Enhancement of Particle Phenomena by High-Spin Black Holes

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Why High-Spin Black Holes?

- Astrophysical BHs only by their mass M and spin aM
- BH masses measured using gas dynamics [Ghez et al. 2008; Gravity Collaboration 2020], stellar orbits [Wash, Barth, Ho, Sarzi 2013], accretion disk imaging [EHT 2019, 2022]
- BH spin is harder to measure
  - use fact that observable features get exaggerated with increasing spin
  - ~30 BH spins have been constrained [Reynold 2020]
  - are there real world BH close to maximally spinning?

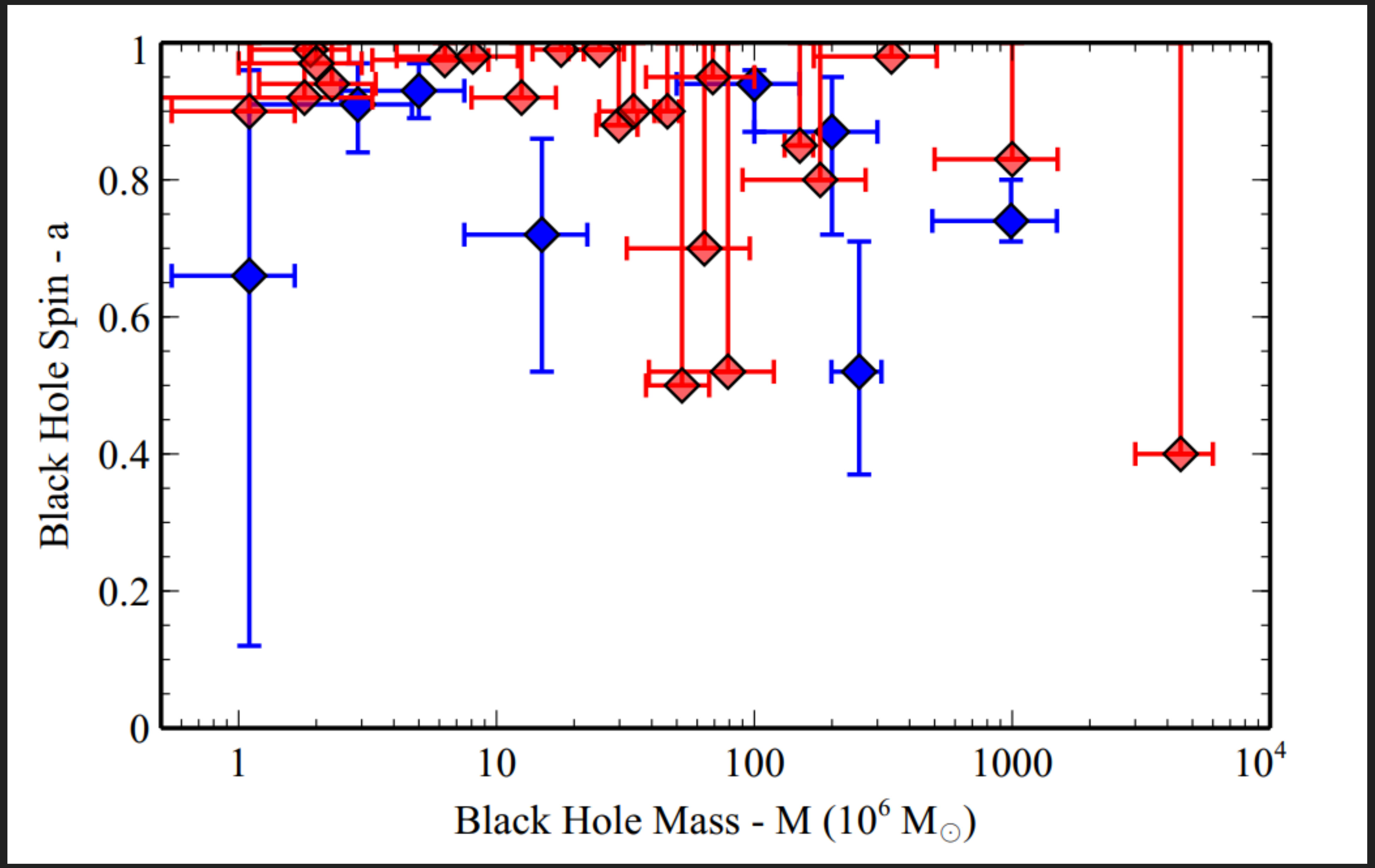


Image Ref: Reynold 2020

Why High-Spin Black Holes?

- Mathematically, (near-)maximally rotating BHs are interesting
  - as we spin up a BH, the near-horizon region gets stretched into a throat like geometry of logarithmically diverging proper depth
  - throat is described be its own metric which is more symmetric than Kerr metric
  - emission from matter in the throat can lead to high-spin BH critical phenomena which offer prospective ways for identify high-spin BH in the sky
- Outline:
  - Geometry of high-spin BH
  - Critical Phenomena
    - Adiabatic growth of matter density spike around high-spin BH
    - Observability of high-energy collisions near high-spin BH

Geometry of High-Spin BH

Geometry of Spinning BH

- Kerr metric in coordinates (t, r, θ, ϕ) [Boyer-Lindquist]:

$$ds^2 = -\frac{\Delta}{\Sigma}(\mathrm{d}t - a\sin^2\theta \mathrm{d}\phi)^2 + \frac{\Sigma}{\Delta}\mathrm{d}r^2 + \Sigma\theta^2\mathrm{d}\theta^2 + \frac{\sin^2\theta}{\Sigma}[(r^2 + a^2)\mathrm{d}\phi - a\mathrm{d}t]^2$$

$$\Delta(r) = r^2 - 2Mr + a^2, \qquad \Sigma(r, \theta) = r^2 + a^2\cos^2\theta$$

- Characterized by two parameters:
  mass M and angular momentum $J = aM$

- Asymptotically flat: $r \to \infty$

- Horizons: $r_\pm = M \pm \sqrt{M^2 - a^2}$

- Kerr bound: $|a| \leq M$

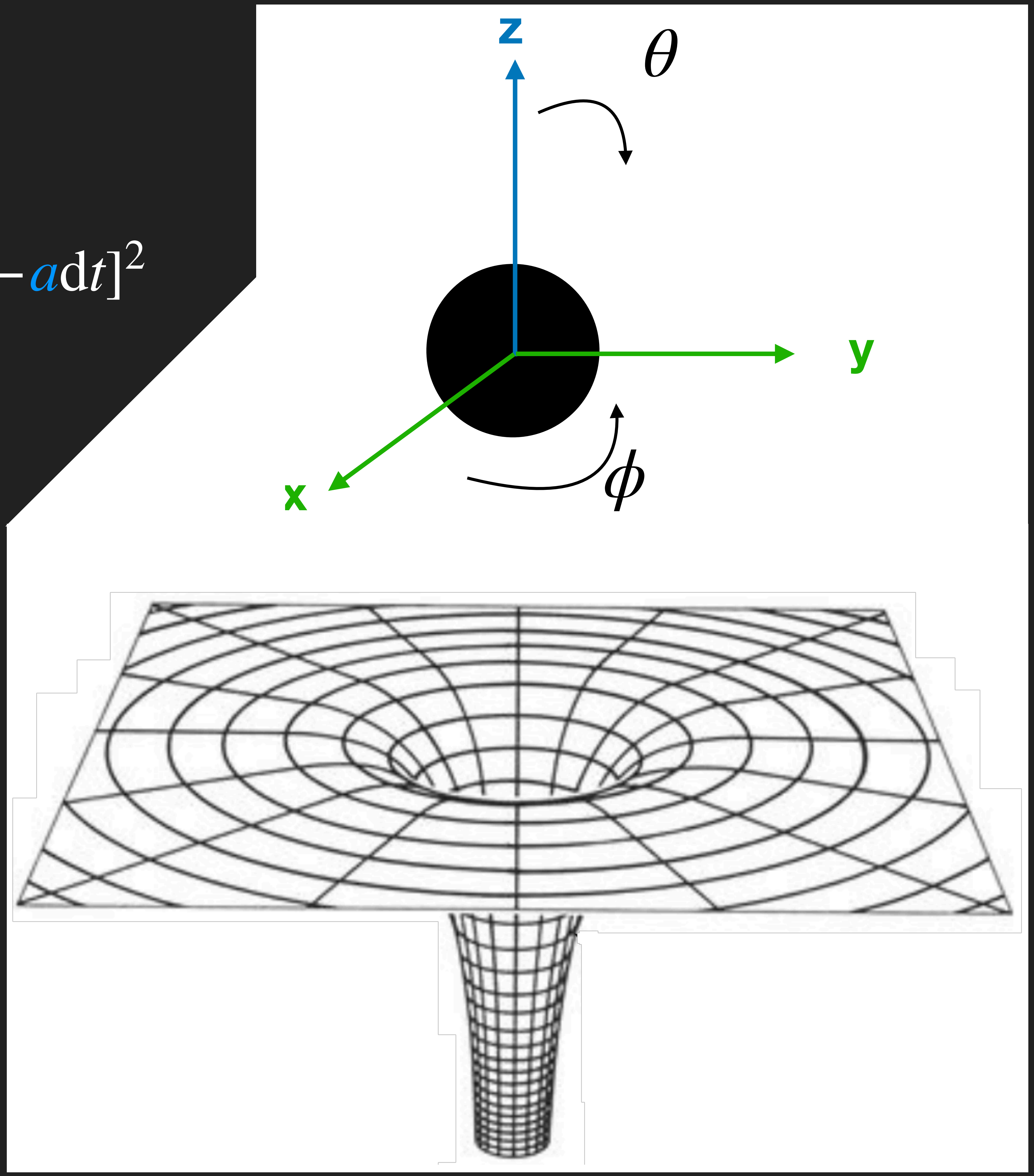
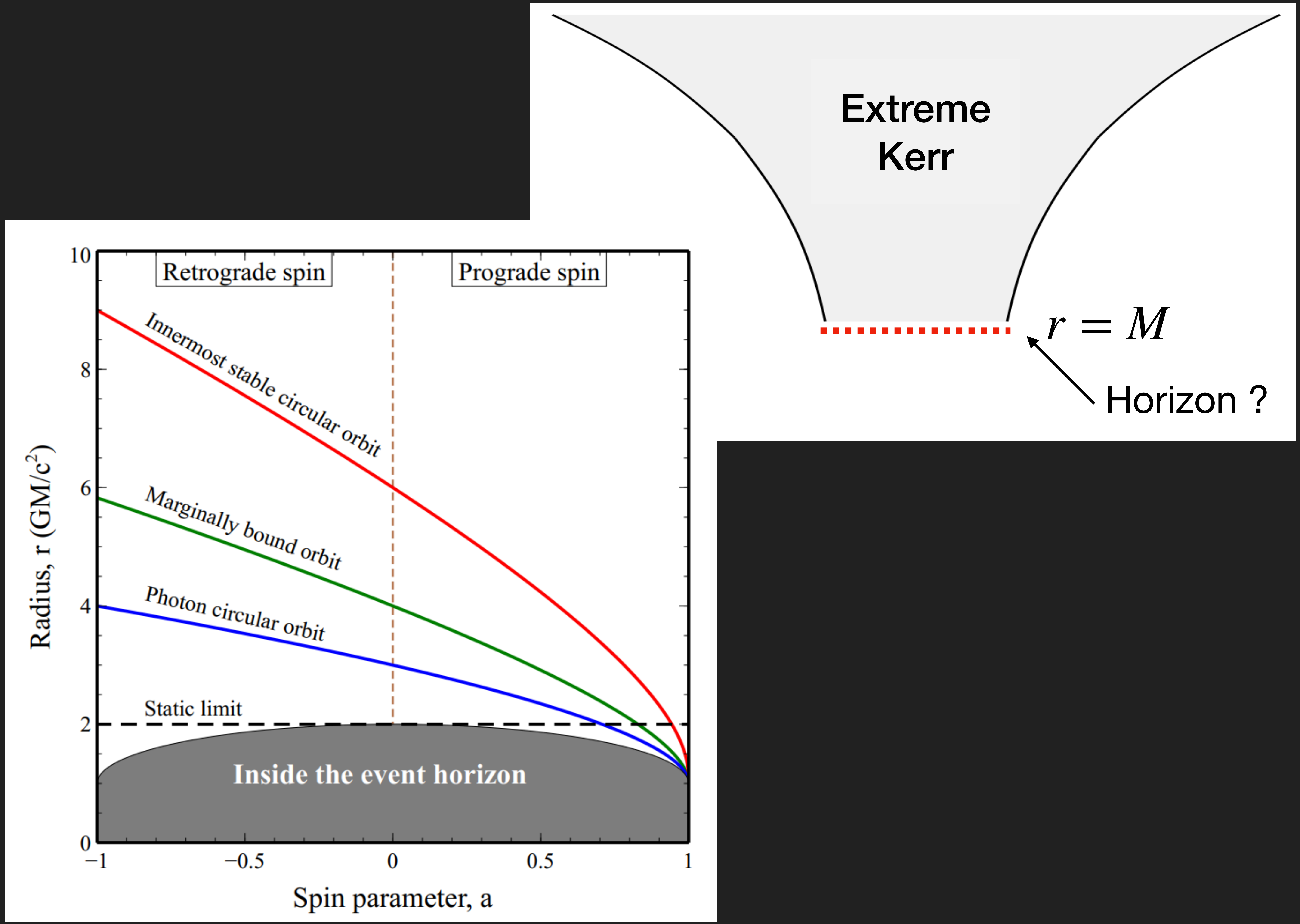


Image Ref: scienceblogs.com/startswithabang/2014/10/09/there-are-no-free-quarks-synopsis

Kerr & Near Horizon Near Extreme Kerr

- The **Extremal Kerr** metric can be found by taking $a \to M$ in the Kerr metric
  - in Boyer-Lindquist coordinates:
    - horizon $r_+ \to M$ (but so do other physical radii distinctly outside horizon)
    - **NOT** we suited to described the near horizon region of a maximally spinning BH properly!

Extreme Kerr (Boyer-Linquist)



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Kerr & Near Horizon Near Extreme Kerr

- Deviation from extremality κ: $a = M\sqrt{1-\kappa^2}$
- Coordinate transformation to (T, R, Φ) [Bardeen-Horowitz]

$$t = \frac{2MT}{\kappa^p}, \quad r = M(1+\kappa^p R), \quad \phi = \Phi + \frac{T}{\kappa^p}$$

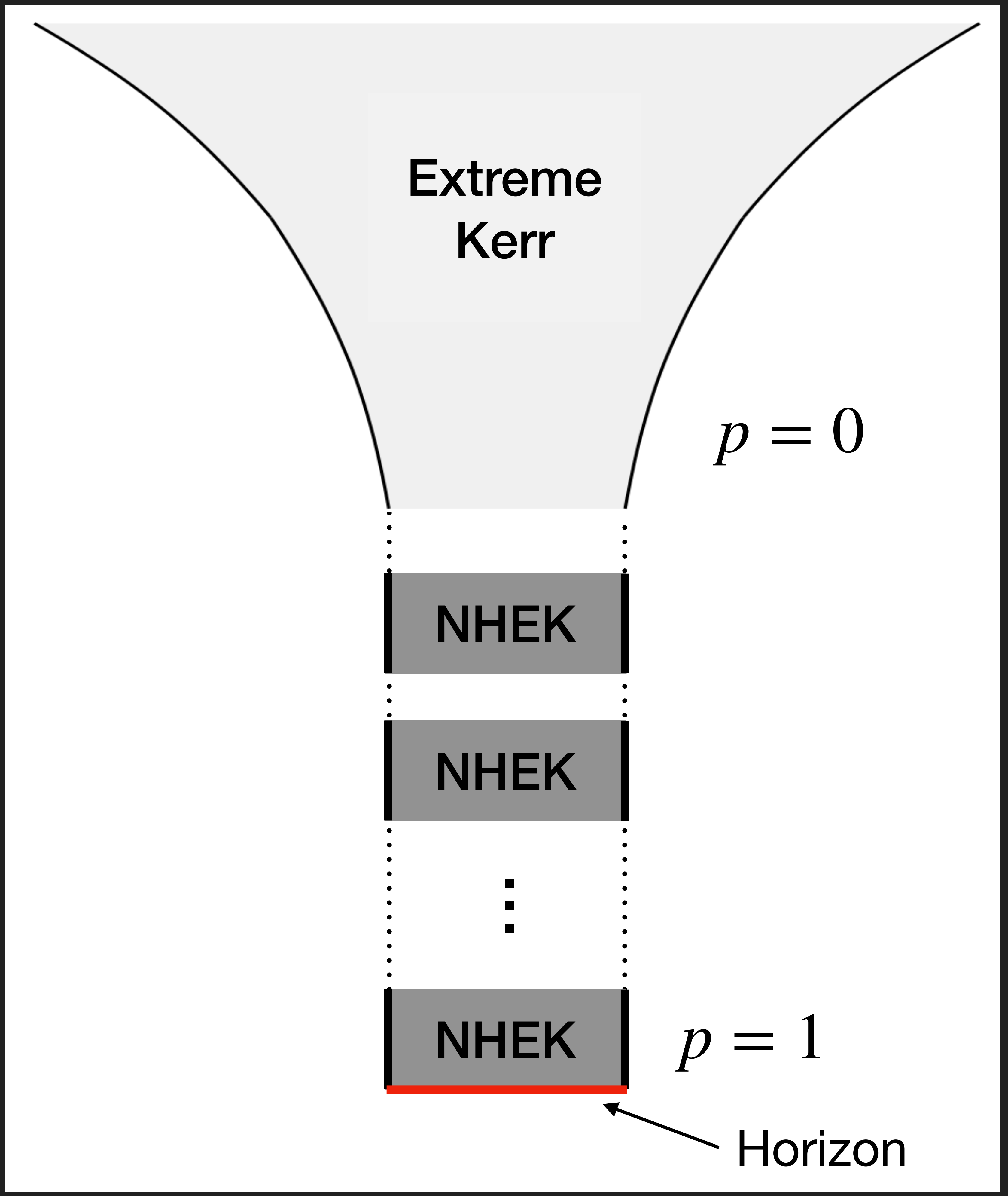
- Apply **near horizon near extremal scaling limit**: $0 < p \leq 1, \quad \kappa \to 0$
- Result: **Near Horizon Extreme Kerr** (NHEK) metric

$$\frac{d\hat{s}^2}{2M^2\Gamma} = -R^2\mathrm{d}T^2 + \frac{\mathrm{d}R^2}{R^2} + \mathrm{d}\theta^2 + \Lambda^2(\mathrm{d}\Phi + R\mathrm{d}T)^2, \quad \Gamma = \frac{1+\cos^2\theta}{2}, \quad \Gamma = \frac{2\sin\theta}{1+\cos^2\theta}$$

which is **not asymptotically flat,** has as **global isometry group** $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$

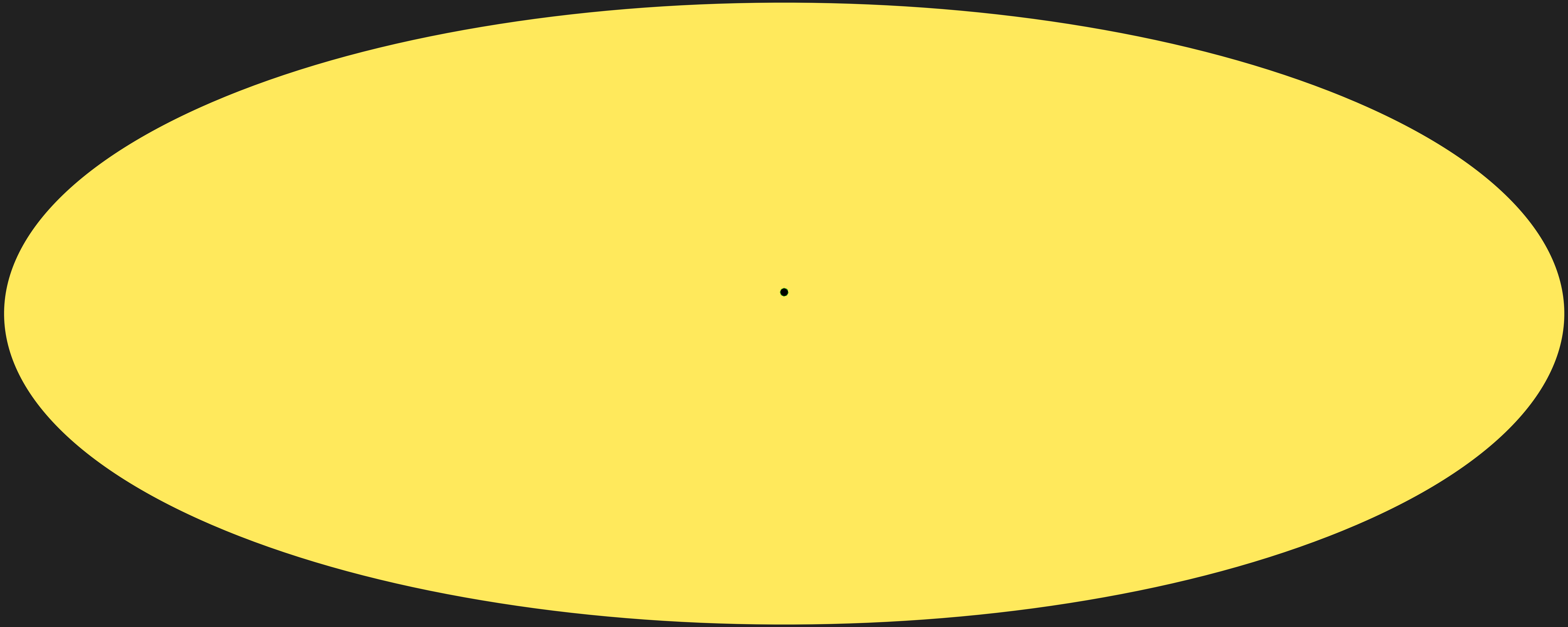
  - axisymmetry $\partial_\Phi = W_0$, stationarity $\partial_T = H_+$,
    NHEK dilation H_0, special conformal symmetry $H\_$

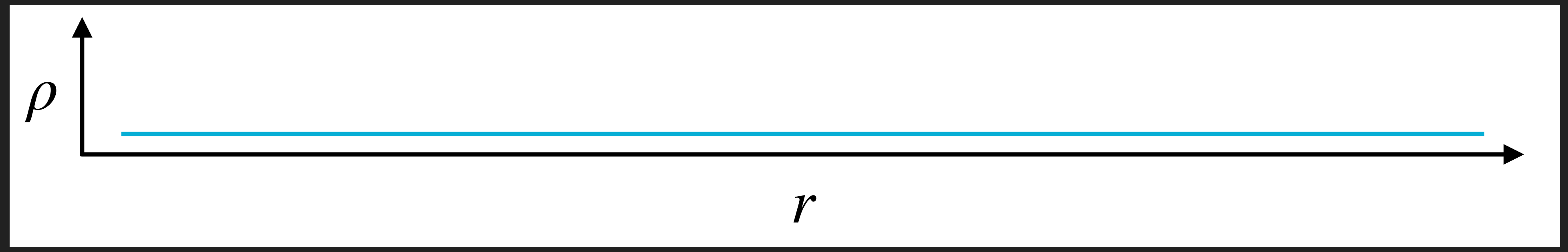
Extreme Kerr (proper-length)



Adiabatic Growth of Matter Density Spike around High-Spin BH

BH Growth in Cloud of Particles





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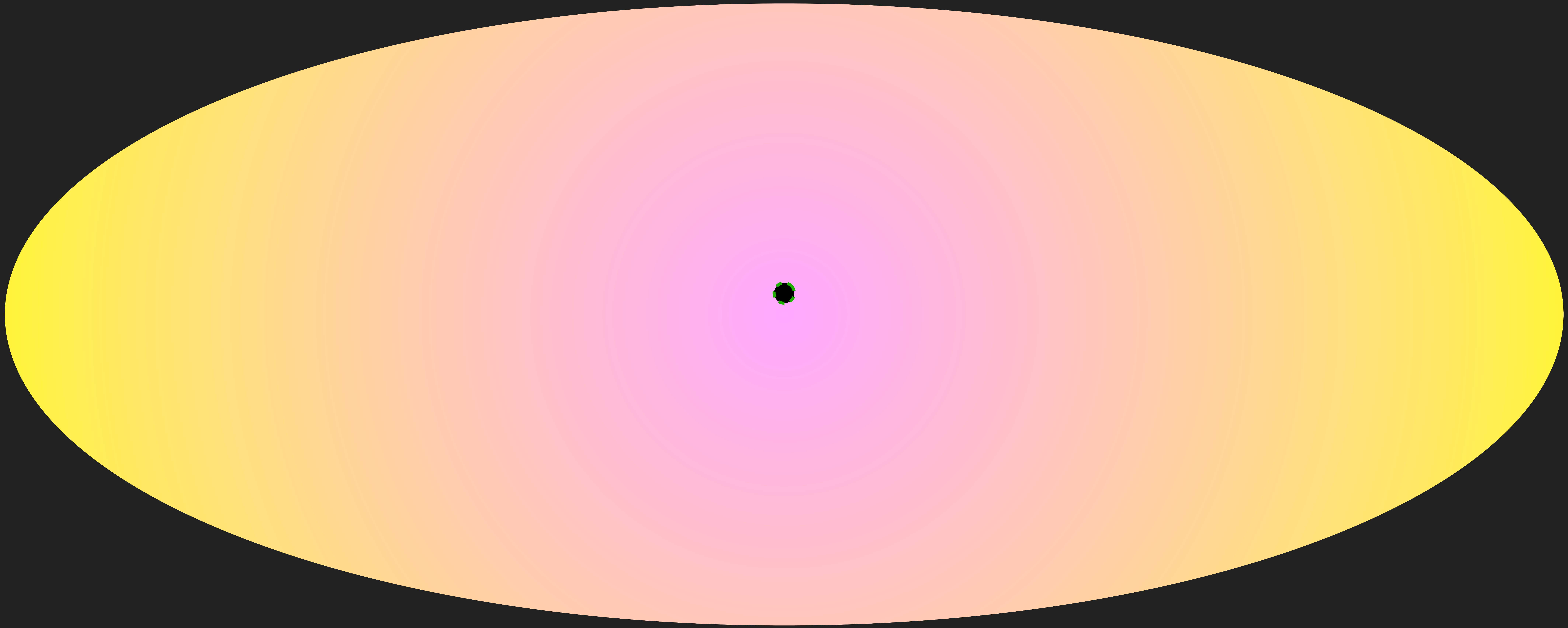

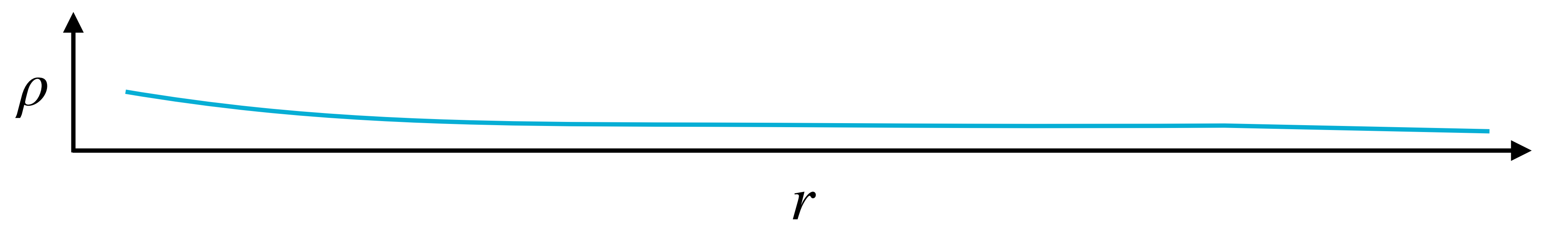

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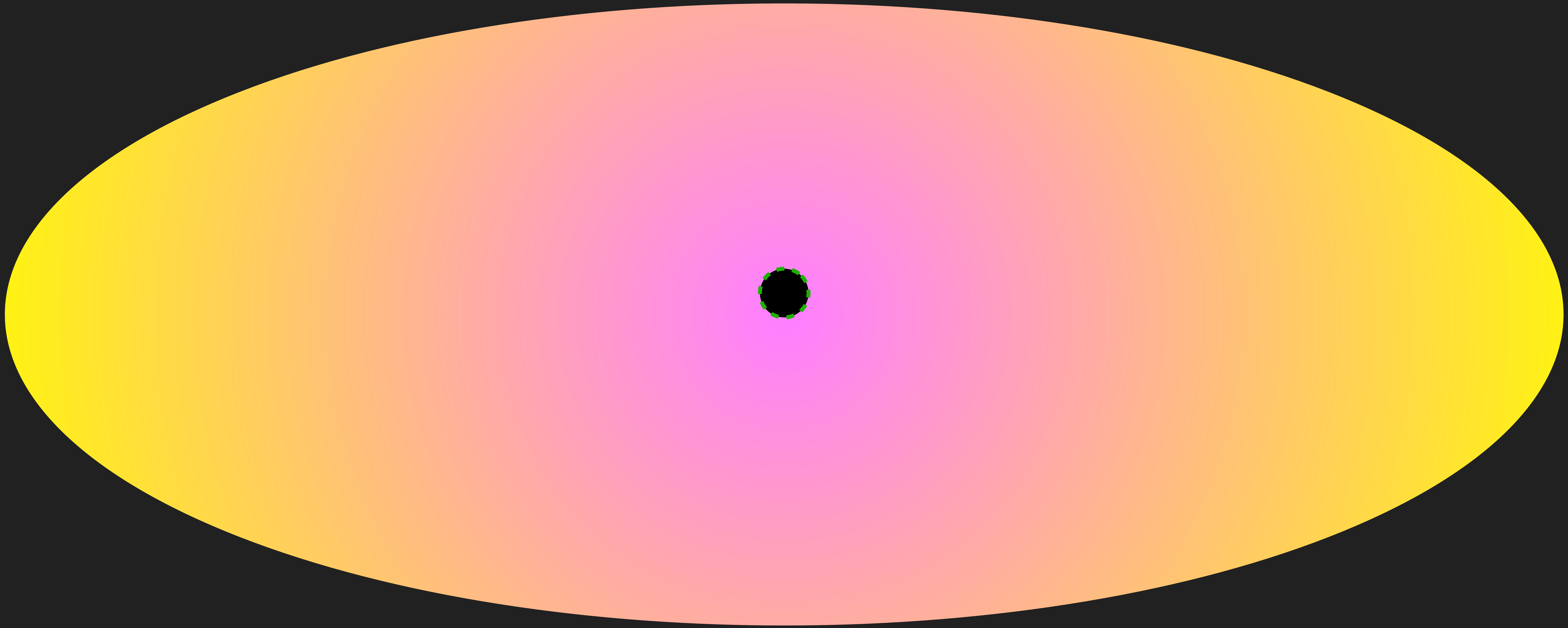

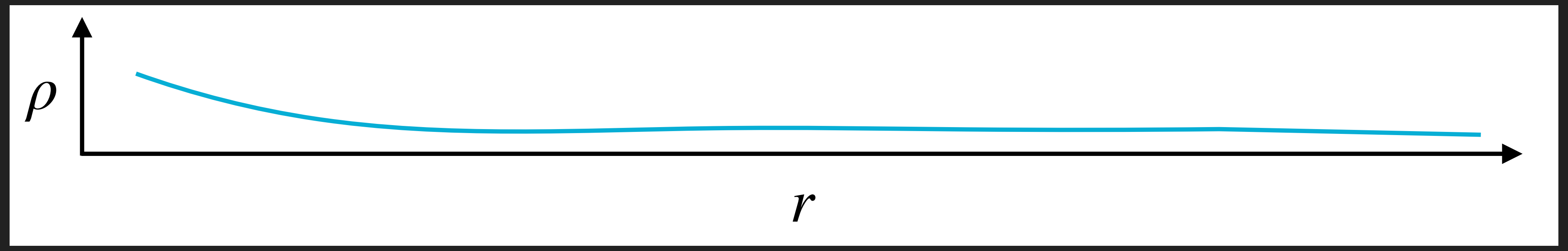

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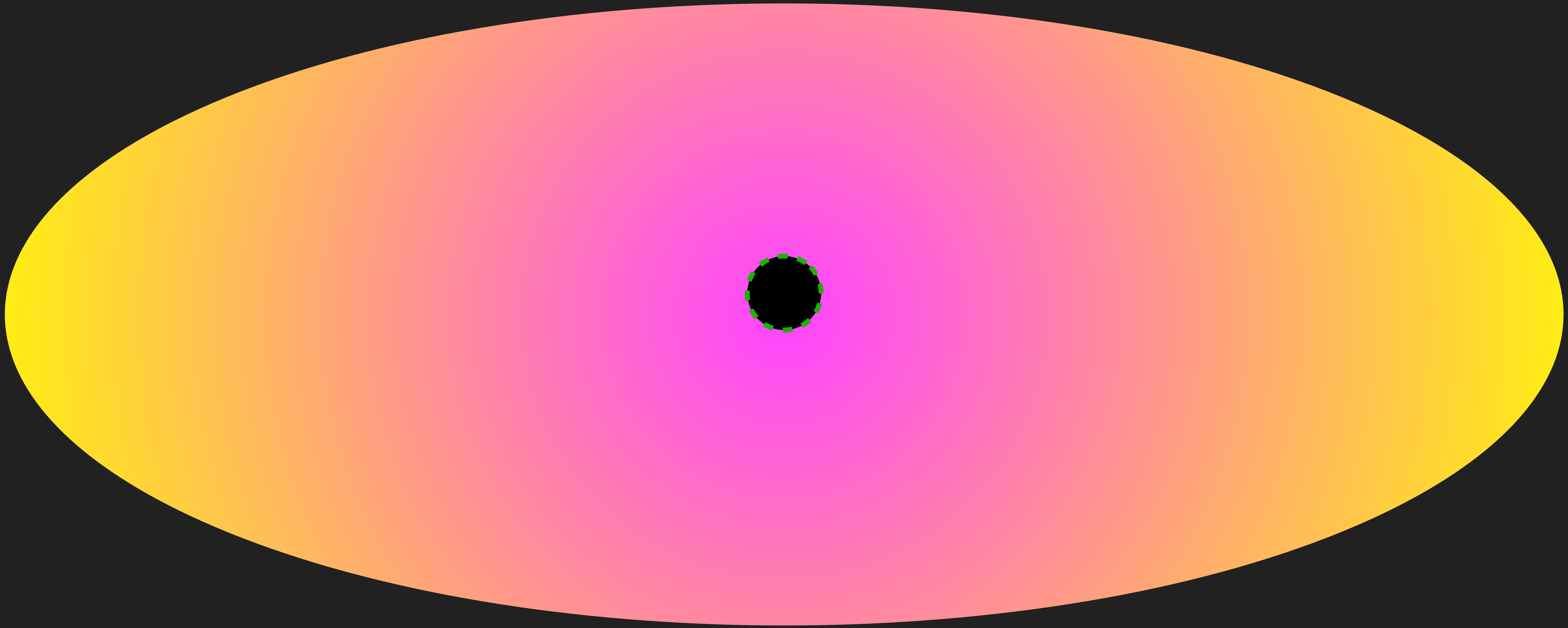
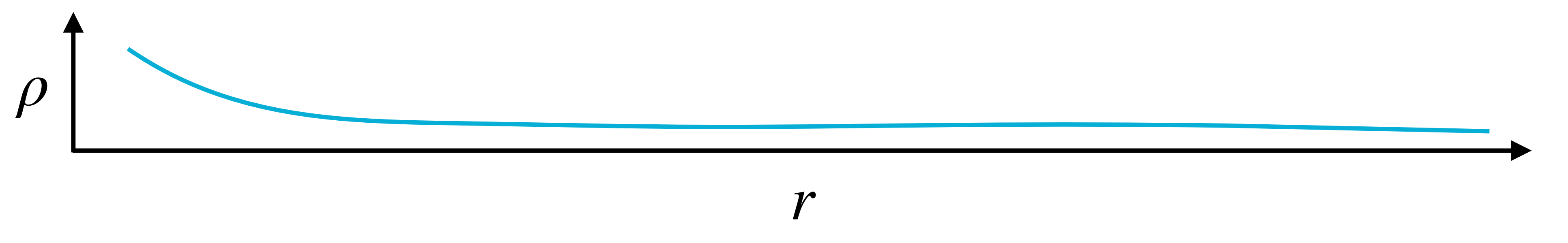


[Peebles 1972]



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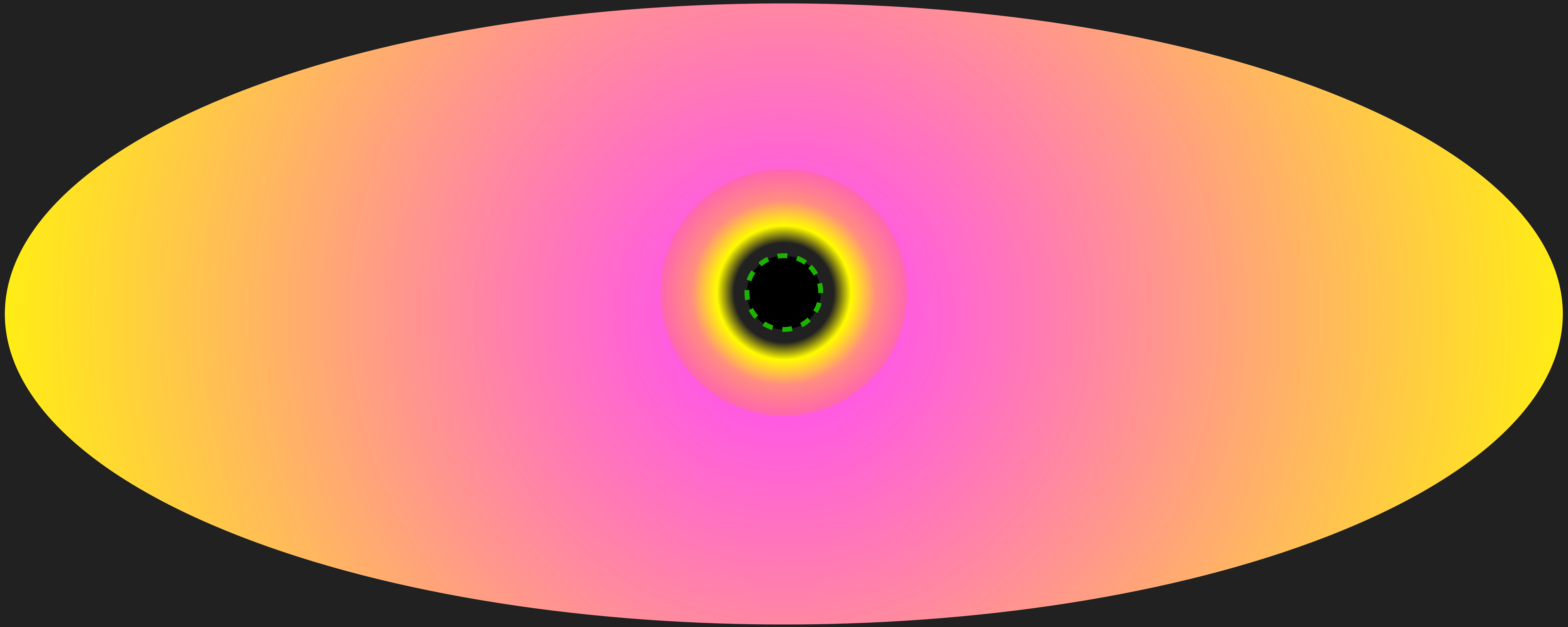
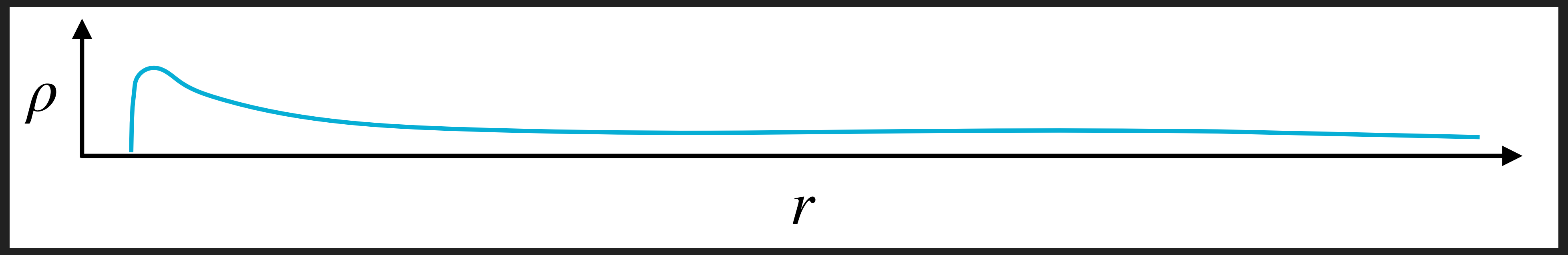


[Gondolo, Silk 1999;
Sadeghian Ferrer, Will 2013;
Ferrer, da Rosa, Will 2017]



Matter Spike in Kerr

- DM Spike around Kerr BH generic spin [Ferrer, da Rosa, Will 2017]

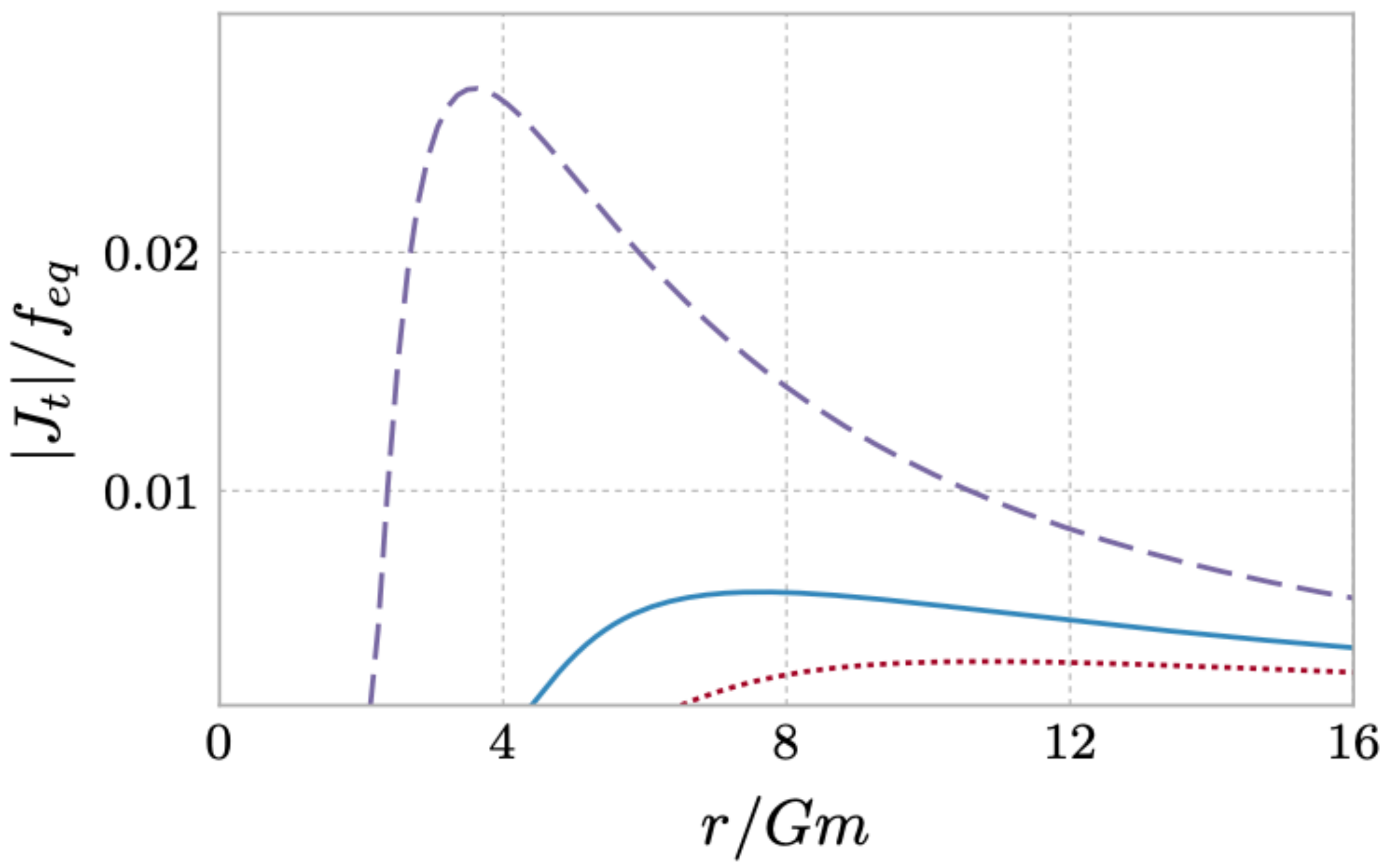


Figure 4. The corotating (dashed) and counter-rotating (dotted) parts of $|J_t|$ for $\tilde{a} = 0.8$, compared with $|J_t|/2$ for the Schwarzschild case (solid).

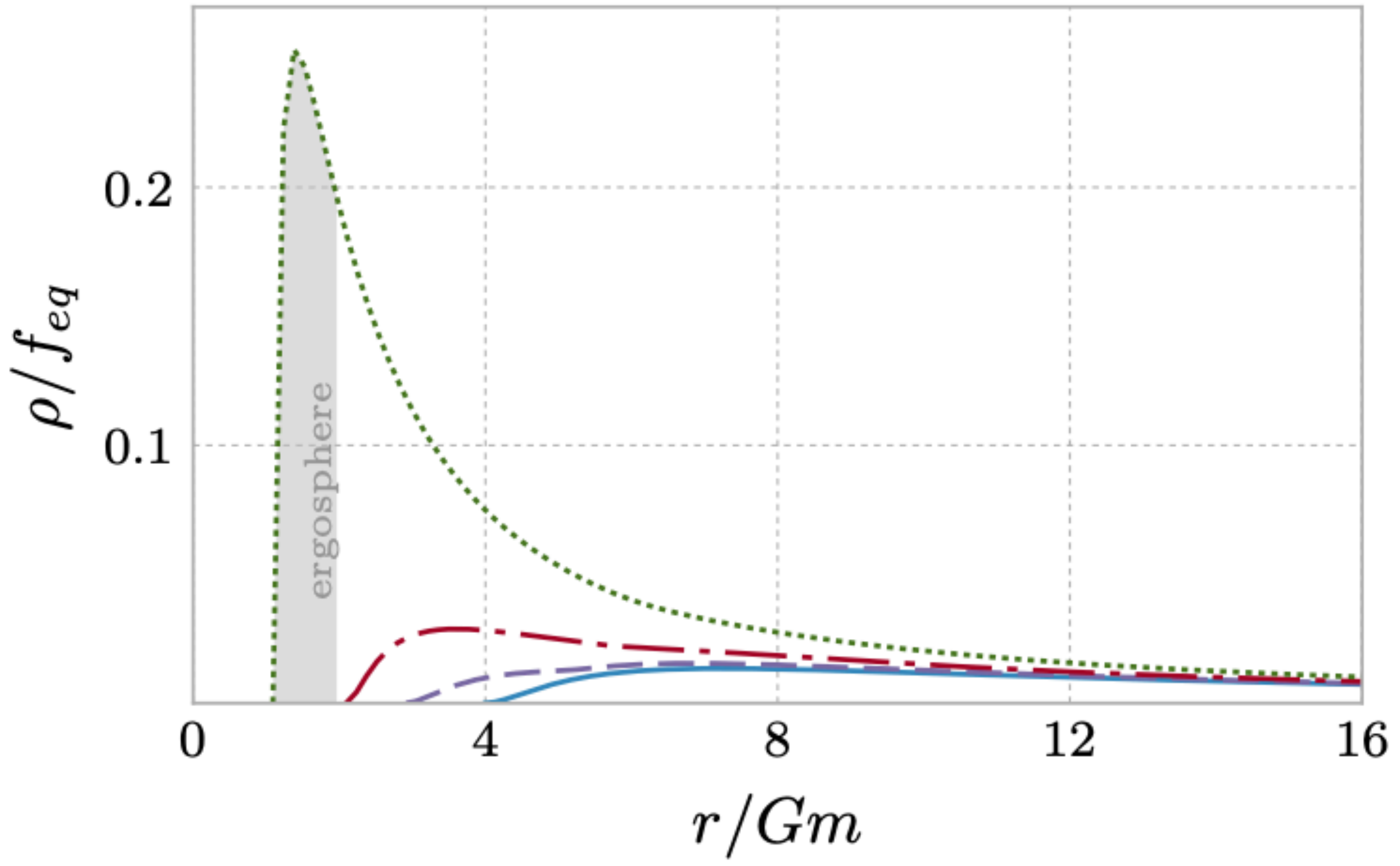


Figure 3. Density profiles for a distribution of equatorial orbits. The density increases as we vary the Kerr parameter $\tilde{a} = 0$ (solid), 0.5 (dashed), 0.8 (dot-dashed) and 0.998 (dotted). Since the latter value is greater than $\tilde{a} > 2(\sqrt{2} - 1)$, the spike extends into the ergosphere (shaded region).

- matter current J
  density $\rho = \sqrt{J \cdot J}$
- density spike is spin dependent
  - peak gets taller and closer to BH with increasing spin
- density closes at prograde innermost bound circular orbit (IBCO)
- what happens as we spin up the BH to extremality?

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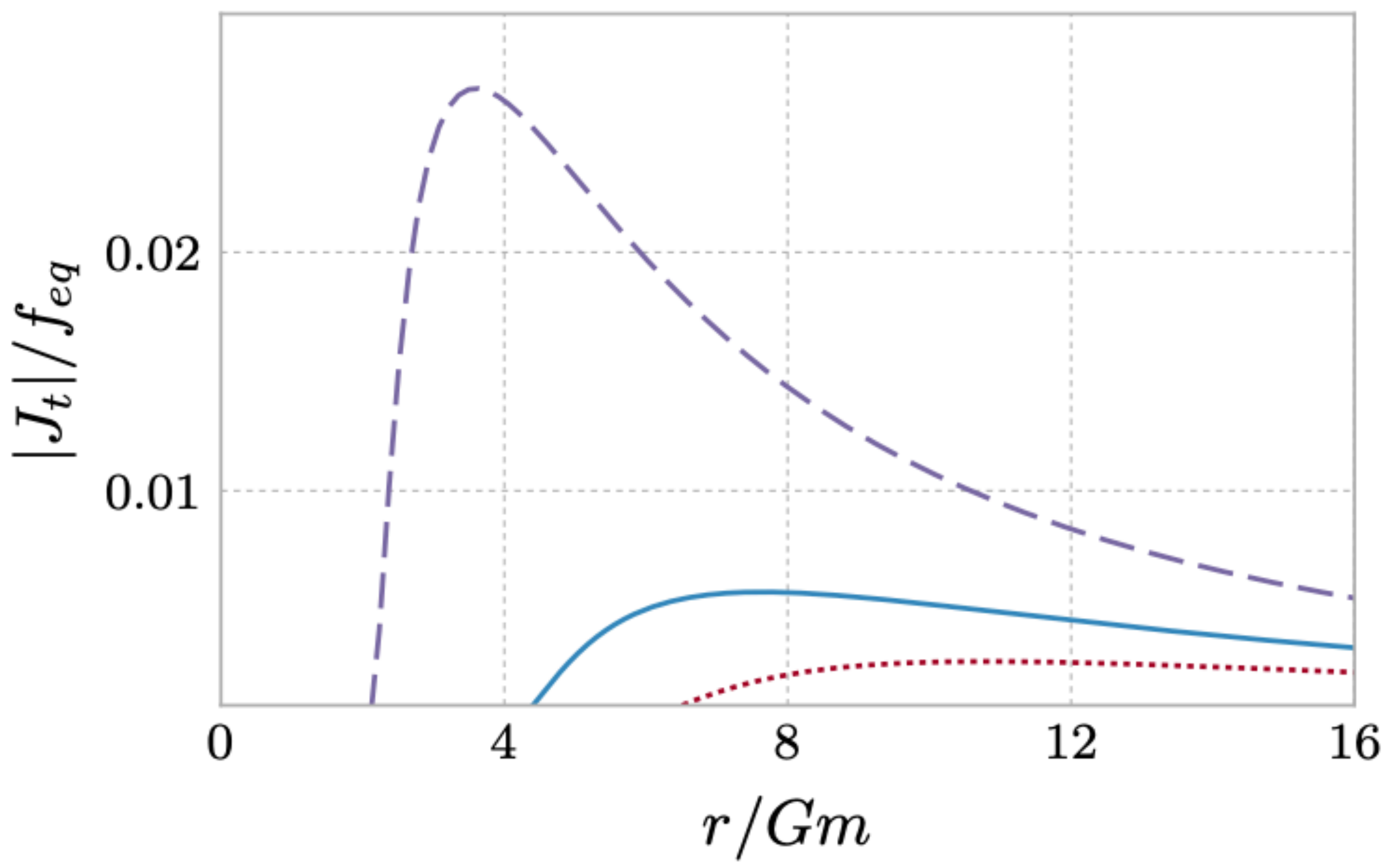

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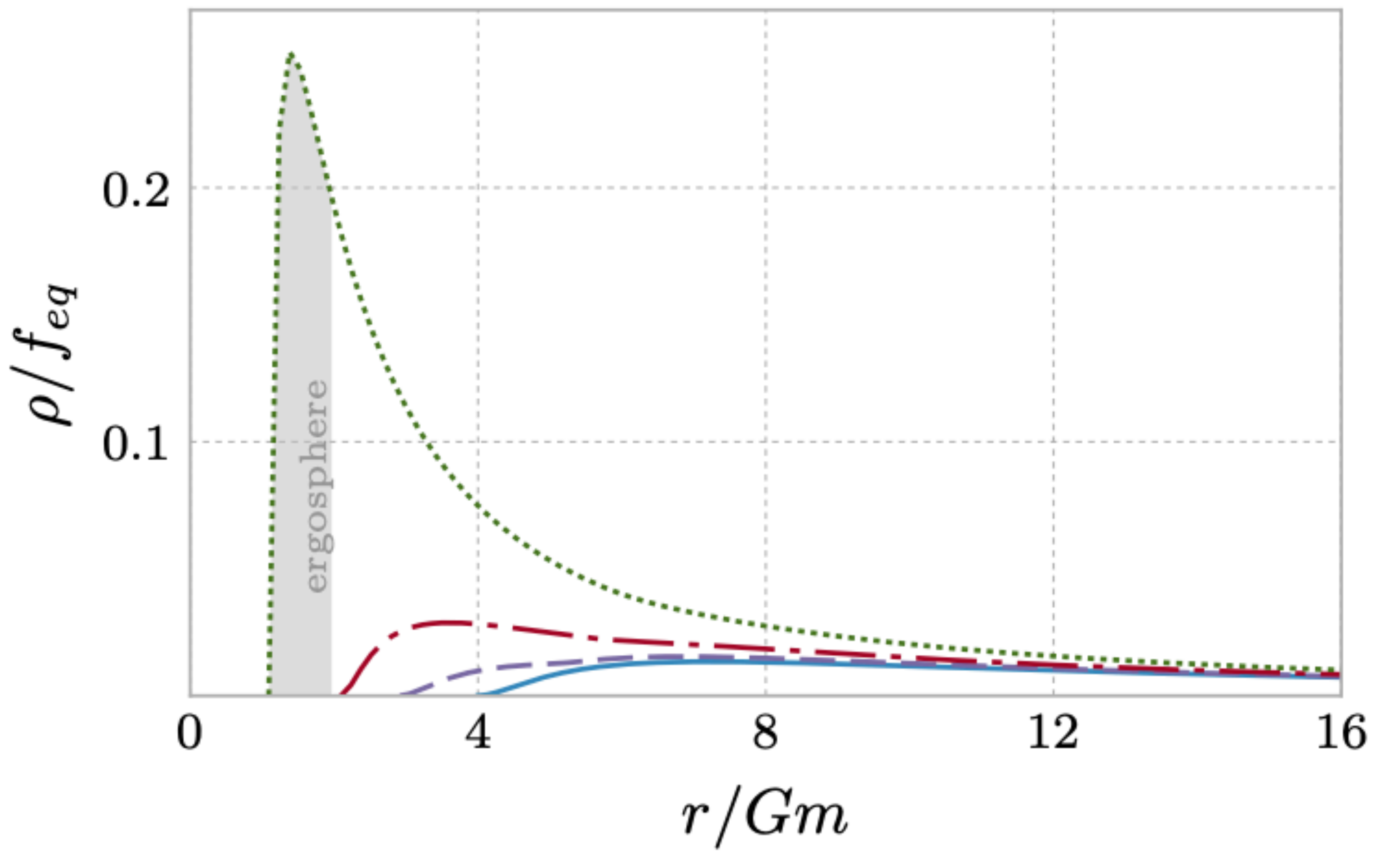

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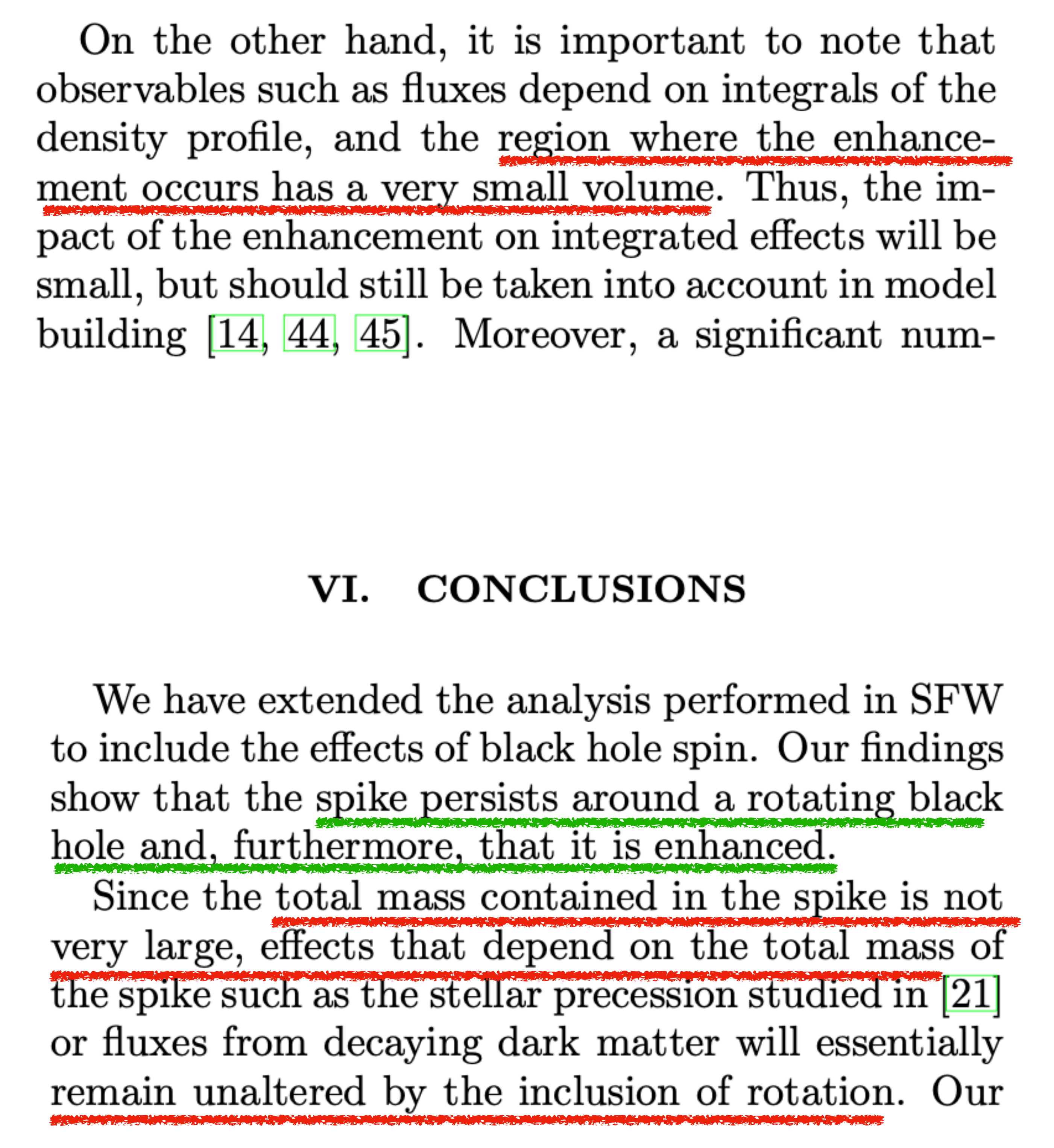


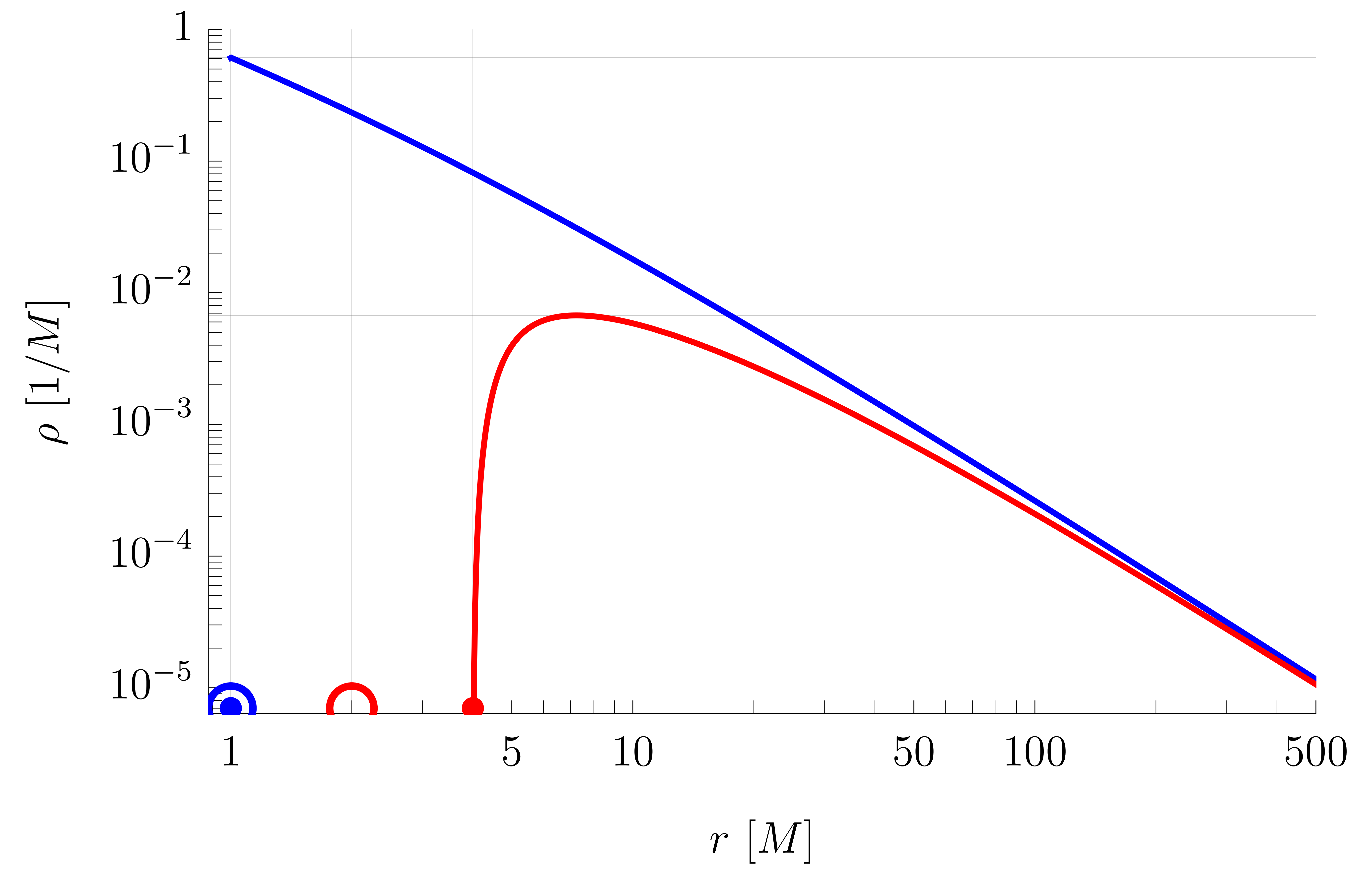
On the other hand, it is important to note that observables such as fluxes depend on integrals of the density profile, and the region where the enhancement occurs has a very small volume. Thus, the impact of the enhancement on integrated effects will be small, but should still be taken into account in model building [14, 44, 45]. Moreover, a significant num-

VI. CONCLUSIONS

We have extended the analysis performed in SFW to include the effects of black hole spin. Our findings show that the spike persists around a rotating black hole and, furthermore, that it is enhanced.

Since the total mass contained in the spike is not very large, effects that depend on the total mass of the spike such as the stellar precession studied in [21] or fluxes from decaying dark matter will essentially remain unaltered by the inclusion of rotation. Our

Density: Schwarzchild & Extreme Kerr (Prograde Orbits)



- BH spin: $a = 0, M$
- horizon: open circle
- IBCO: dot

$$a = 0: \quad r_{\text{peak}} \approx 6.2M,\ \rho_{\text{peak}} \approx \frac{6.1 \times 10^{-3}}{M}$$

$$a = M: \quad r_{\text{peak}} = M,\ \rho_{\text{peak}} \approx \frac{.61}{M}$$

Total Mass in Extreme Kerr

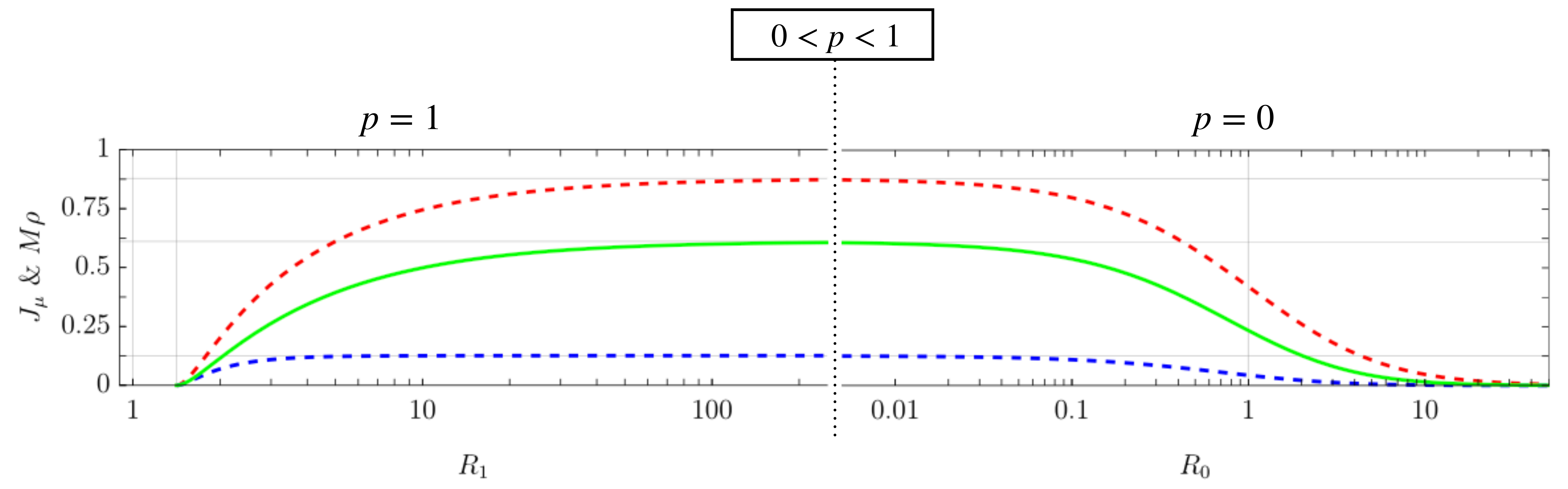
- Total mass of disk out to cutoff radius r_b: $\mathscr{M} = \int_{\text{equatorial plane}} \rho dA = \int_{r_{\text{ibco}}}^{r_b} \rho \sqrt{\gamma} dr d\phi$

  where $\gamma = \dfrac{r(r^3 - a^2(r + 2M))}{(r - r_+)(r - r_-)}$ is the metric induced on fixed $t = t_0$ and on $\theta = \pi/2$

- When $a = M$, $r_\pm = r_{\text{ibco}} = M$
  - mass will diverge unless density at $r = M$ vanishes

Near Extremal Density

- setting $r = M(1 + \kappa^p R_p)$, taking $\kappa = \sqrt{1 - a^2/M^2} \ll 1$ for $0 \leq p \leq 1$
- matter current (J_Φ and J_T) and density ρ are finite, nonzero in the near horizon region



Total Mass in Near-Extremal Limit

- Total mass interior to cutoff radius r_b: $\mathscr{M} = \int_{\text{equatorial plane}} \rho dA = \int_{r_{\text{ibco}}}^{r_b} \rho\sqrt{\gamma} dr d\phi$
- Choose intermediate NHEK radius $r_p = M(1 + \kappa^p R_p)$ with $0 < p < 1$ and split integral in EK and near-NHEK pieces

$$\mathscr{M} = \mathscr{M}_1 + \mathscr{M}_0$$

- $\mathscr{M}_1 = 2\pi \int_{r_{\text{ibco}}}^{r_q} \rho\sqrt{\gamma} dr = 4\pi M^2 \int_{\sqrt{2}}^{R_p \kappa^{p-1}} \frac{\rho(R_0)}{\sqrt{R_1^2 - 1}} dR_1 + \mathrm{O}(\kappa) \sim 4\pi M^2 \rho_p \log(R_p \kappa^{p-1})$
- $\mathscr{M}_0 = 2\pi \int_{r_{\text{ibco}}}^{r_q} \rho\sqrt{\gamma} dr = 4\pi M^2 \int_{R_p \kappa^p}^{\frac{r_b}{M}-1} \rho(R_0)\sqrt{\frac{4}{R_0^2} + \frac{8}{R_0} + 7 + 4R_0^2 + R_0^2}\, dR_0 + \mathrm{O}(\kappa) \sim 4\pi M^2 \rho_p \log(R_p \kappa^p)$

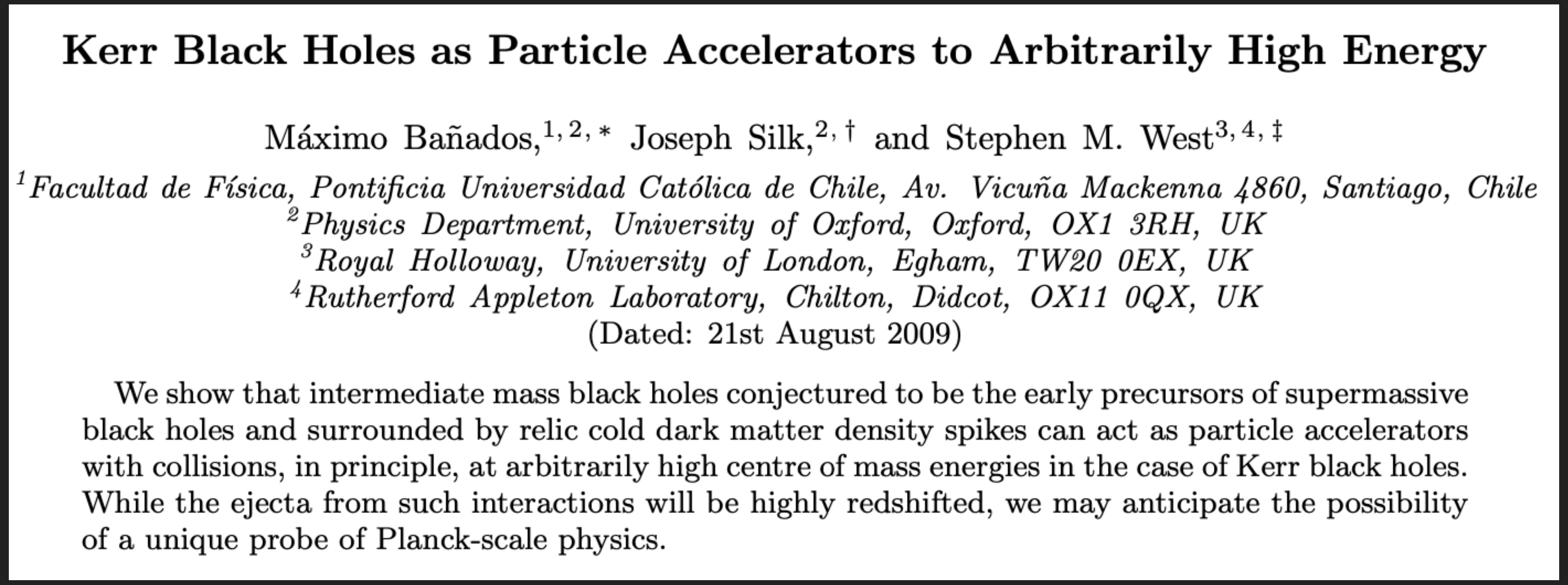
$$\mathscr{M} \sim 4\pi M^2 \rho_p \log(\frac{1}{\kappa})$$

- Total mass is **logarithmically divergent** with deviation from extremality

Observability of High-Energy Collisions near High-Spin BH

Kerr Black Holes as Particle Accelerators

- In the high-spin limit, Kerr BHs admit near-horizon collisions of diverging CM energy if one of particle is tuned to the super radiant bound
  [Bañados, Silk, West 2009]



Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy

Máximo Bañados,[1,2,*] Joseph Silk,[2,†] and Stephen M. West[3,4,‡]
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[2] *Physics Department, University of Oxford, Oxford, OX1 3RH, UK*
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[4] *Rutherford Appleton Laboratory, Chilton, Didcot, OX11 0QX, UK*
(Dated: 21st August 2009)

We show that intermediate mass black holes conjectured to be the early precursors of supermassive black holes and surrounded by relic cold dark matter density spikes can act as particle accelerators with collisions, in principle, at arbitrarily high centre of mass energies in the case of Kerr black holes. While the ejecta from such interactions will be highly redshifted, we may anticipate the possibility of a unique probe of Planck-scale physics.

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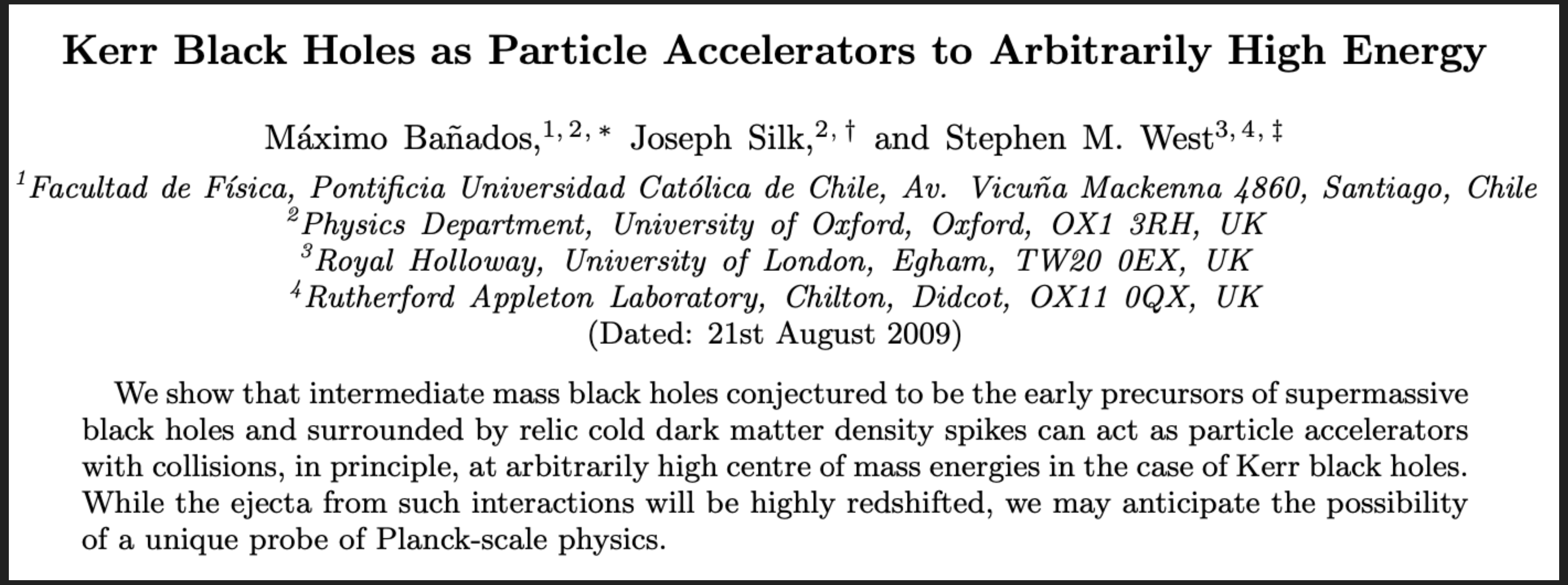
    - hard to observe (large redshift to near-horizon region)



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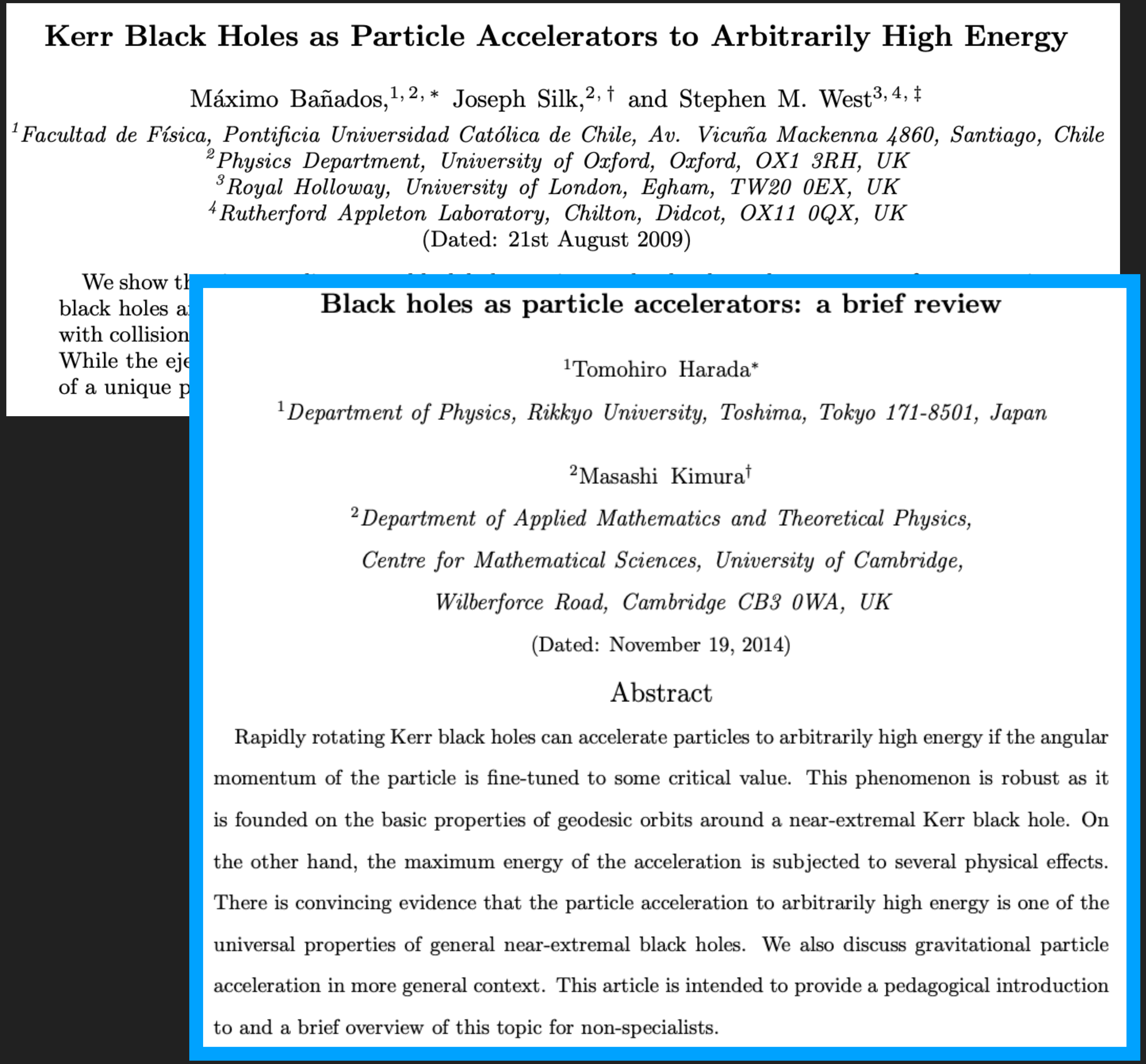

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Black holes as particle accelerators: a brief review

[1]Tomohiro Harada*

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[2]Masashi Kimura†

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Centre for Mathematical Sciences, University of Cambridge,
Wilberforce Road, Cambridge CB3 0WA, UK

(Dated: November 19, 2014)

Abstract

Rapidly rotating Kerr black holes can accelerate particles to arbitrarily high energy if the angular momentum of the particle is fine-tuned to some critical value. This phenomenon is robust as it is founded on the basic properties of geodesic orbits around a near-extremal Kerr black hole. On the other hand, the maximum energy of the acceleration is subjected to several physical effects. There is convincing evidence that the particle acceleration to arbitrarily high energy is one of the universal properties of general near-extremal black holes. We also discuss gravitational particle acceleration in more general context. This article is intended to provide a pedagogical introduction to and a brief overview of this topic for non-specialists.

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    - hard to observe (large redshift to near-horizon region)
      —> emission from ISCO orbiter can be highly blue shifted and lots of emission from ISCO orbiter can reach celestial sphere
      [Gates, Hadar, Lupsasca 2021]

Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy

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Photon Emission from Circular Equatorial Kerr Orbiters

Delilah E. A. Gates,[1,*] Shahar Hadar,[1,†] and Alexandru Lupsasca[2,‡]
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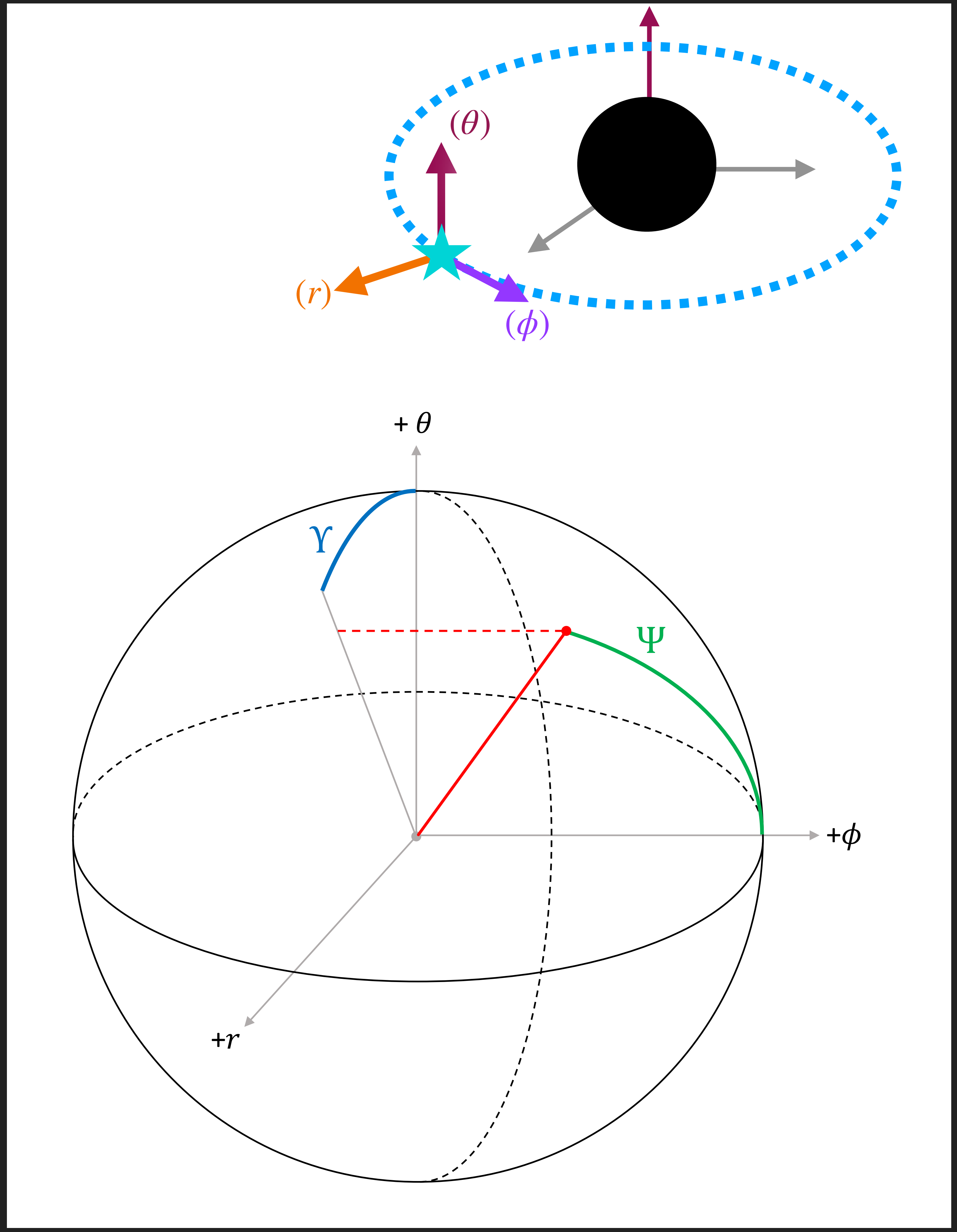
[2] *Princeton Gravity Initiative, Princeton University, Princeton, NJ 08544, USA*

We consider monochromatic and isotropic photon emission from circular equatorial Kerr orbiters. We derive analytic expressions for the photon escape probability and the redshift-dependent total flux collected on the celestial sphere as a function of emission radius and black hole parameters. These calculations crucially involve the critical curve delineating the region of photon escape from that of photon capture in each emitter's sky. This curve generalizes to finite orbital radius the usual Kerr critical curve and displays interesting features in the limit of high spin, which we investigate by developing a perturbative expansion about extremality. Although the innermost stable circular orbit appears to approach the event horizon for very rapidly spinning black holes, we find in this regime that the photon escape probability tends to $5/12+1/(\sqrt{5}\pi)\arctan\sqrt{5/3}\approx 54.65\%$. We also obtain a simple formula for the flux distribution received on the celestial sphere, which is nonzero. This confirms that the near-horizon geometry of a high-spin black hole is in principle observable. These results require us to introduce a novel type of near-horizon double-scaling limit. We explain the dip observed in the total flux at infinity as an imprint of the black hole: the black hole "bite".

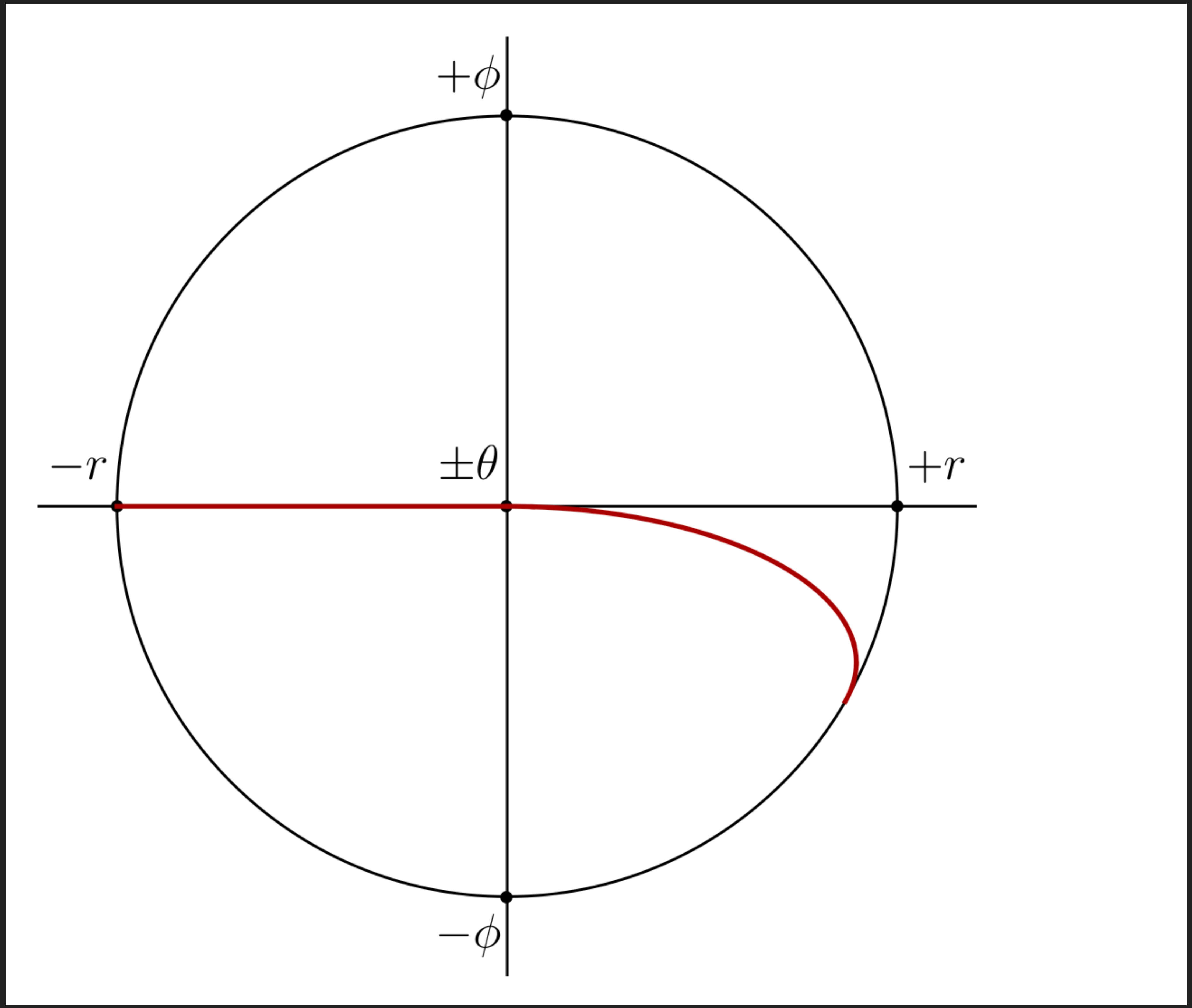
Sky of Circular Orbiter

- We can define **orbiter frame** coordinate $(t), (r), (\theta), (\phi)$ where
  - the (θ) direction is oriented parallel to the BH spin axis
  - and the (ϕ) direction is oriented along the direction of travel
- Define **physical angles** in the **orbiter sky** Ψ, Υ
- Find **redshift** of escape photons is $g(\Psi)$



Universal Kinamatics of NHEK Orbiter Sky

- **critical curve**
  - ~54.65% of emission directions escape



Universal Kinamatics of NHEK Orbiter Sky

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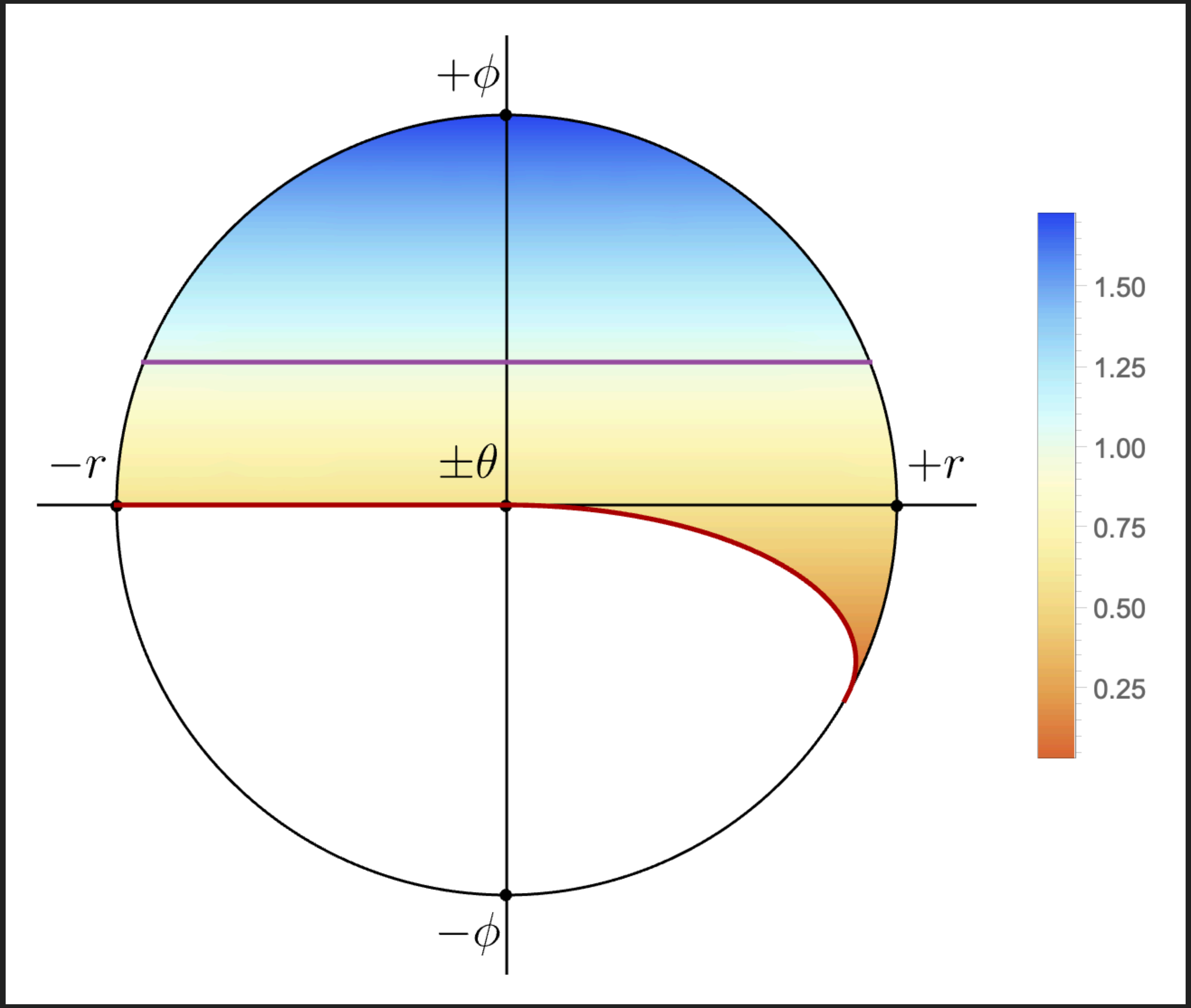
- **redshift factor** of escape photons g
  - photons can reach infinity with redshift between 1 to $\sqrt{3}$
  - ~31.70% of emission directions are blue shifted



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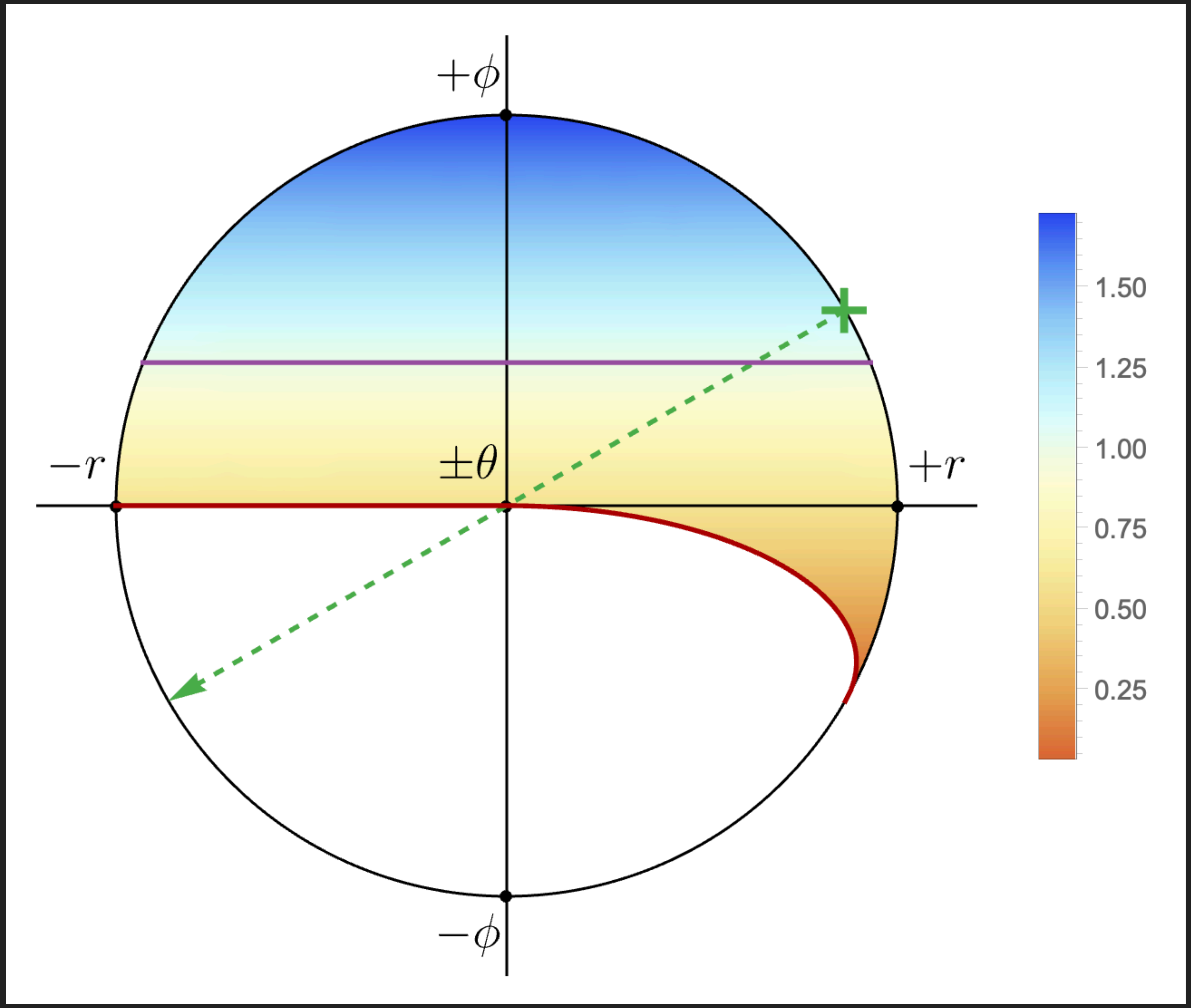
- **generic in-falling particle** angle $\Upsilon = \pi/2, \Psi = 2\pi/3$



Photon Escape Probabilities

- **Cross section** for emission single photon of energy k

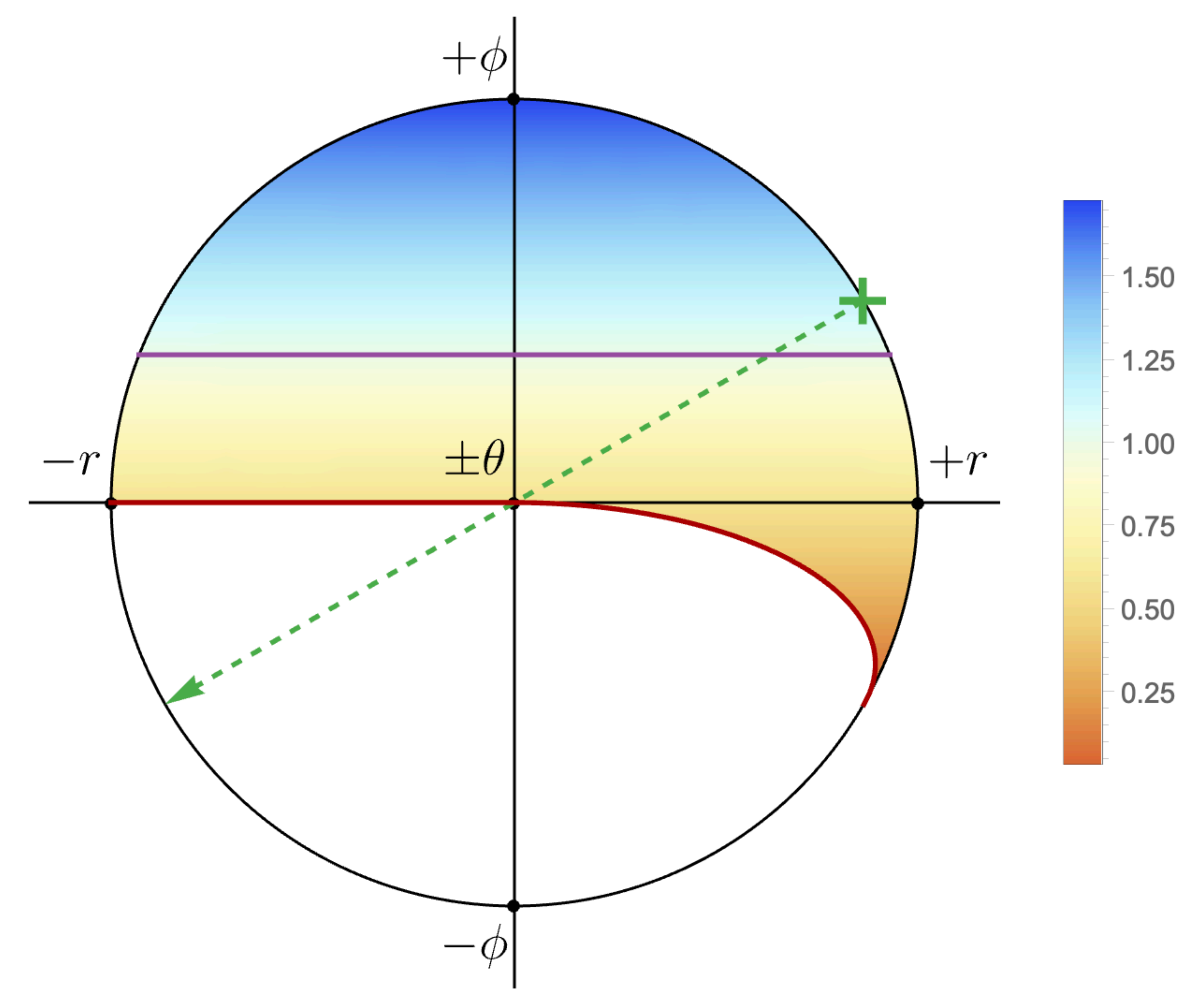
$$\frac{d\sigma}{dkd\Omega}(k,\theta) = \frac{d\sigma}{dkd\Omega}(k,\Theta,\Psi)$$

- Photon **escape probability** and **blueshifted escape probability**

$$\mathrm{P}_e(k) = \left(\frac{d\sigma}{dk}\right)^{-1} \int_{A_e} \frac{d\sigma}{dkd\Omega} d\Omega$$

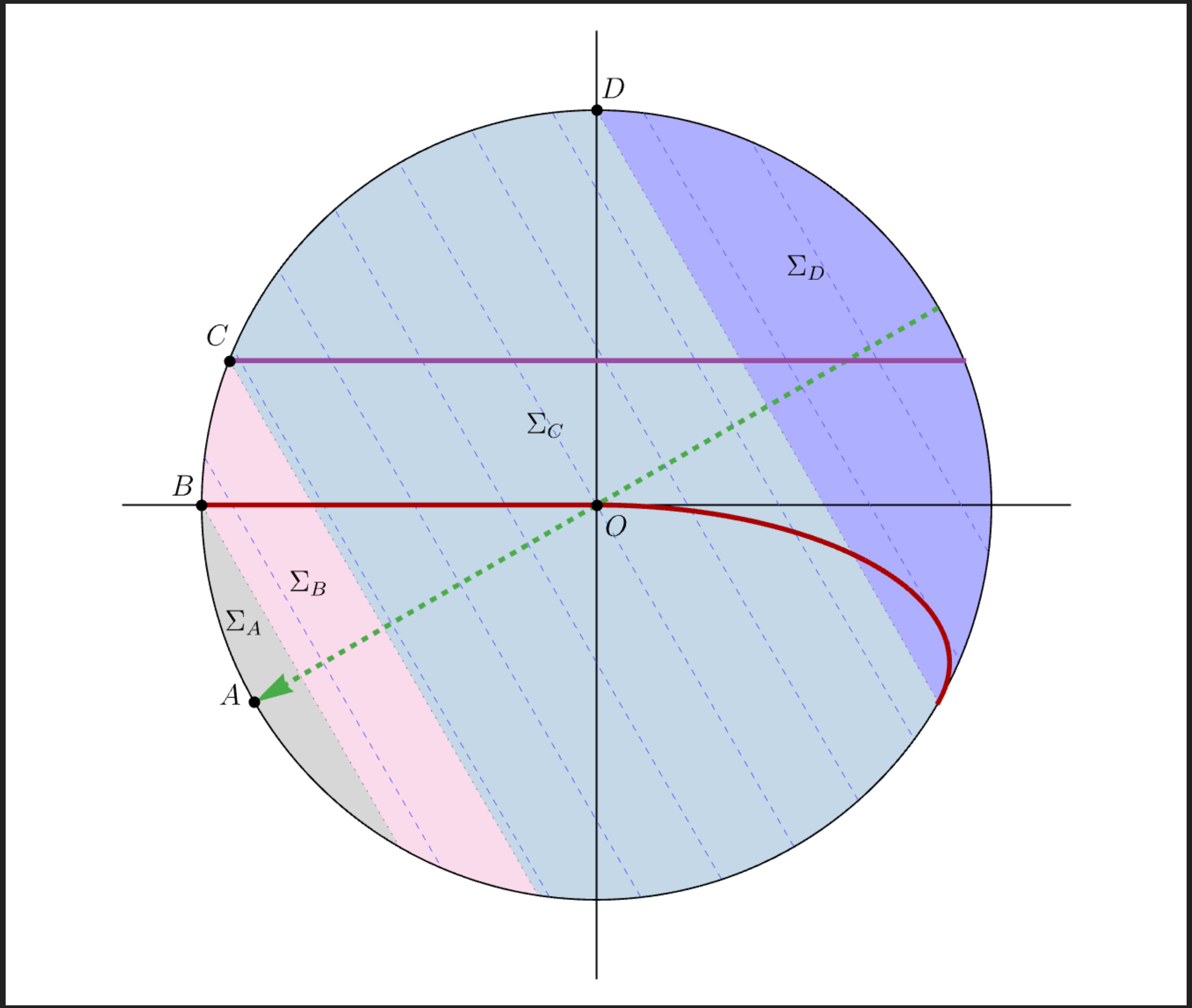
$$\mathrm{P}_b(k) = \left(\frac{d\sigma}{dk}\right)^{-1} \int_{A_b} \frac{d\sigma}{dkd\Omega} d\Omega$$

$$\frac{d\sigma}{dk} = \int_{S^2} \frac{d\sigma}{dkd\Omega} d\Omega$$



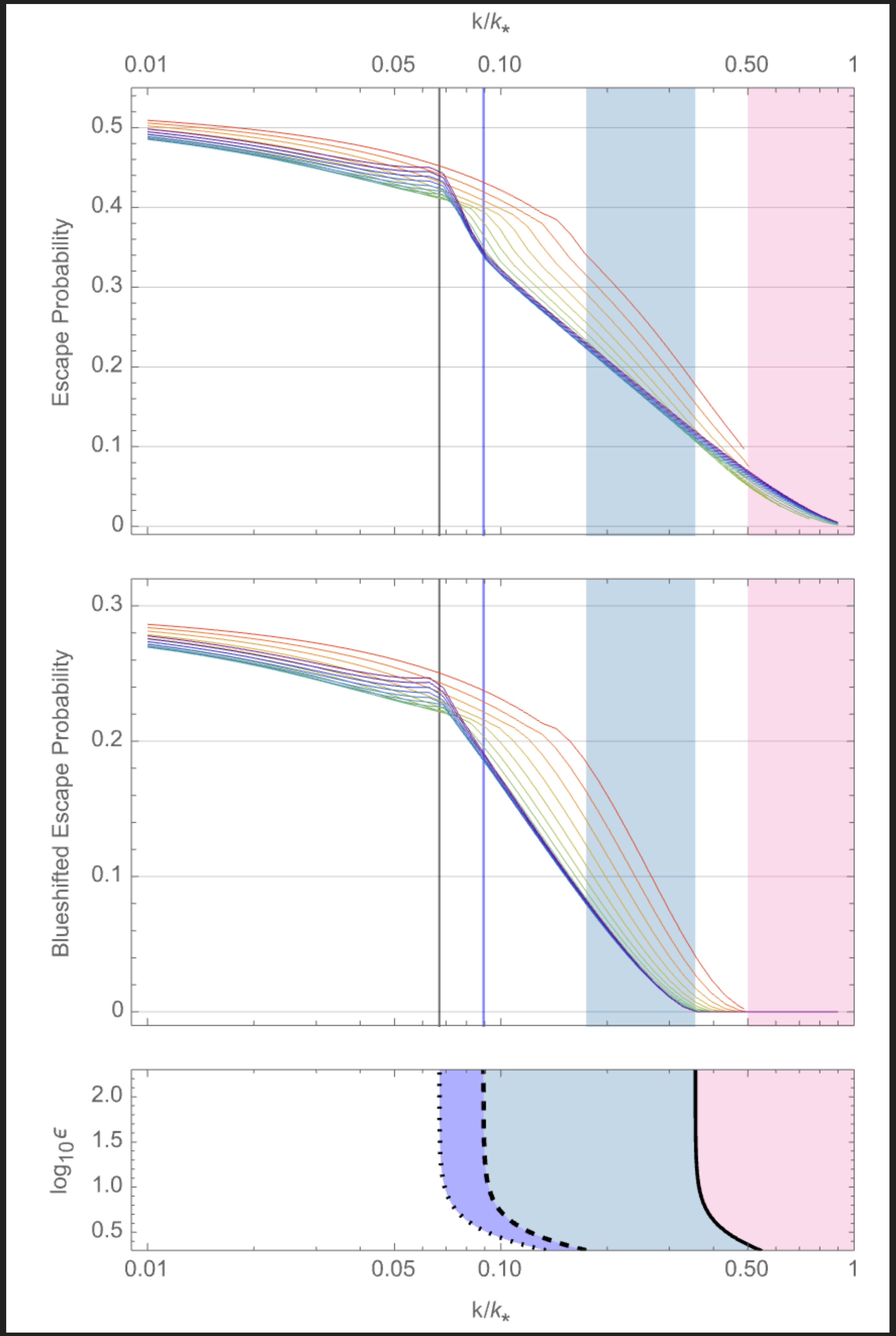
Photon Escape Probabilities

- Cross section may have cutoff $\theta_{\max}$
    - $\cos\theta_{\max} \in \Sigma_A$, $\mathrm{P_e} = 0$
    - $\cos\theta_{\max} \in \Sigma_B$, $\mathrm{P_e} > 0$ but $\mathrm{P_b} = 0$
    - $\cos\theta_{\max} \in \Sigma_C$, $\mathrm{P_e} > \mathrm{P_b} > 0$ but $g < \sqrt{3}$
    - $\cos\theta_{\max} \in \Sigma_D$, $\mathrm{P_e} > \mathrm{P_b} > 0$ and $g \leq \sqrt{3}$
- We need only consider cross sections from which with emission can escape
    - $\theta_* > \arccos(\angle AOB)$



Example: Proton-Electron bremsstrahlung

- Consider collision of
  - electron orbiting at ISCO
  - and in-falling proton (of energy ϵ in units of m_p)
  - emission of single photon (with energy k in units of m_e)
- Restrict to $k < k_*$, where k_* is the photon energy for which the cross section has a cutoff $\theta_\mathrm{max} = \theta_*$
  - For $\epsilon \in [2{,}200]$:
    - $2 \lesssim k_* \lesssim 7.5$, where k_* is the photon energy for which the cross section has a cutoff $\theta_\mathrm{max} = \theta_*$
    - For $\dfrac{k}{k_*} \in [.01,.1]$, $\mathrm{P_e} > 30\,\%$ and $\mathrm{P_b} > 15\,\%$ and emission see by observer at infinity is $\approx 10^4 - 10^5\ \mathrm{eV}$



Conclusions

- High-spin BH has rich geometric structure in the near horizon region
- This structure can have affects on particle phenomena which occur in the near horizon region in the near-extremal limit
  - Adiabatic growth of matter density spike
    - finite, nonzero matter density in near-horizon region
      —> logarithmically divergent total mass in density spike
  - Observability of high-energy collisions
    - universal kinematics of near-horizon circular orbiters
      —> blueshifted emission from high-energy collisions seen on celestial sphere