Recent progress on the black hole information problem

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- In this talk I will attempt to give a brief overview of what I think is the current status.

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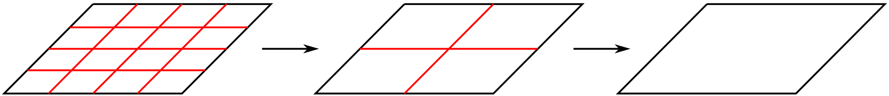

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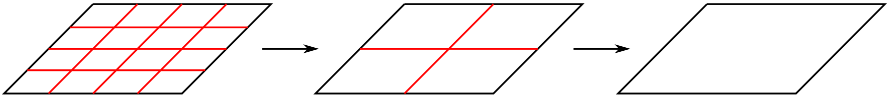

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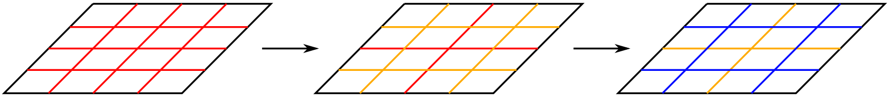

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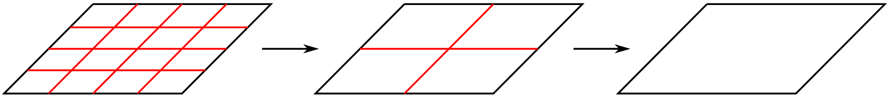

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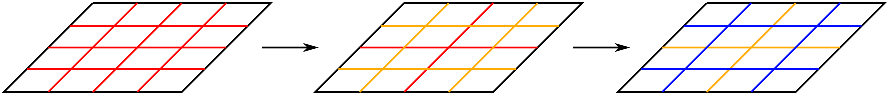
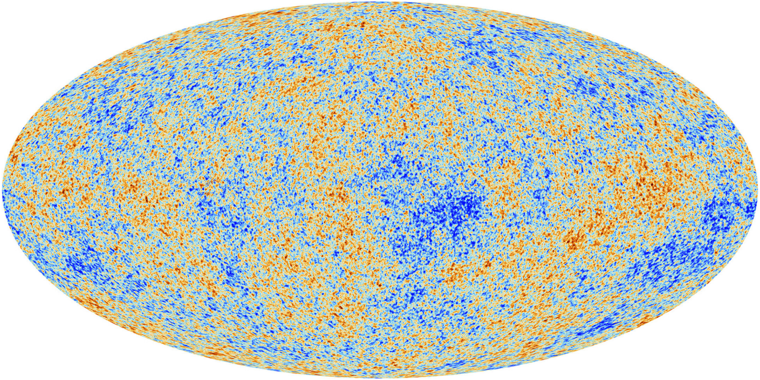


In the presence of some kind of short-distance cutoff (say at the Planck scale), this means that to preserve the cutoff scale we need to introduce new modes at short distances.

We therefore need a rule for what state these new modes are in. The only rule which seems to make any sense is to say that (roughly speaking) these new modes enter in their vacuum state.

This may just sound like some rule that I made up, but in fact it has been confirmed by observation:



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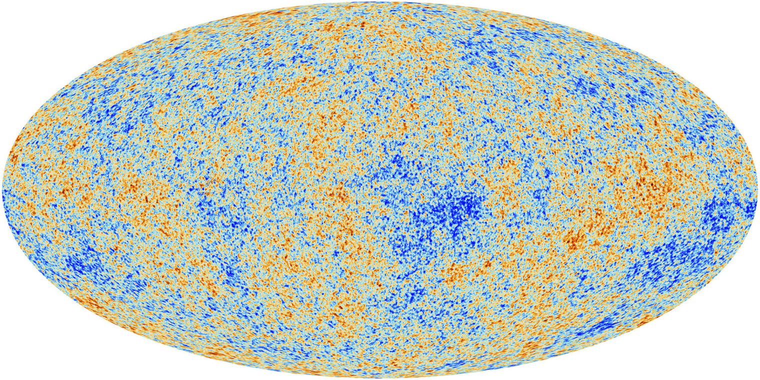

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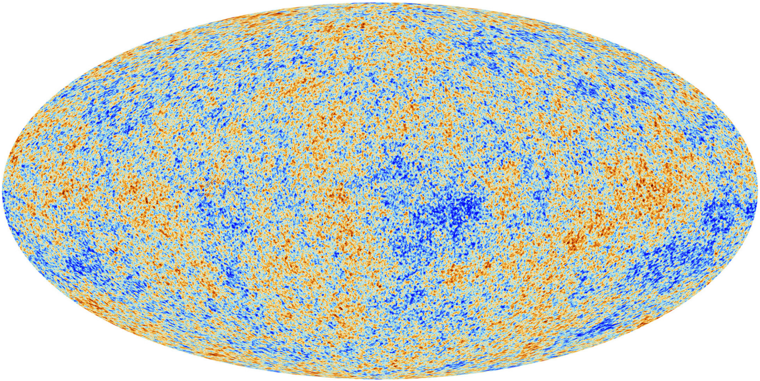

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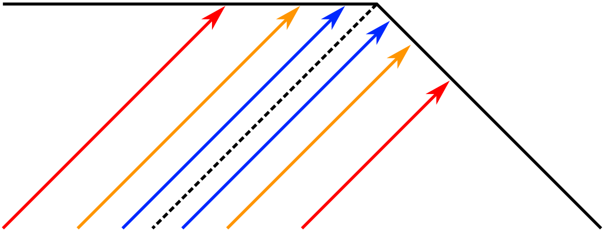

As we evolve forward in time, entangled modes straddling the horizon move away from the horizon. The interior partner falls into the singularity, while the exterior partner makes it out to infinity as Hawking radiation.

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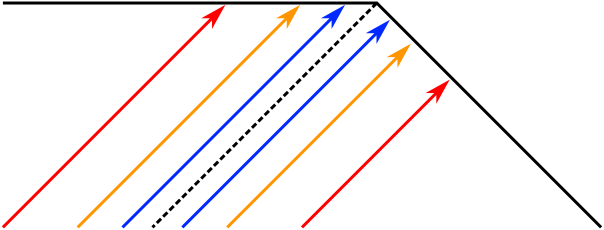

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$$t \sim \frac{\beta}{2\pi} \log S \equiv t_{scr},$$

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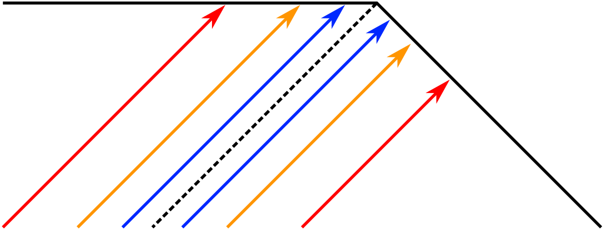

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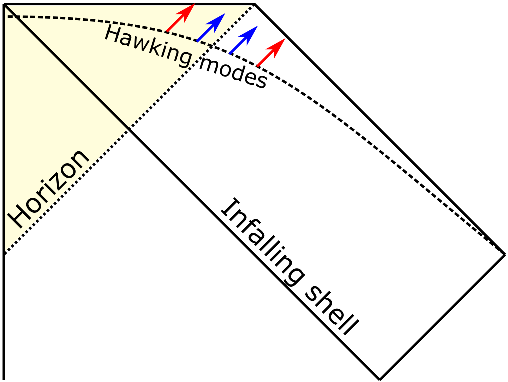
For Sagittarius A* we have $t_{scr} \sim 1000\mathrm{s}$.

Now let's recall how this leads to Hawking's paradox:



Entanglement between interior and exterior modes causes the black hole to radiate, losing energy, but this radiation cannot carry information about the infalling shell since these new modes enter in vacuum and the shell is still deep inside. By the time the black hole reaches Planckian size, it doesn't have enough energy left to return this information to the exterior.

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These are all things we really would like to be true, so any resolution of the paradox will teach us something deep!

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Our challenge is thus to understand what replaces (3).

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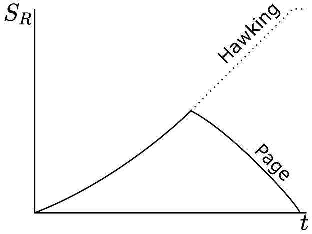
- It is not clear however why this should happen for black holes but not for the CMB, and anyways we shouldn't accept this unless we really have no other choice.
- On the other hand if we think Hawking's picture is valid until times of order the evaporation time, then at these late times the infalling shell is spacelike-separated at great distance from the Hawking radiation. For information to get out, *severe* non-locality is necessary: at distances of order $10^{97}\mathrm{m}$ for Sagittarius A*!

Another illustration of the level of modification which is necessary is the "Page curve", which plots the von Neumann entropy of the radiation as a function of time:



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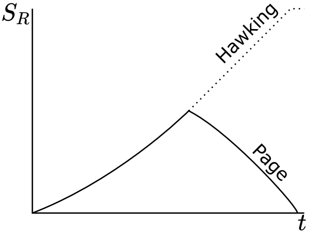

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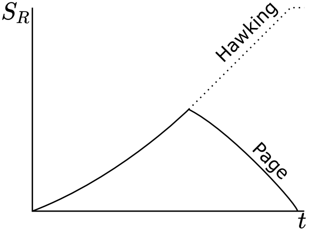

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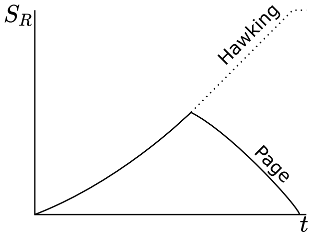

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At this time the entropy in the interior modes (which purify the Hawking radiation) exceeds the entropy of the black hole, which seems to present a serious obstruction to the idea that $S = \frac{A}{4G}$.

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What we seem to be learning is that the effective field theory description of the black hole interior is good until t_{evap}/t_{exp}, and in the remainder of this talk I will try to give a sense of how this works.

Akers/Engelhardt/Harlow/Penington/Vardhan 2022

Emergent spacetime and AdS/CFT

In understanding what might replace (3) (the validity of EFT away from singularities), it is useful to note that in our best theory of quantum gravity so far, AdS/CFT, it not obvious that *any* version of (3) holds:

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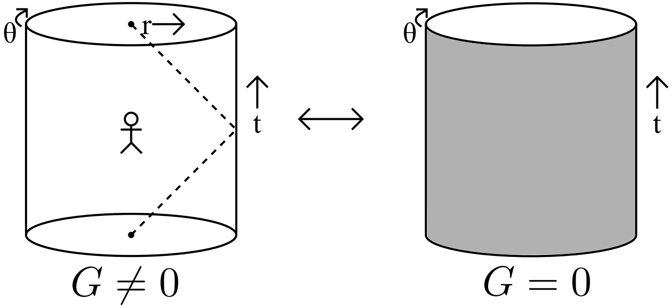

Quantum gravity in asymptotically-AdS space is equal to quantum field theory living on the asymptotic boundary, so the bulk spacetime is at best *emergent*: it makes sense only in certain situations and only in some approximation. Maldacena 1998

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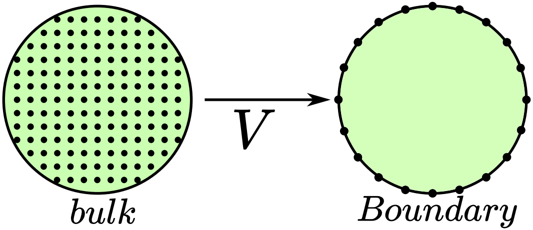

In particular for sufficiently low-energy states (no black holes) there is a holographic encoding map $V : \mathcal{H}_{bulk} \to \mathcal{H}_{boundary}$, where $\mathcal{H}_{bulk}$ is the set of low-energy bulk states and V is approximately (up to $O(e^{-N^2})$) an isometry.

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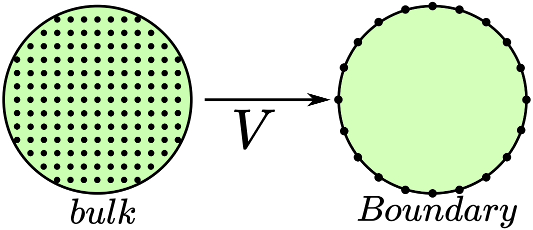

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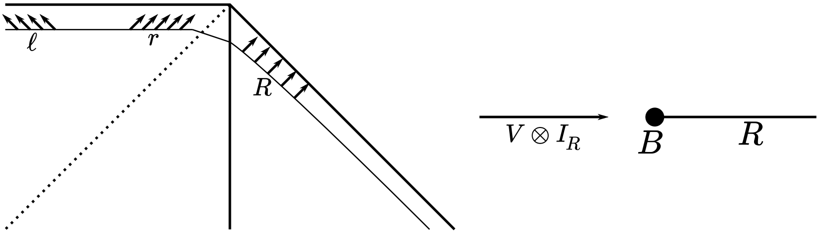
(Recall that an isometry is a linear map $V : \mathcal{H}_A \to \mathcal{H}_B$ that preserves the inner product. Isometries can exist only if $|A| \leq |B|$.)

An encoding map for the BH interior

We'd like to construct a similar holographic encoding map $V : \mathcal{H}_\ell \otimes \mathcal{H}_r \to \mathcal{H}_B$ for the black hole interior:



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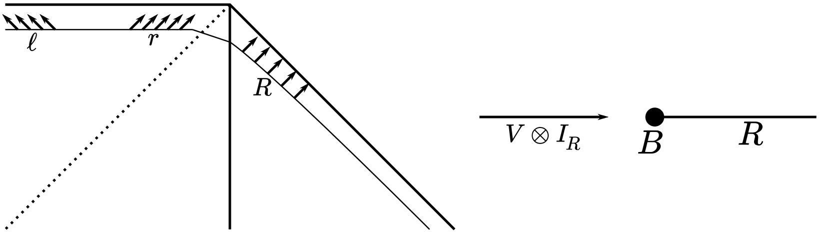

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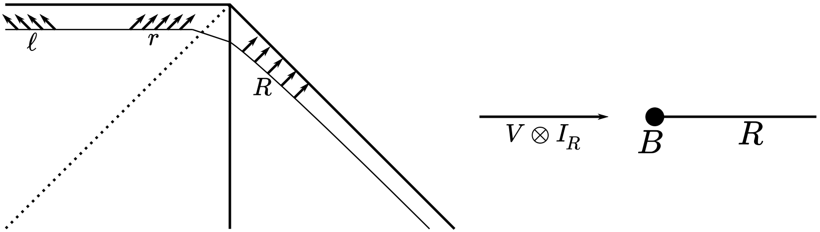

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The main thing we have learned in the last few years is that we should embrace this non-isometric nature of V: it may sound scary, but understood properly it explains the difference between Page and Hawking without giving up too much on EFT!

In more detail the essence of our proposal is the following:

There is a large set of "null states" in the Hilbert space of effective field theory inside a black hole, each of which is annihilated by the holographic map to the fundamental degrees of freedom. This however cannot be detected by any observer who does not perform an operation of exponential complexity.

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In the previous language we advocate the following replacement:

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Indeed we can construct models where appropriate analogous of (1), (2), and (3*) are all proveably true. They are thus compatible, and so if we are willing to accept (3*) then the information problem is resolved in these models.

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- Here is another one, which is also nearly orthogonal to $|\psi\rangle$:

$$|\phi\rangle = e^{-S/2} \sum_{n=1}^{e^S} (-1)^n |n\rangle.$$

More generally we can make a toy encoding map of the black hole interior ℓr into the microstates B:

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You can fit a lot more nearly orthogonal states into Hilbert space than you might have thought!

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Thanks for listening!