Modular Invariance and the Strong CP problem

$$\tau \to \frac{a\,\tau + b}{c\,\tau + d}$$

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Modular invariance and the QCD angle

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the strong CP problem

$$\mathcal{L}_{QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4g_3^2}\mathcal{G}^a_{\mu\nu}\mathcal{G}^{a\mu\nu} + \frac{\theta_{QCD}}{32\pi^2}\mathcal{G}^a_{\mu\nu}\tilde{\mathcal{G}}^{a\mu\nu}$$

$\bar{\theta} = \theta_{QCD} + \arg\det m$ $\quad$ $d_n \approx 1.2\times10^{-16}\,\bar{\theta}\; e\cdot cm$

➡ $|\bar{\theta}| \lesssim 10^{-10}$ & $\delta_{CKM} \approx \mathcal{O}(1)$

solutions

- $\bar{\theta}$ dynamically relaxed to zero by the axion, would-be GB of a global, anomalous $U(1)_{PQ}$ symmetry
- CP (or P) is a symmetry of the UV, spontaneously broken to get $\bar{\theta} = 0$ & $\delta_{CKM} = \mathcal{O}(1)$

A.E. Nelson, 'Naturally Weak CP Violation', Phys.Lett.B 136 (1984) 387.

S.M. Barr, 'Solving the Strong CP Problem Without the Peccei-Quinn Symmetry', Phys.Rev.Lett. 53 (1984) 329.

our solution: CP symmetry of the UV theory

1. CP → $\theta_{QCD} = 0$

2. $\arg\det m = 0$ ← automatic from modular invariance, absence of gauge anomalies

3. CP spontaneously broken by the VEV of a single complex field τ

↓

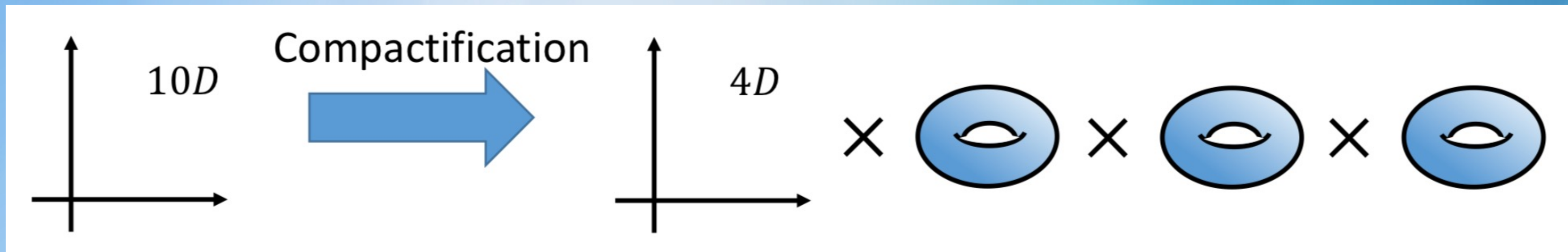
4. $\delta_{CKM} = \mathcal{O}(1)$

5. quark mass hierarchies naturally reproduced

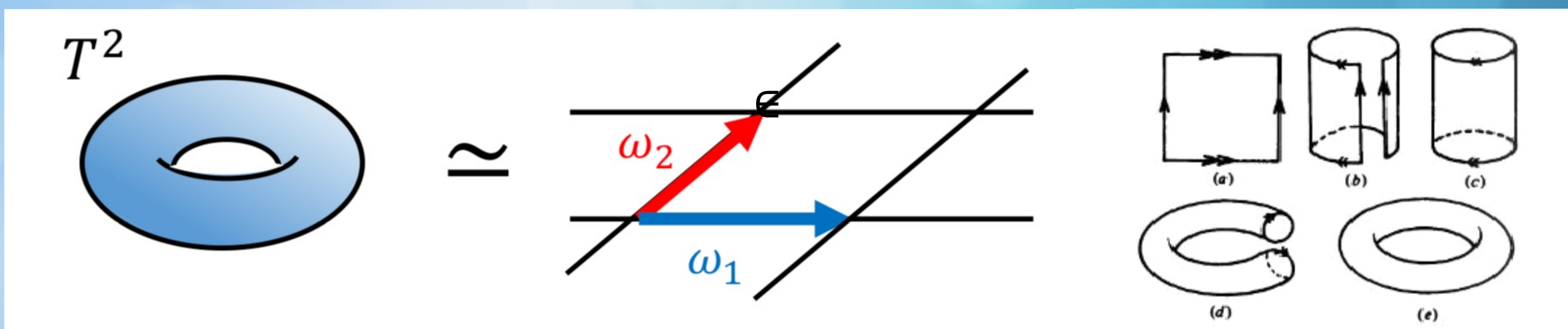
6. no corrections from higher-dimensional operators
radiative corrections

modular invariance

string theory in d=10 need 6 compact dimensions



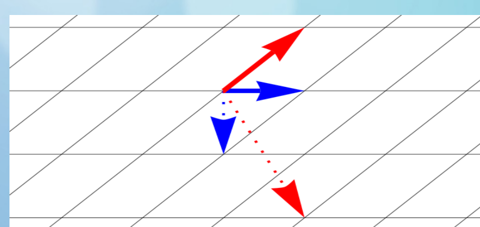
simplest compactification: 3 copies of a torus T^2



tori parametrized by

$$\mathcal{M} = \left\{ \tau = \frac{\omega_2}{\omega_1} \quad Im(\tau) > 0 \right\}$$

lattice left invariant by modular transformations:

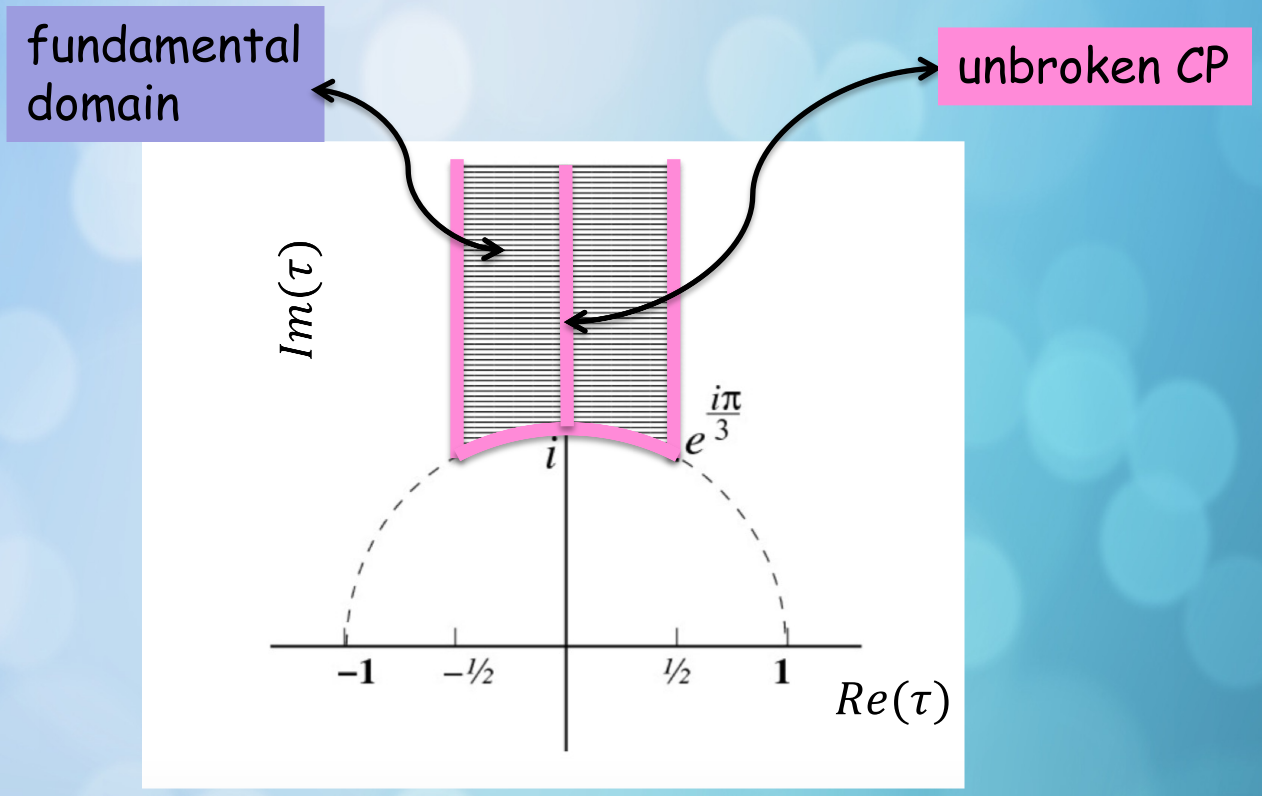


$$\tau \rightarrow \frac{a\tau + b}{c\tau + d} \in SL(2, Z)$$

a, b, c, d integers

$ad - bc = 1$

- τ promoted to a field. Through a gauge choice we can restrict τ to the fundamental domain



- CP

$$\tau \to -\tau^*$$

[up to modular transformations]

[Novichkov, Penedo, Petcov and Titov 1905.11970
Baur, Nilles, Trautner and Vaudrevange, 1901.03251]

$\mathcal{N}$=1 SUSY CP & modular-invariant theories

$$\mathcal{L} = \int d^2\theta d^2\bar{\theta}\, K(\tau, e^{2V}\varphi, \bar{\tau}, \bar{\varphi}) + \left[\int d^2\theta\, w(\tau,\varphi) + \frac{1}{16}\int d^2\theta\, f\, WW + h.c\right]$$

kinetic terms

Yukawa couplings $\mathcal{Y}(\tau)$

gauge kinetic function

$$f_3 = \frac{1}{g_3^2} - i\frac{\theta_{QCD}}{8\pi^2}$$

$$\arg\det m(\tau) = \arg\det Y(\tau)v$$

G. Hiller, M. Schmaltz, 'Solving the Strong CP Problem with Supersymmetry', Phys.Lett.B 514 (2001) 263 [arXiv:hep-ph/0105254].

$$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y(\tau)v$$

no dependence on K

$\bar{\theta}$ holomorphic

A note on the predictions of models with modular flavor symmetries

Mu-Chun Chen (UC, Irvine), Saúl Ramos-Sánchez (Mexico U. and Munich, Tech. U.), Michael Ratz (UC, Irvine)
15, 2019)

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CP & modular-invariance

- CP $\leftrightarrow$ real coupling constants
- modular invariance

$$\tau \to \frac{a\tau + b}{c\tau + d}$$

$$\varphi \to (c\,\tau + d)^{-k_\varphi}\ \varphi$$ matter multiplets

$$V \to V$$ vector multiplets

- $$K = -h^2 \log(-i\tau + i\bar{\tau}) + \sum_\varphi (-i\tau + i\bar{\tau})^{-k_\varphi} \varphi^+ e^{2V} \varphi$$
- $$f_3 = \frac{1}{g_3^2} \qquad (\theta_{QCD} = 0)$$
- $$w(\tau, \varphi) = U_i^c\, Y_{ij}^u(\tau)\, Q_j H_u + D_i^c\, Y_{ij}^d(\tau)\, Q_j H_d + E_i^c\, Y_{ij}^e(\tau)\, L_j H_d + \cdots$$

$$Y_{ij}^q(\tau) \to (c\tau + d)^{k_{ij}^q}\, Y_{ij}^q(\tau)$$

$$k_{ij}^u = k_{Q_j} + k_{U_i^c} + k_{H_u}$$

$$k_{ij}^d = k_{Q_j} + k_{D_i^c} + k_{H_d}$$

assuming no singularities: $Y_{ij}^q(\tau)$ are modular forms of weight k_{ij}^q

$k_{ij}^q < 0$: no modular forms

$k_{ij}^q = 0$: modular forms are constants

$k_{ij}^q > 0$: modular forms polynomials in $E_4(\tau), E_6(\tau)$

Modular weight k	0	2	4	6	8	10	12	14
Number of forms	1	0	1	1	1	1	2	1
Modular forms	1	–	E_4	E_6	$E_8 = E_4^2$	$E_{10} = E_4 E_6$	E_4^3, E_6^2	$E_{14} = E_4^2 E_6$

$$\det Y(\tau) \equiv \det Y^u(\tau) \det Y^d(\tau)$$

$$\det Y(\tau) \to (c\tau + d)^{k_{\det}} \quad \det Y(\tau)$$

$$k_{\det} = \sum_{i=1}^{3} \left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) + 3k_{H_u} + 3k_{H_d}$$

$k_{\det} = 0$ ⇒ $\det Y(\tau) =$ (real) constant

cancellation of modular anomalies

$$\psi_{can} \to \left(\frac{c\tau + d}{c\tau^{+} + d}\right)^{-\frac{k_{\varphi}}{2}} \psi_{can}$$

conditions for gauge-modular anomaly cancellation

$SU(3)$

$$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) = 0$$

$SU(2)$

$$\sum_{i=1}^{3}\left(3k_{Q_i} + k_{L_i}\right) + k_{H_u} + k_{H_d} = 0$$

$U(1)$

$$\sum_{i=1}^{3}\left(k_{Q_i} + 8k_{U_i^c} + 2k_{D_i^c} + 3k_{L_i} + 6k_{E_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

cancellation of modular anomalies

$$\psi_{can} \to \left(\frac{c\tau + d}{c\tau^+ + d}\right)^{-\frac{k_\varphi}{2}} \psi_{can}$$

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$SU(3)$ $$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c}\right) = 0$$

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$U(1)$ $$\sum_{i=1}^{3}\left(k_{Q_i} + 8k_{U_i^c} + 2k_{D_i^c} + 3k_{L_i} + 6k_{E_i^c}\right) + 3\left(k_{H_u} + k_{H_d}\right) = 0$$

simplest solution:

$$k_{H_u} + k_{H_d} = 0 \qquad k_{Q_i} = k_{U_i^c} = k_{D_i^c} = k_{L_i} = k_{E_i^c} = (-k, 0, k)$$

$$k_{\det} = 0 \qquad \det Y(\tau)\ \&\ f \text{ real constants}$$

$$\bar{\theta} = -8\pi^2 Im\, f + \arg\det Y = 0$$

holds also after SUSY breaking, if no new phases in SUSY breaking sector

Example $k_{Q_i} = k_{U_i^c} = k_{D_i^c} = (-6, 0, +6)$

$$Y_q(\tau) = \begin{pmatrix} 0 & 0 & c_{13}^q \\ 0 & c_{22}^q & c_{23}^q E_6 \\ c_{31}^q & c_{32}^q E_6 & c_{33}^q E_4^3 + c_{33}^{q\prime} E_6^2 \end{pmatrix}$$

$\tan\beta = 10 \quad \tau = 0.125 + i$

$$c_{ij}^u \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.56 \\ 0 & -1.86 & 0.87 \\ 1.29 & 4.14 & 3.51, 1.40 \end{pmatrix}, \qquad c_{ij}^d \approx 10^{-3} \begin{pmatrix} 0 & 0 & 1.55 \\ 0 & -2.59 & 4.59 \\ 0.378 & 0.710 & 0.734, 1.76 \end{pmatrix}$$

reproduce quark masses, mixing angles and CKM phase

$\delta_{CKM} \neq 0$ ⟺ $Im\,\det[Y_u^+ Y_u, Y_d^+ Y_d] \neq 0$ non-holomorphic

Leptons: $k_{L_i} = k_{E_i^c} = (-6, 0, +6)$

$$c_{ij}^e = 10^{-3} \begin{pmatrix} 0 & 0 & 1.29 \\ 0 & 5.95 & 0.35 \\ -2.56 & 1.47 & 1.01, 1.32 \end{pmatrix}, \qquad c_{ij}^\nu = \frac{1}{10^{16}\,\text{GeV}} \begin{pmatrix} 0 & 0 & 3.4 \\ 0 & 7.1 & 1.2 \\ 3.4 & 1.2 & 0.19, 0.95 \end{pmatrix}$$

heavy quarks and singularities

- heavy quarks not needed, but they can exist in the UV

  example $k_\varphi = (-6, -2, 0, +2, +6)$ $\qquad k_{H_u} + k_{H_d} = 0$

  chiral: $(-6, -2, 0)$ — heavy vector-like quark: $(+2, +6)$

- UV theory $\bar{\theta} = -8\pi^2 Im\, f_{UV} + \arg\det Y_{UV} = 0$

- IR theory has an anomalous field content, anomaly cancelled by:

$$f_{IR} = f_{UV} - \frac{1}{8\pi^2} \log\det Y_{Heavy}(\tau)$$

$$\bar{\theta} = -8\pi^2 Im\, f_{IR}(\tau) + \arg\det Y_{Light}(\tau) =$$

$$= +\arg\det Y_{Heavy}(\tau) + \arg\det Y_{Light}(\tau)$$

$$= \arg\det Y_{UV} = 0$$

- $Y_{Light}(\tau)$ is singular at τ values such that $\det Y_{Heavy}(\tau) = 0$

$\mathcal{N} = 1$ supergravity

$$K = -h^2 \log(-i\tau + i\tau^+) + \cdots$$

$$k_W = \frac{h^2}{M_{Pl}^2} \to 0$$

back to the rigid case

corrections of $\mathcal{O}(k_W)$?

- K and w no more independent

$$\mathcal{G} = \frac{K}{M_{Pl}^2} + \log\left|\frac{w}{M_{Pl}^3}\right|^2$$

$$w(\tau) \to (c\tau + d)^{-k_W}\, w(\tau)$$

$$k_W > 0$$

no negative weight modular forms, $w(\tau)$ singular somewhere

- modular-QCD anomaly modified into

$$\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) + 3k_W$$

quarks

gluino

can be rotated away if gluino is massless

V. Kaplunovsky, J. Louis, 'On Gauge couplings in string theory', Nucl.Phys.B 444 (1995) 191 [arXiv:hep-th/9502077].

J.P. Derendinger, S. Ferrara, C. Kounnas, F. Zwirner, 'On loop corrections to string effective field theories: Field dependent gauge couplings and sigma model anomalies', Nucl.Phys.B 372 (1992) 145.

L.J. Dixon, V. Kaplunovsky, J. Louis, 'Moduli dependence of string loop corrections to gauge coupling constants', Nucl.Phys.B 355 (1991) 649.

spontaneously broken supergravity

- $\bar{\theta} = -8\pi^2 Im\, f + \arg\det M_{quark} + 3\arg M_3 = 0$

$\arg\det M_{quark} = 0$ ⇐ $\sum_{i=1}^{3}\left(2k_{Q_i} + k_{U_i^c} + k_{D_i^c} - 2k_W\right) = k_{H_u} + k_{H_d} = 0$

$\arg M_3 = -\arg w$ ⇐ if no other phases from SUSY breaking $\quad M_3 = \frac{1}{2} e^{\frac{K}{2M_{Pl}^2}} K^{i\bar{j}} D_{\bar{j}} w^+ f_i$

- assume unique singularity at $\tau = i\infty$

$$w(\tau) = \cdots + c_0 M_{Pl}^3\, \eta(\tau)^{-2k_W}$$

$$f = \cdots + 3\frac{k_W}{4\pi^2}\log\eta(\tau)$$

$\eta(\tau)$ Dedekind eta function

H. Rademacher, H.S. Zuckeman, 'On the Fourier coefficients of certain modular forms of positive dimensions', Annals of Mathematics 39 (1938) 433.

cancels the gluino anomaly

$$\bar{\theta} = -8\pi^2 Im\, f + 3\arg M_3 = 0$$

deviations from $\bar{\theta} = 0$

SUSY unbroken

no corrections from K
no corrections from nonrenormalizable operators: $SL(2, \mathbb{Z})$
no corrections from additional moduli/singlets under reasonable assumptions

SUSY breaking corrections

potentially big if soft terms violate flavour in a generic way

minimized if $\Lambda_{CP} \gg \Lambda_{SUSY}$ (as e.g. in gauge mediation)

and soft breaking terms respect the flavour structure of the SM

$$\bar{\theta} \lesssim \frac{M_t^4 M_b^4 M_c^2 M_s^2}{v^{12}} J_{\rm CP} \tan^6 \beta \sim 10^{-28} \tan^6 \beta.$$

SM corrections

negligible: $\bar{\theta} \leq 10^{-18}$ at four loops

J.R. Ellis, M.K. Gaillard, 'Strong and Weak CP Violation', Nucl.Phys.B 150 (1979) 141.

I.B. Khriplovich, 'Quark Electric Dipole Moment and Induced θ Term in the Kobayashi-Maskawa Model', Phys.Lett.B 173 (1986) 193.

Ingredients

1. CP in the UV
2. Yukawa couplings are field-dependent quantities
3. the vacuum has a redundant description: vacua related by $SL(2,\mathbb{Z})$ are equivalent
4. CP and $SL(2,\mathbb{Z})$ are unified in a gauge flavour symmetry
5. absence of anomalies
6. no singularities in the UV theory

Ingredients	String Theory

1. CP in the UV

   the four-dimensional CP symmetry is a gauge symmetry in most string theory compactifications.

2. Yukawa couplings are field-dependent quantities

   string theory has no free parameters anc Yukawa couplings are set by moduli VEVs

3. the vacuum has a redundant description: vacua related by $SL(2,\mathbb{Z})$ are equivalent

   modular invariance is a key ingredient of string theory compactifications

4. CP and $SL(2,\mathbb{Z})$ are unified in a gauge flavour symmetry

   Unification of Flavor, CP, and Modular Symmetries

   Alexander Baur (Munich, Tech. U.), Hans Peter Nilles (Bonn U. and Bonn U., HISK Trautner (Heidelberg, Max Planck Inst.), Patrick K.S. Vaudrevange (Munich, Tech

   Published in: *Phys.Lett.B* 795 (2019) 7-14 • e-Print: 1901.03251 [hep-th]

5. absence of anomalies

   mandatory in string theory

6. no singularities in the UV theory

   string theory is free of singularities. These arise in the IR when some UV modes become massless

THANK YOU!

back-up slides

The elusive τ phenomenology

- couplings to matter suppressed by $1/h$ $(1/M_{Pl}$ in SUGRA)
- no couplings to gauge vector bosons, when SUSY exact

↓

inaccesible experimental test?

- τ should be heavy, not to spoil nucleosynthesis

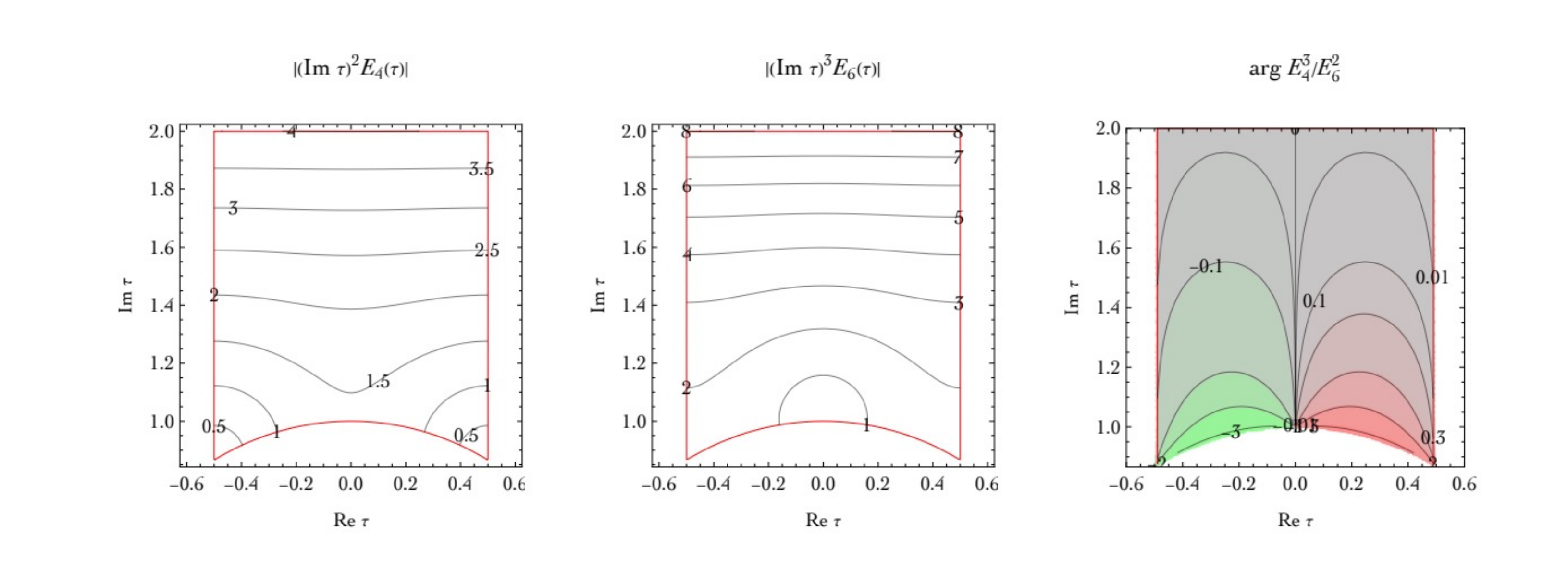
$$m_\tau \approx m_{3/2} > 10 \text{ TeV}$$

Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings

B. de Carlos (Madrid, Inst. Estructura Materia), J.A. Casas (CERN and Madrid, Inst. Estructura Materia), F. Quevedo (Neuchatel U.), E. Roulet (CERN) (Jul, 1993)

Published in: *Phys.Lett.B* 318 (1993) 447-456 • e-Print: hep-ph/9308325 [hep-ph]

- domain walls separating patches with opposite CP: inflated away if CP breaking occurs before inflation.



|(Im τ)²E₄(τ)|
|(Im τ)³E₆(τ)|
arg E₄³/E₆²
Im τ
Re τ

axion solution

- $\bar{\theta}$ dynamically relaxed to zero by the axion, would-be GB of a global, anomalous $U(1)_{PQ}$ symmetry

- provides a candidate for DM
- many axion candidates in e.g. superstring theories

- axion quality problem

minimum of $V(a)$ should be at $a = 0$

$$V(a) = V_{QCD}(a) - M^4 e^{-S} \cos(\frac{a}{f_a} + \delta)$$

$M = M_P$
$\delta = \mathcal{O}(1)$

⇒ $S \geq 200$

- axion undetected, so far

Nelson-Barr solution | our solution

CP ia a symmetry of the UV,
SB to get $\bar{\theta} = 0$ & $\delta_{CKM} = \mathcal{O}(1)$

CP ⇒ $\theta_{QCD} = 0$

Nelson-Barr solution	our solution
heavy vector-like quark sector	no extra matter
CP spontaneously broken by $\langle \eta_a \rangle$ complex [one is not enough]	CP spontaneously broken by τ alone
$\mu \approx \lambda_a \eta_a$ [tuning]	no tuning

Nelson-Barr quark sector (Q | q):

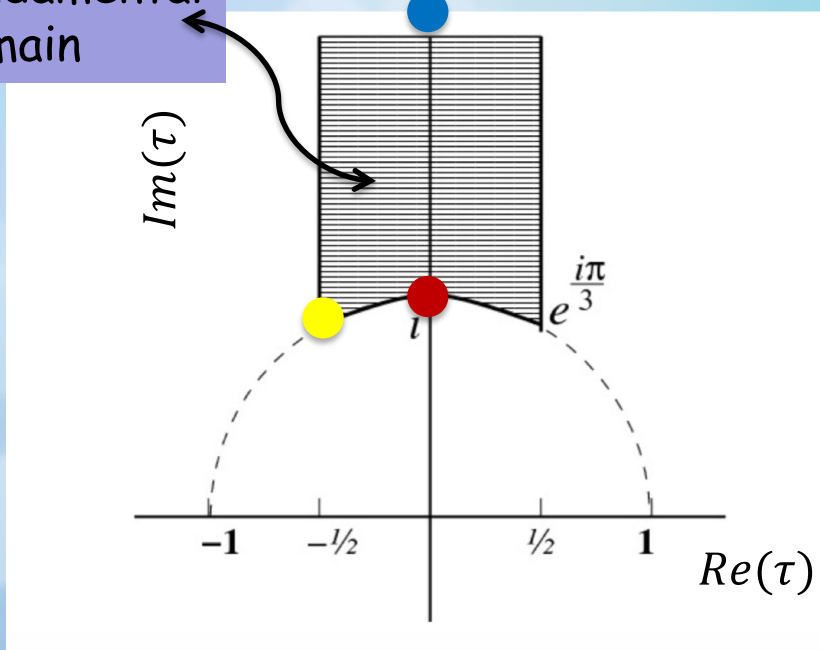
$$m = \begin{pmatrix} \mu & \lambda_a \eta_a \\ 0 & y\, v \end{pmatrix}$$

Yukawa matrices $Y_{u,d}$	Modular weights $(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$	Alternative bigger weights $(u_L, d_L)_{1,2,3}$	$u_{R1,2,3}$	$d_{R1,2,3}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_6 \\ 1 & E_6 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ -2 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4 & E_4^3 + E_6^2 \end{pmatrix}$	$\begin{pmatrix} -6 \\ -2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -6 \\ 2 \\ 6 \end{pmatrix}$	$\begin{pmatrix} -4 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 4 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^2 \\ 1 & E_4^2 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 0 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4 E_6 \\ 1 & E_6 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 2 \\ 8 \end{pmatrix}$
$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & E_4^3 + E_6^2 \\ 1 & E_4 & E_4(E_4^3 + E_6^2) \end{pmatrix}$				$\begin{pmatrix} -8 \\ -4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$	$\begin{pmatrix} -8 \\ 4 \\ 8 \end{pmatrix}$

Table 2: *Simplest modular weights that lead to Yukawa matrices such that $\bar{\theta} = 0$ and $\delta_{\rm CKM} \neq 0$. The list is complete up to permutations and transpositions, and assumes vanishing modular weights of the Higgs doublets and of the super-potential. Real constants c_{ij}^q are here omitted.*

fixed point		symmetry	residual symmetry
$\tau = i$	S: $\tau \to -\frac{1}{\tau}$		$\mathbb{Z}_4^S$
$\tau = e^{i\,2\pi/3}$	ST: $\tau \to -\frac{1}{\tau+1}$		$\mathbb{Z}_2^{ST} \times \mathbb{Z}_2^{S^2}$
$\tau = i\,\infty$	T: $\tau \to \tau + 1$		$\mathbb{Z}^T \times \mathbb{Z}_2^{S^2}$

fundamental domain



modular invariance completely broken everywhere but at three fixed points

$SL(2,Z)$ generated by

$$S : \tau \to -\frac{1}{\tau} \quad , \qquad T : \tau \to \tau + 1$$