delta)=(1,7,1)$

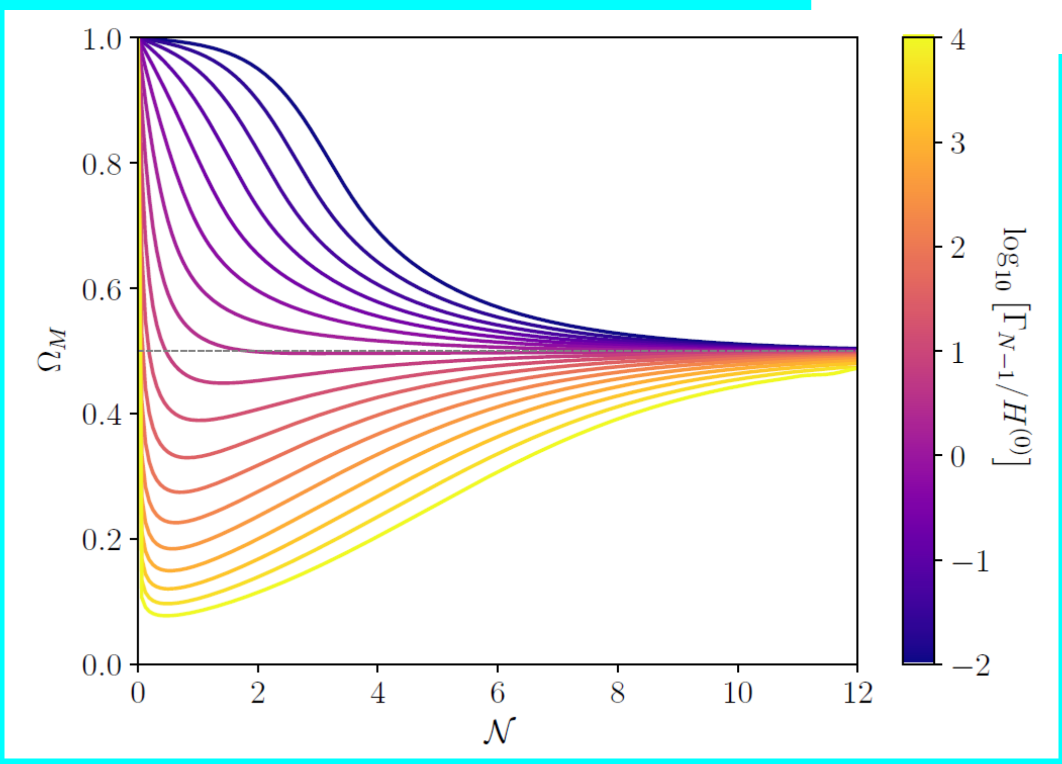
$N=300$
$m_0/\Delta m=1$
$\Gamma_{N-1}/H^{(0)} = 0.1$

Exact numerical solution using Boltzmann code, no approximations.

Note interleaving of energies of dominant components.

Indeed, we obtain stasis *regardless* of values of (α,γ,δ) within the allowed range! For δ=1 and different (α,γ) we find

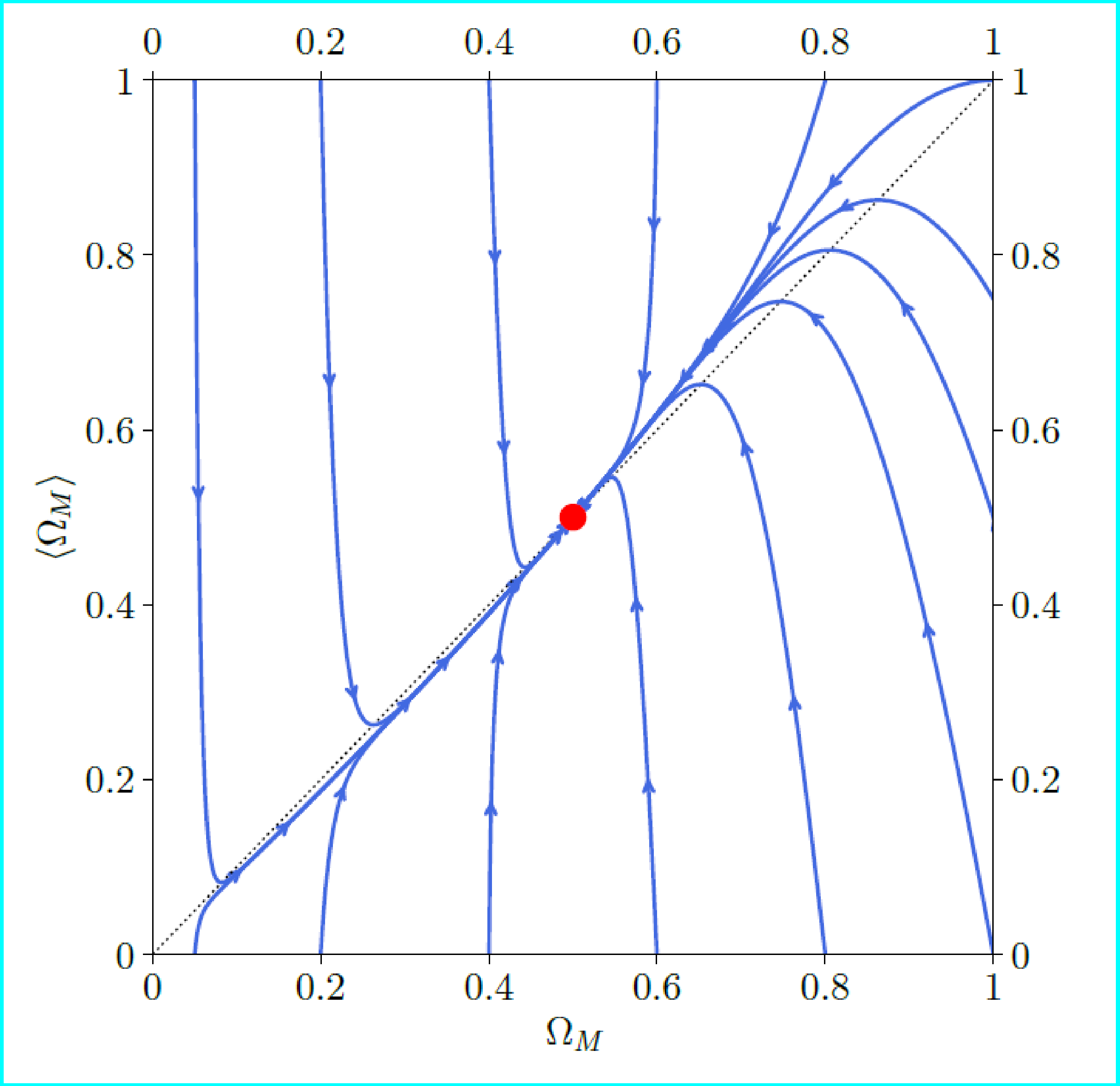




Can also vary $\Gamma_{N\text{-}1}/H^{(0)}$ = rate of decays relative to cosmological expansion. Affects initial behavior but stasis always emerges!

Indeed, matter/radiation stasis is a *global* attractor within such cosmologies…

time-averaged *history*
of abundance



instantaneous abundance

Once we're in stasis, how long does it last?

Lasts until the last (lightest) tower component decays.
Then our stasis runs out of fuel and the universe exits from stasis.

$$\begin{aligned}
\mathcal{N}_s &\equiv \log\left[\frac{a(t=\tau_0)}{a(t=t_{N-1})}\right] = \frac{2}{4-\overline{\Omega}_M}\log\left(\frac{\Gamma_{N-1}}{\Gamma_0}\right) \\
&= \frac{2\gamma}{4-\overline{\Omega}_M}\log\left(\frac{m_{N-1}}{m_0}\right) \\
&= \frac{2\gamma}{4-\overline{\Omega}_M}\log\left[1+\frac{\Delta m}{m_0}(N-1)^\delta\right] \\
&\approx \frac{2\gamma\delta}{4-\overline{\Omega}_M}\log N
\end{aligned}$$

⇨ **Number of *e*-folds of stasis goes as *log(N)*.**

Thus far we've focused on stasis between matter (M) and radiation (γ). What about vacuum energy (Λ)?

w= -1 w=0 w=1/3

Vacuum energy density ρ_Λ scales as a^0 = constant.

Therefore cosmological expansion tends to push

Radiation → Matter

Matter → Vacuum energy

Counterbalancing "pumps" transfer abundance back again...

← decay

← ??

How can vacuum energy convert to matter?

Consider the coherent state consisting of the zero-momentum modes of a scalar field φ of mass *m*.

- At early times, when the Hubble parameter is large (with $3H > 2m$), this field is overdamped and thus has no kinetic energy. The energy of the field is pure potential energy (vacuum energy), with $w = -1$.

- However, as the universe expands, the Hubble parameter generally drops. As a result, the field eventually becomes underdamped (with $3H < 2m$) and begins to experience damped oscillations. These quickly virialize, with field energy split equally between potential and kinetic energy. We then have $w = 0$, with energy density behaving as matter as far as cosmic expansion is concerned.

Underdamping ("turn-on") transition at $3H(t)=2m$: Converts vacuum energy → matter!

So, just as before, let us begin with a tower of ϕ_ℓ = vacuum-energy components/species, and wait until they sequentially become underdamped and behave as matter.

Problem: Nothing ever happens!

Since this universe is initially vacuum-dominated,

Hubble = constant.

Therefore all fields remain overdamped and there is no dynamics!

How to get around this problem? Several ideas ---

- Introduce additional non-vacuum energy component. This then introduces a non-trivial time-dependence for Hubble which in turn eventually triggers the cascading overdamped/ underdamped transitions that we require.

- Is arbitrary and non-minimal, with new parameters to govern extra components.

- Introduce a *regulator* for vacuum energy: consider w slightly bigger than -1. Now Hubble evolves and dynamics emerges naturally. Then consider $w \to -1$ limit as the vacuum-energy limit.

- Minimal. System can be solved analytically. Can even study stasis as function of w.

- Consider full scalar-field dynamics: damped driven harmonic oscillator in which the Hubble damping terms carry a non-trivial time dependence.

- True scalar field → has UV completion. But if universe not yet in stasis, EOMs lack analytical solutions → must study system numerically.

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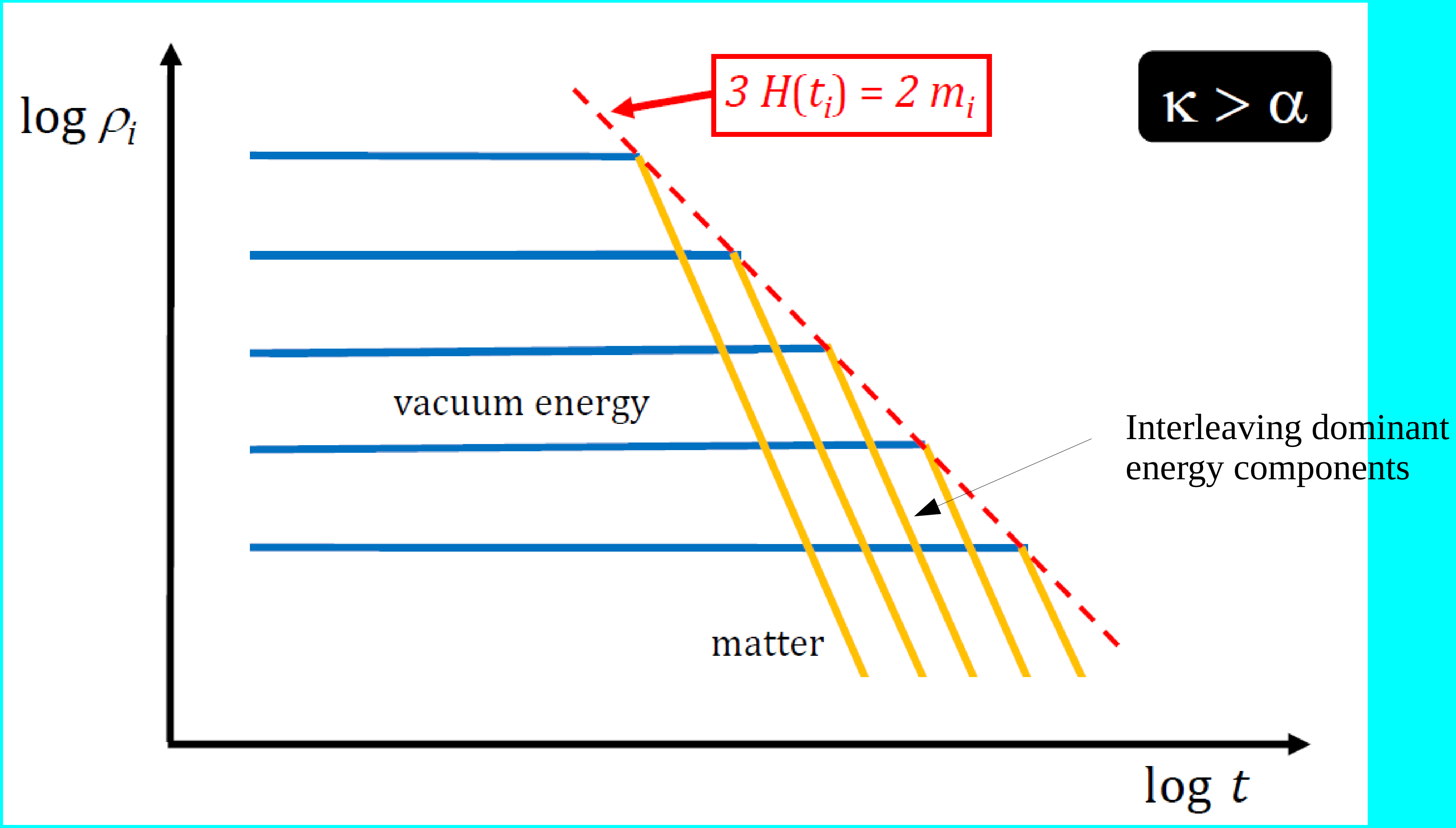
- True scalar field → has UV completion. But if universe not yet in stasis, EOMs lack analytical solutions → must study system numerically.

In the w→ -1 limit, our system has an interesting dynamics



But does this give rise to a stasis between vacuum energy and matter?

Perform similar analysis as for matter/radiation stasis, and find

Density of states per unit turn-on time

Abundance turning on at time t.

$$
\begin{aligned}
n_{\hat{t}}(t)\,\Omega(t) &= \left[-w\,\overline{\kappa}\,\overline{\Omega}_\Lambda(1-\overline{\Omega}_\Lambda)\right]\frac{1}{t} \\
&= \left[2-(1+w)\overline{\kappa}\right]\overline{\Omega}_\Lambda\,\frac{1}{t}\,.
\end{aligned}
$$

1. Realize proper 1/t scaling
2. Avoid potential *logarithmic time dependence* in abundances
3. Match *prefactors* as well.

$$
\alpha + \frac{1}{\delta} = 2 - (1+w)\,\overline{\kappa}\ ,
$$

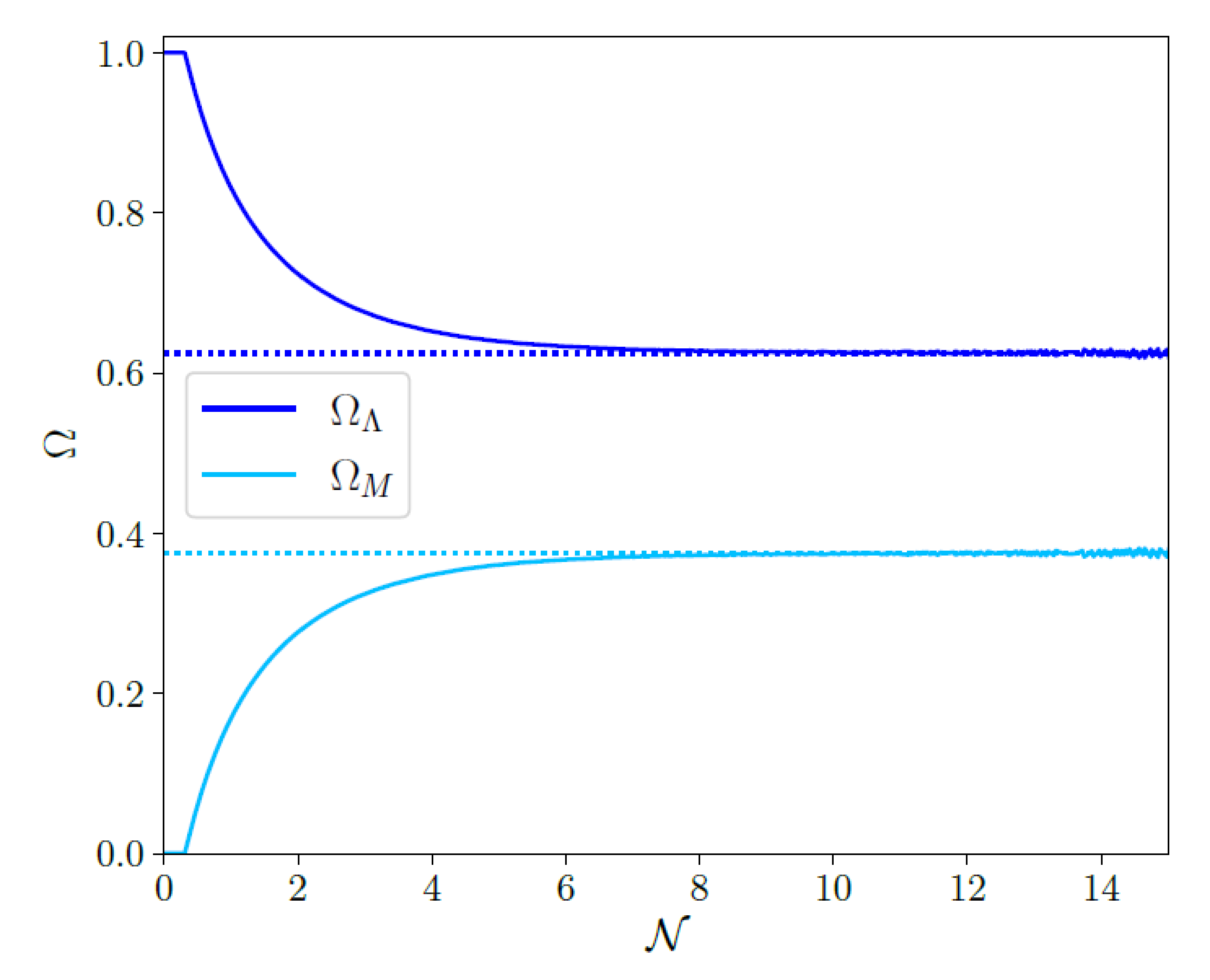
Once again, this is not a constraint on (α,δ) so much as a *prediction for* κ during stasis!

Thus, so long as $0 < \alpha + \frac{1}{\delta} < -2w$

we will *always* have stasis (!),
and indeed the corresponding stasis abundance will be given by

$$
\overline{\Omega}_\Lambda = -\frac{\alpha + 1/\delta + 2w}{(\alpha + 1/\delta - 2)w}
$$

Example: Vacuum-energy/matter stasis



$\delta = 2,\ \alpha = 0.7,\ \overline{\kappa} = 4,\ w = -0.8.$

Vacuum-energy/radiation stasis.

Perform similar analysis as for matter/radiation stasis, find

$$\sum_\ell \Gamma_\ell \Omega_\ell(t) = \left[\left(\frac{1}{3} - w\right) \overline{\kappa}\, \overline{\Omega}_\Lambda (1 - \overline{\Omega}_\Lambda)\right] \frac{1}{t}$$
$$= \left[2 - (1+w)\overline{\kappa}\right] \overline{\Omega}_\Lambda \frac{1}{t} .$$

1. Realize proper 1/t scaling
2. Avoid potential *logarithmic time dependence* in abundances
3. Match *prefactors* as well.

→

$$\frac{1}{\gamma}\left(\alpha + \frac{1}{\delta}\right) = 2 - (1+w)\overline{\kappa} .$$

This too is not a constraint on (α,γ,δ)

so much as a *prediction for* κ during stasis!

Thus, so long as $0 < \frac{1}{\gamma}\left(\alpha + \frac{1}{\delta}\right) < \frac{1-3w}{2}$

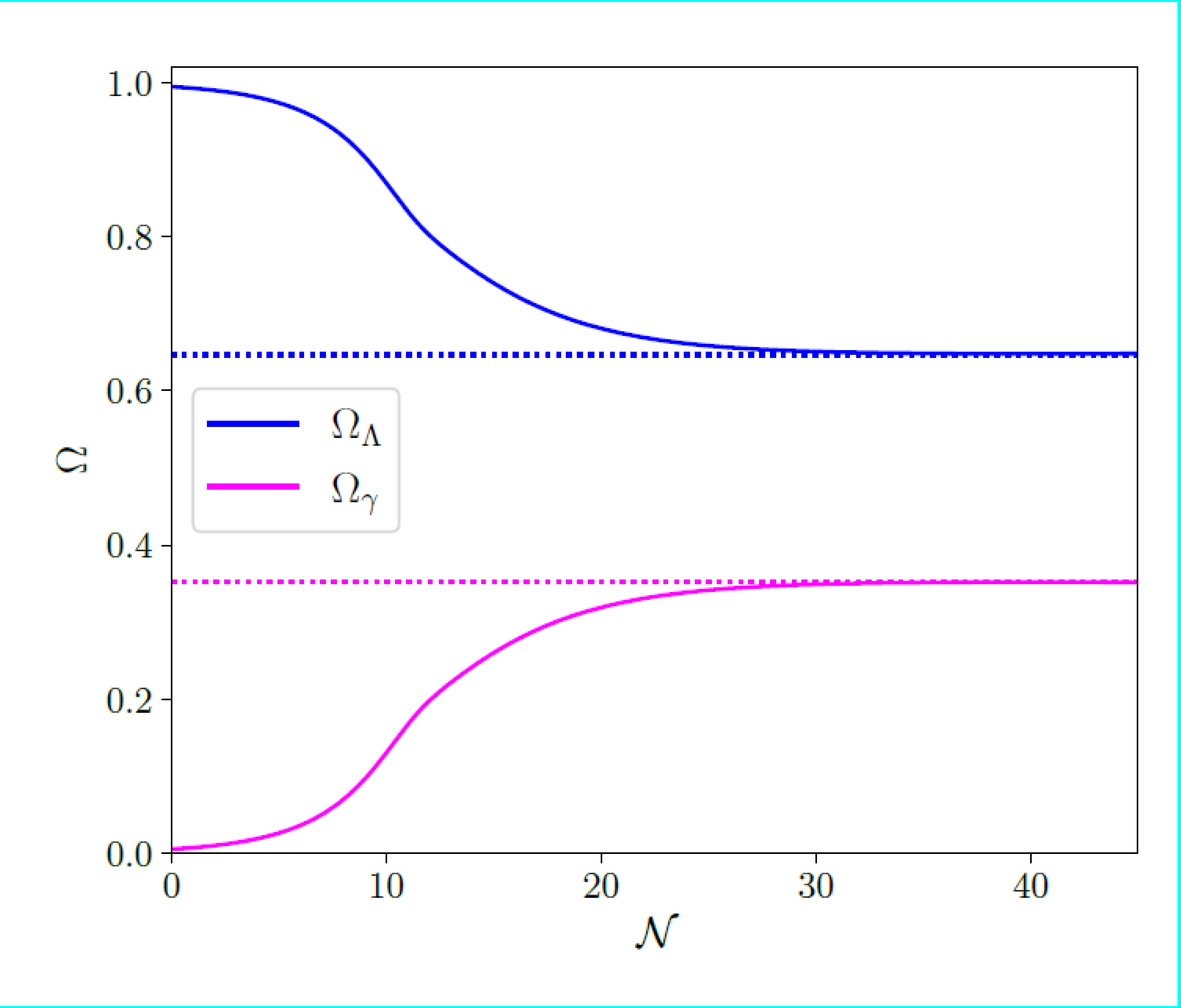
we will *always* have stasis (!),

and indeed the corresponding stasis abundance will be given by

$$\overline{\Omega}_\Lambda = \frac{2(2X + 3w - 1)}{(1-3w)(X-2)}$$

where $X \equiv (\alpha + 1/\delta)/\gamma$

Example: Vacuum-energy/radiation stasis



α=3.5, γ=3, δ=2, w=-0.8, κ=4/3

Triple stasis

Thus far, we have shown the existence of three different kinds of pairwise stasis:

- Matter with radiation
- Vacuum energy with matter
- Vacuum energy with radiation

Each occurs in a universe consisting of only those two types of energy components.

Obvious next step:

Can we have a universe in which all three are in stasis with each other??

This is highly non-trivial!

Just because A comes into stasis with B in an A/B universe,
… and just because A comes into stasis with C in an A/C universe,
… and just because B comes into stasis with C in a B/C universe,
does *not* imply A, B, and C will all come into stasis in an A/B/C universe!

This is because each of our previous energy-transfer processes (decay and underdamping) would now need to operate in a universe which also contains a *third* energy component. This third component also affects Hubble and the overall expansion rates whose effects would need to cancel for a triple stasis.

In other words, both processes (decay and underdamping) must occur *simultaneously* while embedded in a **common cosmology**!
Triple stasis can arise only if these processes can co-exist with each other, potentially placing new constraints on each!

We can nevertheless proceed the same way as before, find

General algebraic structure of triple stasis

Let P_{ij} = "pump" that converts energy components *i*-type → *j*-type

e.g., $P_{M\gamma}$ = particle decay

$P_{\Lambda M}$ = underdamping transition

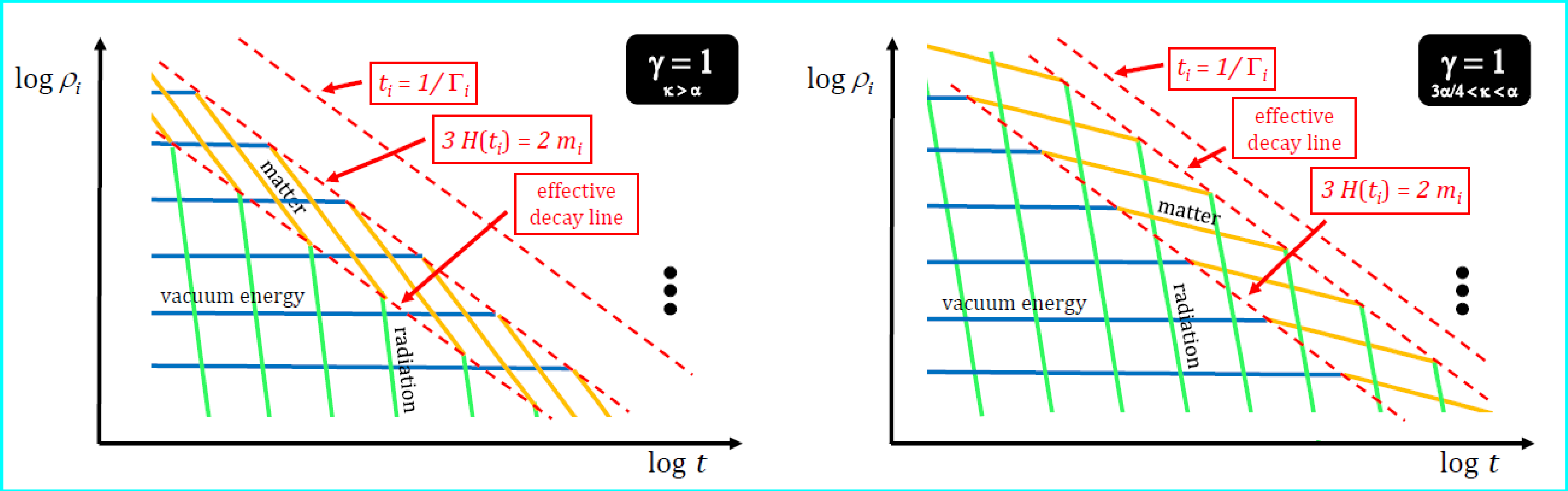
pump = abundance conversion rate (per unit time)

Then our stasis conditions now take the form

$$\begin{aligned} P_{\Lambda M} + P_{\Lambda\gamma} &= \left[2 - (1+w)\overline{\kappa}\right] \overline{\Omega}_\Lambda \frac{1}{t} \\ -P_{\Lambda M} + P_{M\gamma} &= \left[2 - \overline{\kappa}\right] \overline{\Omega}_M \frac{1}{t} \\ -P_{\Lambda\gamma} - P_{M\gamma} &= \left[2 - \frac{4\overline{\kappa}}{3}\right] \overline{\Omega}_\gamma \frac{1}{t} \,. \end{aligned}$$

Pumping actions balanced against **cosmological expansion** within a common cosmology (κ)

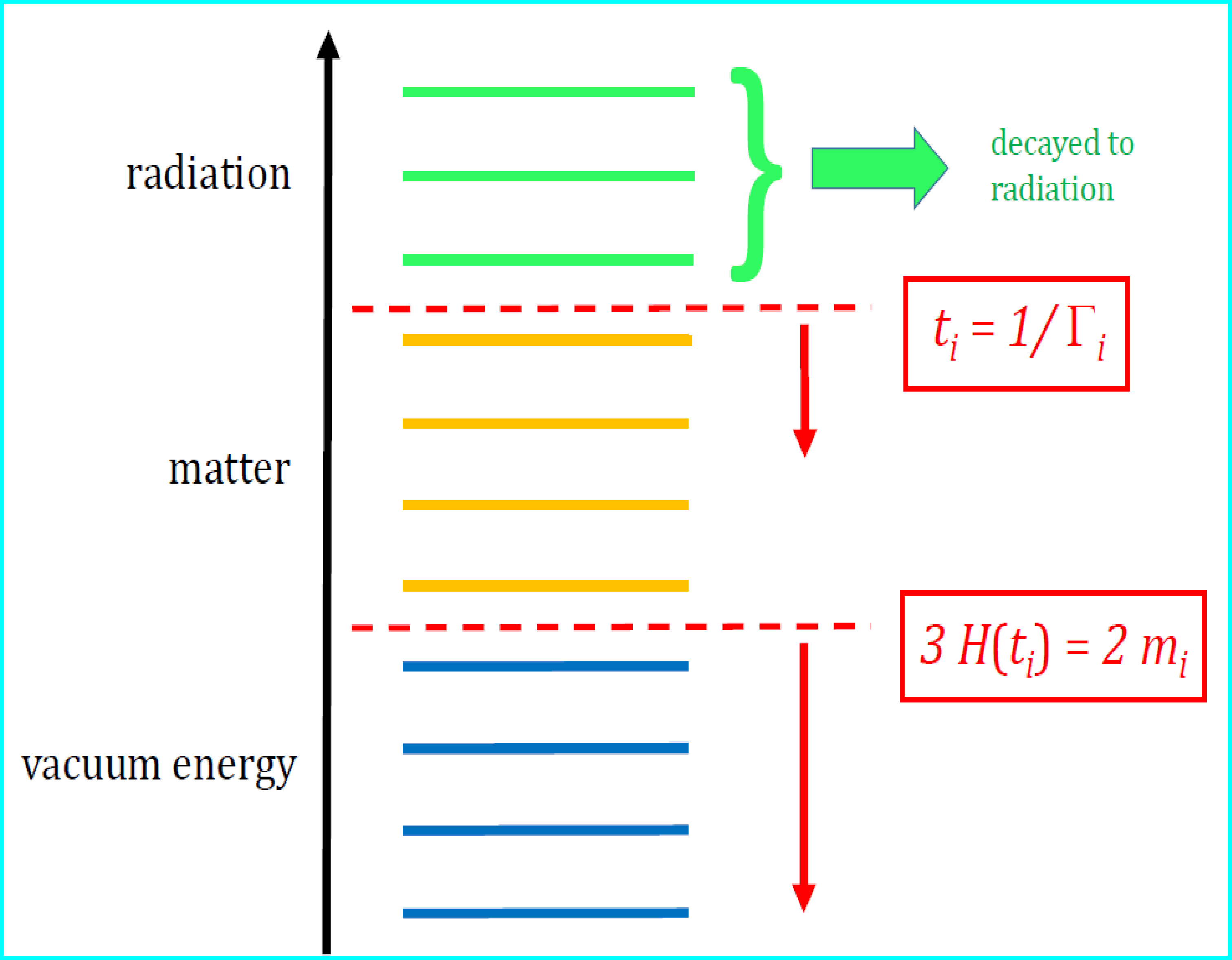
Such a triple stasis would have an even more complex dynamics.
Depending on relevant parameters, there are two possible varieties...



Each case exhibits
the interleaving
characteristic of stasis.

Each state in the tower starts as **vacuum energy**
… then becomes underdamped, behaves as **matter**
… and then decays and becomes **radiation**.

Indeed, taking time-slices through the previous figures, we find that the dynamics takes the form



This is triple stasis in action!

Each phase transition proceeds down the tower, counterbalancing the effects of cosmic expansion.

But the fundamental question remains:
Can this system come into balance??   Is a true triple stasis possible?

Must satisfy the triple-stasis algebraic constraints, yielding relations for (α,γ,δ).

$$\begin{aligned} P_{\Lambda M} + P_{\Lambda\gamma} &= \left[2-(1+w)\overline{\kappa}\right]\overline{\Omega}_\Lambda \frac{1}{t} \\ -P_{\Lambda M} + P_{M\gamma} &= \left[2-\overline{\kappa}\right]\overline{\Omega}_M \frac{1}{t} \\ -P_{\Lambda\gamma} - P_{M\gamma} &= \left[2-\frac{4\overline{\kappa}}{3}\right]\overline{\Omega}_\gamma \frac{1}{t} . \end{aligned}$$

Recall that this boils down to three kinds of constraints

1. Realize proper 1/t scaling
2. Avoid potential *logarithmic time dependence* in abundances
3. Match *prefactors* as well.

For the case of triple stasis, these constraints now give different parts of the puzzle.

#1: gives relations for (α,γ,δ), as before
#2: is now an independent constraint, will rule out certain cases allowed by #1
#3: allows us to determine the final stasis abundances for each component!

So what do we find?

#1: Now gives *two* constraints since there are now *two* pumps!

$$\alpha + \frac{1}{\delta} \; = \; 2 - (1+w)\overline{\kappa} \; ,$$

$$\alpha + \frac{1}{\delta} \; = \; 2\gamma - (\gamma+w)\overline{\kappa} \; .$$

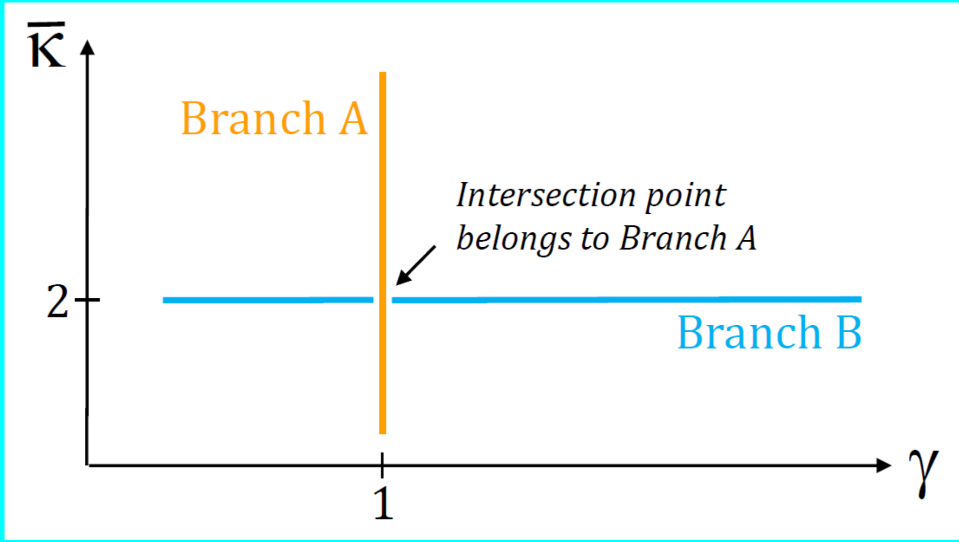
These separate constraints cannot be satisfied simultaneously unless

$$(2-\overline{\kappa})\left(1-\frac{1}{\gamma}\right) \; = \; 0 \; .$$

This is the condition that correlates the two pumps, allowing them to co-exist!

Thus, our system splits into two disjoint branches:

Branch A :	$\gamma = 1,$	any $\overline{\kappa}$
Branch B :	$\overline{\kappa} = 2,$	any $\gamma \neq 1$



#2: Avoiding spurious logarithmic time-evolution for abundances imposes an additional constraint: must have

$$\text{either} \quad \gamma = 1 \quad \text{or} \quad \overline{\kappa} \neq 2$$

This kills Branch B! (However, may come back later …)

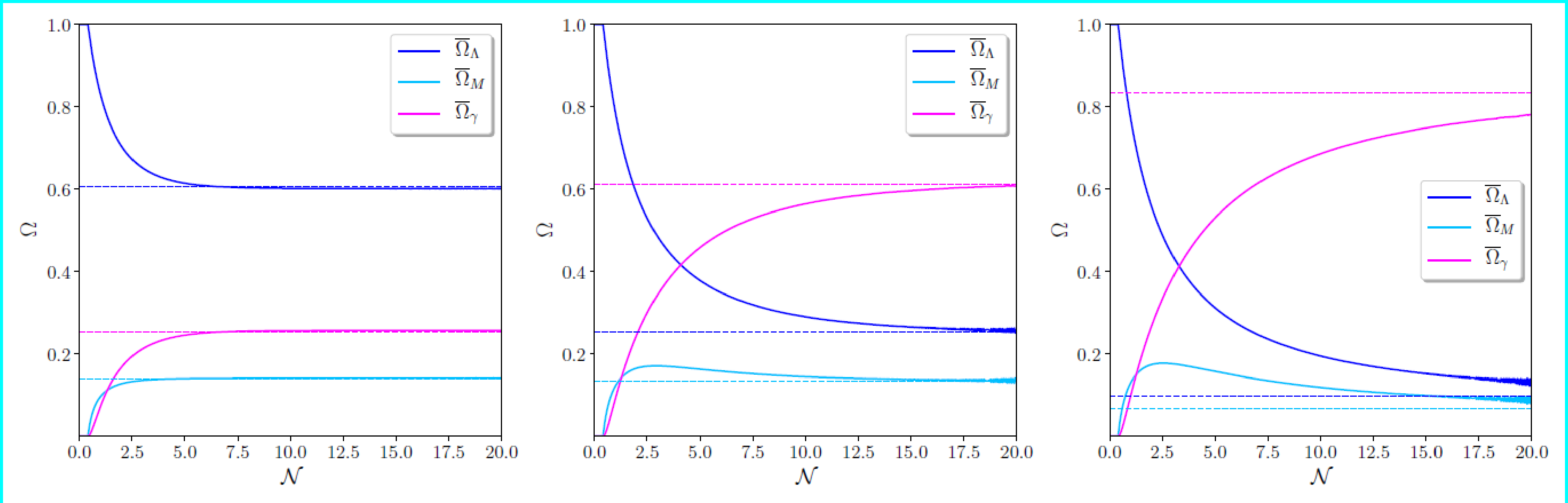
Henceforth restrict to Branch A ($\gamma = 1$, κ unrestricted)

#3: This relation imposes no further limitation and even allows us to solve for stasis abundances analytically!

Thus, once again, our triple-stasis conditions are satisfied for all (α,δ). We have only one new constraint, namely $\gamma = 1$, and this is completely natural for fermions decaying to photons!

→ Like its pairwise cousins, triple stasis is also fairly generic!

Triple-stasis solutions for various underlying parameters ---



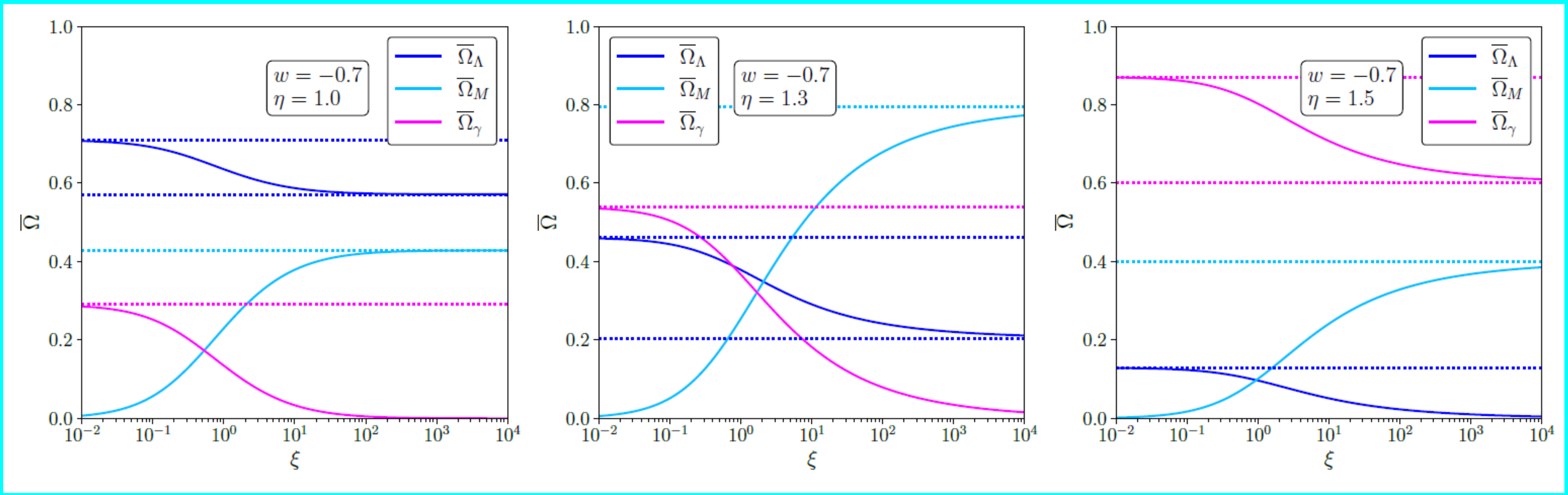
$\kappa > 2$ $\kappa = 2$ $\kappa < 2$

- In all cases shown, system approaches a triple-stasis configuration.
- As resulting κ decreases, it takes longer to approach triple stasis.
- Triple stasis can have abundances with various relative orderings of magnitudes.

While important in its own right, triple stasis also *interpolates* between different *pairwise* stases!

Define parameter $$\xi \equiv \frac{2}{\overline{\kappa}} \frac{m_0}{\Gamma_0}$$ ← Indicates duration of time interval between underdamping and decay (more ξ, more matter).

Plot final stasis abundances Ω_i versus ξ for various values of $\alpha + 1/\delta$, obtain



Λγ ← triple stasis → ΛM	Λγ ← triple stasis → ΛM	Λγ ← triple stasis → Mγ

Triple stasis connects different pairwise stases through parameter-space interiors.

Given the forms of the constraint equations, there is also a very "geometric" way of understanding the triple-stasis phenomenon.

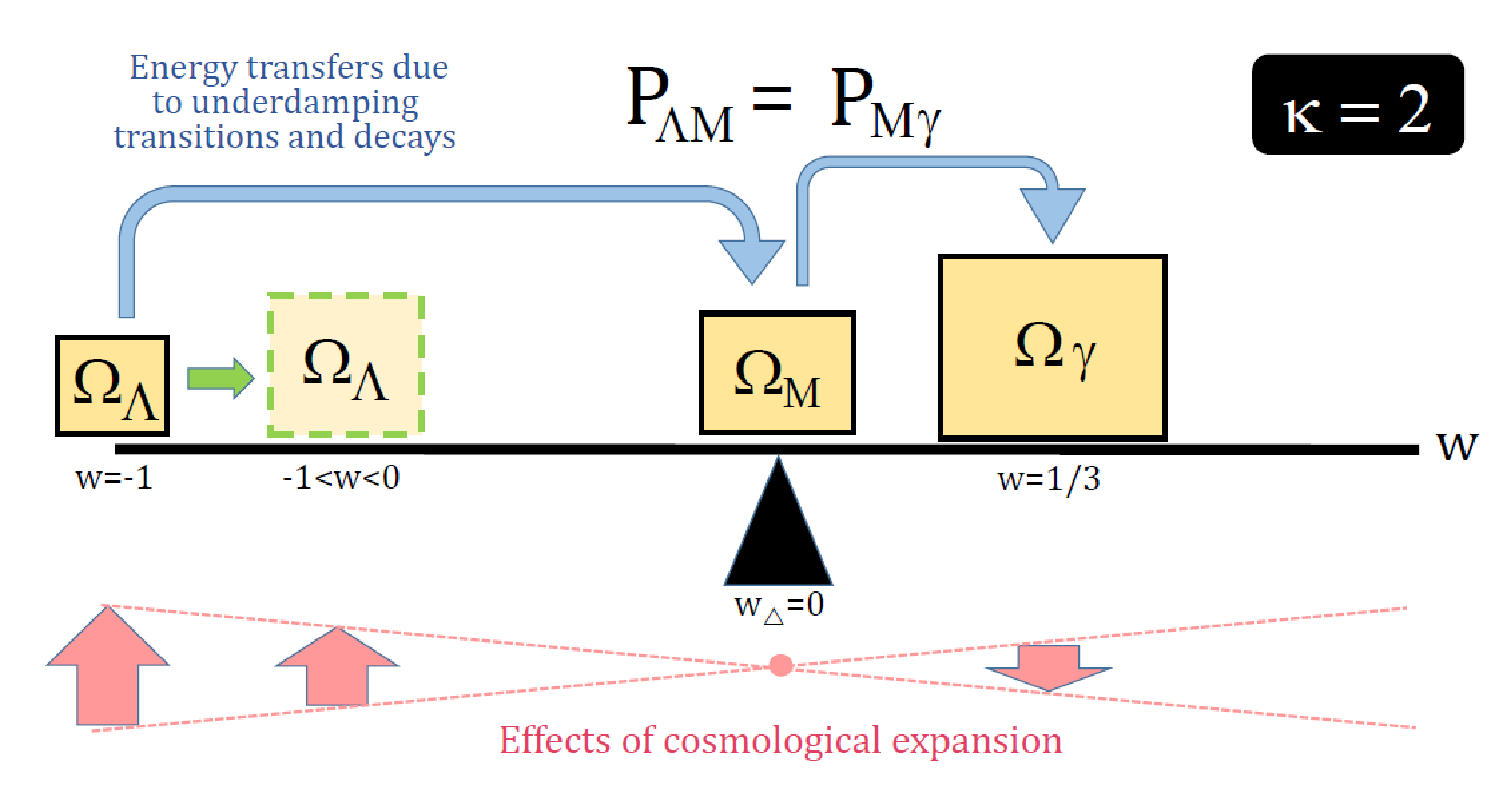
This depends on the values of κ that are being realized in each case.

Recall

$$\alpha + \frac{1}{\delta} = 2 - (1+w)\overline{\kappa} \ ,$$

Let us first study the case with $\kappa = 2$...

The w-seesaw!



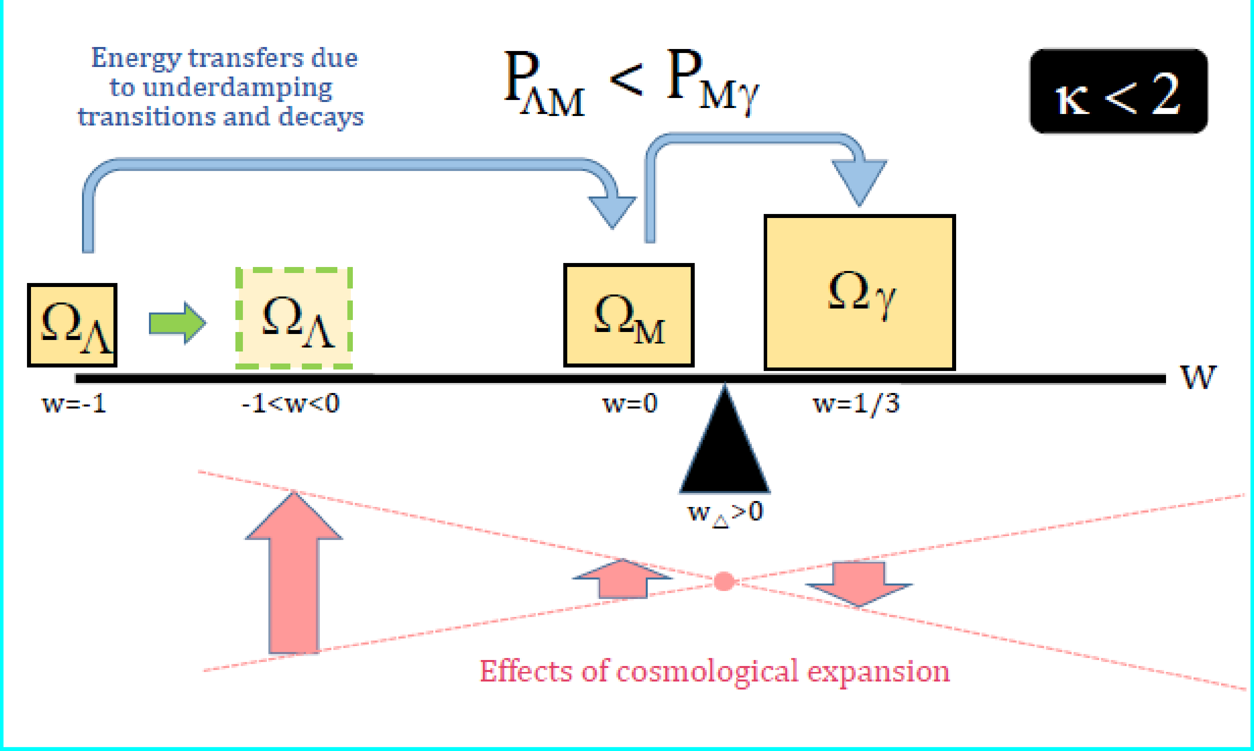
Triple stasis with $\kappa = 2$ is effectively matter-dominated.
This is the most "symmetric" situation,
with equal pumps and a fulcrum location at $w_\Delta=0$.
Shifting w does not destroy the stasis because the abundances compensate.

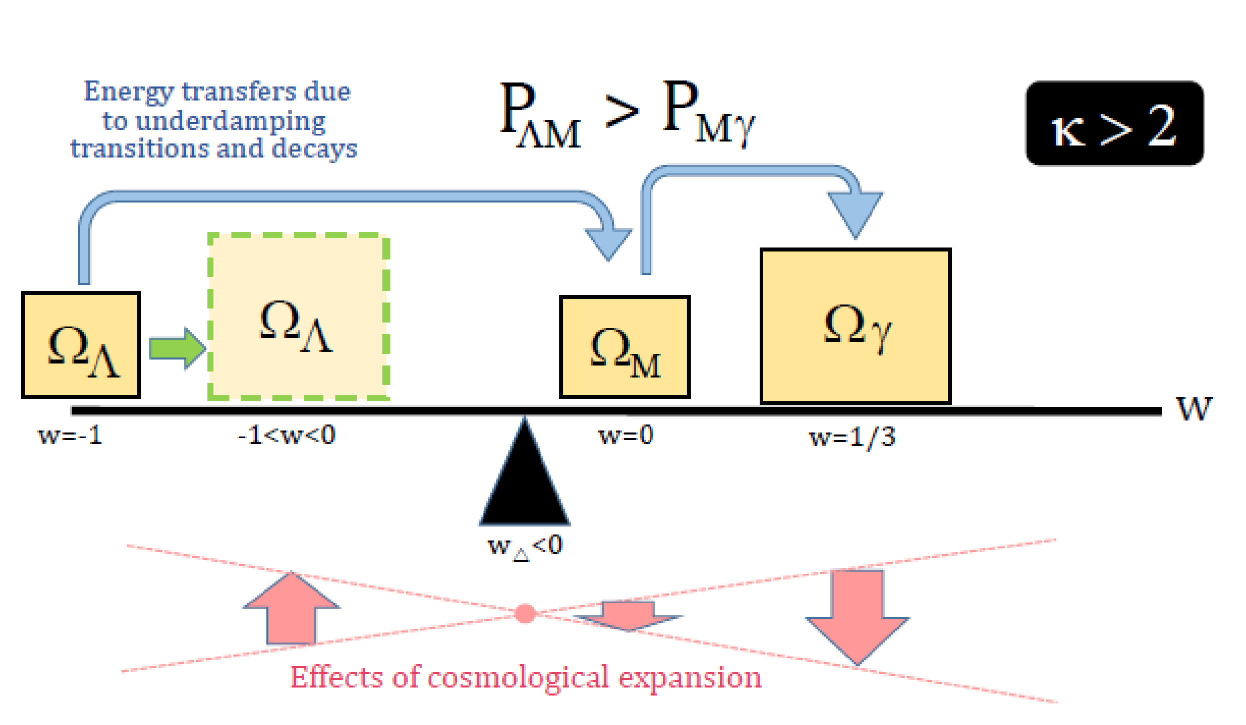
Other values of κ correspond to shifting the location of the fulcrum!

In general, we have

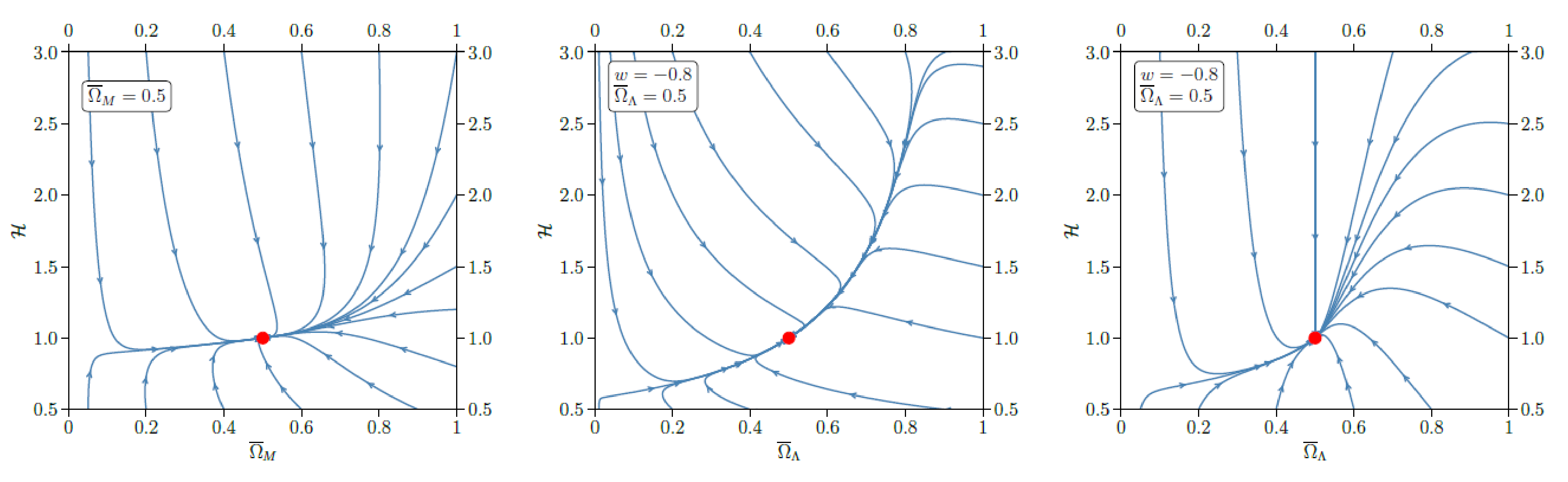
$$w_{\triangle} \equiv \frac{2}{\kappa} - 1$$

Pumps become unequal but stasis is always maintained!





Just like its pairwise cousins, triple stasis is also a global attractor!



Each of the three abundances flows to its triple-stasis value.

Even more stasis!

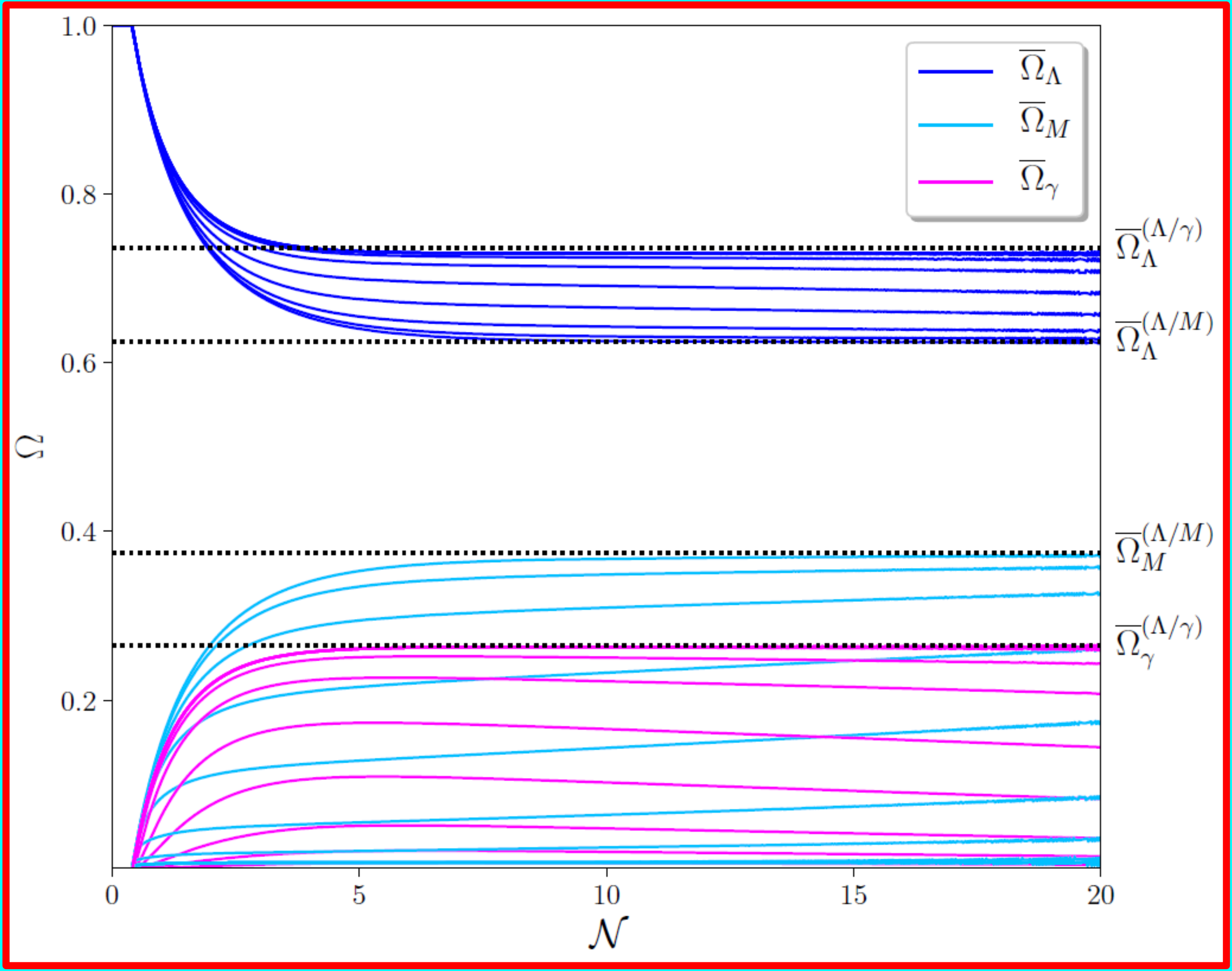
- Quadruple and higher stasis?
- Other energy components (kination, etc.)? – much is already captured by our general formalisms.

Stasis also has its variants!

There are also lots of versions of "quasi-stasis" which are worthy of exploration. Each of these illustrates the fundamental robustness of the stasis phenomenon.

- "Near-stasis" --- When scaling exponents deviate slightly from their stasis values (*e.g.*, due to radiative effects), abundances evolve slowly, but may still be interesting / important on cosmological timescales.
- "Log-stasis" --- What happens when our system satisfies all of the constraints needed for stasis except the log-avoidance constraints? Abundances might then develop a very slow, logarithmic-like time-evolution. Might still be relevant for phenomenology over cosmological timescales.

For example, consider "near-stasis", with γ =1.05 rather than γ =1.
For different values of ξ, curves are still approximately constant across many *e*-folds...



Oscillatory Stasis

Another option is to consider what happens if Δm is large.

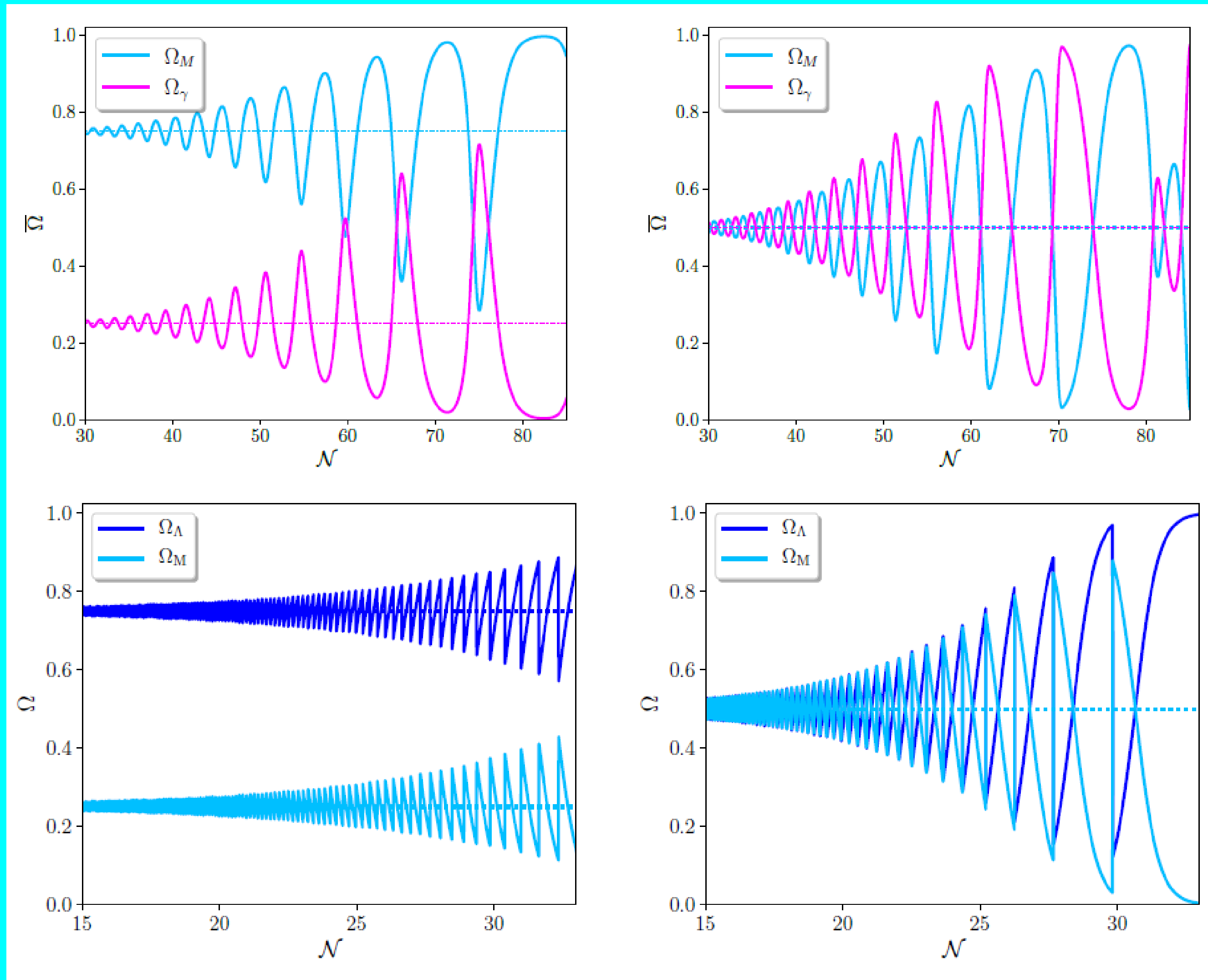
- Spacing between successive components of tower becomes significant
- This increases the time intervals between underdamping and/or decay transitions.
- During this time interval, cosmological expansion can work its magic before another "infusion" of abundance occurs from the next transition along the tower.

This results in an "oscillatory" stasis!
This effect is normally seen near the end of stasis,
but this can now persist over many *e*-folds.
It's still a form of stasis because the abundance oscillations
occur around *fixed* central values!
As such, the time-average of oscillatory stasis is ordinary stasis!

Oscillatory stasis

M/γ

Λ/M

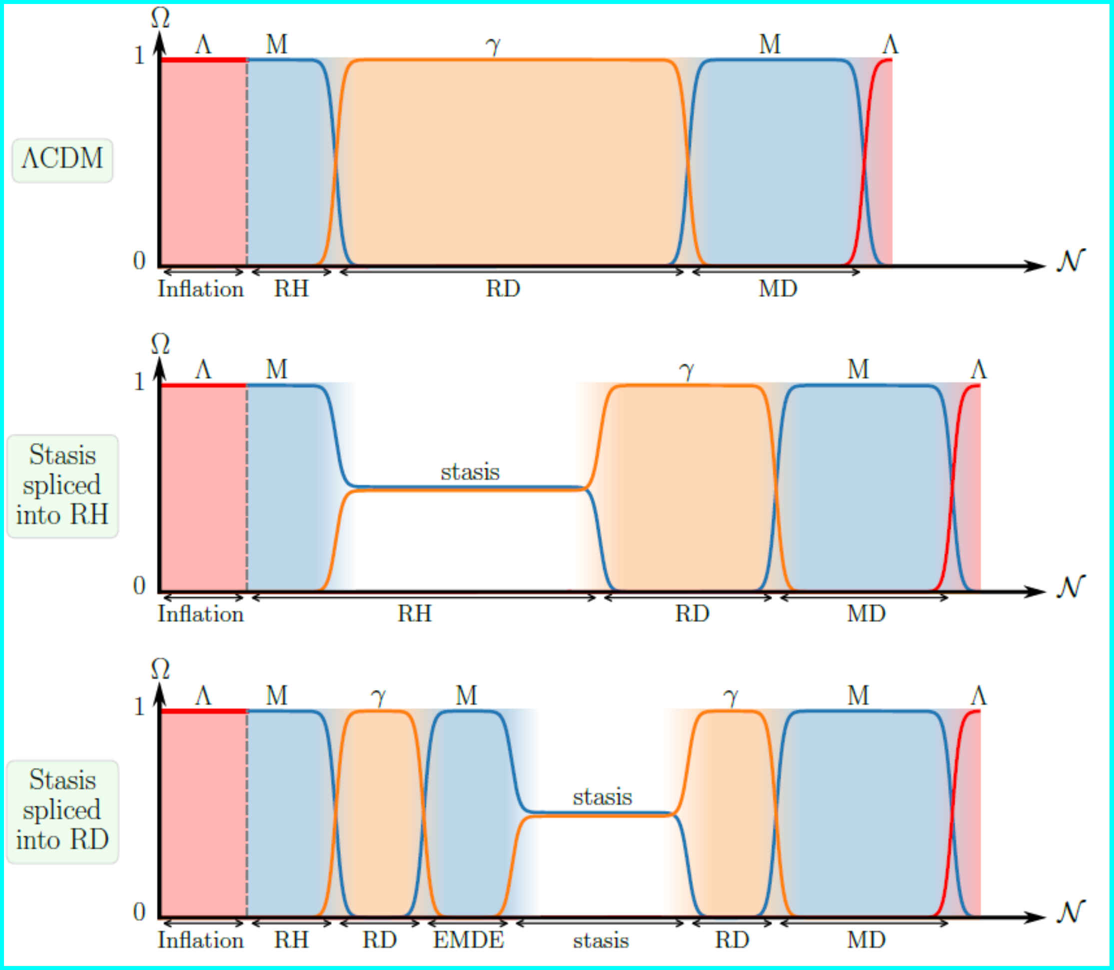


when stasis abundances
are well separated

when stasis abundances
are relatively close…
abundances are “braided”

Conclusions

Stasis is nothing less than a new kind of cosmological epoch!
The implications are likely to be profound...



Stasis epoch can find itself "spliced" into various points during the ΛCDM history...

The existence of a stasis epoch within BSM cosmologies is likely to give rise to a host of new theoretical possibilities across the entire cosmological timeline, ranging from potential implications for primordial density perturbations, dark-matter production, and structure formation all the way to early reheating, early matter-dominated eras, and even the age of the universe.

[See Brooks Thomas' talk in parallel session this afternoon!]

We are only at the tip of the iceberg! Lots of implications to be explored!